DOUBLE BETA DECAY AND NEUTRINO MASSES

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Lecture 1

Unanswered questions in neutrino physics (2015):

- What is the absolute mass scale of neutrinos?
- Are neutrinos Dirac or Majorana particles?
- How many neutrino species are there?

An answer to these questions can be obtained from neutrinoless double-beta decay (DBD)

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-} $$

INTRODUCTION

Double beta decay is a process in which a nucleus (A,Z) decays to another nucleus (A,Z±2) by emitting two electrons or positrons, and, usually, other light particles:

$$(A,Z) \rightarrow (A,Z\pm 2) + 2e^{\mp} + anything$$

The processes where two neutrinos (or antineutrinos) are emitted

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\overline{\nu} \qquad (2\nu\beta^{-}\beta^{-})$$

$$(A,Z) \rightarrow (A,Z-2) + 2e^{+} + 2\nu \qquad (2\nu\beta^{+}\beta^{+})$$

are predicted by the standard model. Indeed, the study of this process was suggested by Maria Goeppert-Meyer[§] in 1935, shortly after the Fermi theory of beta decay[¶] appeared (1934).

[¶] E. Fermi, Z. Phys. 88, 161 (1934).

[§] M. Goeppert-Meyer, Phys. Rev. 48, 512 (1935).

Versuch einer Theorie der β-Strahlen. I[1]).

Von **E. Fermi** in Rom.

Mit 3 Abbildungen. (Eingegangen am 16. Januar 1934.)

Eine quantitative Theorie des β-Zerfalls wird vorgeschlagen, in welcher man die Existenz des Neutrinos annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β-Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β-Strahlenspektrums werden abgeleitet und mit der Erfahrung verglichen.

1. Grundannahmen der Theorie.

Bei dem Versuch, eine Theorie der Kernelektronen sowie der β-Emission aufzubauen, begegnet man bekanntlich zwei Schwierigkeiten. Die erste ist durch das kontinuierliche β-Strahlenspektrum bedingt. Falls der Erhaltungssatz der Energie gültig bleiben soll, muß man annehmen, daß ein Bruchteil der beim β-Zerfall frei werdenden Energie unseren bisherigen Beobachtungsmöglichkeiten entgeht. Nach dem Vorschlag von W. Pauli kann man z. B. annehmen, daß beim β-Zerfall nicht nur ein Elektron, sondern auch ein neues Teilchen, das sogenannte „Neutrino" (Masse von der Größenordnung oder kleiner als die Elektronenmasse; keine elektrische Ladung) emittiert wird. In der vorliegenden Theorie werden wir die Hypothese des Neutrinos zugrunde legen.

Eine weitere Schwierigkeit für die Theorie der Kernelektronen besteht darin, daß die jetzigen relativistischen Theorien der leichten Teilchen (Elektronen oder Neutrinos) nicht imstande sind, in einwandfreier Weise zu erklären, wie solche Teilchen in Bahnen von Kerndimensionen gebunden werden können.

Es scheint deswegen zweckmäßiger, mit Heisenberg[2]) anzunehmen, daß ein Kern nur aus schweren Teilchen, Protonen und Neutronen, besteht. Um trotzdem die Möglichkeit der β-Emission zu verstehen, wollen wir versuchen, eine Theorie der Emission leichter Teilchen aus einem Kern in Analogie zur Theorie der Emission eines Lichtquants aus einem angeregten Atom beim gewöhnlichen Strahlungsprozeß aufzubauen. In der Strahlungstheorie ist die totale Anzahl der Lichtquanten keine Konstante: Lichtquanten entstehen, wenn sie von einem Atom emittiert werden, und verschwinden, wenn sie absorbiert werden. In Analogie hierzu wollen wir der β-Strahlentheorie folgende Annahmen zugrunde legen:

[1]) Vgl. die vorläufige Mitteilung: La Ricerca Scientifica **2**, Heft 12, 1933. — [2]) W. Heisenberg, ZS. f. Phys. **77**, 1, 1932.

wir von *verbotenen β-Übergängen*. Man muß natürlich nicht erwarten, daß die verbotenen Übergänge überhaupt nicht vorkommen, da (32) nur eine Näherungsformel ist. Wir werden in Ziffer 9 etwas über diesen Typ von Übergängen sprechen.

7. Die Masse des Neutrinos.

Durch die Übergangswahrscheinlichkeit (32) ist die Form des kontinuierlichen β-Spektrums bestimmt. Wir wollen zuerst diskutieren, wie diese Form von der Ruhemasse μ des Neutrinos abhängt, um von einem Vergleich mit den empirischen Kurven diese Konstante zu bestimmen. Die Masse μ ist in dem Faktor p_σ^2/v_σ enthalten. Die Abhängigkeit der Form der Energieverteilungskurve von μ ist am meisten ausgeprägt in der Nähe des Endpunktes der Verteilungskurve. Ist E_0 die Grenzenergie der β-Strahlen, so sieht man ohne Schwierigkeit, daß die Verteilungskurve für Energien E in der Nähe von E_0 bis auf einen von E unabhängigen Faktor sich wie

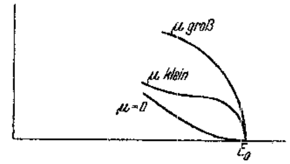


Fig. 1.

$$\frac{p_\sigma^2}{v_\sigma} = \frac{1}{c^3}(\mu c^2 + E_0 - E)\sqrt{(E_0 - E)^2 + 2\mu c^2 (E_0 - E)} \qquad (36)$$

verhält.

In der Fig. 1 ist das Ende der Verteilungskurve für $\mu = 0$ und für einen kleinen und einen großen Wert von μ gezeichnet. Die größte Ähnlichkeit mit den empirischen Kurven zeigt die theoretische Kurve für $\mu = 0$.

Wir kommen also zu dem Schluß, daß die Ruhemasse des Neutrinos entweder Null oder jedenfalls sehr klein in bezug auf die Masse des Elektrons ist[1]). In den folgenden Rechnungen werden wir die einfachste Hypothese $\mu = 0$ einführen. Es wird dann (30)

$$v_\sigma = c; \quad K_\sigma = p_\sigma c; \quad p_\sigma = \frac{K_\sigma}{c} = \frac{W - H_s}{c}. \qquad (37)$$

Die Ungleichungen (33), (34) werden jetzt:

$$H_s \leqq W; \quad W \geqq m c^2. \qquad (38)$$

Und die Übergangswahrscheinlichkeit (32) nimmt die Form an:

$$P_s = \frac{8\pi^3 g^2}{c^3 h^4} \left| \int v_m^* u_n \, d\tau \right|^2 \widetilde{\psi}_s \psi_s (W - H_s)^2. \qquad (39)$$

[1]) In einer kürzlich erschienenen Notiz kommt F. Perrin, C. R. **197**, 1625, 1933, mit qualitativen Überlegungen zu demselben Schluß.

Double Beta-Disintegration

M. GOEPPERT-MAYER, *The Johns Hopkins University*

(Received May 20, 1935)

From the Fermi theory of β-disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over 10^{17} years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.

1. INTRODUCTION

In a table showing the existing atomic nuclei it is observed that many groups of isobars occur, the term isobar referring to nuclei of the same atomic weight but different atomic number. It is unreasonable to assume that all isobars have exactly the same energy; one of them therefore will have the lowest energy, the others are unstable. The question arises why the unstable nuclei are in reality metastable, that is, why, in geologic time, they have not all been transformed into the most stable isobar by consecutive β-disintegrations.

The explanation has been given by Heisenberg[1] and lies in the fact that the energies of nuclei of fixed atomic weight, plotted against atomic number, do not lie on one smooth curve, but, because of the peculiar stability of the α-particle are distributed alternately on two smooth curves, displaced by an approximately constant amount against each other (the minimum of each curve is therefore at, roughly, the same atomic number). For even atomic weight the nuclei of even atomic number lie on the lower curve, those with odd atomic number on the higher one. One β-disintegration then brings a nucleus from a point on the lower curve into one of the upper curve, or *vice versa*. The nuclei on the upper curve are all of them unstable. But it may happen that a nucleus on the lower curve, in the neighborhood of the minimum, even though it is not the most stable one, cannot emit a single β-particle, since the resultant isobar, whose energy lies on the upper curve, has higher energy. This nucleus would then be metastable, since it cannot go over into a more stable one by consecutive emission of two electrons. This explanation is borne out by the fact that almost only isobars of even difference in atomic number occur.

A metastable isobar can, however, change into a more stable one by simultaneous emission of two electrons. It is generally assumed that the frequency of such a process is very small. In this paper the propability of a disintegration of that kind has been calculated.

The only method to attack processes involving the emission of electrons from nuclei is that of Fermi[2] which associates with the emission of an electron that of a neutrino, a chargeless particle of negligible mass. Thereby it is possible to explain the continuous β-spectrum and yet to have the energy conserved in each individual process by adjusting the momentum of the neutrino. In this theory the treatment of a β-disintegration is very similar to that of the emission of light by an excited atom.

A disintegration with the simultaneous emission of two electrons and two neutrinos will then be in strong analogy to the Raman effect, or, even more closely, to the simultaneous emission of two light quanta,[3] and can be calculated in essentially the same manner, namely, from the second-order terms in the perturbation theory. The process will appear as the simultaneous occurrence of two transitions, each of which does not fulfill the law of conservation of energy separately.

The following investigation is a calculation of the second-order perturbation, due to the interaction potential introduced by Fermi between neutrons, protons, electrons and neutrinos. As far as possible the notation used is that of Fermi. For a more detailed discussion and justification of this mathematical form and the assumptions involved reference must be made to Fermi's paper.

[1] W. Heisenberg, Zeits. f. Physik **78**, 156 (1932).

[2] E. Fermi, Zeits. f. Physik **88**, 161 (1934).

[3] M. Goeppert-Mayer, Ann. d. Physik (V) **9**, 273 (1931).

It took however more than 50 years to observe it (Elliott *et al.*, 1987)[§] in view of its very long half-life

$$\tau_{1/2}^{2\nu}(^{100}Mo) = \left(7.1 \pm 0.4\right) \times 10^{18}\, yr$$

Now (2015) $2\nu\beta^-\beta^-$ has been observed in 10 nuclei[¶] .
[The positron emitting and related processes $2\nu\beta^+\beta^+$, $2\nu\beta^+EC$, $2\nu ECEC$ has been observed only in 1 nucleus (^{130}Ba).]
The measured half-lives are

$$\tau_{1/2}^{2\nu} \sim (10^{18} - 10^{21})\, yr$$

[§] S. R. Elliott, A.A. Hahn, and M.K. Moe, Phys. Rev. Lett. 59 (1987) 2020.

[¶] A review of all observed $2\nu\beta\beta$ decays is given in:
A.S. Barabash, Nucl. Phys. A935, 52 (2015).

The processes where no neutrinos are emitted

$$(A, Z) \rightarrow (A, Z+2) + 2e^{-} \qquad (0\nu\beta^{-}\beta^{-})$$

$0\nu\beta^-\beta^-$, and $0\nu\beta^+\beta^+$,$0\nu\beta^+EC$,$0\nu ECEC$, are forbidden by the standard model, and, if observed, will provide evidence for physics beyond the standard model, in particular will determine whether or not the neutrino is a Majorana particle and will measure its (average) mass.

Majorana[§] (1937) suggested that neutral particles could be their own antiparticles and Racah[¶] (1937) pointed out that the neutron cannot be its own antiparticle since it has a magnetic moment, while the neutrino could be such a particle.

§ E. Majorana, Nuovo Cimento 14, 171 (1937).

¶ G. Racah, Nuovo Cimento 14, 322 (1937).

E. Majorana, Nuovo Cimento 14, 171 (1937).

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di Ettore Majorana

Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di* Dirac *ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di « antiparticelle » corrispondenti ai « vuoti » di energia negativa.*

L'interpretazione dei cosidetti « stati di energia negativa » proposta da Dirac (1) conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accordi con il suo contenuto, non sono del tutto soddisfacenti; sia perchè si parte sempre da una impostazione asimmetrica, sia perchè la simmetrizzazione viene in seguito ottenuta mediante tali procedimenti (come la cancellazione di costanti infinite) che possibilmente dovrebbero evitarsi. Perciò abbiamo tentato una nuova via che conduce più direttamente alla meta.

Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

(1) P. A. M. Dirac, « Proc. Camb. Phil. Soc. », **30**, 150, 1924. V. anche W. Heisenberg, « ZS. f. Phys. », **90**, 209, 1934.

SULLA SIMMETRIA TRA PARTICELLE E ANTIPARTICELLE

Nota di GIULIO RACAH

Sunto. - *Si mostra che la simmetria tra particelle e antiparticelle porta alcune modificazioni formali nella teoria di* FERMI *sulla radioattività β, e che l'identità fisica tra neutrini ed antineutrini porta direttamente alla teoria di* E. MAJORANA.

Nella prima parte del presente lavoro si pone in rilievo una certa arbitrarietà che ancora sussiste nella trasformazione delle autofunzioni di DIRAC associata a un cambiamento di assi nello spazio-tempo, e si mostra come si possa eliminare questa arbitrarietà aggiungendo al postulato dell'invarianza relativistica quello della simmetria tra particelle e antiparticelle. Si perviene così ad una legge di trasformazione che differisce in alcuni casi da quella generalmente ammessa (1), e ad una conseguente modificazione dell'interazione proposta da FERMI nella sua teoria dei raggi β (2). Gli effetti di tale modificazione non sono verificabili sperimentalmente, perchè tendono a zero con la massa del neutrino, ma hanno una certa importanza teorica, perchè eliminano una dissimmetria che era stata rilevata da KONOPINSKI e UHLENBECK (3).

Nella seconda parte si considera l'ipotesi (che dovrà essere un giorno verificata sperimentalmente) che nel caso particolare dei neutrini non si abbia una semplice simmetria, ma addirittura una identità fisica tra neutrini ed antineutrini, e si mostra come questa ipotesi porti automaticamente al formalismo di E. MAJORANA (4). Si rende così evidente il contenuto fisico assolutamente nuovo della teoria di E. MAJORANA, e si indica come l'esperienza potrà decidere della sua validità.

(1) W. PAULI, « Handbuch der Physik », vol. XXIV/1, pp. 220-224.
(2) E. FERMI, « Nuovo Cimento », **11**, 1, 1934.
(3) E. J. KONOPINSKI e G. E. UHLENBECK, « Phys. Rev. », **48**, 7, 1935.
(4) E. MAJORANA, « Nuovo Cimento », **14**, 171, 1937.

Da un punto di vista più fisico possiamo riassumere queste considerazioni dicendo che la teoria di E. MAJORANA equivale a identificare le particelle con le antiparticelle, e che se tale identificazione può farsi per i neutrini, essa non sembra possibile per i neutroni, perchè l'antineutrone dovrebbe differire dal neutrone e per il segno del momento magnetico e per la capacità di trasformarsi per processo β in antiprotone anzichè in protone. Ricordando l'ipotesi di WICK (1) sull'origine del momento magnetico del neutrone, si vede che le due difficoltà non sono indipendenti.

Desidero ringraziare il prof. W. PAULI per interessanti e proficue discussioni sugli argomenti di questa nota.

Firenze, Istituto Fisico di Arcetri, Luglio 1937-XV.

(1) G. C. WICK, « Rend. Lincei », **21**, 170, 1935.

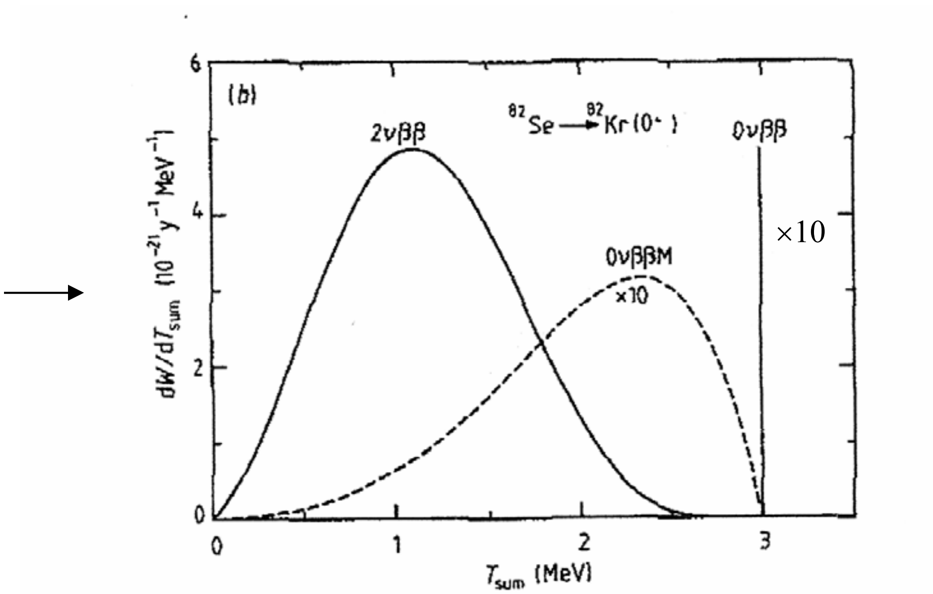
G. Racah, Nuovo Cimento 14, 322 (1937).

A major experimental effort started a few years ago to detect neutrinoless DBD. All experiments so far have given negative results, with exception of Klapdor- Kleingrothaus *et al*., 2004. This result has however been very recently (2013) disproved.

Neutrino less DBD remains therefore one of the most fundamental problems in physics today. Its detection will be crucial for understanding whatever physics is beyond the standard model (SM) and is currently the subject of many experiments.

In addition to the fact that the expected half-life is very long, a major problem is the concomitance of the 2ν process

Summed energy spectra of the two emitted electrons →



In order to be able to extract the neutrino mass if DBD is observed, or to put a limit on its value if it is not observed, one needs a theory of $0\nu\beta\beta$ and of its concomitant process $2\nu\beta\beta$.

For processes allowed by the standard model, the half-life can be, to a good approximation, factorized in the form

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\left|M_{2\nu}\right|^2$$

For processes not allowed by the standard model, the half-life can be factorized as

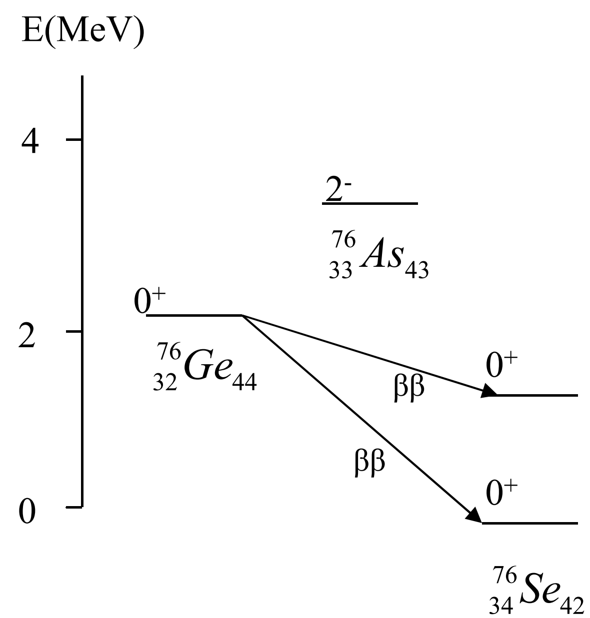
$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2\left|f(m_i, U_{ei})\right|^2$$

except for 0vECEC, which is forbidden by energy and momentum conservation but can occur under resonance conditions. In this case the inverse half-life is given by

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2\left|f(m_i, U_{ei})\right|^2\frac{(m_e c^2)\Gamma}{\Delta^2 + (\Gamma^2/4)}$$

For processes not allowed by the standard model one needs to derive the function $f(m_i,U_{ei})$ (lecture 1).
For all processes one needs to calculate the NME (lecture 2) and the PSF (lecture 3).

BRIEF THEORY OF 0νββ



$$^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-}$$

Half-life for the process:

$$\left[\tau_{1/2}^{0\nu\beta\beta}(0^{+} \rightarrow 0^{+})\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^{2}\left|f(m_i, U_{ei})\right|^{2}$$

Beyond the standard model
(Particle physics)

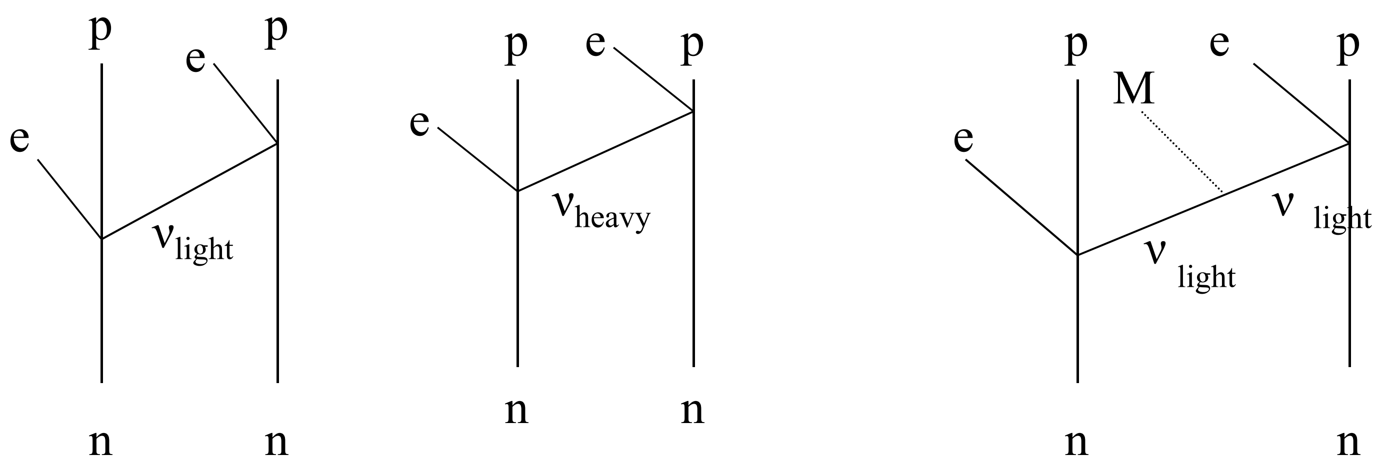
Matrix elements
(Nuclear physics)

Phase-space factor
(Atomic physics)

TRANSITION OPERATOR

The transition operator *T(p)* depends on the model of $0\nu\beta\beta$ decay. Three scenarios have been considered [#,¶,§].

1 2 3



$M_{\nu light} \ll 1MeV$ $M_{\nu heavy} \gg 1GeV$

[#] M. Doi *et al*, Prog. Theor. Phys. 66, 1739 (1981); 69, 602 (1983).

[¶] T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

[§] F.Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

The formulations of Doi, Tomoda and Šimkovic, as well as previous formulations by Furry, Primakoff and Rosen, and Haxton and Stephenson, differ by factors of 2, by the number of terms retained in the non-relativistic expansion of the current and by their contribution.

The formulation currently adopted in most calculations is that of Šimkovic¶.

¶ F. Šimkovic *et al.*, loc. cit.

To derive the expression for T(p) one starts from the weak interaction Hamiltonian

$$H^{\beta} = \frac{G_F}{\sqrt{2}}\left[\bar{e}\gamma_{\mu}\left(1-\gamma_5\right)\nu_{eL}\right]J_L^{\mu\dagger} + h.c.$$

and the nucleon current §

$$J_L^{\mu\dagger} = \bar{\Psi}\tau^{+}\left[g_V(q^2)\gamma^{\mu} - ig_M(q^2)\frac{q_{\nu}}{2m_p}\sigma^{\mu\nu} - g_A(q^2)\gamma^{\mu}\gamma_5 + g_P(q^2)\frac{q^{\mu}}{2m_p}\gamma_5\right]\Psi$$

vector — weak-magnetism — axial vector — induced pseudo-scalar

HOC (weak-magnetism, induced pseudo-scalar)

q^{μ} = momentum transferred from hadrons to leptons

§ F. Šimkovic *et al.*, loc.cit.

[Tomoda ¶ also considered right-handed couplings]

¶ T. Tomoda, loc. cit.

From the weak interaction Hamiltonian, H^β, and the weak nucleon current, J^μ , one finds the transition operator, $T(p)$, which can be written as ($p = |\vec{q}|$)

$$T(p) = H(p) f(m_i, U_{ei})$$

In momentum space and including higher order corrections (HOC), *H(p)* can be written as §

$$H(p) = \sum_{n,n'} \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p)\vec{\sigma}_n \bullet \vec{\sigma}_{n'} + h^T(p) S_{nn'}^p]$$

with

$$S_{nn'}^p = 3\left(\vec{\sigma} \cdot \hat{p}\right)\left(\vec{\sigma} \cdot \hat{p}\right) - \vec{\sigma}_n \cdot \vec{\sigma}_{n'}$$

[The general formulation of Tomoda ¶ includes more terms, nine in all, 3GT, 3F, 1T, one pseudoscalar (P) and one recoil (R). This formulation is no longer used but it will have to be revisited if a very accurate description of 0νββ is needed.]

§ F. Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

¶ T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

The Fermi (F), Gamow-Teller (GT) and Tensor (T) contributions are given by

$$h^F(p) = h^F_{VV}(p)$$
$$h^{GT}(p) = h^{GT}_{AA}(p) + h^{GT}_{AP}(p) + h^{GT}_{PP}(p) + h^{GT}_{MM}(p)$$
$$h^T(p) = h^T_{AP}(p) + h^T_{PP}(p) + h^T_{MM}(p)$$

The terms AP, PP, and MM are higher order corrections (HOC) arising from weak magnetism (M) and induced pseudoscalar terms (P) in the weak current.

The form factors $h^{F,GT,T}(p)$ can be further factorized into

$$h^{F,GT,T}(p) = v(p)\tilde{h}^{F,GT,T}(p)$$

where v(p) is called the neutrino "potential" and $\tilde{h}(p)$ is given by

Term	$\tilde{h}(p)$
$\tilde{h}^{F}_{VV}$	$g_A^2 \frac{(g_V^2/g_A^2)}{\left(1+p^2/M_V^2\right)^4}$
$\tilde{h}^{GT}_{AA}$	$\frac{g_A^2}{\left(1+p^2/M_A^2\right)^4}$
$\tilde{h}^{GT}_{AP}$	$g_A^2 \left[-\frac{2}{3}\frac{1}{\left(1+p^2/M_A^2\right)^4}\frac{p^2}{p^2+m_\pi^2}(1-\frac{m_\pi^2}{M_A^2})\right]$
$\tilde{h}^{GT}_{PP}$	$g_A^2 \left[\frac{1}{\sqrt{3}}\frac{1}{\left(1+p^2/M_A^2\right)^2}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right]^2$
$\tilde{h}^{GT}_{MM}$	$g_A^2 \left[\frac{2}{3}\frac{g_V^2}{g_A^2}\frac{1}{\left(1+p^2/M_V^2\right)^4}\frac{\kappa_\beta^2 p^2}{4m_p^2}\right]$
$\tilde{h}^{T}_{AP}$	$-\tilde{h}^{GT}_{AP}$
$\tilde{h}^{T}_{PP}$	$-\tilde{h}^{GT}_{PP}$
$\tilde{h}^{T}_{MM}$	$\frac{1}{2}\tilde{h}^{GT}_{MM}$

The finite nucleon size (FNS) is taken into account by taking the coupling constants, g_V and g_A, momentum dependent

Primitive (V-A)

$$g_V(p^2) = g_V \frac{1}{\left(1 + \frac{p^2}{M_V^2}\right)^2} \qquad g_V = 1; M_V^2 = 0.71\left(GeV/c^2\right)^2$$

$$g_A(p^2) = g_A \frac{1}{\left(1 + \frac{p^2}{M_A^2}\right)^2} \qquad g_A = 1.269; M_A^2 = 1.09\left(GeV/c^2\right)^2$$

Induced (HOC)

$$g_M(p^2) = \left(\mu_p - \mu_n\right) g_V(p^2)$$

$$g_P(p^2) = \left(2m_p\right)^2 g_A(p^2) \frac{\left(1 - \frac{m_\pi^2}{M_A^2}\right)}{\left(p^2 + m_\pi^2\right)}$$

Short range correlations (SRC) are taken into account by convoluting the "potential" *v(p)* with the Jastrow function *j(p)* parametrized in various forms (Miller-Spencer, MS/ Argonne/CD Bonn) or by other methods (UCOM)

$$u(p) = \int v(p - p') j(p') dp'$$

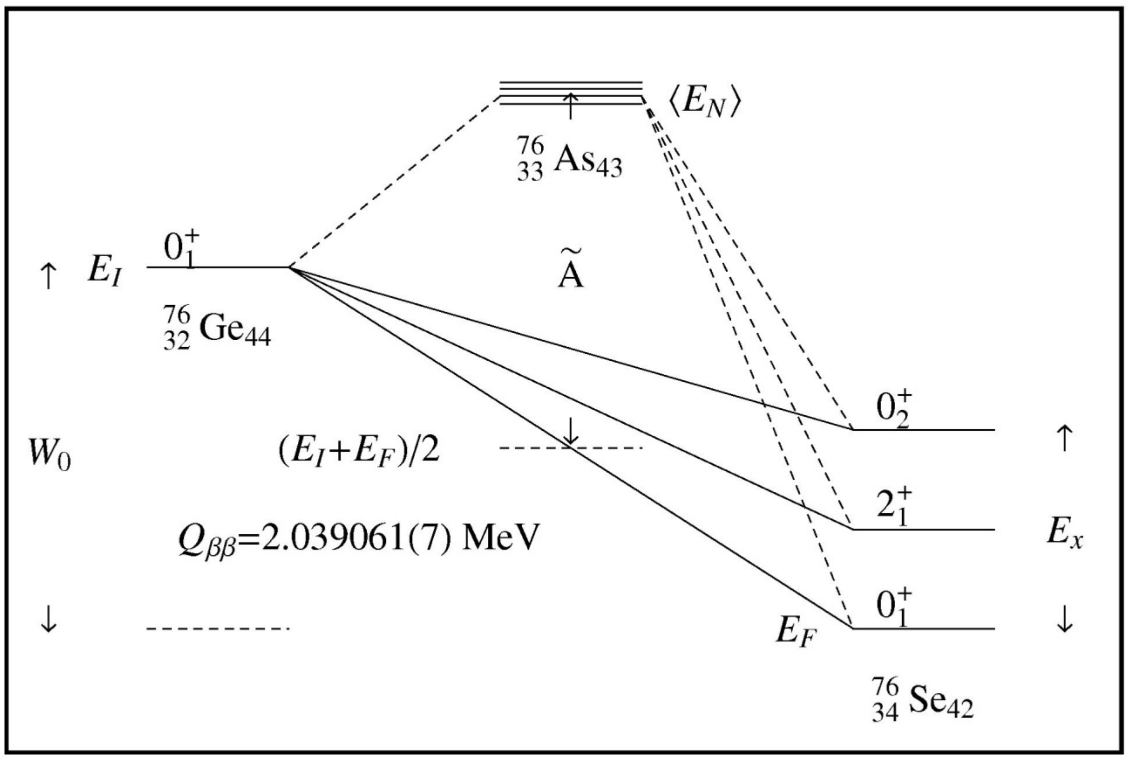
The Jastrow function in configuration space is

$$f_J(r) = 1 - c e^{-ar^2} \left(1 - br^2\right)$$

with

a=1.10 fm^{-2} , b=0.68 fm^{-2} , c=1	MS	soft
a=1.59 fm^{-2} , b=1.45 fm^{-2} , c=0.92	Argonne	hard
a=1.52 fm^{-2} , b=1.88 fm^{-2} , c=0.46	CD Bonn	hard

Finally, half-lives are usually calculated in the closure approximation. This approximation is good for 0νββ where the momentum of the virtual neutrino is of the order of 100 MeV/c, and the scale of the energy levels in the intermediate nucleus is ~0.1MeV.



F FUNCTION AND NEUTRINO POTENTIAL

Scenario 1: LIGHT NEUTRINO EXCHANGE

The function f for this scenario is

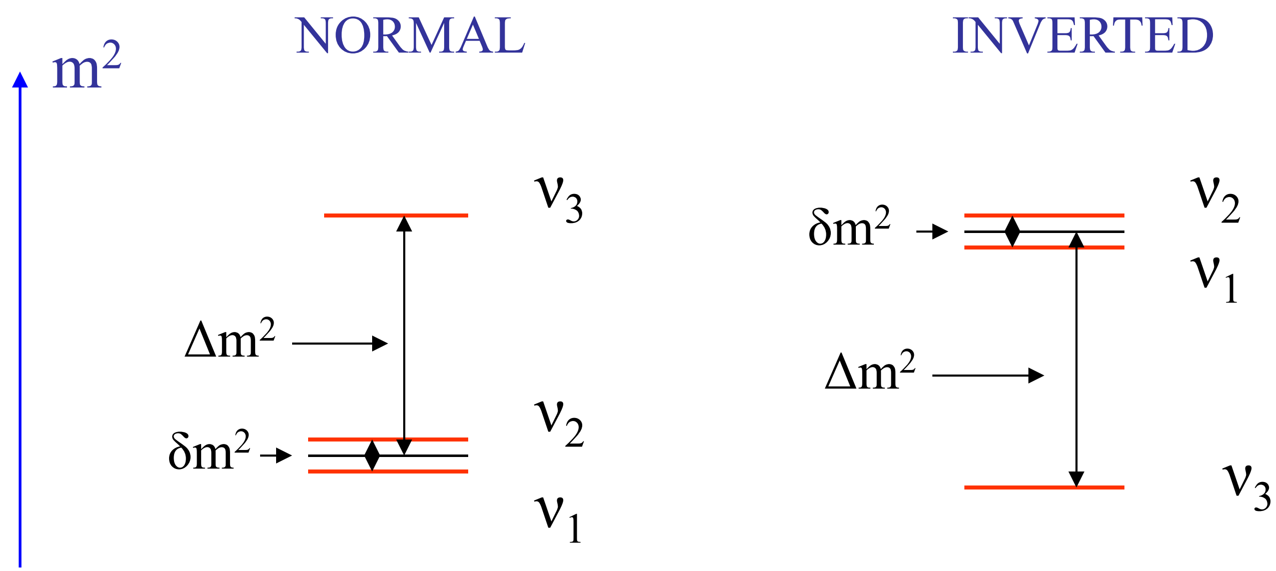
$$f = \frac{\langle m_\nu \rangle}{m_e} \qquad \langle m_\nu \rangle = \sum_{k=light} (U_{ek})^2 m_k$$

and the neutrino potential is

$$v(p) = \frac{2}{\pi} \frac{1}{p\left(p + \tilde{A}\right)}$$

$\tilde{A}$=closure energy=$1.12A^{1/2}$(MeV)

In the last few years atmospheric, solar, reactor and accelerator neutrino oscillation experiments have provided information on light neutrino mass differences and their mixings. Two possibilities, normal and inverted hierarchy, are consistent with experiment.



The average light neutrino mass can be written as

$$\langle m_\nu \rangle = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3} \right|$$

$$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}, \varphi_{2,3} = [0, 2\pi]$$

$$\left(m_1^2, m_2^2, m_3^2\right) = \frac{m_1^2 + m_2^2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right)$$

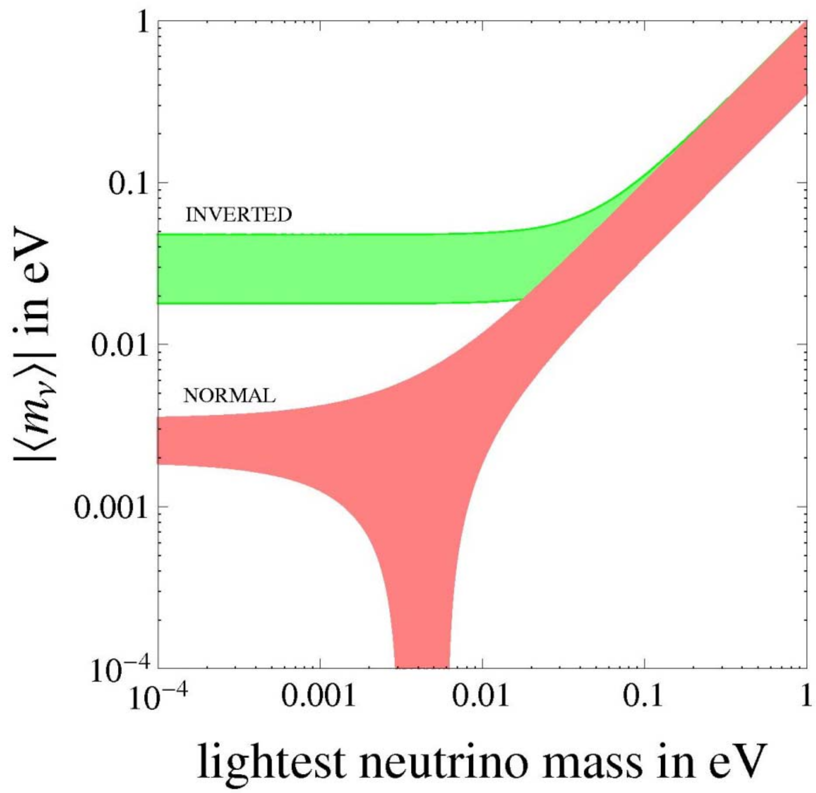
A fit to oscillation experiments gives §

$$\sin^2\theta_{12} = 0.308, \sin^2\theta_{13} = 0.024, \sin^2\theta_{23} = 0.455$$

$$\delta m^2 = 7.54\times10^{-5}\,eV^2, \Delta m^2 = 2.43\times10^{-3}\,eV^2$$

§ F. Capozzi, G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, Phys. Rev. D89, 093018 (2014).

Variation of the phases φ_2 and φ_3 from 0 to 2π gives the values of $<m_\nu>$ consistent with oscillation experiments



Vissani-Strumia plot ¶

¶ F. Vissani, J. High Energy Phys. 06, 022 (1999).

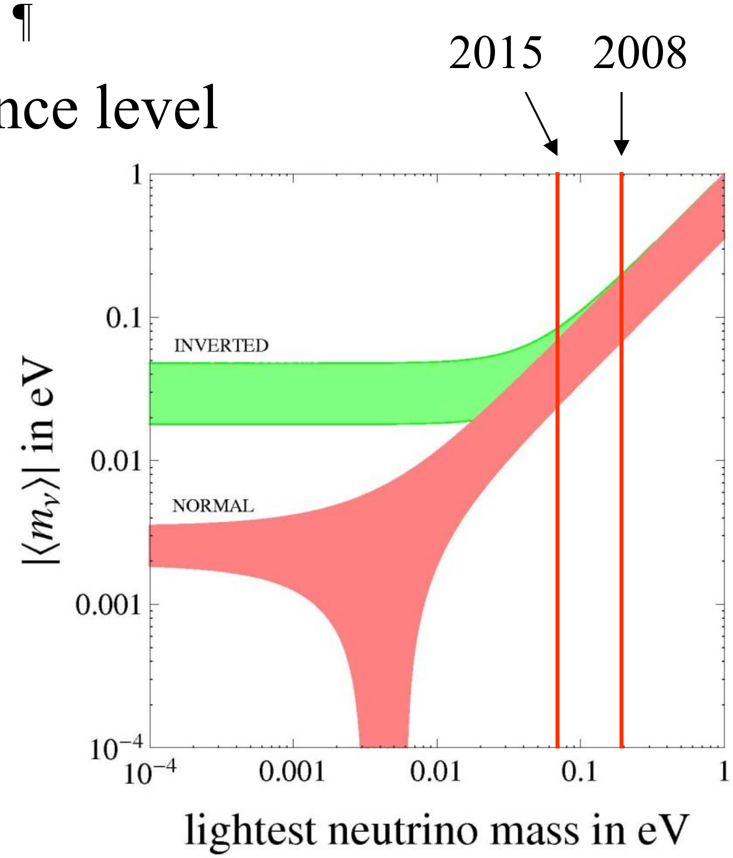
In addition there is a (model dependent) bound from cosmology on the sum of the masses

$$\sum_i m_i \leq 0.6eV \qquad (2008)$$

$$\sum_i m_i \leq 0.230eV \qquad \text{(2015) Plank ¶}$$

68% confidence level

Cosmological bound



¶ S. Matarrese for the Plank collaboration, Proc. XVI Int. Workshop NEUTEL 2015, Venice, Italy.

Scenario 2: HEAVY NEUTRINO EXCHANGE

In recent years, scenario 2 has again become of interest. The transition operator for this scenario is the same as for 1, but with

$$T_h(p) = H_h(p) f(m_{ih}, U_{eih})$$

$$f = m_p \left\langle \frac{1}{m_{\nu_h}} \right\rangle$$

$$\left\langle m_{\nu_h}^{-1} \right\rangle = \sum_{k=heavy} \left(U_{ek_h}\right)^2 \frac{1}{m_{k_h}}$$

and neutrino "potential"

$$v(p) = \frac{2}{\pi} \frac{1}{m_p m_e}$$

Constraints on the average inverse heavy neutrino mass are model dependent. V. Tello *et al.* ¶ have recently (2011) worked out constraints from lepton flavor violating processes and (potentially LHC experiments). In this model

$$f \equiv \eta = \frac{M_W^4}{M_{WR}^4} \sum_{k=heavy} \left(V_{ek_h}\right)^2 \frac{m_p}{m_{k_h}} \equiv \frac{M_W^4}{M_{WR}^4} \frac{m_p}{\left\langle m_{\nu_h} \right\rangle}$$

$$M_W = 80.41 \pm 0.10 GeV; M_{WR} = 3.5 TeV$$

η=lepton violating parameter.

Constraints on η can then be converted into constraints on the average heavy neutrino mass as

$$\left\langle m_{\nu_h} \right\rangle = m_p \left(\frac{M_W}{M_{WR}} \right)^4 \frac{1}{\eta}$$

¶ V. Tello, M. Nemevšek, F. Nesti, G. Senjanović, and F. Vissani, Phys. Rev. Lett. 106, 151801 (2011).

If both light and heavy neutrino exchange contribute, the half-lives are given by

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\frac{\langle m_{\nu}\rangle}{m_e} + M_{0\nu_h}\eta\right|^2$$

The two contributions could add or subtract depending on their relative phase.

Scenario 3: MAJORON EMISSION

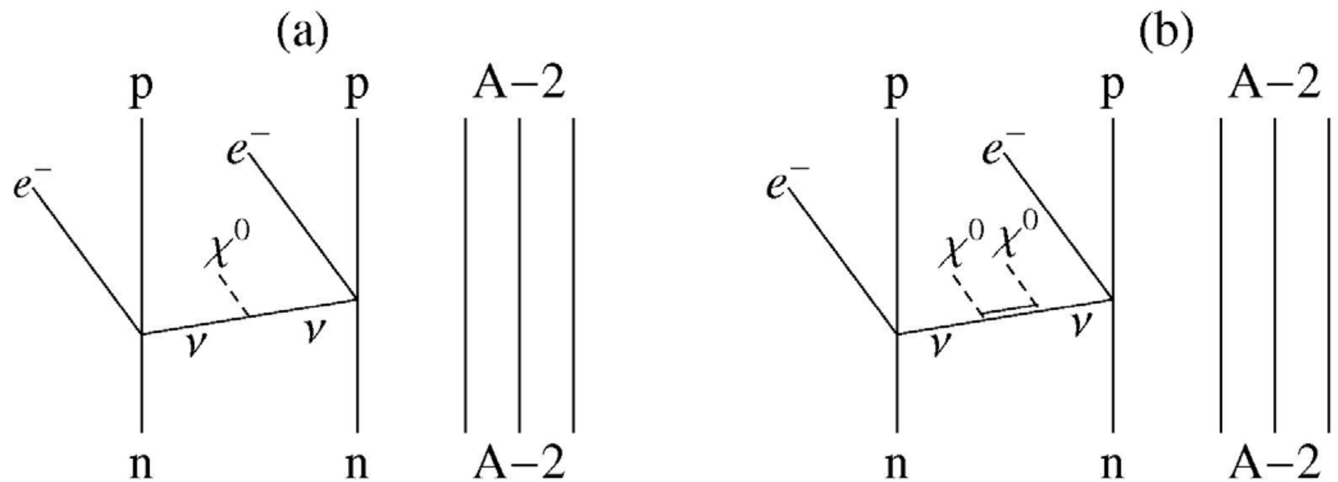
This scenario ($0\nu\beta\beta M$ decay) was very much of interest a few years ago (1980's), it was not much studied since, but it has become again of interest in very recent years.
Majorons were introduced as massless Nambu-Goldstone bosons arising from a global B-L symmetry broken spontaneously in the low-energy regime [#,§,¶].

[#] Y. Chikashige, R.N. Mohapatra, and R.D. Peccei, Phys. Rev. Lett. 45, 1926 (1980).
[§] G.B. Gelmini and M. Roncadelli, Phys. Lett. B99, 411 (1981).
[¶] H.M. Georgi, S.L. Glashow, and S. Nussinov, Nucl. Phys. B193, 297 (1981).

Although these older models are disfavored by precise measurements of the width of the Z boson decay, several other models have been proposed in which one or two Majorons are emitted

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-} + \chi_0$$

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\chi_0$$



Different models are distinguished by the nature of the emitted Majorons, i.e. whether it is a Nambu-Goldstone boson or not (NG), the leptonic charge of the emitted Majorons (L), and the spectral index of the model, n.

Model	Decay Mode	NG boson	L	n
IB	$0\nu\beta\beta\chi_0$	No	0	1
IC	$0\nu\beta\beta\chi_0$	Yes	0	1
ID	$0\nu\beta\beta\chi_0\chi_0$	No	0	3
IE	$0\nu\beta\beta\chi_0\chi_0$	Yes	0	3
IIB	$0\nu\beta\beta\chi_0$	No	-2	1
IIC	$0\nu\beta\beta\chi_0$	Yes	-2	3
IID	$0\nu\beta\beta\chi_0\chi_0$	No	-1	3
IIE	$0\nu\beta\beta\chi_0\chi_0$	Yes	-1	7
IIF	$0\nu\beta\beta\chi_0$	Gauge boson	-2	3
"Bulk"	$0\nu\beta\beta\chi_0$	Bulk field	0	2

The transition operator for this scenario can be written as [#,§,¶]

$$T(p) = H(p)\langle g \rangle$$

effective Majoron coupling constant

The inverse half-life is given by

$$\left[\tau_{1/2}^{0\nu\beta\beta M}\right]^{-1} = G_{0\nu M}\left|M_{0\nu}\right|^{2}\langle g \rangle^{2}$$

[#] M. Doi *et al.,* loc.cit.
¶ T. Tomoda, loc. cit.
§ F. Šimkovic *et al.*, loc.cit.

Scenario 4: STERILE NEUTRINOS

In addition, another scenario is currently being considered, namely the mixing of one, two or three additional "sterile" neutrinos, 4, 5 and 6, with masses in the eV-GeV range. Sterile neutrinos were introduced by Pontecorvo (1968) [¶] as neutrinos with no standard model interactions. In very recent years several suggestions have been made for sterile neutrinos in the eV range [a], in the keV range [b], in the MeV-GeV range [c], and in the TeV range [d], in order to account for various anomalies in neutrino physics.

¶ B. Pontecorvo, Phys. Lett. B26, 630 (1968).

a C. Giunti and M. Laveder, Phys. Rev. D82, 053005 (2010).
J. Barry, W. Rodejohann, and H. Zhang, J. High Energy Phys. 07 (2011) 91.
b T. Asaka and M. Shaposhnikov, Phys. Rev. B620, 17 (2005).
c T. Asaka, S. Eijima, and H. Ishida, J. High Energy Phys. 04 (2011) 11.
M. Shaposhnikov and I. Tkachev, Phys. Lett. B639, 414 (2006).
d V. Tello *et al.*, loc.cit.

This scenario can be investigated by using a transition operator, for a neutrino with mass m_N, as in scenario 1 and 2 but with

$$f = \frac{m_N}{m_e}$$

$$v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_N^2}\left(\sqrt{p^2 + m_N^2} + \tilde{A}\right)}$$

$\tilde{A}$=closure energy

When the mass m_N is intermediate, and especially when it is of the order of the Fermi momentum, p_F~100MeV/c, the factorization of τ^{-1} into PSF, NME and F function is not possible and physics beyond the standard model is entangled with nuclear physics. In this case the half-life can be written as

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu} \left| \sum_N (U_{eN})^2 M_{0\nu}(m_N) \frac{m_N}{m_e} \right|^2$$

These formulas apply for a single additional neutrino with mass m_N

The product fv(p)

$$fv(p) = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{\sqrt{p^2+m_N^2}\sqrt{p^2+m_N^2}+\tilde{A}}$$

has the limits:

$$m_N \to 0 \qquad fv = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$$

$$m_N \to \infty \qquad fv = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{m_N^2} = \frac{2}{\pi}\frac{1}{m_e m_N}$$

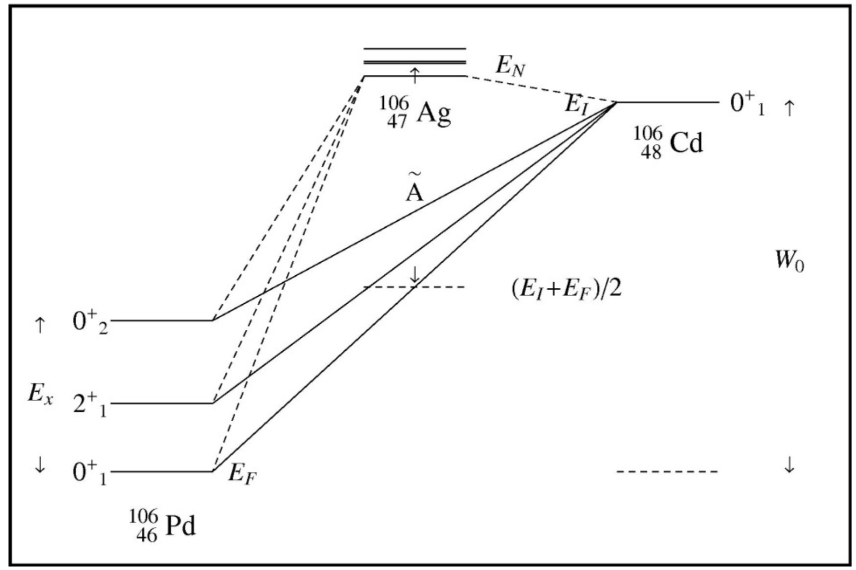
as in scenarios 1 and 2.

BRIEF THEORY OF 0νβ⁺β⁺ AND 0νECβ⁺

These processes are:

$$(A,Z)\rightarrow(A,Z-2)+2e^{+}$$

$$(A,Z)+e^{-}\rightarrow(A,Z-2)+e^{+}$$



The theory for these processes is identical to that of $0\nu\beta^-\beta^-$ with half lives still given by

$$\left[\tau_{1/2}^{0\nu}\right]^{-1}=G_{0\nu}\left|M_{0\nu}\right|^{2}\left|f\left(m_i,U_{ei}\right)\right|^{2}$$

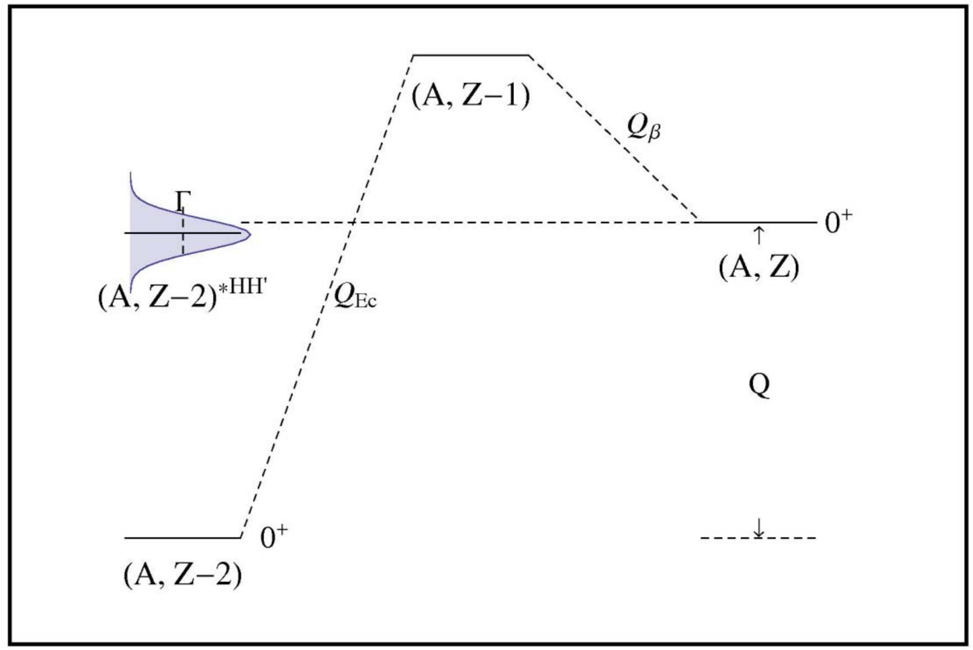
but with PSF appropriate for the process, $G_{0\nu}^{\beta^+\beta^+}$ $G_{0\nu}^{\beta^+EC}$

BRIEF THEORY OF 0νECEC

The process

$$(A, Z) + 2e^{-} \rightarrow (A, Z-2)$$

cannot in general occur because of energy and momentum conservation.



If however the energy of the initial state matches precisely the energy of the final state the process can occur and is termed resonant double electron capture or R0νECEC

For this process the half-life can be factorized as

$$\left[\tau_{1/2}^{ECEC}\right]^{-1} = G_{0\nu}^{ECEC}\left|M_{ECEC}^{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 \frac{\left(m_e c^2\right)\Gamma}{\Delta^2 + \Gamma^2/4}$$

Here Δ is the degeneracy parameter

$$\Delta = \left|Q - B_{2h} - E\right|$$

Energy of the two-holes in the daughter atom

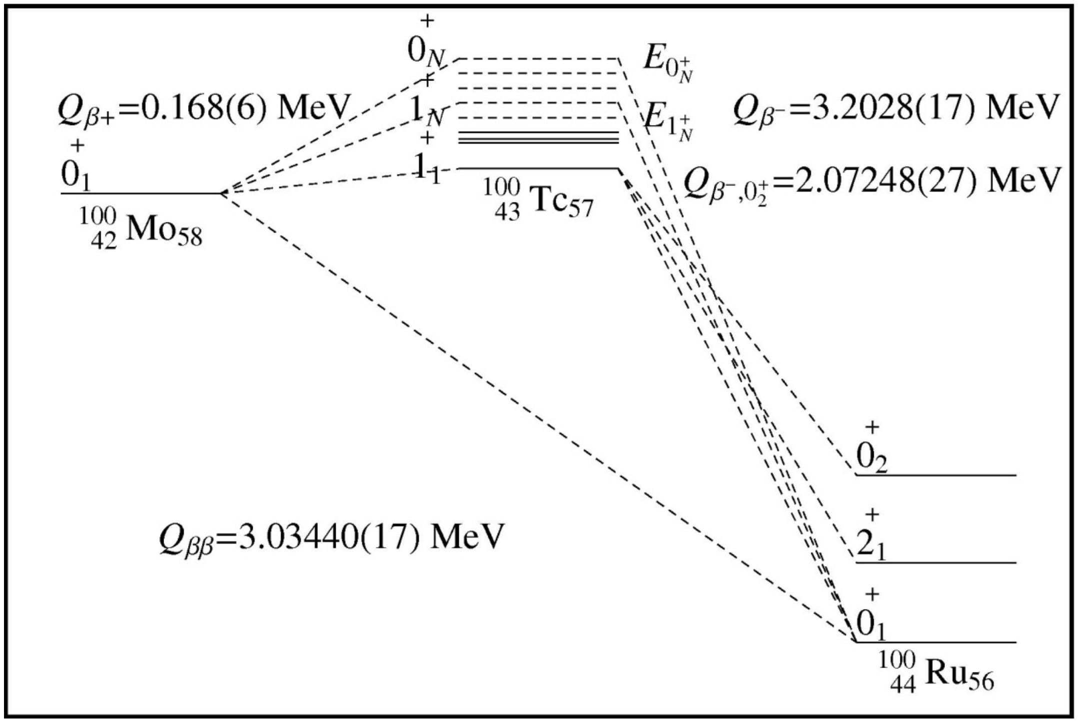
and Γ is the two-hole width in the daughter atom

Calculation of this process heavily relies on atomic physics and on nuclear physics

BRIEF THEORY OF 2νββ

The theory of 2νββ is more complicated than that of 0νββ because in general the closure approximation may not be good and the separation between PSF and NME may not be good.

One needs therefore to calculate NME and PSF for each individual state and sum over them.



To apply this procedure, one needs to calculate states in the intermediate odd-odd nucleus and then

$$M_{GT,N}^{(2\nu)} = \frac{\left\langle 0_F^+ \left\| \tau^\dagger \sigma \right\| 1_N^+ \right\rangle \left\langle 1_N^+ \left\| \tau^\dagger \sigma \right\| 0_I^+ \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{1_N^+} - E_I}$$

$$M_{F,N}^{(2\nu)} = \frac{\left\langle 0_F^+ \left\| \tau^\dagger \right\| 0_N^+ \right\rangle \left\langle 0_N^+ \left\| \tau^\dagger \right\| 0_I^+ \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{0_N^+} - E_I}$$

This calculation is daunting and has been done only in a selected number of cases

The separation between PSF and NME can be done in two cases:
(i) Closure approximation (CA)
(ii) Single state dominance (SSD)

In both cases the inverse half-life can be written as

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\left|m_e c^2 M_{2\nu}\right|^2$$

For these cases, the calculation of the NME in IBM-2 is done in the same way as for 0νββ, except that the neutrino potential is

$$v_{2\nu}(p) = \frac{\delta(p)}{p^2}$$

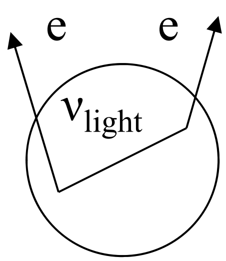
which is the Fourier-Bessel transform of V(r)=1.

Most calculation that attempt a simultaneous calculation of 0νββ and 2νββ are done in this way to avoid possible sources of systematic or accidental errors.

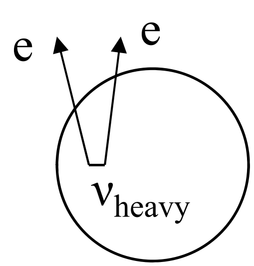
SUMMARY

All processes light-neutrino exchange, heavy-neutrino exchange, 2ν decay, Majoron emission, sterile-neutrino exchange, …., can then calculated simultaneously by just changing the neutrino potential.

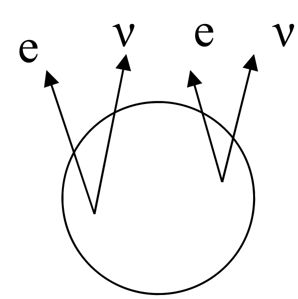
Light-neutrino	Heavy-neutrino	2ν
$v(p)=\frac{2}{\pi}\frac{1}{p(p+\tilde{A})}$	$v(p)=\frac{2}{\pi}\frac{1}{m_p m_e}$	$v(p)=\frac{\delta(p)}{p^2}$



Long-range



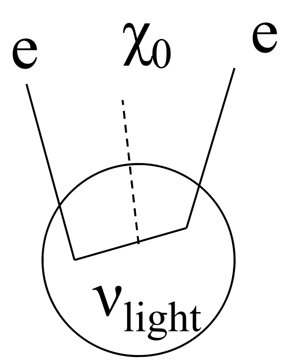
Short-range



Constant

Majoron

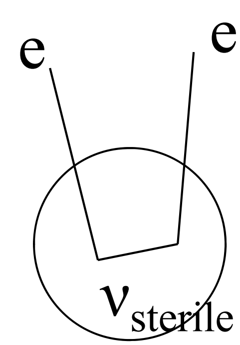
$$v(p) = \frac{2}{\pi} \frac{1}{p\left(p + \tilde{A}\right)}$$



Long-range

Sterile

$$v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_N^2}\,\sqrt{p^2 + m_N^2} + \tilde{A}}$$



Intermediate-range