# DOUBLE BETA DECAY AND NEUTRINO MASSES

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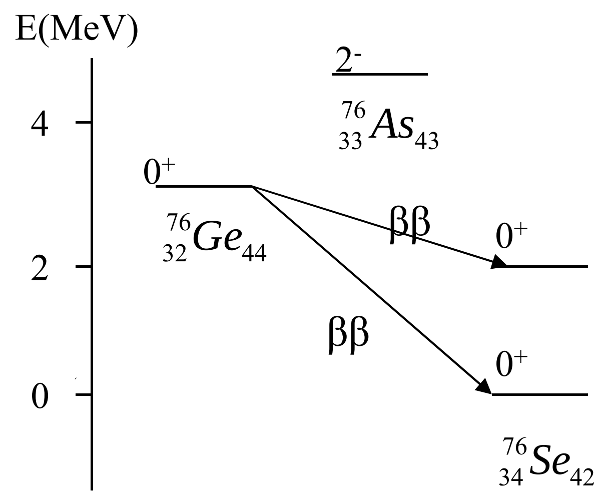
Lecture 2

# EVALUATION OF THE NUCLEAR MATRIX ELEMENTS: $0\nu\beta\beta$

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z\pm 2}Y_{N\mp 2} + 2e^{\mp} $$




Half-life for the process:

$$ \left[ \tau_{1/2}^{0\nu\beta\beta}(0^{+} \rightarrow 0^{+}) \right]^{-1} = G_{0\nu} \left| M_{0\nu} \right|^{2} \left| f(m_{i}, U_{ei}) \right|^{2} $$

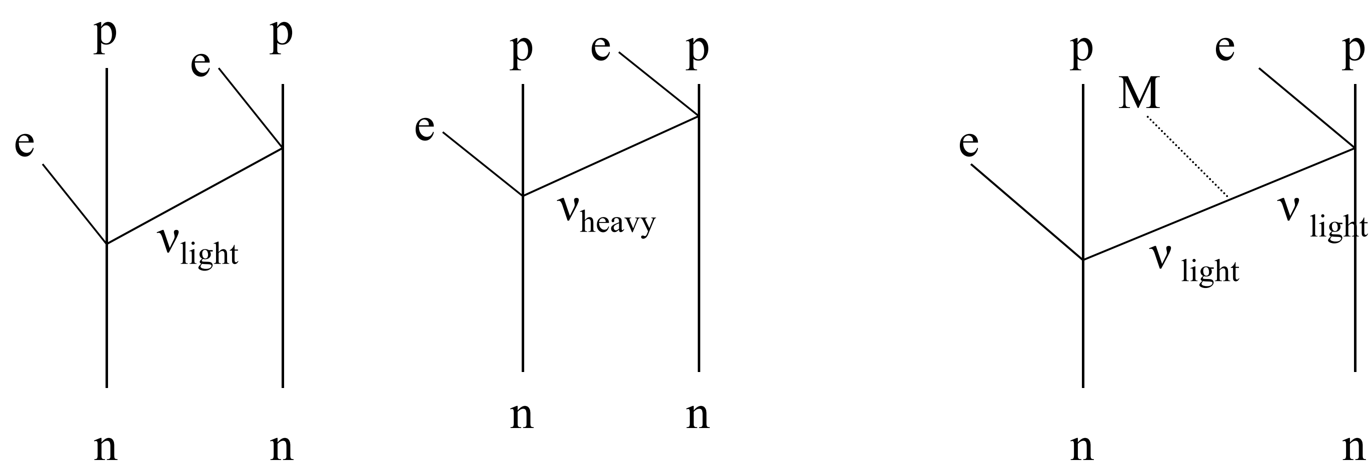
Beyond the standard model (Particle physics)

Matrix elements (Nuclear physics)

Phase-space factor (Atomic physics)

As discussed in lecture 1, the transition operator *T(p)* depends on the model of 0νββ decay and three scenarios have been considered [#,¶,§].

1 2 3




$M_{\nu light} \ll 1\text{MeV}$ $M_{\nu heavy} \gg \text{GeV}$

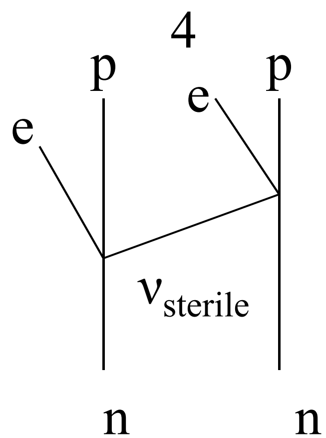
[#] M. Doi *et al*, Prog. Theor. Phys. 66, 1739 (1981); 69, 602 (1983).

[¶] T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

[§] F.Šimkovic *et al*., Phys. Rev. C60, 055502 (1999).

In scenario 3, if the Majoron couples only to light neutrino, the NME needed to calculate the half-life are the same of scenario 1 and will not be considered further.

In recent years, a fourth scenario is being considered




For this scenario, the NME need to be calculated as a function of the mass of the exchanged neutrino, $m_N$.

The NME can be written as:

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

Several methods have been used to evaluate $M_{0\nu}$:
QRPA (Quasiparticle Random Phase Approximation)
ISM (Shell Model)
IBM-2 (Interacting Boson Model)
EDF (Density Functional Theory)

# EVALUATION OF MATRIX ELEMENTS IN IBM-2 ¶

All matrix elements, F, GT and T, can be calculated at once using the compact expression:

$$V_{s_1,s_2}^{(\lambda)} = \frac{1}{2}\sum_{n,n'} \tau_n^+ \tau_{n'}^+ \left[\Sigma_n^{(s_1)} \times \Sigma_{n'}^{(s_2)}\right]^{(\lambda)} \cdot V(r_{nn'}) C^{(\lambda)}(\Omega_{nn'})$$

$$\lambda = 0, s_1 = s_2 = 0 (F)$$

$$\lambda = 0, s_1 = s_2 = 1 (GT)$$

$$\lambda = 2, s_1 = s_2 = 1 (T)$$

In second quantized form:

$$V_{s_1,s_2}^{(\lambda)} = -\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2}\sum_{J} (-1)^J \sqrt{1+(-1)^J \delta_{j_1 j_2}}\sqrt{1+(-1)^J \delta_{j'_1 j'_2}}$$

$$\times G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1 j'_2; J)\left[\left(\pi_{j_1}^\dagger \times \pi_{j_2}^\dagger\right)^{(J)} \cdot \left(\tilde{\nu}_{j'_1} \times \tilde{\nu}_{j'_2}\right)^{(J)}\right]$$

Creates a pair of protons with angular momentum J

Annihilates a pair of neutrons with angular momentum J

¶ J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).

The fermion operator V is mapped onto the boson space by using:

$$\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)} \mapsto A_\pi(j) s_\pi^\dagger$$

$$\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)_M^{(2)} \mapsto B_\pi(j, j') d_{\pi,M}^\dagger$$

$$V_{s_1 s_2}^{(\lambda)} \mapsto -\frac{1}{2} \sum_{j_1} \sum_{j'_1} G_{s_1 s_2}^{(\lambda)} \left( j_1 j_1 j'_1\, j'_1; 0 \right) A_\pi(j_1) A_\nu(j'_1) s_\pi^\dagger \cdot \tilde{s}_\nu$$

$$-\frac{1}{4} \sum_{j_1 j_2} \sum_{j'_1\, j'_2} \sqrt{1+\delta_{j_1 j_2}} \sqrt{1+\delta_{j'_1\, j'_2}}\, G_{s_1 s_2}^{(\lambda)} (j_1 j_2 j'_1\, j'_2; 2) B_\pi(j_1, j_2) B_\nu(j'_1, j'_2) d_\pi^\dagger \cdot \tilde{d}_\nu$$

The coefficients A, B are obtained by equating fermionic matrix elements in the Generalized Seniority (GS) basis with bosonic matrix elements, the so-called OAI mapping procedure ¶.

The basis

$$\left(S^\dagger\right)^{\frac{n-\nu}{2}} \left(D^\dagger\right)^{\frac{\nu}{2}} |0\rangle$$

is constructed with operators:

$$S_\pi^\dagger = \sum_j \alpha_j \sqrt{\frac{j+\frac{1}{2}}{2}} \left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)}$$

$$D_\pi^\dagger = \sum_{j \le j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)^{(2)}$$

¶ T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309, 1 (1978).

The structure coefficients $\alpha_j$, $\beta_{jj'}$ are obtained by diagonalizing the surface delta interaction (SDI). The strength of the interaction, $A_T$, is chosen as to reproduce the 0-2 separation in the two-particle system.

The fermion matrix elements are calculated using the commutator method of Frank and Van Isacker and Lipas *et al.* [¶,§].

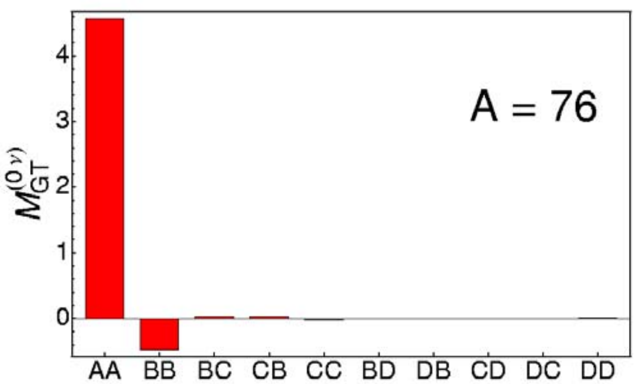
¶ A. Frank and P. Van Isacker, Phys. Rev. C26, 1661 (1982).

§ P.O. Lipas, M. Koskinen, H. Harter, R. Nojarov, and A. Faessler, Nucl. Phys. A508, 509 (1990).

Expansion to next to leading order (NLO) has been considered

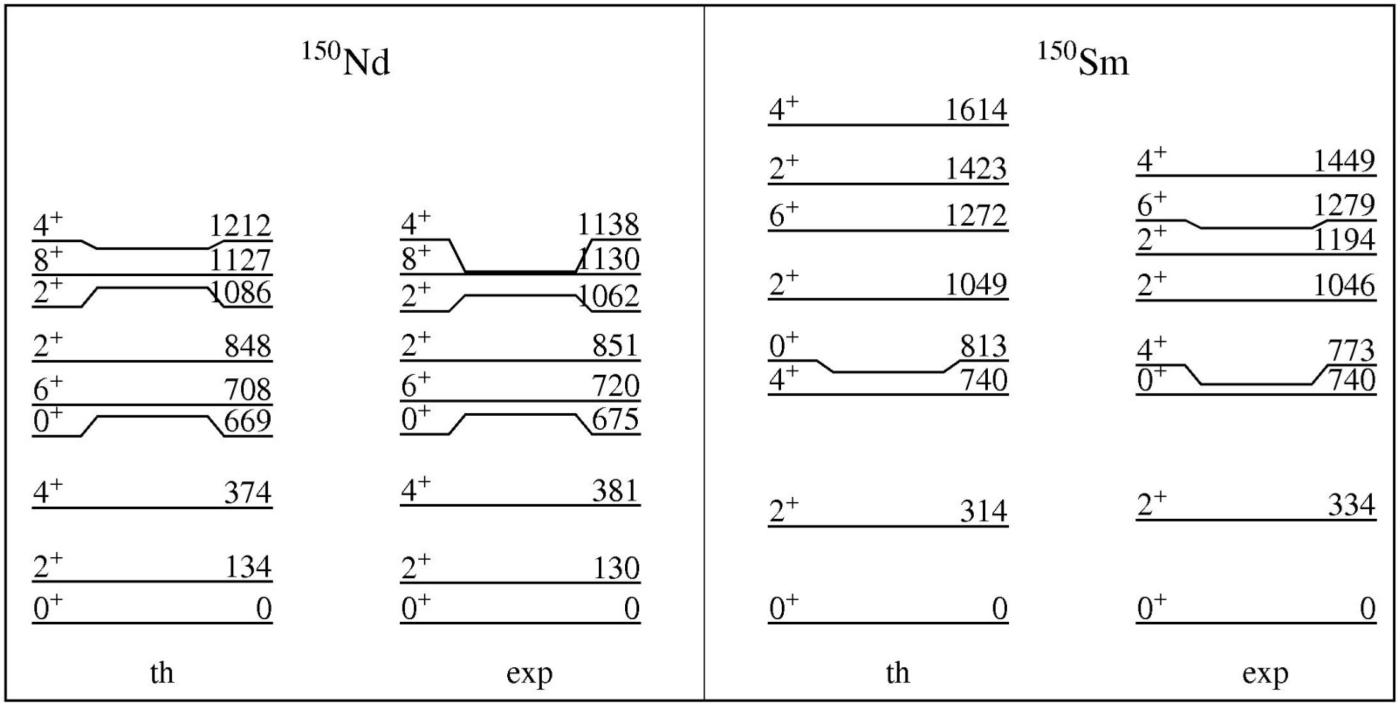
$$(\pi_j^\dagger \times \pi_{j'}^\dagger)_M^{(2)} \mapsto B_\pi(j,j')\left(d_\pi^\dagger\right)_M + C_\pi(j,j')s_\pi^\dagger\left(s_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)} + D_\pi(j,j')s_\pi^\dagger\left(d_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)}$$

Effect small <5%. Will be neglected henceforth.



Matrix elements of the mapped operators are then evaluated with realistic wave functions of the initial and final nuclei taken from the literature. They fit all experimental data for excitation energies, B(E2) values and quadrupole moments, B(M1) values and magnetic moments, etc., very well.

Example:



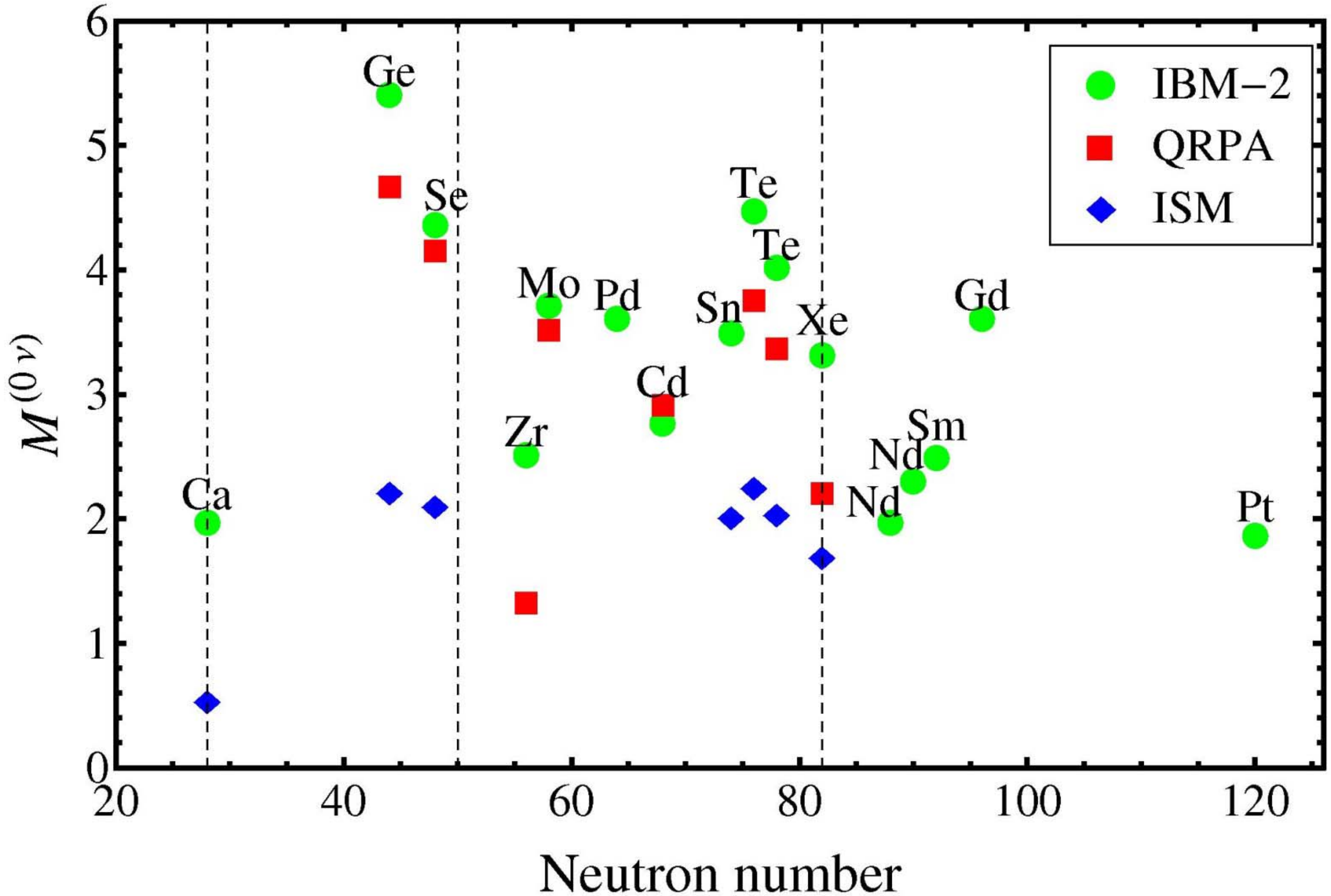
Matrix elements for light and heavy neutrino exchange have been evaluated in

- ISM (2008)
- QRPA-Tü (2008)
- IBM-2 (2009)

As well as in other models

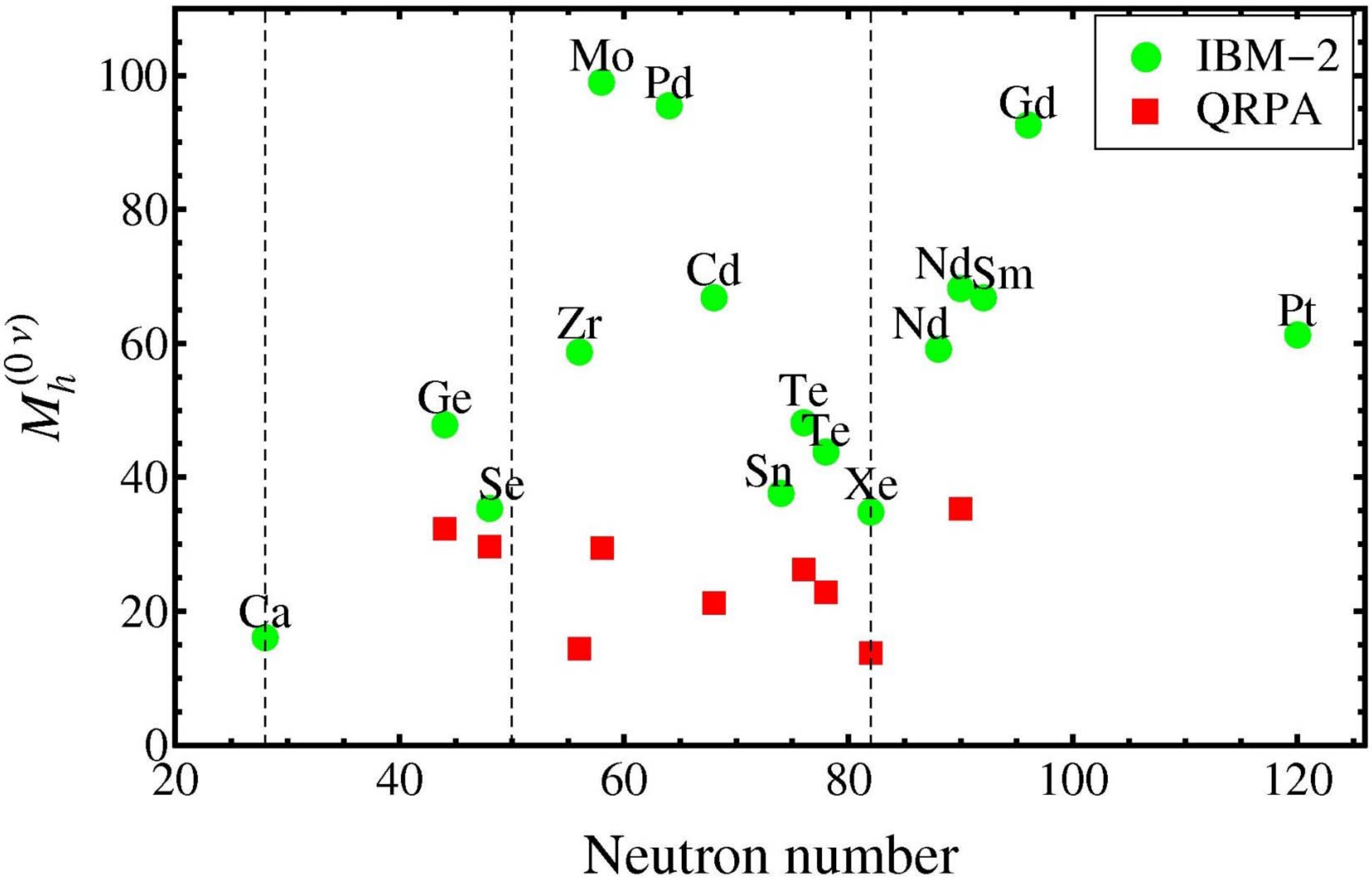
- QRPA-Jy (2008)
- PHFB (2008)
- EDF-DFT (2010)

# RESULTS (2013)
# LIGHT NEUTRINO EXCHANGE




IBM-2 from J. Barea and F. Iachello, Phys. Rev. C 79, 044301 (2009); J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013). MS-SRC.
QRPA from F. Šimkovic *et al.*, Phys. Rev. C 77, 045503 (2008).
ISM from E. Caurier *et al.*, Phys. Rev. Lett. 100, 052503 (2008).

# RESULTS (2013)
# HEAVY NEUTRINO EXCHANGE




IBM-2 from J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013). MS-SRC.
QRPA from Šimkovic *et al.*, Phys. Rev. C 60, 055502 (1999). MS-SRC.

All models make assumptions. The results differ by as much as factors of 2.

It is of importance therefore to study the dependence on the assumptions made in different models.

## SENSITIVITY ANALYSIS (IBM-2): LIGHT NEUTRINO

Estimated sensitivity to input parameter changes:

| | |
|---|---|
| 1. Single-particle energies [¶,§] | 10% |
| 2. Strength of surface delta interaction | 5% |
| 3. Oscillator parameter | 5% |
| 4. Closure energy | 5% |

Estimated sensitivity to model assumptions:

| | |
|---|---|
| 1. Truncation to S, D space | 1% (spherical)-10% (deformed) |
| 2. Isospin purity | 1%(GT)-20%(F)-1%(T) |

¶ This point has been emphasized by J. Suhonen and O. Civitarese, Phys. Lett. B668, 277 (2008).

§ New experiments are being done to check the single particle levels in Ge, Se and Te, J.P. Schiffer *et al.*, Phys. Rev. Lett. **100**, 112501 (2008).

Estimated sensitivity to operator assumptions:

1. Form of the operator 5%
2. Finite nuclear size (FNS) 2%
3. Short range correlations (SRC) [#] 10%

Total: 44%-55% (addition) or 16%-19% (quadrature).

[#] This point is discussed in many articles, for example, M. Kortelainen and J. Suhonen, Phys. Rev. C 75, 051303 (R) (2007).

# SENSITIVITY ANALYSIS (IBM-2): HEAVY NEUTRINO

Heavy-neutrino exchange (short-range) is particularly sensitive to short-range correlations.

Short range correlations (SRC) are taken into account by convoluting the "potential" *v(p)* with the Jastrow function *j(p)* parametrized in various forms (Miller-Spencer, MS/ Argonne/CD Bonn) or by other methods (UCOM)

$$u(p) = \int v(p-p')\, j(p')\, dp'$$

The Jastrow function in configuration space is

$$f_J(r) = 1 - c e^{-ar^2}\left(1 - br^2\right)$$

with

| | | |
|---|---|---|
| a=1.10 $fm^{-2}$ , b=0.68 $fm^{-2}$ , c=1 | MS | soft |
| a=1.59 $fm^{-2}$ , b=1.45 $fm^{-2}$ , c=0.92 | Argonne | hard |
| a=1.52 $fm^{-2}$ , b=1.88 $fm^{-2}$ , c=0.46 | CD Bonn | hard |

The sensitivity to SRC has been in recent years the subject of many investigations ¶.

For light neutrino exchange going from Miller-Spencer (MS)-soft to Argonne (CCM)-hard correlations introduces a factor of ~1.2.

For heavy neutrino exchange going from MS to CCM has a major effect introducing a factor ~2.5!

This is due to the fact that as $r \to 0$

$$f_J(r) \to (1-c) + (a+b)r^2 + \ldots$$

| | (1-c) | (a+b) |
|---|---|---|
| → MS | 0 | 1.78 |
| → Argonne | 0.08 | 3.04 |
| → CD Bonn | 0.54 | 3.40 |

For infinitely heavy neutrinos, the matrix elements calculated with MS-SRC vanish!

¶ From J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013)

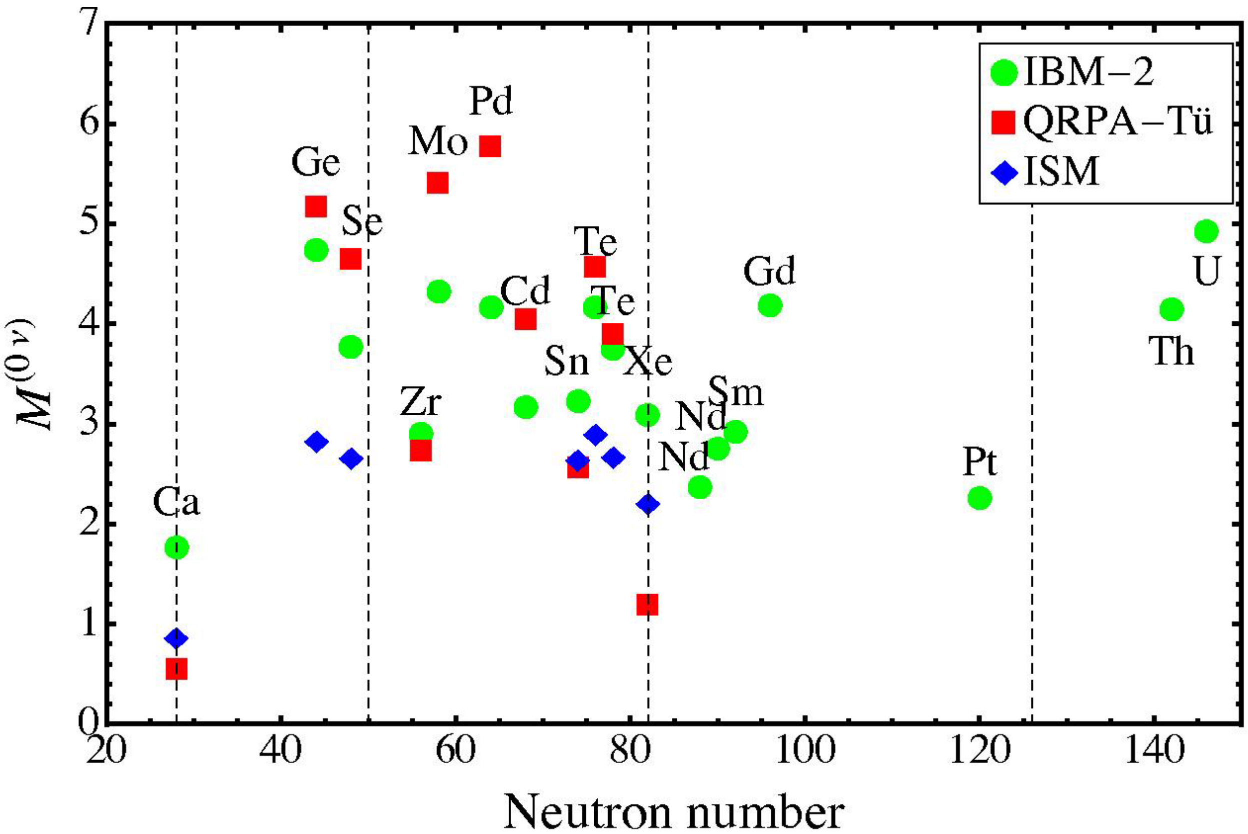
Therefore, for heavy neutrino exchange, the sensitivity to operator assumptions should be increased. A detailed study indicates that SRC in nuclei are hard . The sensitivity can be estimated by comparing results for Argonne with CD Bonn SRC. Going from one to the other introduces a factor 1.2 in the NME for heavy neutrino exchange, with a sensitivity of 20%. The sensitivity for light neutrino exchange is only 2%.

Another source of error is the amount of isospin purity in the wave functions. This error affects QRPA and IBM-2 but not ISM.

In order to take into account this effect, calculations in QRPA-Tü and IBM-2 have been recently updated.

Isospin projection mostly affects the Fermi matrix elements (F).

Most recent (2015) results for $0\nu\beta^-\beta^-$ (light neutrino exchange)

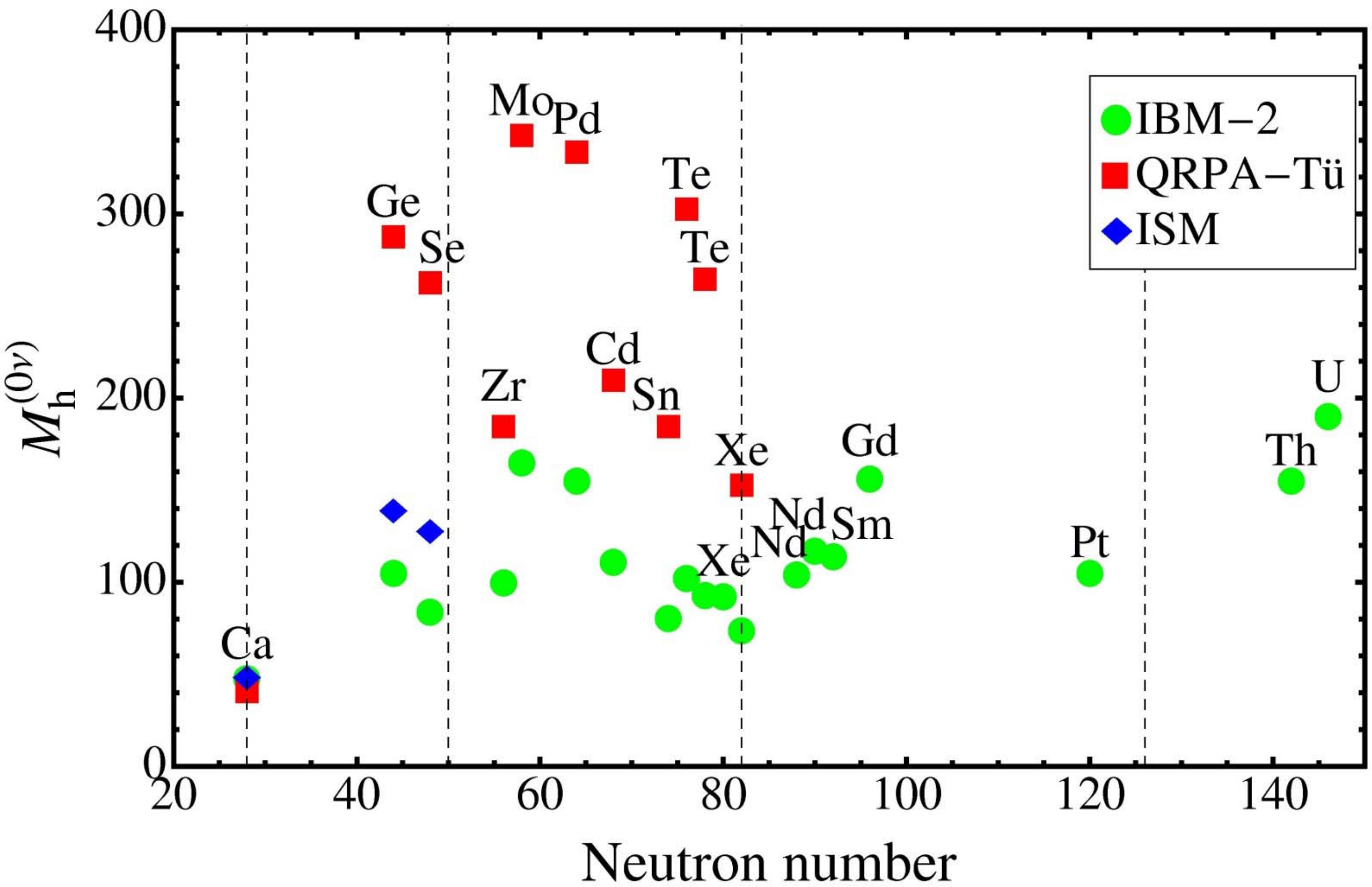



$g_A$=1.269

IBM-2 *: J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 91, 034304 (2015).
QRPA-Tu *: F. Simkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C 87, 045501 (2013).
ISM: J. Menendez, A. Poves, E. Caurier, and F. Nowacki, Nucl. Phys. A 818, 139 (2009).

* With isospin restoration and Argonne SRC

# Most recent (2015) results for $0\nu\beta^-\beta^-$ (heavy neutrino exchange)




$g_A$=1.269

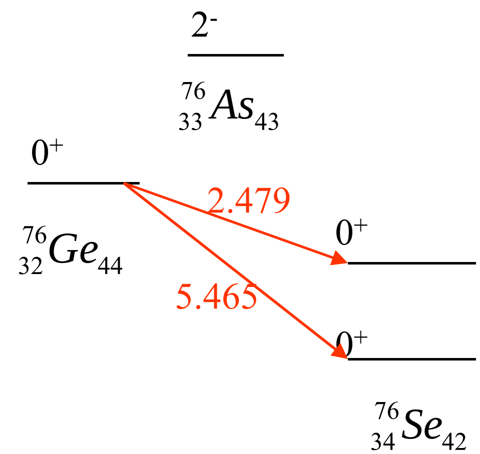
* With isospin restoration and Argonne SRC

# FINAL IBM-2 RESULTS WITH ERROR (2015)

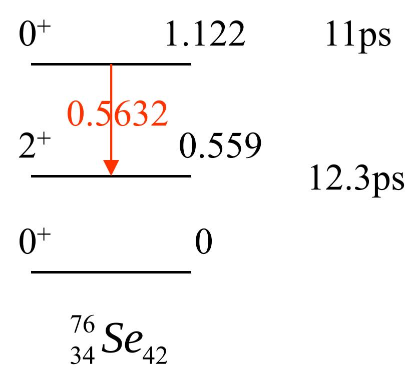
| Decay | Light neutrino exchange | Heavy neutrino exchange |
|---|---|---|
| $^{48}$Ca | 1.75(28) | 47(13) |
| $^{76}$Ge | 4.68(75) | 104(29) |
| $^{82}$Se | 3.73(60) | 83(23) |
| $^{96}$Zr | 2.83(45) | 99(28) |
| $^{100}$Mo | 4.22(68) | 164(46) |
| $^{110}$Pd | 4.05(65) | 154(43) |
| $^{116}$Cd | 3.10(50) | 110(31) |
| $^{124}$Sn | 3.19(51) | 79(22) |
| $^{128}$Te | 4.10(66) | 101(28) |
| $^{130}$Te | 3.70(59) | 92(26) |
| $^{134}$Xe | 4.05(65) | 91(26) |
| $^{136}$Xe | 3.05(59) | 73(20) |
| $^{148}$Nd | 2.31(37) | 103(29) |
| $^{150}$Nd | 2.67(43) | 116(32) |
| $^{154}$Sm | 2.82(45) | 113(32) |
| $^{160}$Gd | 4.08(65) | 155(43) |
| $^{198}$Pt | 2.19(35) | 104(29) |
| $^{232}$Th | 4.04(65) | 159(45) |
| $^{238}$U | 4.81(77) | 189(53) |

# MATRIX ELEMENTS TO EXCITED STATES

In some cases, the matrix elements to the first excited $0^+$ state are large. Although the kinematical factor hinders the decay to the excited state, large matrix elements offer the possibility of a direct detection, by looking at the $\gamma$-ray de-exciting the $0^+$ level.




[On the contrary, matrix elements to the excited $2^+$ state are zero in lowest order since with two leptons in the final state we cannot form angular momentum 2.]



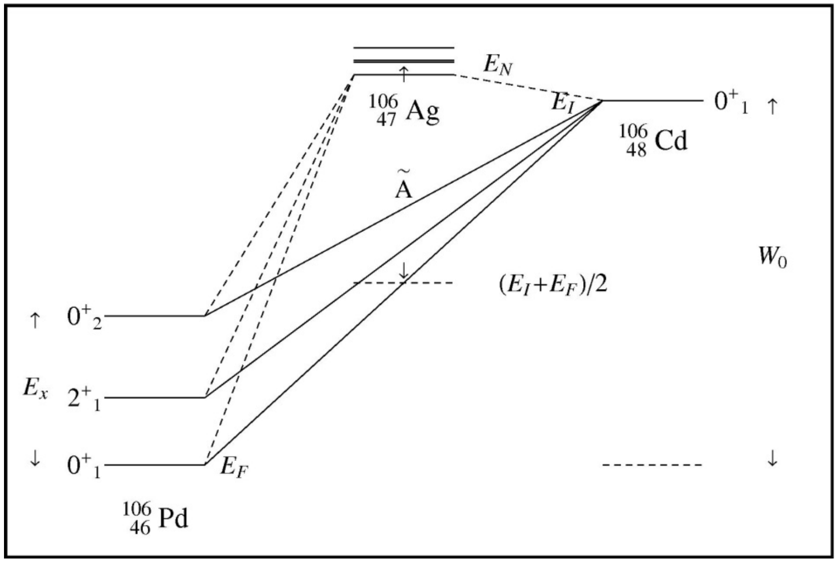
# NUCLEAR MATRIX ELEMENTS TO $0_2$ (2015)

| | $M^{(0\nu)}$ | | |
|---|---|---|---|
| | IBM-2[§] | QRPA[¶] | ISM[*] |
| $^{48}Ca \rightarrow {}^{48}Ti$ | 3.82 | | 0.68 |
| $^{76}Ge \rightarrow {}^{76}Se$ | 2.02 | 1.28 | 1.49 |
| $^{82}Se \rightarrow {}^{82}Kr$ | 0.95[a] | 1.34 | 0.28 |
| $^{96}Zr \rightarrow {}^{96}Mo$ | 0.05 | | |
| $^{100}Mo \rightarrow {}^{100}Ru$ | 1.12 | 1.27 | |
| $^{110}Pd \rightarrow {}^{110}Cd$ | 0.52 | | |
| $^{116}Cd \rightarrow {}^{116}Sn$ | 0.93 | | |
| $^{124}Sn \rightarrow {}^{124}Te$ | 2.38 | | 0.80 |
| $^{128}Te \rightarrow {}^{128}Xe$ | 2.85[a] | | |
| $^{130}Te \rightarrow {}^{130}Xe$ | 2.71 | | 0.19 |
| $^{136}Xe \rightarrow {}^{136}Ba$ | 1.60 | 4.42 | 0.49 |
| $^{148}Nd \rightarrow {}^{148}Sm$ | 0.29 | | |
| $^{150}Nd \rightarrow {}^{150}Sm$ | 0.45 | | |
| $^{154}Sm \rightarrow {}^{154}Gd$ | 0.41 | | |
| $^{160}Gd \rightarrow {}^{160}Dy$ | 0.87 | | |
| $^{198}Pt \rightarrow {}^{198}Hg$ | 0.10[a] | | |

[a] negative Q-value

[§] J. Barea, J. Kotila and F. Iachello, Phys. Rev. C91 (2015) 034304. With isospin restoration and Argonne-SRC.
[¶] F. Šimkovic, M. Nowak, W.A. Kaminsky, A.A. Raduta, and A. Faessler, Phys. Rev. C64 (2001) 035501(QRPA-RCM).
[*] J. Menendez, A. Poves, E. Caurier, F. Nowacki, Nucl. Phys. A818 (2009) 139.

# DOUBLE POSITRON DECAY




Nuclear matrix elements for $0\nu\beta^+\beta^+/0\nu\beta^+EC$ decay have been calculated. The matrix elements are of the same order of magnitude of $0\nu\beta^-\beta^-$ decay.

| | $0_1^+$ | | | $0_2^+$ | |
|---|---|---|---|---|---|
| Decay | IBM-2 | QRPA[a] | | IBM-2 | QRPA |
| $^{58}$Ni | 2.61 | 1.55 | | 2.44 | |
| $^{64}$Zn | 5.44 | | | 0.70 | |
| $^{78}$Kr | 3.92 | 4.16 | | 0.90 | |
| $^{96}$Ru | 2.85 | 3.23 | 4.29[b] | 0.04 | 2.31[b] |
| $^{106}$Cd | 3.59 | 4.10 | 7.54[c] | 1.72 | 0.61[c] |
| $^{124}$Xe | 4.74 | 4.76 | | 0.80 | |
| $^{130}$Ba | 4.67 | 4.95 | | 0.34 | |
| $^{136}$Ce | 4.54 | 3.7 | | 0.38 | |
| $^{156}$Dy | 3.17 | | | 1.75 | |
| $^{164}$Er | 3.95 | | | 1.13 | |
| $^{180}$W | 4.67 | | | 0.31 | |

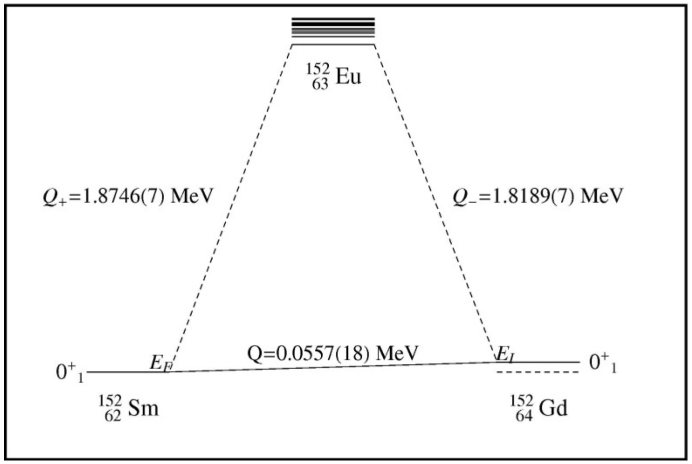
[a] M. Hirsch *et al.*, Z. Phys. A **347**, 151 (1994). No isospin restoration.
[b] J. Suhonen, Phys. Rev. C **86**, 024301 (2012). (UCOM SRC). No isospin restoration.
[c] J. Suhonen, Phys. Lett. B **701**, 490 (2011). (UCOM SRC). No isospin restoration.

IBM-2: J. Barea, J. Kotila and F. Iachello, Phys. Rev. C87, 057301 (2013).

QRPA: M. Hirsch, K. Muto, T. Oda, and H.V. Klapdor-Kleingrothaus, Z. Phys. A347, 151 (1994).

QRPA-Jy: J. Suhonen, Phys. Rev. C86, 024301 (2012); J. Suhonen, Phys. Lett. B701, 490 (2011).

# RESONANT DOUBLE ELECTRON CAPTURE




Nuclear matrix elements for double electron capture 0νECEC have been calculated

J. Kotila, J. Barea and F. Iachello, Phys. Rev. C 89, 064319 (2014).

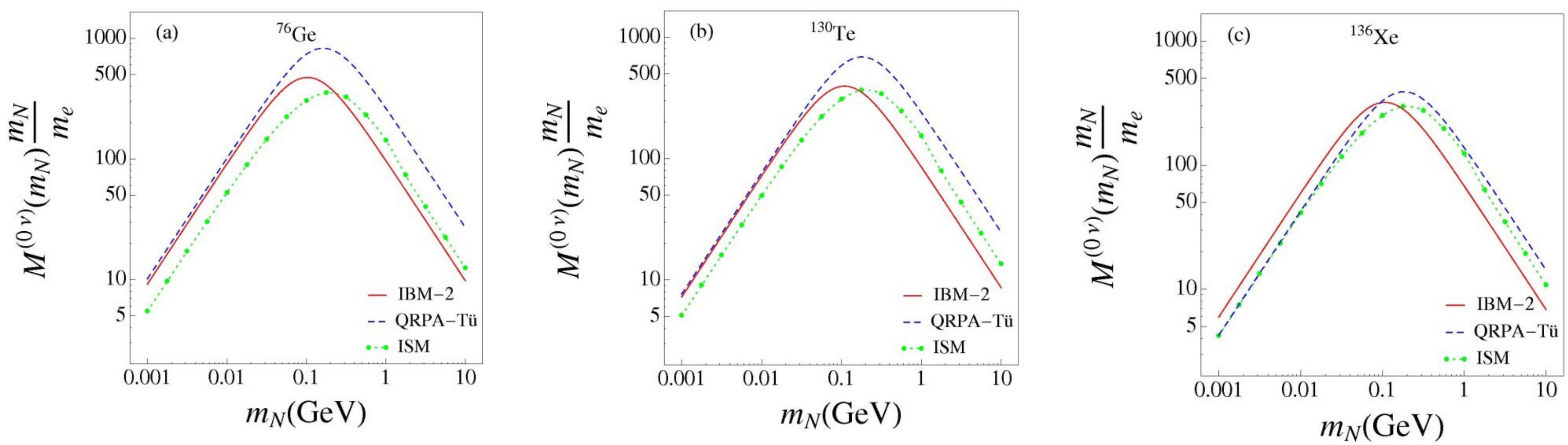
TABLE I. Comparison between IBM-2 matrix elements with Argonne SRC for $0\nu ECEC$ decay and QRPA and EDF.

| Decay | $M^{(0\nu)}$ (light) | | | |
|---|---|---|---|---|
| | IBM-2 | spherical QRPA[a] | deformed QRPA[a] | EDF[b] |
| $^{152}_{64}Gd_{88} \rightarrow ^{152}_{62}Sm_{90}$ | 2.44 | 7.59 | 3.23-2.67 | 1.07-0.89 |
| $^{164}_{68}Er_{96} \rightarrow ^{164}_{66}Dy_{98}$ | 3.95 | 6.12 | 2.64-1.79 | 0.64-0.50 |
| $^{180}_{74}W_{106} \rightarrow ^{180}_{72}Hf_{108}$ | 4.67 | 5.79 | 2.05-1.79 | 0.58-0.38 |

[a] D.-L Fang *et al.*, Phys. Rev. C **85**, 035503 (2012).
[b] T. R. Rodríguez and G. Martínez-Pinedo, Phys. Rev. C **85**, 044310 (2012).

# STERILE NEUTRINOS

The NME depend in this case on the mass of the exchanged neutrino. For a neutrino of mass $m_N$, the NME are $\frac{m_N}{m_e} M_{0\nu}(m_N)$

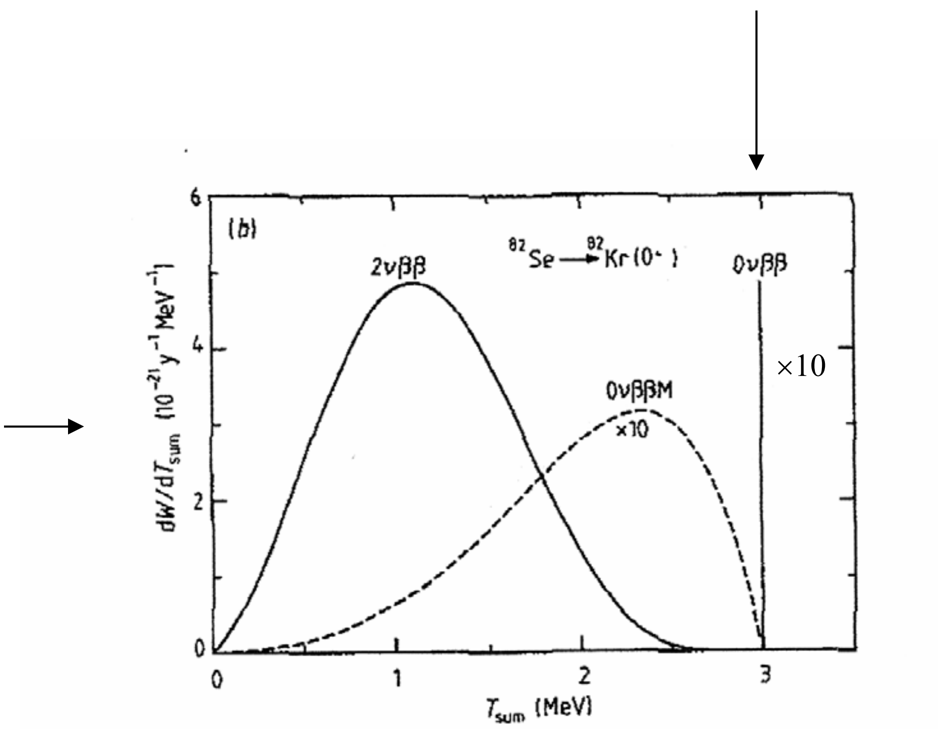


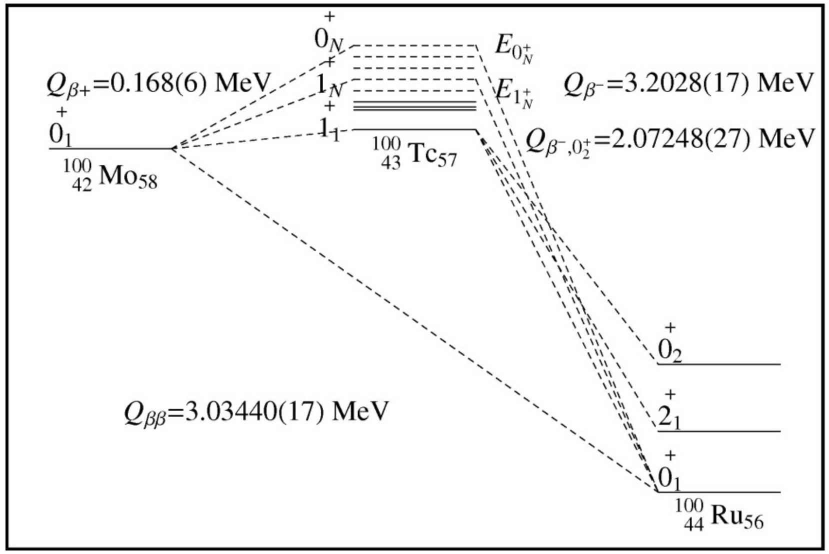

Note the resonant behavior at $m_N$~100MeV, the Fermi momentum, $p_F$, of nucleons in the nucleus.

# EVALUATION OF THE NUCLEAR MATRIX ELEMENTS: 2νββ

2νββ decay is concomitant to 0νββ decay. It has been measured in several cases. Its calculation is needed for comparing with experiments and for estimating its tail at the location of 0νββ

Summed energy spectra of the two emitted electrons






The evaluation of 2νββ NME is more difficult than 0νββ because in this case the closure approximation may not be good. Therefore one needs to evaluate the individual matrix elements

$$M^{(2\nu)}_{GT,N} = \frac{\left\langle 0^+_F \left\| \tau^\dagger \sigma \right\| 1^+_N \right\rangle \left\langle 1^+_N \left\| \tau^\dagger \sigma \right\| 0^+_I \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{1^+_N} - E_I}$$

$$M^{(2\nu)}_{F,N} = \frac{\left\langle 0^+_F \left\| \tau^\dagger \right\| 0^+_N \right\rangle \left\langle 0^+_N \left\| \tau^\dagger \right\| 0^+_I \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{0^+_N} - E_I}$$

From the individual NME and PSF one then obtains the matrix elements by summing over all the intermediate states in the odd-odd nucleus.
The full evaluation has been done in selected cases in QRPA and ISM [¶,§,#] and very recently in IBFM-2 [*].

[¶] pnQRPA: J. Suhonen, Phys. At. Nucl. 61, 1186 (1998); 65, 2176 (2002).
[§] pnMAVA: J. Kotila, J. Suhonen, and D.S. Delion, J. Phys. G36, 045106 (2009)
[#] ISM: E. Caurier, F. Nowacki, and A. Poves, Int. J. Mod. Phys. E16, 552 (2007).

[*] N. Yoshida and F. Iachello, Prog. Theor. Exp. Phys. 2013, 043D01 (2013).

The separation between PSF and NME can be done in two cases: (1) closure approximation (CA) and (2) single-state dominance (SSD). In both cases

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}^{(0)} \left| m_e c^2 M_{2\nu} \right|^2$$

In CA

$$M_{2\nu} = g_A^2 M^{(2\nu)}$$

$$M^{(2\nu)} = -\left[ \frac{M_{GT}^{(2\nu)}}{\tilde{A}_{GT}} - \left( \frac{g_V}{g_A} \right)^2 \frac{M_F^{(2\nu)}}{\tilde{A}_F} \right]$$

where

$$M_{GT}^{(2\nu)} = \left\langle 0_F^+ \left| \sum_{nn'} \tau_n^\dagger \tau_{n'}^\dagger \vec{\sigma}_n \cdot \vec{\sigma}_{n'} \right| 0_I^+ \right\rangle$$

$$M_F^{(2\nu)} = \left\langle 0_F^+ \left| \sum_{nn'} \tau_n^\dagger \tau_{n'}^\dagger \right| 0_I^+ \right\rangle$$

and

$$\tilde{A}_{GT} = \frac{1}{2}\left( Q_{\beta\beta} + 2m_e c^2 \right) + \left\langle E_{1^+,N} \right\rangle - E_I$$

$$\tilde{A}_F = \frac{1}{2}\left( Q_{\beta\beta} + 2m_e c^2 \right) + \left\langle E_{0^+,N} \right\rangle - E_I$$

The Fermi matrix elements for $2\nu\beta\beta$ decay are zero if isospin is a good quantum number and can therefore be neglected.

In the SSD approximation, the matrix elements are given by

$$M^{(2\nu)}_{GT,SSD} = \frac{\left\langle 0^+_F \left\| \tau^\dagger \sigma \right\| 1^+_1 \right\rangle \left\langle 1^+_1 \left\| \tau^\dagger \sigma \right\| 0^+_I \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{0^+_1} - E_I}$$

$$M^{(2\nu)}_{F,SSD} = \frac{\left\langle 0^+_F \left\| \tau^\dagger \right\| 0^+_1 \right\rangle \left\langle 0^+_1 \left\| \tau^\dagger \right\| 0^+_I \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{0^+_1} - E_I}$$

from which one has

$$M^{(2\nu)}_{SSD} = -\left[ M^{(2\nu)}_{GT,SSD} - \left(\frac{g_V}{g_A}\right)^2 M^{(2\nu)}_{F,SSD} \right]$$

$$M_{2\nu,SSD} = g_A^2 M^{(2\nu)}_{SSD}$$

In the case of CA and SSD the NME for 2νββ decay can be calculated in the same way as for 0νββ but with neutrino potential

$$v_{2\nu}(p) = \frac{\delta(p)}{p^2}$$

Results for 2νββ

| | $0_1^+$ | | $0_2^+$ | |
|---|---|---|---|---|
| Nucleus | $M_{GT}^{(2\nu)}$ | $M_F^{(2\nu)}$ | $M_{GT}^{(2\nu)}$ | $M_F^{(2\nu)}$ |
| $^{48}$Ca | 1.64 | -0.01 | 5.07 | -0.01 |
| $^{76}$Ge | 4.44 | -0.01 | 2.02 | -0.00 |
| $^{82}$Se | 3.59 | -0.01 | 1.05 | -0.00 |
| $^{96}$Zr | 2.28 | -0.00 | 0.04 | -0.00 |
| $^{100}$Mo | 3.05 | -0.00 | 0.81 | -0.00 |
| $^{110}$Pd | 3.08 | -0.00 | 0.38 | -0.00 |
| $^{116}$Cd | 2.38 | -0.00 | 0.83 | -0.00 |
| $^{124}$Sn | 2.86 | -0.01 | 2.19 | -0.00 |
| $^{128}$Te | 3.71 | -0.01 | 2.70 | -0.00 |
| $^{130}$Te | 3.39 | -0.01 | 2.64 | -0.00 |
| $^{134}$Xe | 3.69 | -0.01 | 2.34 | -0.00 |
| $^{136}$Xe | 2.82 | -0.01 | 1.65 | -0.00 |
| $^{148}$Nd | 1.31 | -0.00 | 0.18 | -0.00 |
| $^{150}$Nd | 1.61 | -0.00 | 0.31 | -0.00 |
| $^{154}$Sm | 1.95 | -0.00 | 0.35 | -0.00 |
| $^{160}$Gd | 3.08 | -0.00 | 0.53 | -0.00 |
| $^{198}$Pt | 1.06 | -0.00 | 0.03 | -0.00 |
| $^{232}$Th | 2.75 | -0.00 | 0.08 | -0.00 |
| $^{238}$U | 3.35 | -0.00 | 0.24 | -0.00 |

# QUENCHING OF $g_A$

Results in the previous slides are obtained with $g_A$=1.269.

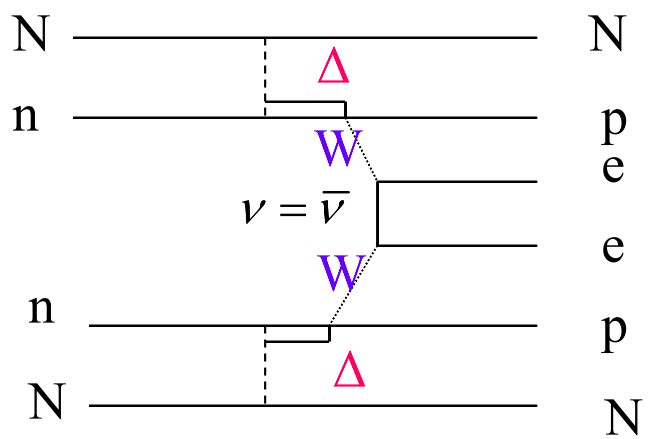
It is well-known from single β-decay/EC ¶ and from 2νββ that $g_A$ is renormalized in models of nuclei. Two reasons:

(i) Limited model space

(ii) Omission of non-nucleonic degrees of freedom (Δ,…)

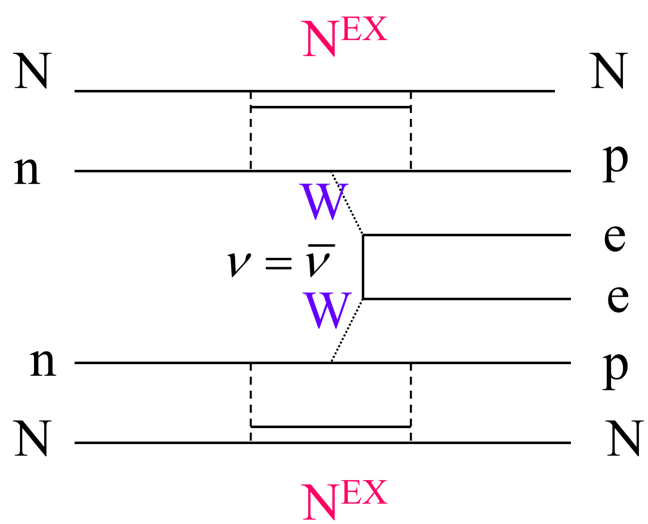
¶ J. Fujita and K. Ikeda, Nucl. Phys. 67, 145 (1965).
D.H. Wilkinson, Nucl. Phys. A225, 365 (1974).

# ORIGIN OF QUENCHING OF $g_A$ IN DBD




Quenching factor $q_\Delta \cong 0.7$

(Δ means excited states of the nucleon)




Quenching factor $q_{N^{EX}} \cong 0.7$
(nuclear model dependent)

($N^{EX}$ means excited states of the nucleus not included explicitly)

Maximal quenching:

$$Q = q_\Delta q_{N^{EX}} \cong 0.5$$

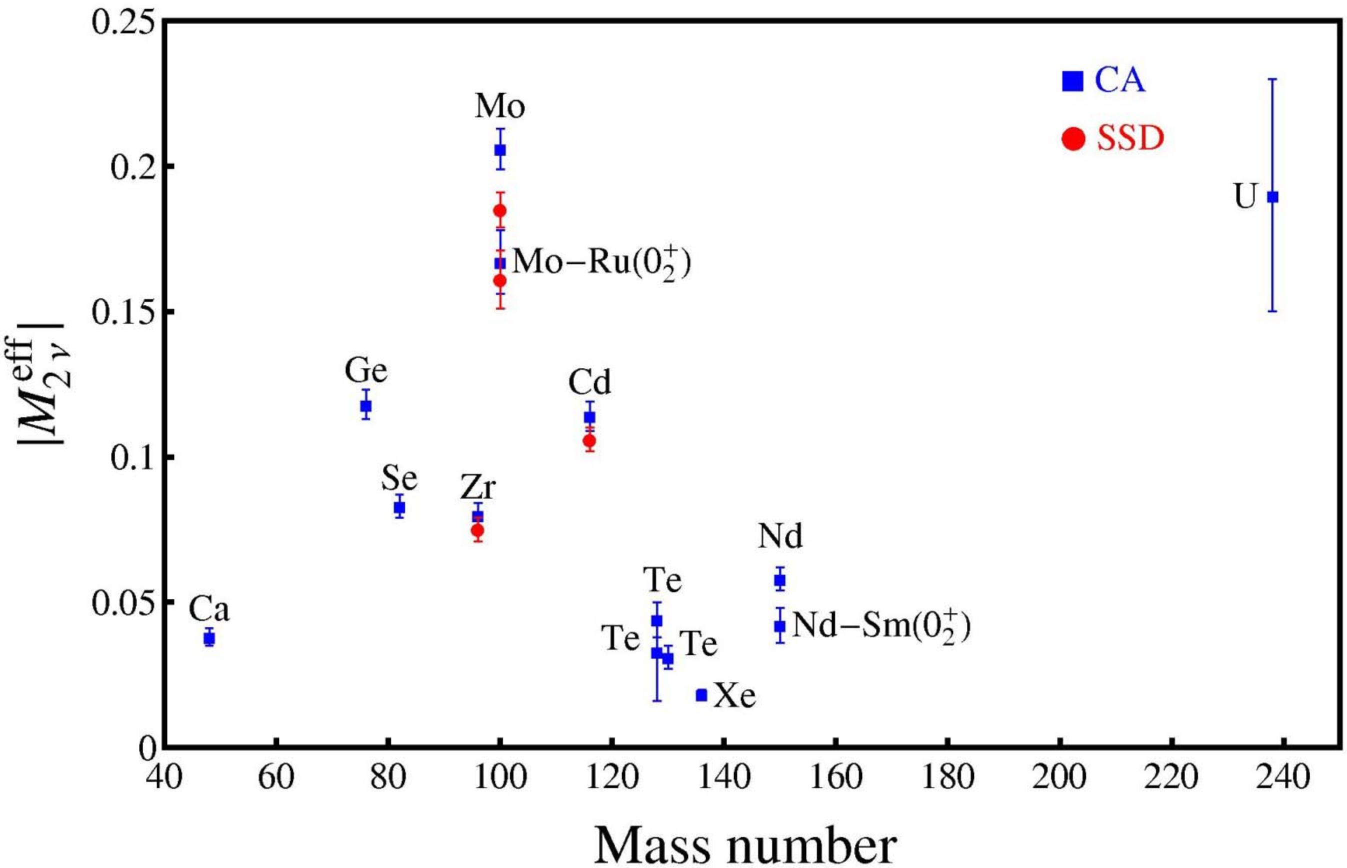
For each model (ISM/QRPA/IBM-2) one can define an effective $g_{A,eff}$ by writing

$$M_{2\nu}^{eff} = \left( \frac{g_{A,eff}}{g_A} \right)^2 M_{2\nu}$$

$$M_{\beta/EC}^{eff} = \left( \frac{g_{A,eff}}{g_A} \right) M_{\beta/EC}$$

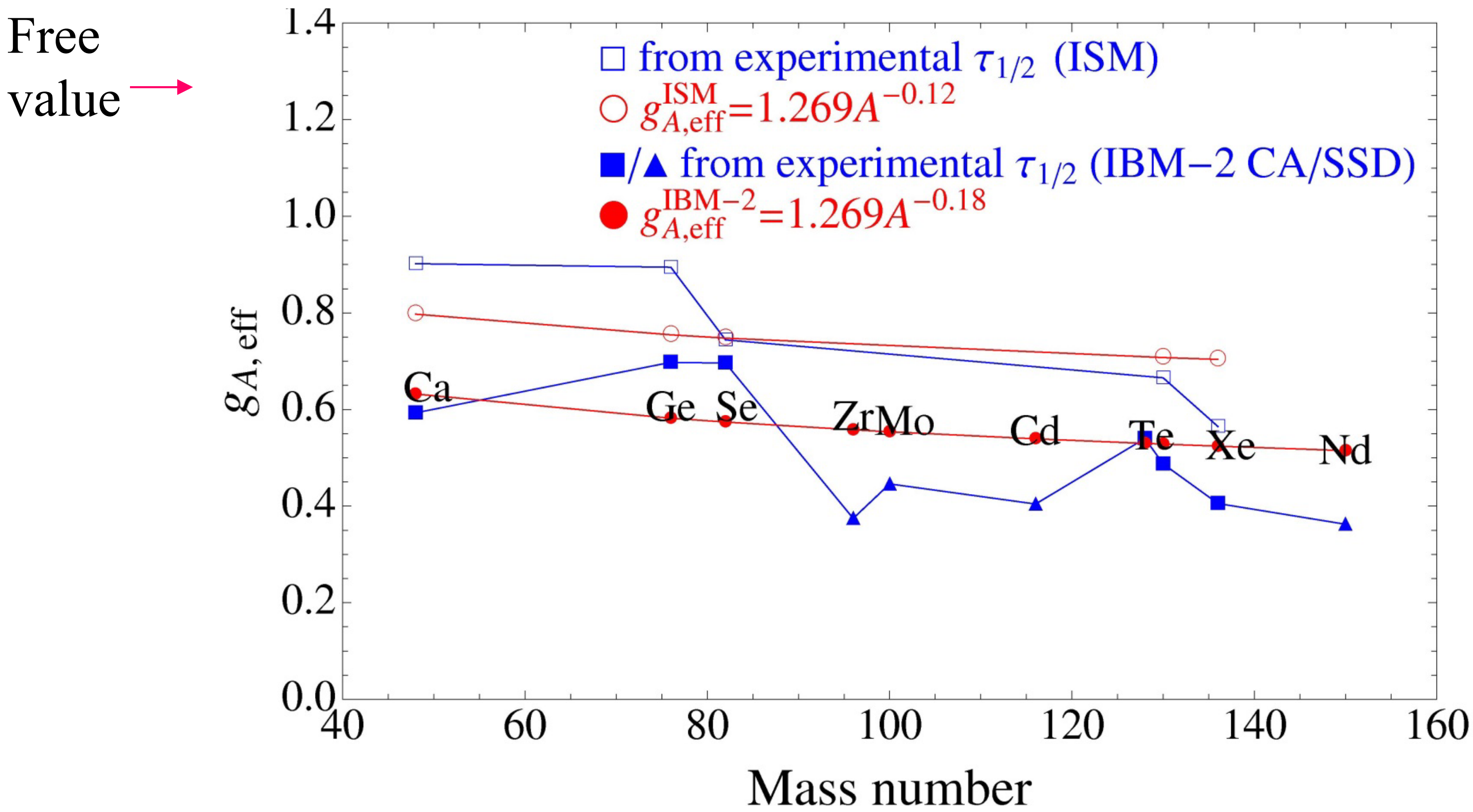
The value of $g_{A,eff}$ in each nucleus can then be obtained by comparing the calculated and measured half-lives for β/EC and for 2νββ.

Values of $|M_{2\nu}^{eff}|$ obtained from experimental half-lives ¶




¶ From a compilation by A.S. Barabash, Phys. Rev. C 81, 035501 (2010). For $^{136}$Xe, N. Ackerman *et al.* (EXO Collaboration), Phys. Rev. Lett. 107, 212501 (2011).

# Effective axial vector coupling constant in nuclei from $2\nu\beta\beta$ ¶




One obtains $g_{A,eff}^{IBM\text{-}2} \sim 0.6\text{-}0.5$.
The extracted values can be parametrized as
A similar analysis can be done for the ISM
for which $g_{A,eff}^{ISM} \sim 0.8\text{-}0.7$.

$$g_{A,eff}^{IBM2} = 1.269A^{-0.18}$$

$$g_{A,eff}^{ISM} = 1.269A^{-0.12}$$

¶ J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013).

$g_{A,eff}$, has been extracted also from single β/EC in QRPA, very recently by Suhonen and Civitarese (QRPA-Jy), $g_{A,eff}^{QRPA} \sim 0.8\text{-}0.4$ [§], and a few years ago by Faessler *et al*. (QRPA-Tü) $\sim 0.7$ [*].

[In some earlier (1989) QRPA papers[¶] , it is claimed that no renormalization of $g_A$ is needed. However, this claim is based on results where the renormalization of $g_A$ is transferred to a renormalization of the free parameter $g_{pp}$ used in the calculation and adjusted to the experimental 2νββ half-life.]

[§] J. Suhonen and O. Civitarese, Phys. Lett. B 725, 153 (2013).

[*] A. Faessler, G.L. Fogli, E. Lisi, V. Rodin, A.M. Rotunno, and F. Šimkovic, J. Phys. G: Nucl. Part. Phys. 35, 075104 (2008).

[¶] K. Muto, E. Bender, H.V. Klapdor, Z. Phys. A334, 177 (1989); 187 (1989).

An "exact" extraction of $g_{A,eff}$ has also recently been done¶ in IBFM-2 both from single β/EC and from 2νββ decay in $^{128}$Te and $^{130}$Te and is given in Appendix B. The extracted values of $g_A$ are ~0.4!

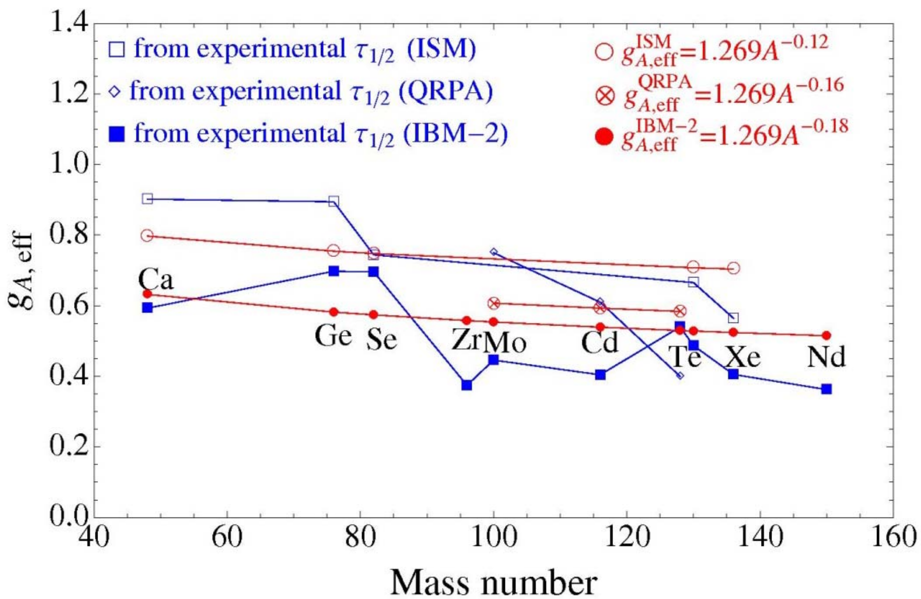
¶ N. Yoshida and F. Iachello, Prog. Theor. Exp. Phys. 2013, 043D01 (2013).

Very recently Pirinen and Suhonen[§] have done a systematic analysis of $g_{A,eff}^{QRPA}$ from single β/EC. A parametrization of these results is

$$g_{A,eff}^{QRPA} = 1.269 A^{-0.16}$$

[§] P. Pirinen and J. Suhonen, Phys. Rev. C91, 054309 (2015).

A combined parametrization of $g_{A,eff}$ including IBM-2, QRPA, and ISM is



# IMPACT OF THE RENORMALIZATION

The axial vector coupling constant, $g_A$, appears to the second power in the NME

$$M_{2\nu} = g_A^2 M^{(2\nu)}$$

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

and hence to the fourth power in the half-life!

Therefore, the results of the previous slides should be multiplied by 6-34 to have realistic estimates of expected half-lives. [See also, H. Robertson ¶, and S. Dell'Oro, S. Marcocci, F. Vissani#.]

¶ R.G.H. Robertson, Modern Phys. Lett. A 28, 1350021 (2013).

# S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D90, 033005 (2014).

The question of whether or not $g_A$ in 0νββ is renormalized as much as in 2νββ is of much debate. In 2νββ only the $1^+$ (GT) multipole contributes. In 0νββ all multipoles $1^+$, $2^-$,…; $0^+$, $1^-$ … contribute. Some of these could be unquenched. However, even in 0νββ, $1^+$ intermediate states dominate. Hence, our current understanding is that $g_A$ is renormalized in 0νββ as much as in 2νββ.

This problem is currently being addressed from various sides. Experimentally by measuring the matrix elements to and from the intermediate odd-odd nucleus in 2νββ decay [§]. Theoretically, by using effective field theory (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents) [¶].

[§] P. Puppe *et al.*, Phys. Rev. C 86, 044603 (2012).

[¶] J. Menendez, D. Gazit, and A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011).

Another question is whether or not the vector coupling constant, $g_V$, is renormalized in nuclei.
Because of CVC, the mechanism (ii) omission of non-nucleonic degrees of freedom cannot contribute.
However, the mechanism (i), limited model space, can contribute, and, if so, the ratio $g_V/g_A$ may remain the same as the non-renormalized ratio 1/1.269.
No experimental information is available, but is could be obtained by measuring with ($^3$He,t) and (d,$^2$He) reactions the F matrix elements to and from the intermediate odd-odd nucleus.
Also, measurements of double charge exchange reactions with heavy ions at LNS (Catania) could help understand this question, especially the relative role of F versus GT matrix elements.

# SUMMARY

NME have been calculated for

- $0\nu\beta^-\beta^-$,
- $0\nu\beta^+\beta^+$ , $0\nu\beta^+EC$, $R0\nu ECEC$

and

- $2\nu\beta^-\beta^-$ , ← Measured
- $2\nu\beta^+\beta^+$ , $2\nu\beta^+EC$, $2\nu ECEC$

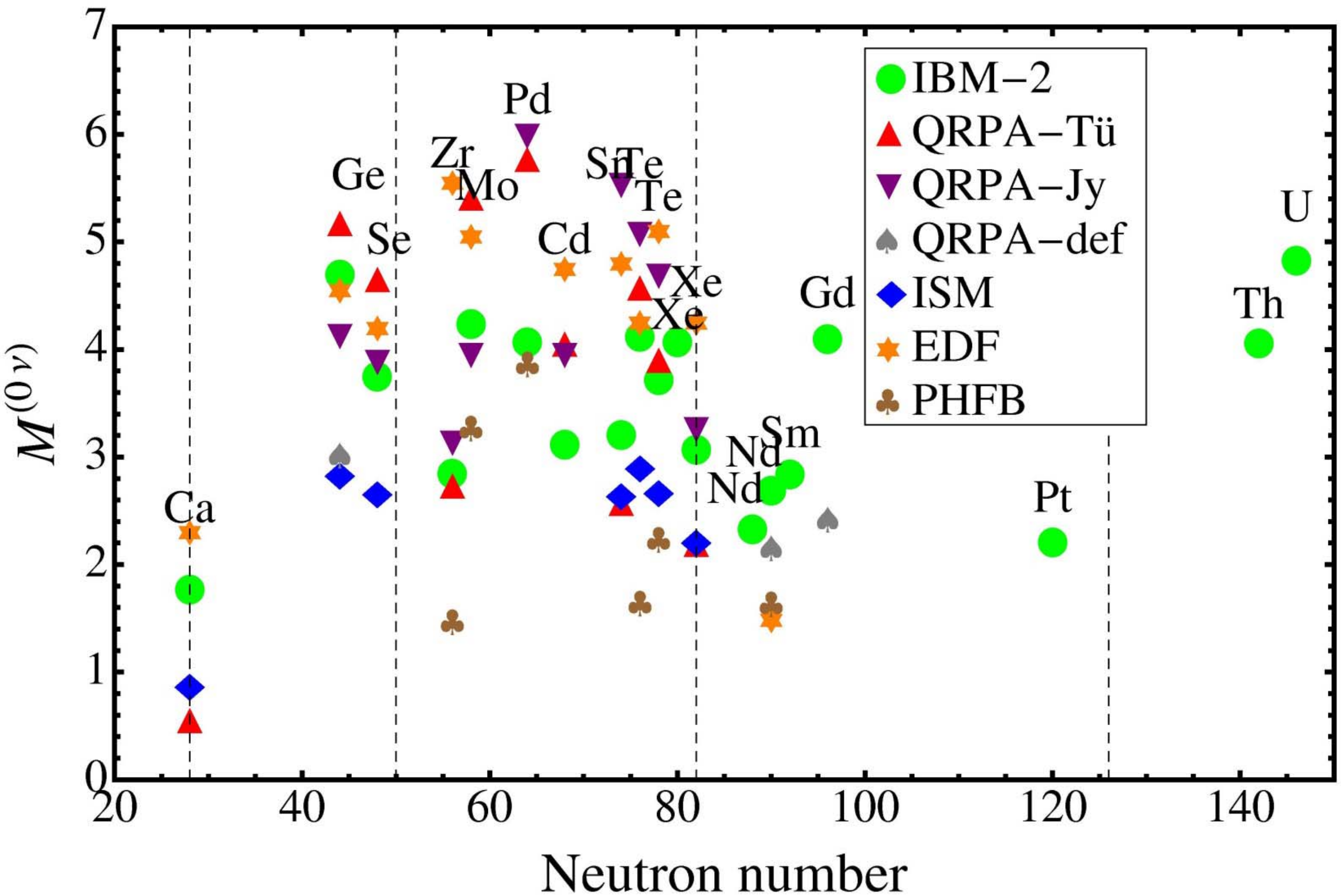
For

- light-neutrino exchange
- heavy-neutrino exchange
- sterile-neutrino exchange
- Majoron emission

← Hypothetical

They are available upon request from jenni.kotila@yale.edu

# APPENDIX A: SUMMARY OF MATRIX ELEMENTS (2015)



## APPENDIX B: ESTIMATE FROM $2\nu\beta\beta$ IN THE “EXACT” NON-CLOSURE CALCULATION

A program has been written to calculate $2\nu\beta\beta$ “exactly” in IBFFM-2 by summing over intermediate states in the odd-odd nucleus (Yoshida, 2012).

Steps in this calculation are:

1. Calculation of spectra of the initial and the final even-even nuclei, in IBM-2.
2. (Calculation of spectra of adjacent odd-even and even-odd nuclei, in IBFM-2, to determine the strength of the boson-fermion interaction).
3. Calculation of spectra of the intermediate odd-odd nuclei, in IBFFM-2.
4. Calculation of GT and F matrix elements from even-even to odd-odd and from odd-odd to even-even.
5. Sum of product with PSF over states in the intermediate nucleus. Approximately 150 states are included.

# Results for $^{128}$Te → $^{128}$Xe and $^{130}$Te → $^{130}$Xe decay¶ .

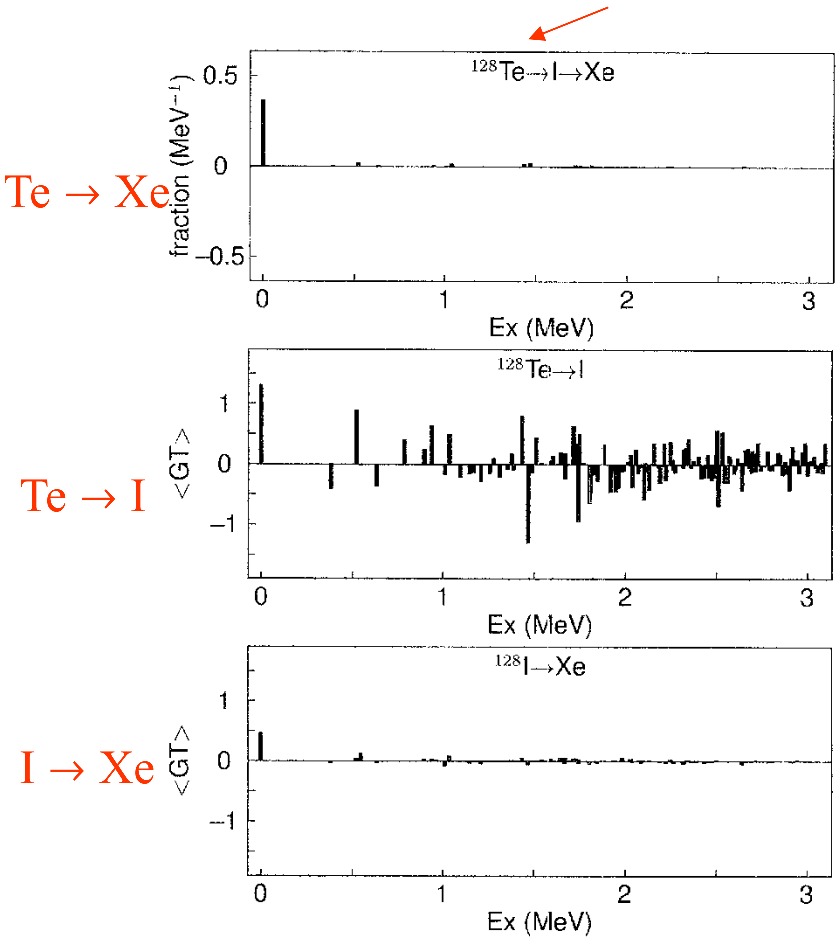



Figure 2: The values of $\langle 0_1^+||t^+\sigma||1_N^+\rangle\langle 1_N^+||t^+\sigma||0_1^+\rangle/(\frac{1}{2}W_0+E_N-E_I)$ (top), $\langle 1_N^+||t^+\sigma||0_1^+\rangle$ (center), and $\langle 0_1^+||t^+\sigma||1_N^+\rangle$ (bottom), for the double $\beta$ decay from the lowest $0^+$ in $^{128}$Te to the lowest $0^+$ in $^{128}$Xe through the intermediate $1^+$ in $^{128}$I, plotted as a function of the excitation energy of $1^+$.

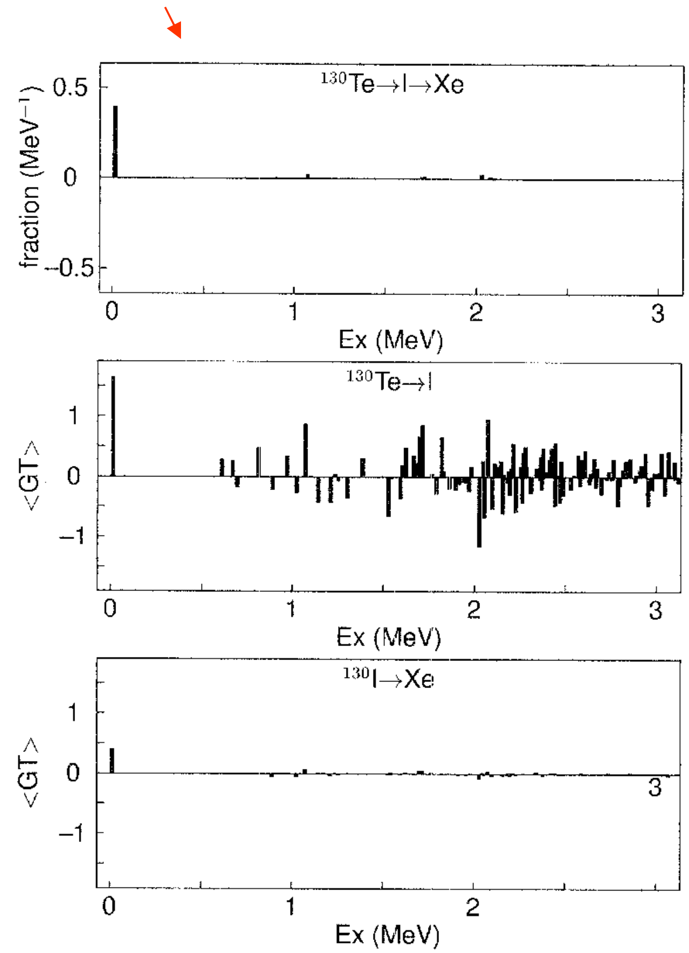
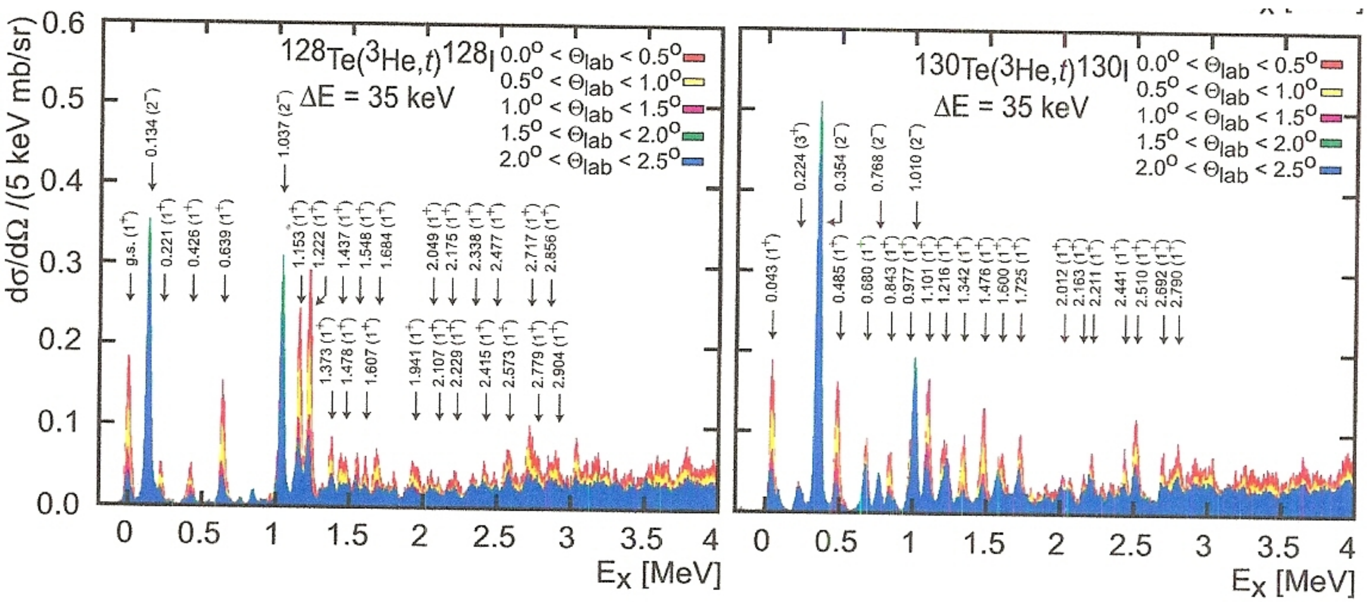



Figure 3: The same plots as Fig. 2 for the decay $^{130}$Te→$^{130}$Xe through $^{130}$I.

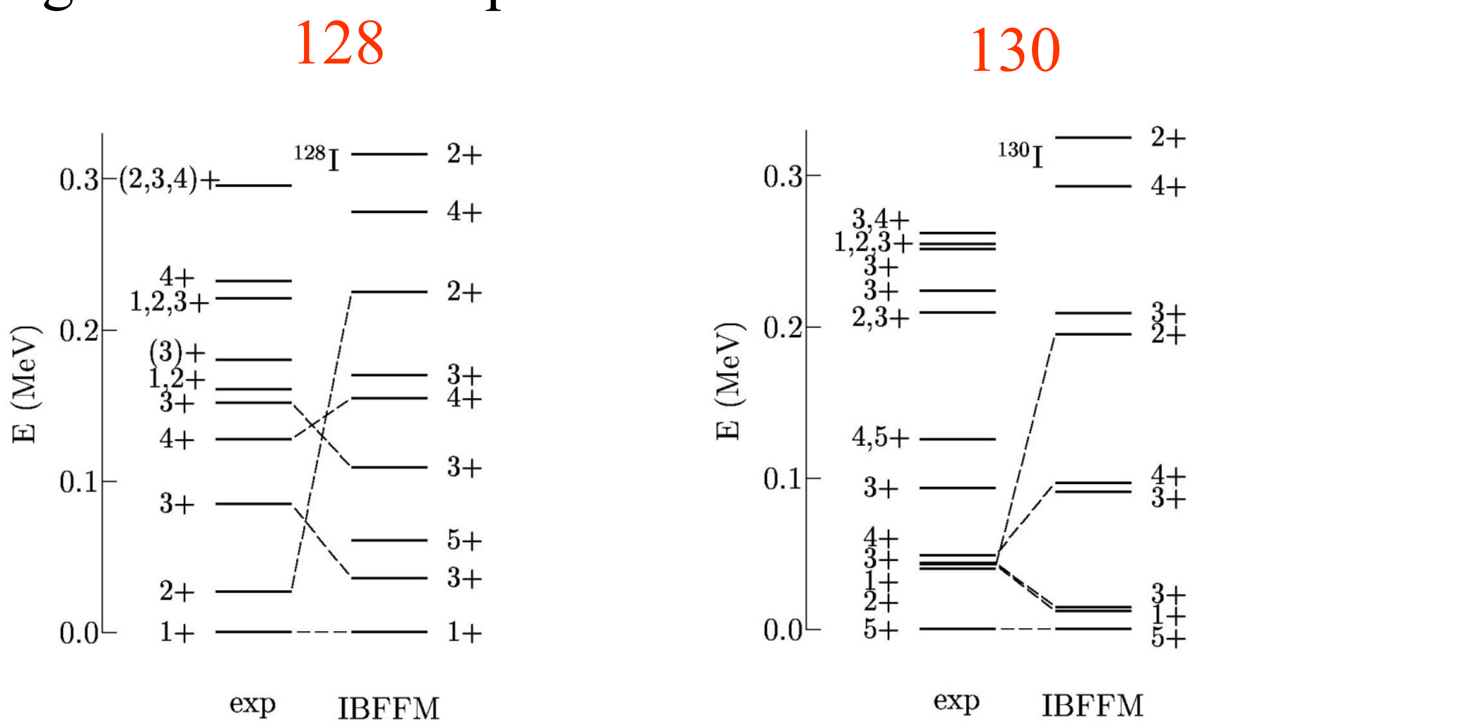
¶ N. Yoshida and F. Iachello, Prog. Theor. Exp. Phys. 2013, 043D01 (2013).

# The calculation Te → I can be compared with recent experiment §




§ P. Puppe *et al.*, Phys. Rev. C 86, 044603 (2012); D. Frekers, private communication

Properties of the strength distribution are "robust", but its details depend on the actual values of the single particle energies and of the strength of the interactions. The calculated odd-odd spectra are in fair agreement with experiment.




Note that Yoshida correctly calculates the g.s. of $^{130}$I to be $5^+$. He also calculates correctly its magnetic moment.

$\mu(5_1^+)_{th} = 3.12$

$\mu(5_1^+)_{exp} = 3.349(7)$

The extracted values of $g_{A,eff}$ are of order ~0.4.