Theory of the Trojan-Horse Method –
From the Original Idea to Actual Applications

Stefan Typel

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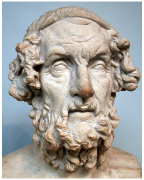

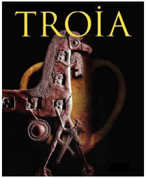

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Outline

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- **Introduction**
- **Theory of Directs Reactions**
- **Conditions and Problems**
- **Experimental Studies**
- **Applications of the Trojan-Horse Method**
- **Further Developments and Outlook**

Introduction

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- **basic idea**
  - study breakup reaction $A + a \rightarrow C + c + b$ to extract cross section of astrophysical reaction $A + x \rightarrow C + c$ with Trojan horse $a = b + x$ and spectator b
  - analyse with help of direct reaction theory

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- introduced by **Gerhard Baur**:

Breakup reactions as an indirect method to investigate low-energy charged-particle reactions relevant for nuclear astrophysics Physics Letters B 178, 135 (1986)
suggested already in his invited talk: **Breakup processes in nuclear reactions**
1985 Varna Int. Summer School on Nuclear Physics (Sept. 22 – Oct. 1), Nuclear Energy 25, 183 (1987)

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- specific features
  - Fermi motion of x inside a compensates relative motion with respect to A
    $\Rightarrow$ small relative energies in $A + x$ system accessible
  - surface dominated reaction $\Rightarrow$ reduction of suppression by Coulomb barrier
  - 'high' relative energy in $A + a$ system $\Rightarrow$ no electron screening

Theory of Direct Reactions I

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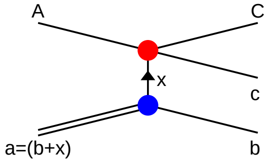
- **general cross section** of reaction $A + a \to B + b$ with $a = b + x$ and $B = C + c$

$$d\sigma = \frac{2\pi}{\hbar} \frac{\mu_{Aa}}{p_{Aa}} \frac{d^3 p_{Bb}}{(2\pi\hbar)^3} \frac{d^3 p_{Cc}}{(2\pi\hbar)^3} \left|T_{fi}\right|^2 \delta(E_{Bb} + E_{Cc} - E_{Aa} - Q)$$

with $Q = m_A + m_a - m_C - m_c - m_b$



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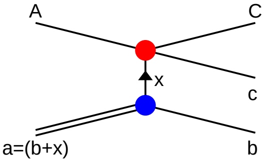
- **T matrix element** in post formulation

  $$T_{fi} = \langle \phi_B \phi_b \exp\left(i\vec{p}_{Bb} \cdot \vec{r}_{Bb}/\hbar\right) | V_{Bb} | \Psi^{(+)}_{Aa} \rangle$$

  with full scattering wave function $\Psi^{(+)}_{Aa}$

- **transfer to continuum state** $B = C + c$

  $\phi_B = \Psi^{(-)}_{Cc} \Rightarrow \ldots \Rightarrow$ scattering matrix elements



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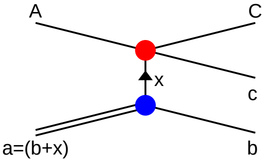

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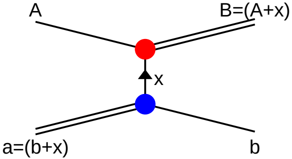
with full scattering wave function $\Psi_{Aa}^{(+)}$

- **transfer to continuum state** $B = C + c$
  $\phi_B = \Psi_{Cc}^{(-)} \Rightarrow \ldots \Rightarrow$ scattering matrix elements
- **transfer to bound state** $B = A + x$
  $\Rightarrow \ldots \Rightarrow$ spectroscopic factors,
  asymptotic normalisation coefficients (ANCs)



Theory of Direct Reactions II

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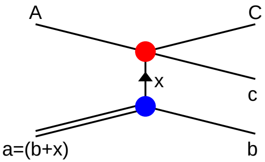
- **various approximations**
  - distorted-wave Born approximation (DWBA)
  - distorted-wave impulse approximation (DWIA)
  - plane-wave impulse approximation (PWIA)
  - ...



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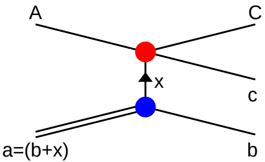

- **factorisation of cross section** in PWIA

$$\frac{d^3\sigma}{dE_{Cc}d\Omega_{Cc}d\Omega_{(Cc)b}} = K\left|\Phi_a(\vec{Q})\right|^2 \frac{d\sigma^{HOES}}{d\Omega_{Cc}}(A + x \to C + c)$$

  - kinematic factor K
  - momentum distribution $|\Phi_a(\vec{Q})|^2$ of Trojan-horse ground state
  - half-off-energy-shell cross section $\frac{d\sigma^{HOES}}{d\Omega_{Cc}}$ of reaction $A + x \to C + c$

Conditions and Problems

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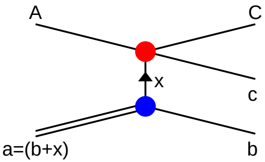
- small momentum transfer to spectator b
  $\Rightarrow$ **quasifree (QF) scattering** $\Rightarrow \vec{Q} \approx 0$
  $\Rightarrow$ selection of kinematical conditions
- small energies in $C + c$ system can be reached (even below Coulomb barrier) with energies in initial $A + a$ channel above Coulomb barrier
  $\Rightarrow$ **no suppression of cross section**
- well **clustered Trojan horse** $a = b + x$ in s-wave ground state
  $\Rightarrow$ maximum of $|\Phi_a(\vec{Q})|^2$ at $\vec{Q} = 0$



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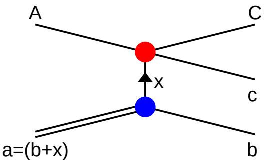

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- well **clustered Trojan horse** $a = b + x$ in s-wave ground state
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- separation of QF reaction from **other reaction mechanisms**
- validity of **factorisation** of cross section?
- **relation** of $\frac{d\sigma^{HOES}}{d\Omega_{Cc}}$ **to astrophysical on-shell cross section**?
- **normalisation** of cross section to direct data

Experimental Studies I

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- **kinematical conditions:**
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- study of momentum distributions in clustered nuclei
  - **Impulse distribution of the α-d motion in ^{6}Li**
    S. Barbarino, M. Lattuada, F. Riggi, **S. Spitaleri**, D. Vinciguerra, Phys. Rev. C 21, 1104 (1980)
  - **Momentum distribution of alpha clusters in light nuclei from ($\alpha, 2\alpha$) reaction at high energy**
    M. Lattuada, F. Riggi, **C. Spitaleri**, D. Vinciguerra, C. M. Sutera, Il Nuovo Cimento 63, 530 (1981)

- test of the cross section factorisation in PWIA
  - **Treimann-Yang criterion as a test of the pole approximation in the ^{9}Be(^{3}He,$\alpha\alpha$)^{4}He reaction**
    P. G. Fallica, M. Lattuada, F. Riggi, **C. Spitaleri**, C. M. Sutera, D. Vinciguerra, Phys. Rev. C 24, 1394 (1981)

Experimental Studies II

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- study of quasifree scattering and reactions
  - **Mechanism of the ^{6}Li+^{6}Li $\rightarrow 3\alpha$ reaction at low energy**
    M. Lattuada, F. Riggi, **C. Spitaleri**, D. Vinciguerra, C. M. Sutera,
    Phys. Rev. C 26, 1330 (1982)
  - **Direct mechanism of ^{6}Li+^{6}Li $\rightarrow 3\alpha$ reaction at low energy**
    M. Lattuada, F. Riggi, **C. Spitaleri**, D. Vinciguerra, G. Vourvopoulos, X. Aslanoglou, Đ. Miljanić.,
    Phys. Rev. C 30, 531 (1984)
  - **The quasi-free ^{9}Be(^{3}He,$\alpha\alpha$)^{4}He reaction between 3 and 12 MeV**
    M. Lattuada, F. Riggi, **C. Spitaleri**, D. Vinciguerra, Đ. Miljanić, M. Zadro, Yao Jinzhang,
    Nucl. Phys. A 458, 493 (1986)
  - **Quasi-free processes in ^{6}Li(^{3}He,pα)^{4}He reaction at low energies**
    M. Zadro, Đ. Miljanić, M. Lattuada, F. Riggi, **C. Spitaleri**,
    Nucl. Phys. A 474, 373 (1987)
  - **Excitation function fo the quasifree contribution in the ^{2}H(^{7}Li,$\alpha\alpha$)n reaction at $E_0 = 28 - 48$ MeV**
    M. Zadro, Đ. Miljanić, **C. Spitaleri**, G. Calvi, M. Lattuada, F. Riggi,
    Phys. Rev. C 40, 181 (1989)
  - **Quasifree reaction mechanism in ^{2}H(^{6}Li,^{3}Heα)n at $E_0 = 21.6 - 33.6$ MeV**
    G. Calvi, M. Lattuada, Đ. Miljanić, F. Riggi, **C. Spitaleri**, M. Zadro,
    Phys. Rev. C 41, 1848 (1990)
  - **^{4}H nucleus and the ^{2}H(t,tp)n reaction**
    S. Blagus, Đ. Miljanić, M. Zadro, G. Calvi, M. Lattuada, F. Riggi, **C. Spitaleri**, C. Blyth, O. Karban,
    Phys. Rev. C 44, 325 (1991)

Application of the Trojan-Horse Method I

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- **first attempts:**
  - **Indirect investigation of the d+^{6}Li reaction at low energies relevant for nuclear astrophysics**
    S. Cherubini, V. N. Kondratyev, M. Lattuada, **C. Spitaleri**, Đ. Miljanić, M. Zadro, G. Baur,
    Ap. J. 457, 855 (1996)
  - **Indirect measurement of nuclear reaction cross sections at astrophysical energies**
    G. Calvi, S. Cherubini, M. Lattuada, S. Romano, **C. Spitaleri**, M. Aliotta, G. Rizzari, M. Sciuto,
    R. A. Zappalà, V. N. Kondratyev, Đ. Miljanić, M. Zadro, G. Baur, O. Yu. Goryunov, A. A. Shvedov,
    Nucl. Phys. A 621, 139c (1997)
  - **Indirect ^{7}Li(p,α)^{4}He reaction at astrophysical energies**
    C. Spitaleri, M. Aliotta, S. Cherubini, M. Lattuada, Đ. Miljanić, S. Romano, N. Soić, M. Zadro,
    R. A. Zappalà,
    Phys. Rev. C 60, 055802 (1999)
  - **Improved information on electron screening in ^{7}Li(p,α)α using the Trojan-horse method**
    M. Aliotta, **C. Spitaleri**, M. Lattuada, A. Musumarra, R. G. Pizzone, A. Tumino, C. Rolfs, F. Strieder,
    Eur. Phys. J. A 9, 435 (2000)

- **problem:**
  relation of $\frac{d\sigma^{HOES}}{d\Omega_{Cc}}$ to astrophysical on-shell cross section?
  "surface approximation" $\Rightarrow$ penetrability correction

Application of the Trojan-Horse Method II

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- **theoretical developments:**
  - **Extraction of astrophysical cross sections in the Trojan-horse method**
    S. Typel, H. H. Wolter, Few-Body Systems 29, 75 (2000)
  - **Theory of the Trojan-horse method**
    S. Typel, G. Baur, Ann. Phys. 305, 228 (2003)

- **extensive experimental studies:**
  - **“Trojan horse” method applied to ^{2}H(^{6}Li,α)^{4}He at astrophysical energies**
    C. Spitaleri, S. Typel, R. G. Pizzone, M. Aliotta, S. Blagus, M. Bogovac, S. Cherubini, P. Figuera, M. Lattuada, M. Milin, Đ. Miljanić, A. Musumarra, M. G. Pellegriti, D. Rendić, C. Rolfs, S. Romano, N. Soić, A. Tumino, H. H. Wolter, M. Zadro, Phys. Rev. C 63, 055801 (2001)
  - **Improved information on the ^{2}H(^{6}Li,α)^{4}He reaction extracted via the “Trojan horse” method**
    A. Musumarra, R. G. Pizzone, S. Blagus, M. Bogovac, P. Figuera, M. Lattuada, M. Milin, Đ. Miljanić, M. G. Pellegriti, D. Rendić, C. Rolfs, N. Soić, **C. Spitaleri**, S. Typel, H. H. Wolter, M. Zadro, Phys. Rev. C 64, 068801 (2001)
  - **Indirect measurement of nuclear reaction cross sections at astrophysical energies**
    M. G. Pellegriti, M. Aliotta, P. Figuera, M. Lattuada, Đ. Miljanić, A. Musumarra, R. G. Pizzone, C. Rolfs, S. Romano, **S. Spitaleri**, S. Tudisco, A. Tumino, S. Typel, H. H. Wolter, Nucl. Phys. 688, 543c (2001)
  - **and many more . . .**

Further Developments and Outlook

- **standard application:**
  - non-resonant rearrangement reactions
- **extensions:**
  - study of resonance properties
  - low-energy elastic scattering
  - electron screening
- **experimental progress:**
  - **Nuclear astrophysics and the Trojan horse method**
    C. Spitaleri, M. La Cognata, L. Lamia,
    A. M. Mukhamedzhanov, R. G. Pizzone,
    Eur. Phys. J. A 52, 77 (2016)
  - more dedicated high-precision experiments needed

- **theoretical progress:**
  - **Indirect techniques in nuclear astrophysics: a review**
    E. R. Tribble, C. A. Bertulani, M. La Cognata, A. M. Mukhamedzhanov, **C. Spitaleri**,
    Rep. Prog. Phys. 77, 106901 (2014)
  - still more detailed calculations required to check validity of approximations

Thank You

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for bringing us to Catania