Neutrinoless Double Beta Decay: An open window to obtaining the neutrino mass

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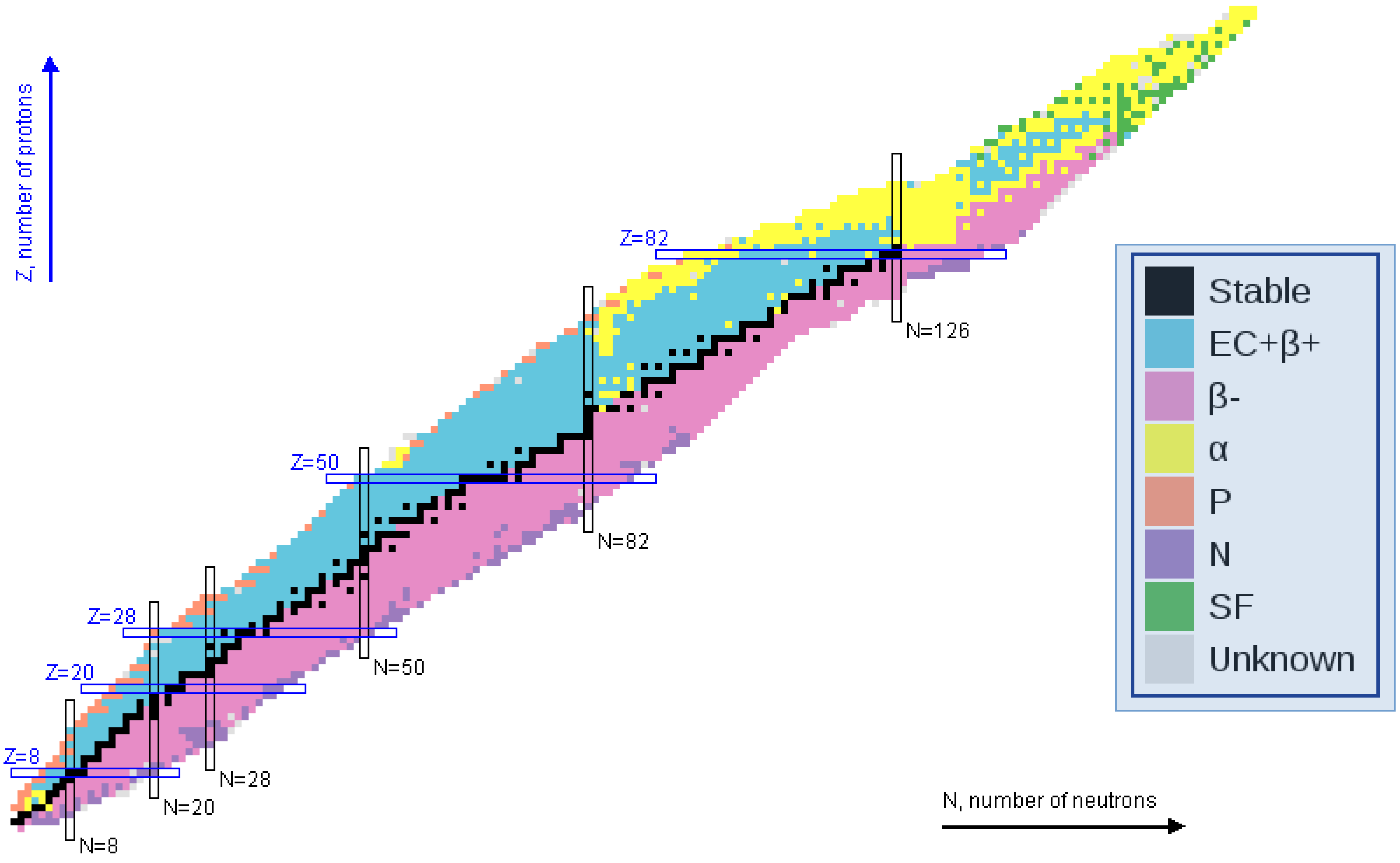
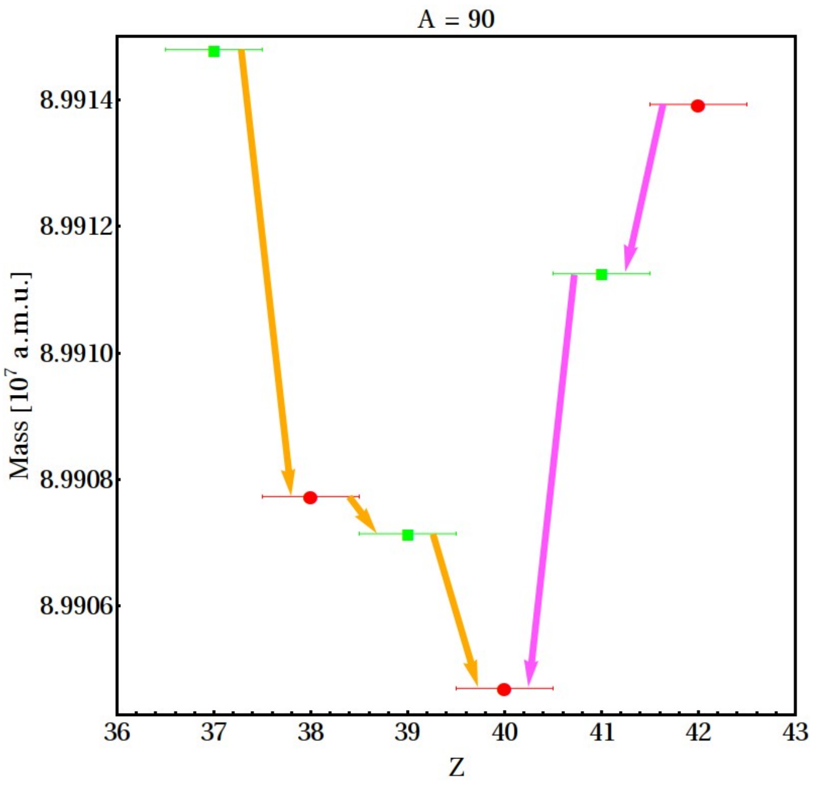
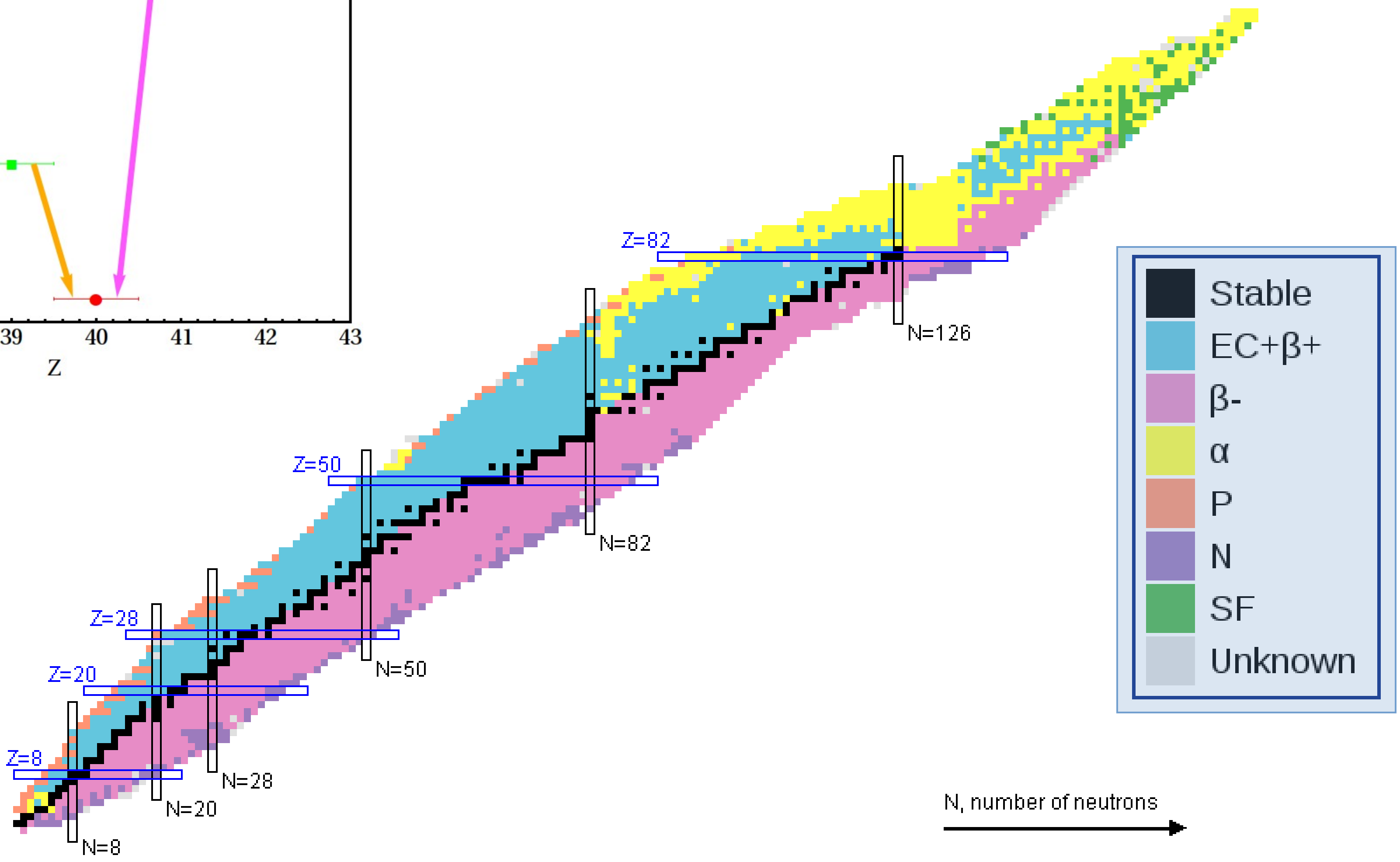
Contents

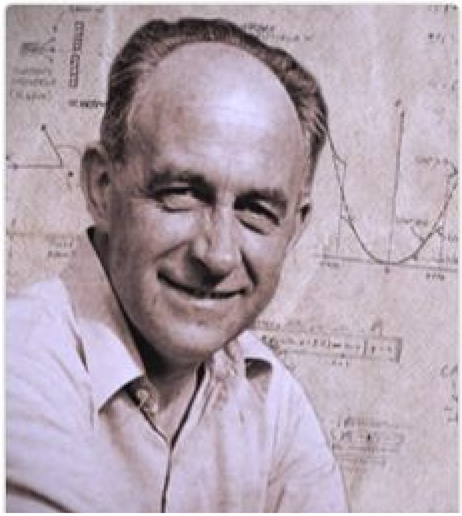
Single beta decay



Single beta decay

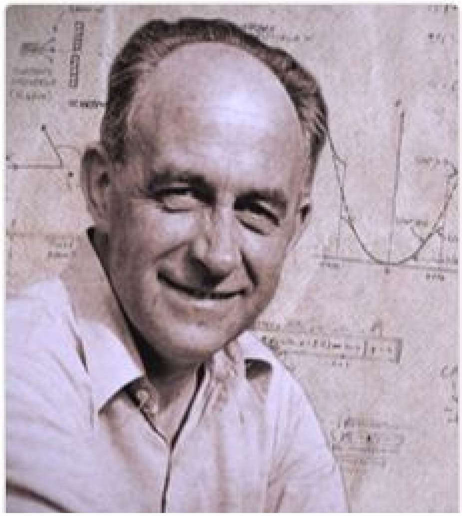






Beta decay theory (1934)

$$\boxed{\begin{array}{l} n \rightarrow p + e^- + \bar{\nu} \\ p \rightarrow n + e^+ + \nu \\ p + e \rightarrow n + \nu \end{array}}$$



Beta decay theory (1934)

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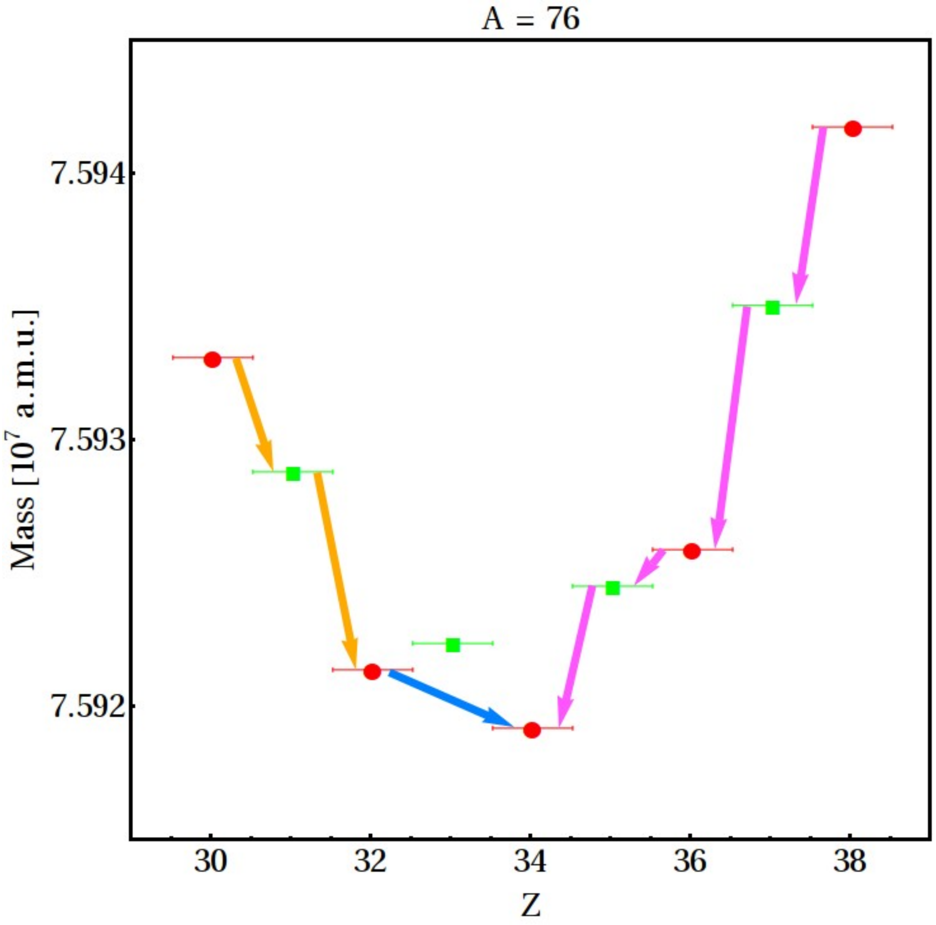
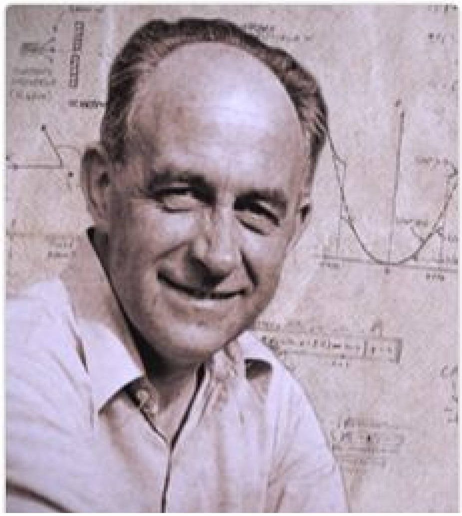


Double beta decay (1935)



Beta decay theory (1934)

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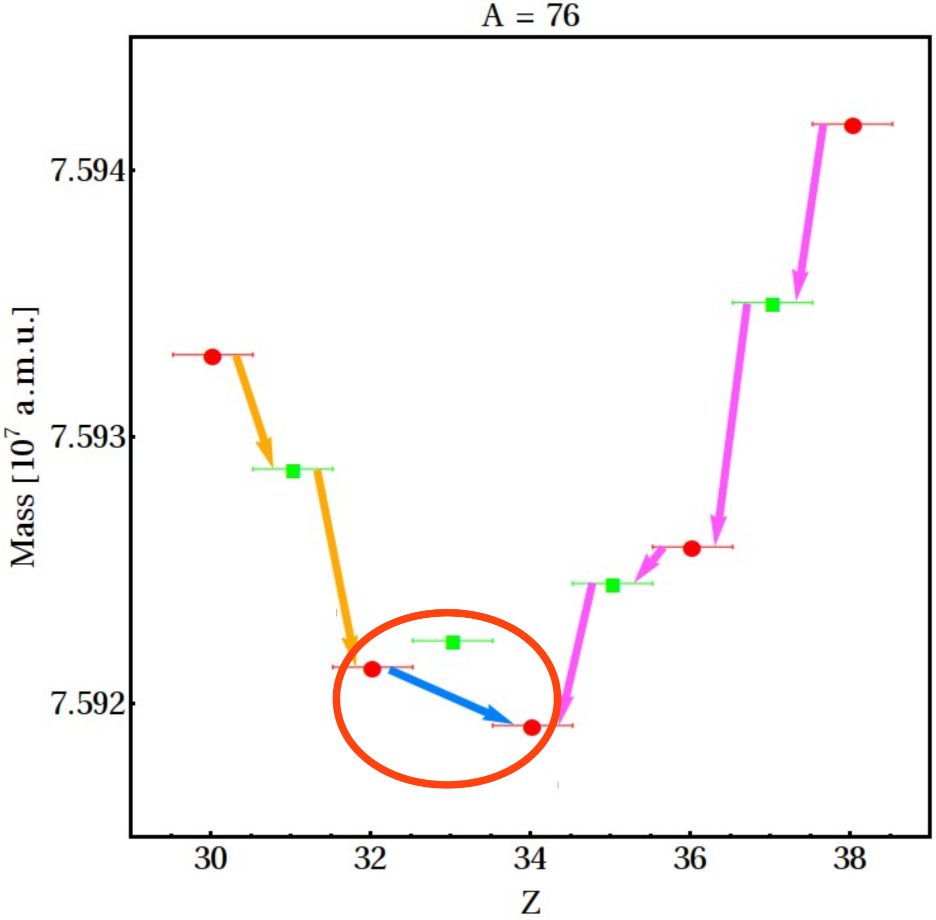


Double beta decay (1935)

$$2n \rightarrow 2p + 2e^- + 2\bar{\nu}$$

Majorana particles (1937)

$$\boxed{\nu = \overline{\nu}}$$

Majorana particles (1937)

$$\boxed{\nu = \bar{\nu}} \longrightarrow \boxed{\begin{array}{l} n \rightarrow p + e^- + \nu \\ \nu + n \rightarrow p + e^- \end{array}}$$

Majorana particles (1937)

$$\boxed{\nu = \bar{\nu}} \longrightarrow \boxed{\begin{array}{l} n \rightarrow p + e^- + \nu \\ \nu + n \rightarrow p + e^- \end{array}}$$

Neutrinoless double beta decay (1937, 1939)

$$\begin{aligned} n_1 + n_2 &\rightarrow n_1 + p_2 + e_2^- + \nu \\ &\rightarrow p_1 + e_1^- + p_2 + e_2^- \end{aligned}$$

Two neutrinos double beta decay

Nuclide	Transition	$T_{1/2}$ (y)
48Ca	$0^+ \rightarrow 0^+$	$(4.39 \pm 0.58)x10^{19}$
76Ge	$0^+ \rightarrow 0^+$	$(1.43 \pm 0.53)x10^{21}$
82Se	$0^+ \rightarrow 0^+$	$(9.19 \pm 0.76)x10^{19}$
96Zr	$0^+ \rightarrow 0^+$	$(2.16 \pm 0.26)x10^{19}$
100Mo	$0^+ \rightarrow 0^+$	$(6.98 \pm 0.44)x10^{18}$
100Mo	$0^+ \rightarrow 0^+$	$(5.70 \pm 1.36)x10^{20}$
116Cd	$0^+ \rightarrow 0^+$	$(2.89 \pm 0.25)x10^{19}$
130Te	$0^+ \rightarrow 0^+$	$(7.14 \pm 1.04)x10^{20}$
136Xe	$0^+ \rightarrow 0^+$	$(2.34 \pm 0.13)x10^{21}$
150Nd	$0^+ \rightarrow 0^+$	$(8.37 \pm 0.45)x10^{18}$

Neutrinoless double beta decay

* Character of the neutrino.

* Lepton number violation.

* Finite mass of the neutrino.

* Absolute scale of the
  neutrino mass.

Halflives and NME

$$\left[\tau_{1/2}^{(0\nu)}\right]^{-1} \simeq G_{0\nu}\left|M_{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 \quad , \quad \left[\tau_{1/2}^{(2\nu)}\right]^{-1} \simeq G_{2\nu}\left|m_e c^2 M_{2\nu}\right|^2 ,$$

$$\begin{aligned} M_{0\nu} &\simeq \left\langle F; J_F \left| -h_{0\nu}^F + h_{0\nu}^{GT} + h_{0\nu}^T \right| I; 0_1^+ \right\rangle \\ M_{2\nu} &\approx \left\langle F; J_F \left| -h_{2\nu}^F + h_{2\nu}^{GT} \right| I; 0_1^+ \right\rangle \end{aligned}$$

$$\begin{aligned} h_X^{F,GT,T} &= -\frac{1}{4} \sum_{\alpha_\pi \alpha'_\pi} \sum_{\alpha_\nu \alpha'_\nu} \sum_J (-1)^J \; G_X^{F,GT,T}\left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu ; J\right) \\ &\times \sqrt{1+(-1)^J \delta_{\alpha_\pi \alpha'_\pi}} \sqrt{1+(-1)^J \delta_{\alpha_\nu \alpha'_\nu}} \\ &\times \left(\pi_{\alpha_\pi}^\dagger \times \pi_{\alpha'_\pi}^\dagger\right)^{(J)} \cdot \left(\tilde{\nu}_{\alpha_\nu} \times \tilde{\nu}_{\alpha'_\nu}\right)^{(J)} \end{aligned}$$

$$\alpha_\rho = (n_\rho l_\rho j_\rho)\,, \;\; \rho = \nu, \pi, \;\; X = 0\nu, 2\nu$$

Source: J. Barea and F. Iachello, Phys. Rev. C **79**, 044301 (2009)

A couple of comments

- The closure approximation have been avoided in computing 2ν NMEs in IBM:

  N. Yoshida and F. Iachello,
  Prog. Theor. Exp. Phys. **2013** 043D01

- IBM is not restricted to just one major shell: the number of single particle levels can be extended to include spin-orbit partners in the formulation.

Two body matrix elements I

$$G_X^{F,GT,T}\left(\alpha_\pi\alpha'_\pi\alpha_\nu\alpha'_\nu;J\right)=\left\langle\alpha_\pi\alpha'_\pi;JM\left|h_X^{F,GT,T}\right|\alpha_\nu\alpha'_\nu;JM\right\rangle$$

$$\begin{aligned}h_X^{F,GT,T} &\equiv h_X^{(s_1,s_2,\lambda)}\\ &= \frac{1}{2}\sum_{n,n'}\tau_n^+\tau_{n'}^+\left(\Sigma_n^{(s_1)}\times\Sigma_{n'}^{(s_2)}\right)^{(\lambda)}\cdot H_X(r_{nn'})C^{(\lambda)}(\Omega_{nn'}),\end{aligned}$$

$$h^F\rightarrow h_X^{(0,0,0)},\;\; h^{GT}\rightarrow h_X^{(1,1,0)},\;\; h^T\rightarrow h_{0\nu}^{(1,1,2)}$$

$$\Sigma_n^{(0)}=1,\;\;\Sigma_n^{(1)}=\vec{\sigma}_n,\;\;C^{(\lambda)}(\Omega)=\sqrt{4\pi/\left(2\lambda+1\right)}Y^{(\lambda)}(\Omega)$$

Two body matrix elements II

$$G_X^{F,GT,T}\left(\alpha_\pi\alpha'_\pi\alpha_\nu\alpha'_\nu;J;J\right)\equiv G_X^{(s_1,s_2,\lambda)}\left(\alpha_\pi\alpha'_\pi\alpha_\nu\alpha'_\nu;J;J\right)=$$

$$\sum_{k_1=|l_\pi-l_\nu|}^{l_\pi+l_\nu}\sum_{k_2=|l'_\pi-l'_\nu|}^{l'_\pi+l'_\nu}\sum_{k=k_{min}}^{k_{max}} i^{k_1-k_2+\lambda}\hat{k}_1^2\hat{k}_2^2\left\langle k_1 0\, k_2 0\mid\lambda 0\right\rangle$$
$$\times(-1)^{s_2+k_1}\left\{\begin{array}{ccc} k_1 & s_1 & k \\ s_2 & k_2 & \lambda \end{array}\right\}(-1)^{j'_\pi+j_\nu+J}\left\{\begin{array}{ccc} j_\pi & j'_\pi & J \\ j'_\nu & j_\nu & k \end{array}\right\}$$
$$\times\hat{k}\hat{j}_\pi\hat{j}_\nu\left\{\begin{array}{ccc} \frac{1}{2} & l_\pi & j_\pi \\ \frac{1}{2} & l_\nu & j_\nu \\ s_1 & k_1 & k \end{array}\right\}\hat{k}\hat{j}'_\pi\hat{j}'_\nu\left\{\begin{array}{ccc} \frac{1}{2} & l'_\pi & j'_\pi \\ \frac{1}{2} & l'_\nu & j'_\nu \\ s_2 & k_2 & k \end{array}\right\}$$
$$\times\left\langle\tfrac{1}{2}\left\|\Sigma^{(s_1)}\right\|\tfrac{1}{2}\right\rangle(-1)^{-k_1}\hat{l}_\pi\left\langle l_\pi 0\, k_1 0\mid l_\nu 0\right\rangle$$
$$\times\left\langle\tfrac{1}{2}\left\|\Sigma^{(s_2)}\right\|\tfrac{1}{2}\right\rangle(-1)^{-k_2}\hat{l}'_\pi\left\langle l'_\pi 0\, k_2 0\mid l'_\nu 0\right\rangle$$
$$\times R_{X,k_1,k_2}^{(s_1,s_2,\lambda)}(n_\pi,l_\pi,n'_\pi,l'_\pi,n_\nu,l_\nu,n'_\nu,l'_\nu),$$

Radial Integrals

$$
\begin{aligned}
&R^{(s_1,s_2,\lambda)}_{X,k_1,k_2}(n_\pi, l_\pi, n'_\pi, l'_\pi, n_\nu, l_\nu, n'_\nu, l'_\nu) = \\
\int_0^\infty h_X^{(s_1,s_2,\lambda)}(p)\, p^2 dp \quad &\times \int_0^\infty R_{n_\pi l_\pi}(r_1)\, R_{n_\nu l_\nu}(r_1)\, j_{k_1}(pr_1)\, r_1^2 dr_1 \\
&\times \int_0^\infty R_{n'_\pi l'_\pi}(r_2)\, R_{n'_\nu l'_\nu}(r_2)\, j_{k_2}(pr_2)\, r_2^2 dr_2
\end{aligned}
$$

$$
h_X^{(s_1,s_2,\lambda)}(p) = v_X(p)\, \underbrace{\tilde{h}^{(s_1,s_2,\lambda)}(p)}_{\substack{+\mathrm{HOC} \\ +\mathrm{FNS} \\ +\mathrm{SRC}}};
$$

$$
v_X(p) = \begin{cases} \frac{\delta(p)}{p^2} & \text{for } 2\nu \\ \frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)} & \text{for light } 0\nu \\ \frac{2}{\pi}\frac{1}{m_e m_p} & \text{for heavy } 0\nu \end{cases}
$$

High Order Corrections

New terms in GT and a Tensor contribution

$$\begin{aligned} h^{F}_{0\nu} &= h^{F}_{VV} \\ h^{GT}_{0\nu} &= h^{GT}_{AA} + h^{GT}_{AP} + h^{GT}_{PP} + h^{GT}_{MM} \\ h^{T}_{0\nu} &= h^{T}_{AP} + h^{T}_{PP} + h^{T}_{MM} \end{aligned}$$

Finite Nucleon Size

Coupling constants become momentum dependent

$$g_V(p^2) = \frac{g_V}{\left(1 + \frac{p^2}{M_V^2}\right)^2},\ M_V^2 = 0.71\ \left(\mathrm{GeV/c^2}\right)^2$$

$$g_A(p^2) = \frac{g_A}{\left(1 + \frac{p^2}{M_A^2}\right)^2},\ M_A = 1,09\ \mathrm{GeV/c^2}$$

HOC + FNS

HOC term	$\tilde{h}(p)$
$\tilde{h}^{F}_{VV}$	$g_A^2 \frac{g_V^2/g_A^2}{\left(1+p^2/M_V^2\right)^4}$
$\tilde{h}^{GT}_{AA}$	$g_A^2 \frac{1}{\left(1+p^2/M_A^2\right)^4}$
$\tilde{h}^{GT}_{AP}$	$g_A^2 \left[-\frac{2}{3}\frac{1}{\left(1+p^2/M_A^2\right)^4}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right]$
$\tilde{h}^{GT}_{PP}$	$g_A^2 \left[\frac{1}{\sqrt{3}}\frac{1}{\left(1+p^2/M_A^2\right)^2}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right]^2$
$\tilde{h}^{GT}_{MM}$	$g_A^2 \left[\frac{2}{3}\frac{g_V^2}{g_A^2}\frac{1}{\left(1+p^2/M_V^2\right)^4}\frac{\kappa_\beta^2 p^2}{4m_p^2}\right]$
$\tilde{h}^{T}_{AP}$	$-h^{GT}_{AP}$
$\tilde{h}^{T}_{PP}$	$-h^{GT}_{PP}$
$\tilde{h}^{T}_{MM}$	$\frac{1}{2}h^{GT}_{MM}$

Short Range Correlations

Jastrow function in coordinate space

$$\left.\begin{array}{r}\psi_{SRC}=f\left(r\right)\psi\\ \psi'_{SRC}=f\left(r\right)\psi'\end{array}\right\}\rightarrow\left\langle\psi_{SRC}\left|H\right|\psi'_{SRC}\right\rangle=\left\langle\psi f\left(r\right)\left|H\right|f\left(r\right)\psi'\right\rangle$$

$$H^{F,GT,T}(r)\rightarrow H^{F,GT,T}(r)f^2(r)$$

$$f\left(r\right)=1-Ce^{-Ar^2}\left(1-Br^2\right)$$

$$\begin{aligned}H^{F,GT}(r) &= \int_0^\infty j_0(pr)h^{F,GT,T}(p)p^2dp\\ H^T(r) &= \int_0^\infty j_2(pr)h^T(p)p^2dp\end{aligned}$$

SRC Parametrizations

Name	A (fm^{-2})	B (fm^{-2})	C
Miller-Spencer	1.10	0.68	1.00
Argonne	1.59	1.45	0.92
CD-Bonn	1.52	1.88	0.46

Source: F. Šimkovic *et al*, Phys. Rev. **C** 79, 055501 (2009)

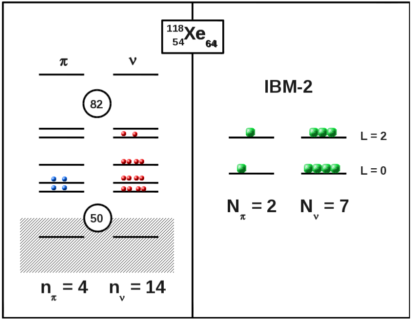
Nuclear models

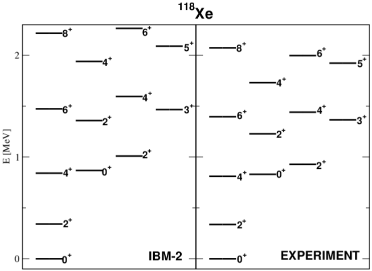
- Interacting shell model (ISM):
    - Truncated Space with increasing number of valence nucleons.
    - Includes all the configurations.
    - Phenomenological character.
- Quasi random phase approximation (QRPA):
    - Includes several major shells.
    - Restricted to configurations np-nh over a BCS vacuum.
    - Effective residual interactions
    - Strong dependence in parameters g_{pp} and g_{ph}.
- Interacting Boson Model (IBM)
    - Strong reduction of the full space.
    - Phenomenological character.
    - Simple formulation with enough physics to reproduce well experimental data.
- Other approaches: Mean Field, HFB and related variants, a.k.a. EDF.

The Interacting Boson Model (IBM)

- Description of the collective degrees of freedom low-lying states of medium and heavy nuclei.
- The states are expressed in terms of a system of N bosons which interact.
- The bosons are allowed to be only in two states:
  - 0^+ (s boson).
  - 2^+ (d boson).
- Phenomenological character: the interaction parameters are fitted to reproduce nuclear properties.
- Different extensions: IBM-1, IBM-2, IBM-3, ..., IBFM-1, ...

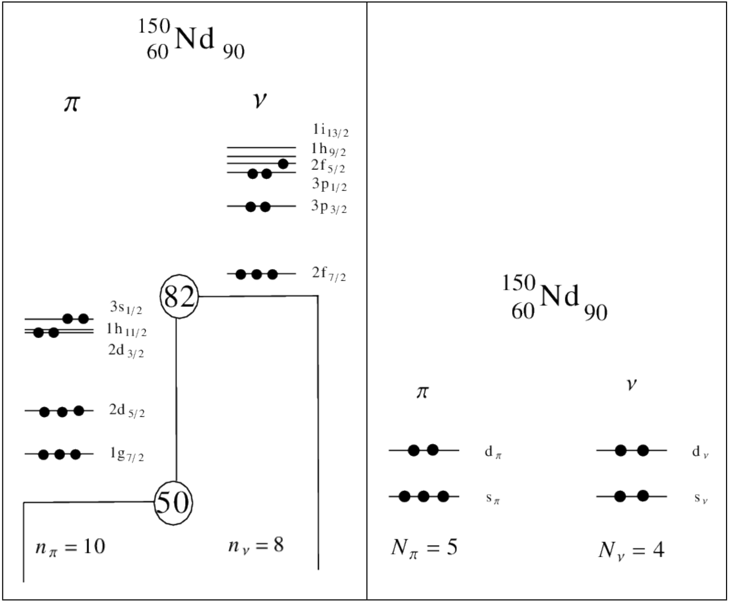
The microscopic IBM: IBM-2



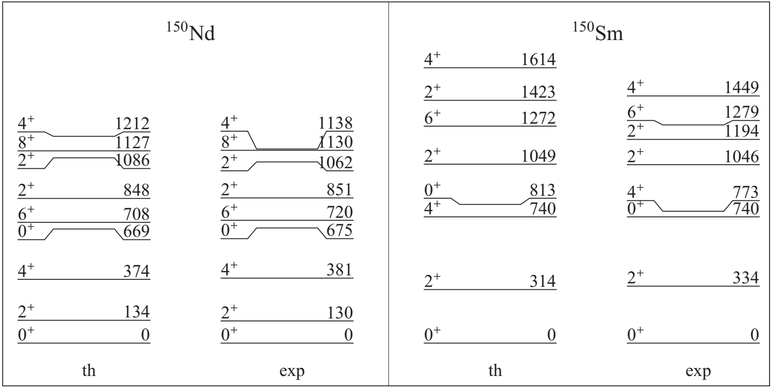


$$
\begin{aligned}
H_{IBM2} &= H_\pi + H_\nu + \kappa\, Q_\pi^{(2)} \cdot Q_\nu^{(2)} + M_{\pi\nu}\left[\xi_1, \xi_2, \xi_3\right] \\
H_\rho &= \epsilon\, \hat{n}_{d_\rho} + \sum_{L=0,2,4} c_L^\rho \left(d_\rho^\dagger d_\rho^\dagger\right)^{(L)} \cdot \left(\tilde{d}_\rho \tilde{d}_\rho\right)^{(L)} \\
Q_\rho^{(2)} &= \left(s_\rho^\dagger \times \tilde{d}_\rho + d_\rho^\dagger \times \tilde{s}_\rho\right)^{(2)} + \chi_\rho \left(d_\rho^\dagger \times \tilde{d}_\rho\right)^{(2)}
\end{aligned}
$$

Another example

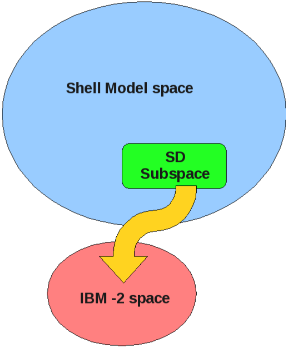


Excitation spectra for $A = 150$



J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013)

The Otsuka-Arima-Iachello Method (OAI)



$$\boxed{\begin{aligned} S_+ &= \sum_j \alpha_j \frac{\sqrt{j+\frac{1}{2}}}{2} \left(C_j^\dagger C_j^\dagger\right)^{(0)}_0 \\ D_+ &= \sum_{j\leq j'} \frac{\beta_{jj'}}{\sqrt{1+\delta_{jj'}}} \left(C_j^\dagger C_{j'}^\dagger\right)^{(2)}_M \end{aligned}}$$

$$\boxed{\begin{array}{rcl} \left|\tilde{\nu} = 0; S^{N_s}; (0,0)\right\rangle & \to & \left|s^{N_s}\right\rangle \\ \left|\tilde{\nu} = 1; S^{N_s} j; (j,m)\right\rangle & \to & \left|s^{N_s} j\right\rangle \\ \left|\tilde{\nu} = 2; S^{N_s} D; (2,M)\right\rangle & \to & \left|s^{N_s} d\right\rangle \\ & \dots & \end{array}}$$

$$\boxed{O_{SM} \to o_{IBM} = \sum_k \gamma_k f_k \left[s, d, j\right]}$$

$$\boxed{\left\langle F \left| O_{SM} \right| I \right\rangle = \left\langle f \left| o_{IBM} \right| i \right\rangle = \left\langle f \left| \sum_k \gamma_k f_k \left[s, d, j\right] \right| i \right\rangle}$$

Boson expansions

Boson expansion of the coupled pairs operators

$$\begin{array}{rcl}
\left(\pi^\dagger_{j_\pi} \times \pi^\dagger_{j_\pi}\right)^{(0)} & \longmapsto & A^{(01)}_{j_\pi} s^\dagger_\pi + A^{(11)}_{j_\pi} s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(0)} + \dots \\
\left(\pi^\dagger_{j_\pi} \times \pi^\dagger_{j'_\pi}\right)^{(2)} & \longmapsto & B^{(01)}_{j_\pi j'_\pi} d^\dagger_\pi \\
 & + & B^{(11)}_{j_\pi j'_\pi} \, s^\dagger_\pi \left(s^\dagger_\pi \tilde{d}_\pi\right)^{(2)} + B^{(12)}_{j_\pi j'_\pi} \, s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(2)} \\
 & + & \dots \\
(\tilde{\nu}_{j_\nu} \times \tilde{\nu}_{j_\nu})^{(0)} & \longmapsto & \tilde{A}^{(01)}_{j_\nu} \tilde{s}_\nu + \tilde{A}^{(11)}_{j_\nu} \tilde{s}_\nu \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(0)} + \dots \\
(\tilde{\nu}_{j_\nu} \times \tilde{\nu}_{j'_\nu})^{(2)} & \longmapsto & \tilde{B}^{(01)}_{j_\nu j'_\nu} \tilde{d}_\nu \\
 & + & \tilde{B}^{(11)}_{j_\nu j'_\nu} \left(d^\dagger_\nu \tilde{s}_\nu\right)^{(2)} \tilde{s}_\nu + \tilde{B}^{(12)}_{j_\nu j'_\nu} \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(2)} \tilde{s}_\nu \\
 & + & \dots
\end{array}$$

The DBD transition operator in IBM-2

$$
\begin{aligned}
h_X^{F,GT,T} &= -\frac{1}{4} \sum_{\alpha_\pi \alpha'_\pi} \sum_{\alpha_\nu \alpha'_\nu} \sum_{J} (-1)^J \, G_X^{F,GT,T} \left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu ; J \right) \\
&\quad \sqrt{1 + (-1)^J \delta_{\alpha_\pi \alpha'_\pi}} \sqrt{1 + (-1)^J \delta_{\alpha_\nu \alpha'_\nu}} \\
&\quad \left(\pi^\dagger_{\alpha_\pi} \times \pi^\dagger_{\alpha'_\pi} \right)^{(J)} \cdot \left(\tilde{\nu}_{\alpha_\nu} \times \tilde{\nu}_{\alpha'_\nu} \right)^{(J)} \\
&\mapsto \quad h_{X,ss}^{F,GT,T} \; s^\dagger_\pi \cdot \tilde{s}_\nu + h_{X,dd}^{F,GT,T} \; d^\dagger_\pi \cdot \tilde{d}_\nu \, , \text{ where}
\end{aligned}
$$

$$
\begin{aligned}
h_{X,ss}^{F,GT,T} &= -\sum_{j_\pi} \sum_{j_\nu} G_X^{F,GT,T} \left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu ; J = 0 \right) A^{(01)}_{j_\pi} \, \tilde{A}^{(01)}_{j_\nu} \\
h_{X,dd}^{F,GT,T} &= -\frac{1}{2} \sum_{j_\pi j'_\pi} \sum_{j_\nu j'_\nu} \sqrt{1 + \delta_{j_\pi j'_\pi}} \sqrt{1 + \delta_{j_\nu j'_\nu}} \\
&\times \quad G_X^{F,GT,T} \left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu ; J = 2 \right) B^{(01)}_{j_\pi j'_\pi} \, \tilde{B}^{(01)}_{j_\nu j'_\nu}
\end{aligned}
$$

Structure of the S and D pairs

$$H_{SDI} = \sum_{j} \varepsilon_j + A_T V_{SDI}$$

- Single particle energies ε_j taken from experiments.
- A_T fitted to reproduce the difference $E[2_1^+] - E[0_{gs}^+]$ in nuclei with 2 nucleons of valence.
- α_j and $\beta_{jj'}$ are extracted from the lowest 0^+ and 2^+ :

$$\left|0_1^+\right\rangle = \sum_{j} A_j \left|j^2; 0\right\rangle \rightarrow \alpha_j = \sqrt{\frac{\sum_j \left(j + \frac{1}{2}\right)}{j + \frac{1}{2}}} A_j$$

$$\left|2_1^+\right\rangle = \sum_{j \leq j'} B_{jj'} \left|jj'; 0\right\rangle \rightarrow \beta_{jj'} = B_{jj'}$$

S. Pittel, P.D. Duval, B.R. Barret, Ann. Phys. **144**, 168 (1982)

The Number Operator Approximation (NOA)

Degenerate orbits

$$S_+ = \sum_j \frac{\sqrt{j+\frac{1}{2}}}{2} \left[C_j^\dagger \times C_j^\dagger\right]_0^{(0)}$$

$$[S_+, S_-] = \underbrace{\sum_j N_j}_{N} - \sum_j \underbrace{\left(j + \frac{1}{2}\right)}_{\Omega_j}$$

Non-degenarate orbits

$$S_+ = \sum_j \alpha_j \frac{\sqrt{j+\frac{1}{2}}}{2} \left[C_j^\dagger \times C_j^\dagger\right]_0^{(0)}$$

$$[S_+, S_-] = \underbrace{\sum_j \alpha_j^2 N_j}_{\neq N} - \underbrace{\sum_j \alpha_j^2 \left(j + \frac{1}{2}\right)}_{\Omega_e}$$

$$\boxed{\sum_j \alpha_j^2 N_j \approx N}$$

Failure of the NOA approximation

The NOA approximation predicts a linear dependence in the number of bosons

$$
\begin{aligned}
v_j^2(\text{NOA}) &= \alpha_j^2 \frac{N}{\Omega_e} \\
v_j^2(\text{EXACT}) &= -\sum_{s=1}^{N} (-1)^s \left[\frac{N!\,\eta_{2(N-s),0,0}\,\alpha_j^s}{(N-s)!\,\eta_{2N,0,0}}\right]^2
\end{aligned}
$$

		50-82 Protons		
Level	E (MeV)	α_j	v_j^2 (NOA)	v_j^2
$1g_{7/2}$	0.00	-1.717	$\boxed{1.290}$	0.827
$2d_{5/2}$	1.00	-0.814	0.290	0.490
$1h_{11/2}$	2.06	0.523	0.120	0.269
$2d_{3/2}$	2.52	-0.453	0.090	0.213
$3s_{1/2}$	2.85	-0.413	0.075	0.182

		82-126 Neutrons		
Level	E (MeV)	α_j	v_j^2 (NOA)	v_j^2
$2f_{7/2}$	0.00	-1.818	$\boxed{1.503}$	0.833
$1h_{9/2}$	0.69	-0.975	0.432	0.576
$1i_{13/2}$	1.75	0.569	0.147	0.304
$2f_{5/2}$	2.18	-0.487	0.108	0.239
$3p_{3/2}$	1.47	-0.640	0.186	0.358
$3p_{1/2}$	2.29	-0.470	0.100	0.226

$$N_\pi = 7,\ N_\nu = 10$$

IBM-2 and isospin

- For heavy nuclei the valence protons occupy orbits full of neutrons

$$T_{-}\left|\psi\right\rangle = 0 \Rightarrow T = |M_T|_{\mathrm{max}} = \frac{1}{2}|N - Z|$$

- For nuclei where protons and neutrons are in the same major shell, but with different character as particles or holes, isospin symmetry is violated only of order

$$\frac{1}{\Omega}$$

- For the rest of the cases (lighter nuclei) IBM-2 does not produce states with definite isospin

$$\Longrightarrow \text{IBM-3 and IBM-4 are required}$$

Source: J. P. Elliot, Prog. Part. Nucl. Phys. **25**, 325 (1990)

Two body matrix elements II

$$G_X^{F,GT,T}\left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu; J; J\right) \equiv G_X^{(s_1,s_2,\lambda)}\left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu; J; J\right) =$$

$$\begin{aligned}
&\sum_{k_1=|l_\pi-l_\nu|}^{l_\pi+l_\nu} \sum_{k_2=|l'_\pi-l'_\nu|}^{l'_\pi+l'_\nu} \sum_{k=k_{min}}^{k_{max}} i^{k_1-k_2+\lambda} \hat{k}_1^2 \hat{k}_2^2 \left\langle k_1 0\, k_2 0 \mid \lambda 0 \right\rangle \\
&\times (-1)^{s_2+k_1} \begin{Bmatrix} k_1 & s_1 & k \\ s_2 & k_2 & \lambda \end{Bmatrix} (-1)^{j'_\pi+j_\nu+J} \begin{Bmatrix} j_\pi & j'_\pi & J \\ j'_\nu & j_\nu & k \end{Bmatrix} \\
&\times \hat{k} \hat{j}_\pi \hat{j}_\nu \begin{Bmatrix} \frac{1}{2} & l_\pi & j_\pi \\ \frac{1}{2} & l_\nu & j_\nu \\ s_1 & k_1 & k \end{Bmatrix} \hat{k} \hat{j}'_\pi \hat{j}'_\nu \begin{Bmatrix} \frac{1}{2} & l'_\pi & j'_\pi \\ \frac{1}{2} & l'_\nu & j'_\nu \\ s_2 & k_2 & k \end{Bmatrix} \\
&\times \left\langle \tfrac{1}{2} \left\| \Sigma^{(s_1)} \right\| \tfrac{1}{2} \right\rangle (-1)^{-k_1} \hat{l}_\pi \left\langle l_\pi 0\, k_1 0 \mid l_\nu 0 \right\rangle \\
&\times \left\langle \tfrac{1}{2} \left\| \Sigma^{(s_2)} \right\| \tfrac{1}{2} \right\rangle (-1)^{-k_2} \hat{l}'_\pi \left\langle l'_\pi 0\, k_2 0 \mid l'_\nu 0 \right\rangle \\
&\times R_{X,k_1,k_2}^{(s_1,s_2,\lambda)}\left(n_\pi, l_\pi, n'_\pi, l'_\pi, n_\nu, l_\nu, n'_\nu, l'_\nu\right),
\end{aligned}$$

Isospin correction of the transition operator

Monopole term removed

$$R^{(s_1,s_2,\lambda)}_{X,k_1,k_2} \to R^{(s_1,s_2,\lambda)}_{X,k_1,k_2} - \delta_{k_1 0}\delta_{k_2 0}\delta_{k 0}\delta_{\lambda 0}\delta_{\alpha_\pi \alpha_\nu}\delta_{\alpha'_\pi \alpha'_\nu} R^{(s_1,s_2,0)}_{X,0,0}$$

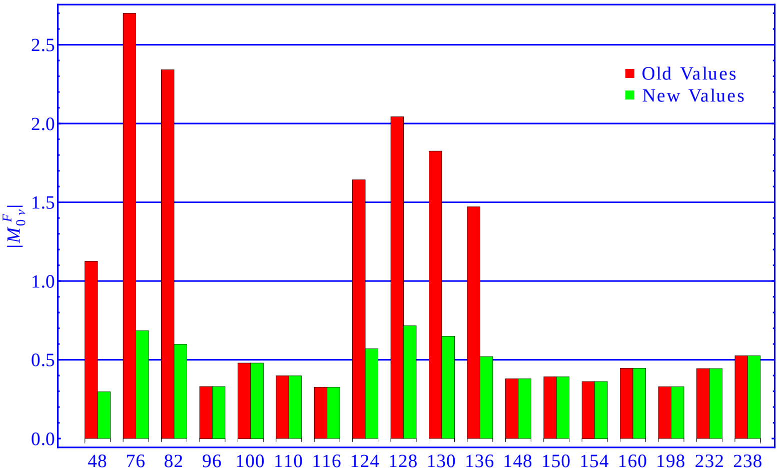
Consequences:

$M^F_{2\nu} \sim 0$ and $M^F_{0\nu}$ are strongly reduced

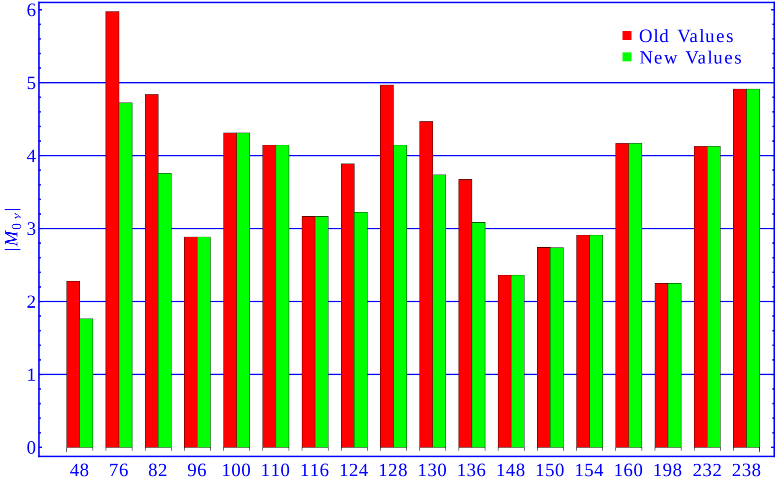
$M^{GT}_{2\nu}$ and $M^{GT,T}_{0\nu}$ does not change

J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2015)

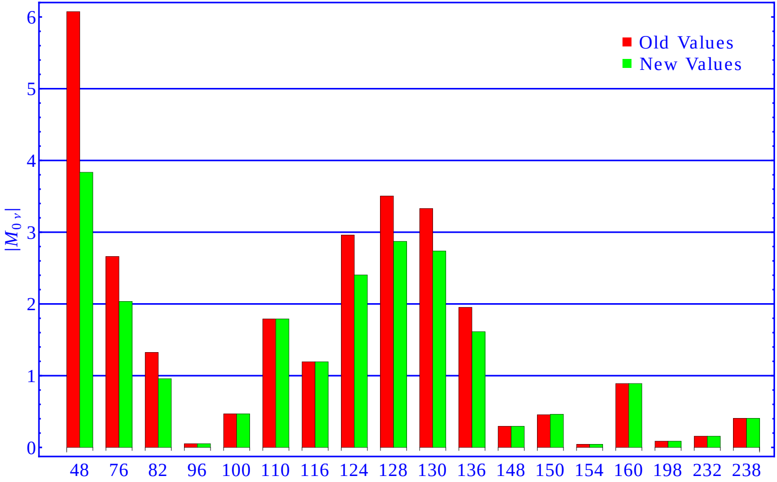
Fermi NMEs for $\beta^-\beta^-$ (0ν) to the ground state



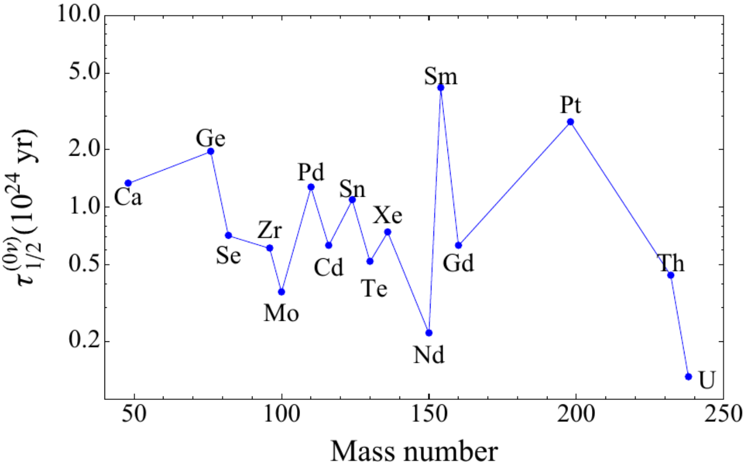
NMEs for $\beta^-\beta^-$ (0ν) to the ground state



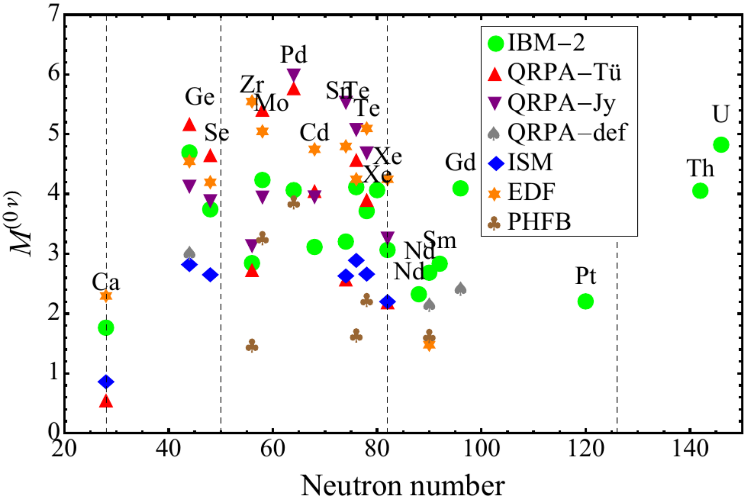
NMEs for $\beta^-\beta^-$ (0ν) to the first excited state



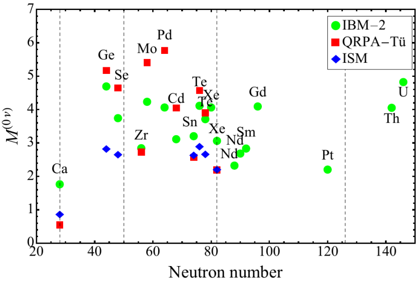
Expected halflives with $m_{\beta\beta} = 1\ eV$

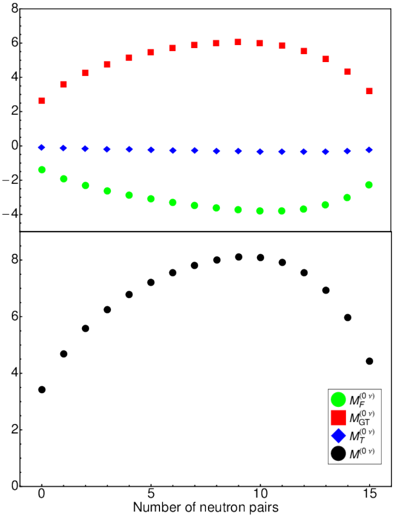


Comparison with other approaches



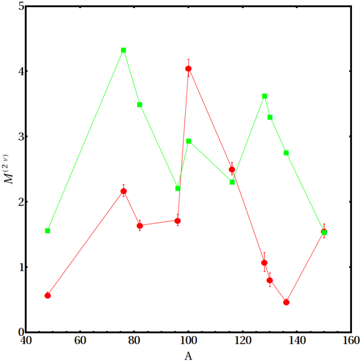
Shell effects





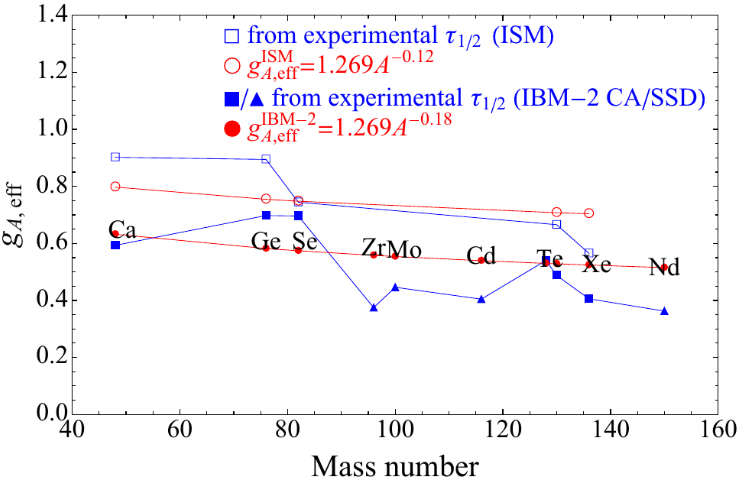
$$\begin{aligned} A_\pi A_\nu s_\pi^\dagger \tilde{s}_\nu &= \alpha_\pi \alpha_\nu s_\pi^\dagger \left(\Omega_\pi - N_\pi\right)^{1/2} \\ &\times \left(\Omega_\nu - N_\nu\right)^{1/2} \tilde{s}_\nu \\ M^{(0\nu)} &\simeq \alpha_\pi \alpha_\nu \sqrt{N_\pi + 1}\sqrt{N_\nu} \\ &\times \sqrt{\Omega_\pi - N_\pi}\sqrt{\Omega_\nu - N_\nu + 1} \end{aligned}$$

Two neutrinos results



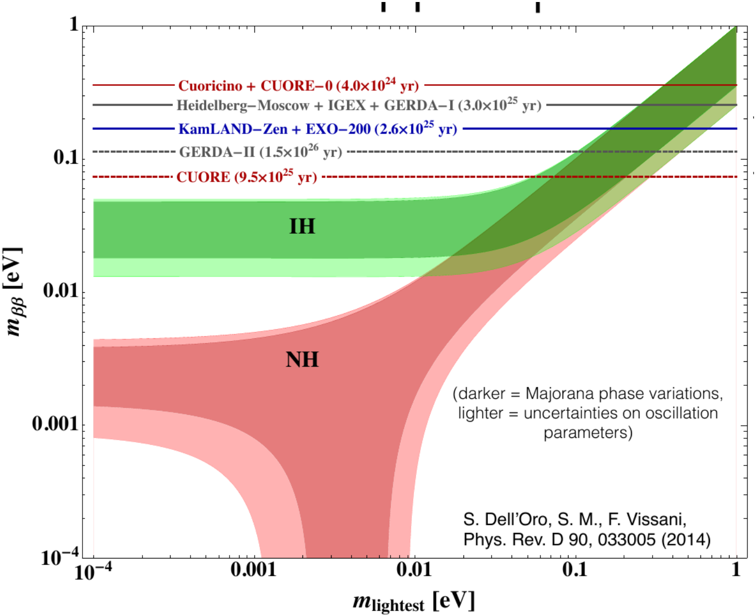
$$M_{2\nu} = g_A^2 M^{(2\nu)}, \quad M^{(2\nu)} = -\left[\frac{M_{GT}^{(2\nu)}}{\tilde{A}_{GT}} - \left(\frac{g_V}{g_A}\right)^2 \frac{M_F^{(2\nu)}}{\tilde{A}_F}\right]$$

The quenching of g_A



$$M_{2\nu}^{\text{eff}} = \left(\frac{g_{A,\text{eff}}}{g_A}\right) M_{2\nu}$$

Experimental situation



Summary

- Neutrinoless double beta decay is a promising tool to unveil neutrino properties.
- Resolving the differences between the nuclear matrix elements is essential.
- Quenching of the coupling constants should be clarified.