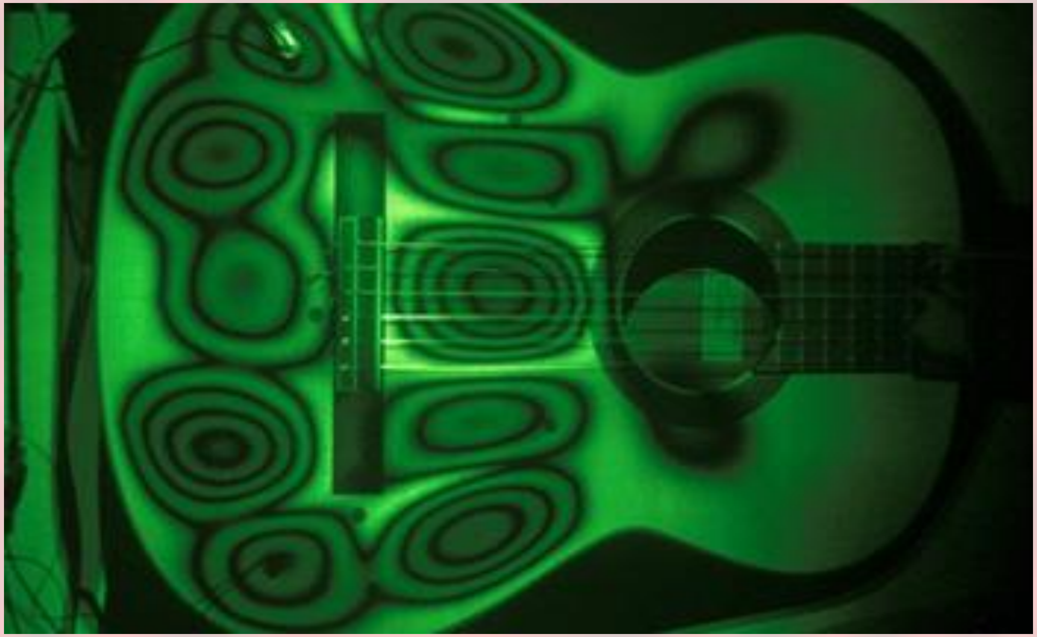
Optical Interferometry



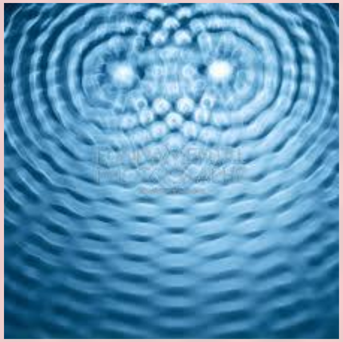
Emiliano Descrovi

Politecnico *di Torino*

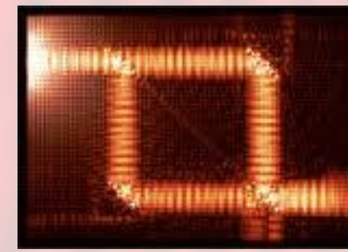
Department of Applied Science and Technology

emiliano.descrovi@polito.it

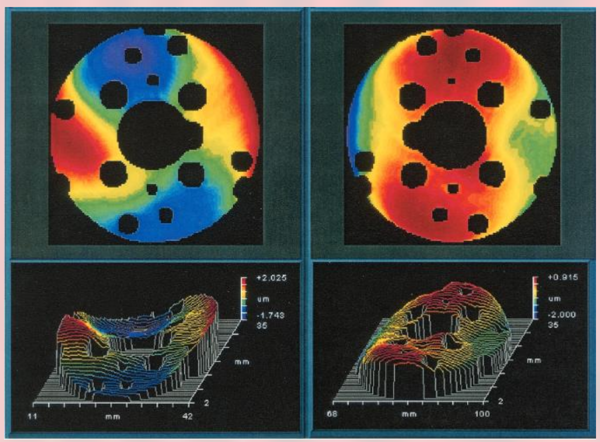
Content



- Interference: Basic principles
- Conventional Interferometers



- Micro-Interferometric imaging



- Adaptive optics: Imaging through scattering media

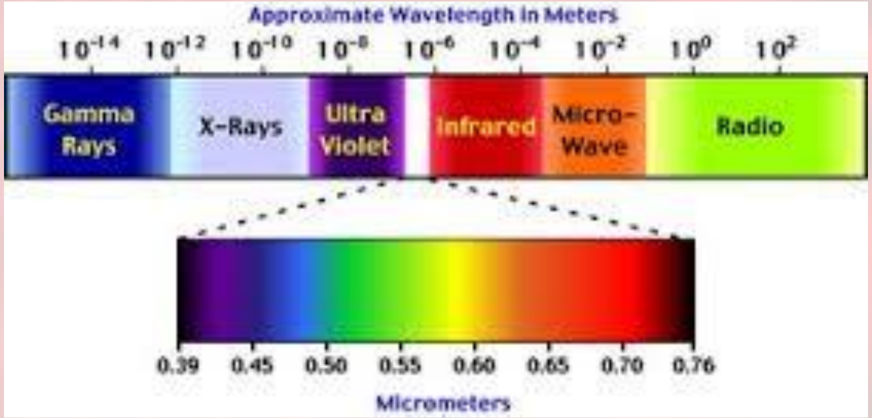
Interference

It is a typical phenomenon associated to **waves**. In general it is the superposition of two or more waves that form a new wave. The intensity of the resulting wave is in general different from the sum of the intensities of the original waves.

Usually we talk about interference when the superposing waves are **coherent,** i.e. if they have a **constant phase relation** between them.

However, it is possible to observe several interference phenomena in nature wherein light is not (fully) coherent to some extent.--.

Light waves: some definitions



υ: $frequency$ (≈ $4.5 \cdot 10^{14}$Hz ÷ $7 \cdot 10^{14}$Hz)

λ: $wavelength$ (≈ 400nm ÷ 800nm)

$c = \upsilon\lambda$: $Speed\ of\ light\ in\ vacuum$ (≈ $3 \cdot 10^{8}$ m/s)

In a medium with refractive index n the speed of light is:

$v = c/n$ → $\lambda_n = \lambda/n$ Optical path: $p = d \cdot n$

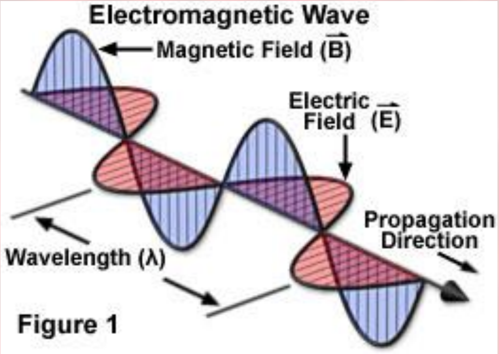
Light waves

Generally speaking, light is a **transverse electromagnetic wave** propagating through free-space.

To describe light propagation it is sufficient to consider the electric field at any point.

$$E(x,y,z,t) = Re[E_0 \cdot e^{2\pi i\left(\upsilon t - \frac{z}{\lambda}\right)}]$$

Amplitude Phase



$2\pi\upsilon$ = **ω**: circular frequency

$2\pi/\lambda$ = **k**: wavevector

$$E(x,y,z,t) = Re[E_0 e^{-ikz} \cdot e^{i\omega t}]$$

$E_0 e^{i\varphi}$ = A: complex amplitude

Intensity in an interference pattern

$$E(x, y, z, t) = Re[A \cdot e^{i\omega t}]$$

$I \propto |A|^2$: Intensity (time-averaged Poynting vector)

Assumptions:

1. Two waves are propagating in the **same direction**
2. They have the **same frequency**
3. They are **polarized with their fields in the same direction**

When the **two waves** superpose the resulting **complex amplitude is the sum of the complex amplitudes**:

$A = A_1 + A_2$

$$I \propto |A|^2 = (A_1 + A_2)(A_1^* + A_2^*) = |A_1|^2 + |A_2|^2 + A_1 \cdot A_2^* + A_1^* \cdot A_2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos(\varphi_1 - \varphi_2)$$

Interference term

Intensity in an interference pattern

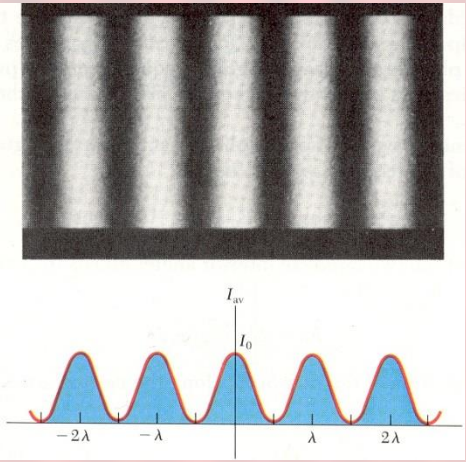
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos(\Delta\varphi)$$

If the two waves have the same phase at the origin, then $\Delta\varphi$ corresponds to a path difference:

$$\Delta p = \left(\frac{\lambda}{2\pi}\right)\Delta\varphi$$

Or equivalently to a time delay:

$$\tau = \frac{\Delta p}{c} = \left(\frac{\lambda}{2\pi c}\right)\Delta\varphi$$



If $\Delta\varphi$ varies linearly in the observation plane, then the intensity varies cosinusoidally, giving rise to alternating bright and dark bands, known as interference fringes.

Fringes: Loci of constant phase difference $\Delta\varphi$

Visibility of the fringes

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos(\Delta\varphi)$$

When **Δφ = 2mπ** and **Δφ = (2m+1)π** the intensity in the interference pattern has its **maximum** and **minimum** values:

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \qquad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

We define the visibility of the fringes V by the relation:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad 0 \leq V \leq 1$$

$$V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} \Rightarrow V = 1 \; when \; I_1 = I_2$$

Coherence of quasi-monochromatic light

Coherence theory is a **statistical description** of the radiation field due to a light source, in terms of the **correlation between the vibrations** at different points in the field.

Quasi-monochromatic light: a source emitting light with a **narrow range of frequencies**.

The electric field at any point radiated by a quasi-monocromatic light can be written as:

$$V(t) = \int_0^{\infty} a(\upsilon) \exp\{i[2\pi\upsilon t - \varphi(\upsilon)]\} \, d\upsilon$$

$$I = \lim_{T\to\infty} 1/2T \int_{-T}^{T} V(t)V^*(t)dt = \langle V(t)V^*(t)\rangle$$

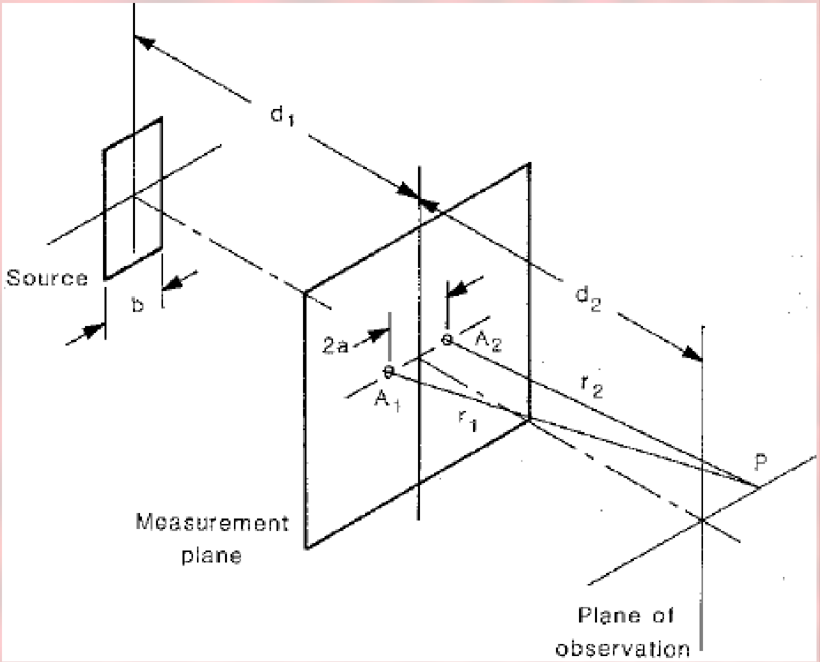
The mutual coherence function

A quasi-monochormatic extended source S illuminates the screen containing two pinholes A_1 and A_2.

$V_1(t)$and $V_2(t)$ are the wave fields produced by S at A_1 and A_2.

The wave field in P will be:

$$V_P(t) = K_1 V_1(t - t_1) + K_2 V_2(t - t_2)$$



K_1 and K_2: geometrical factors

$$t_i = \frac{r_i}{c}$$

Since the wave field is stationary:

$$V_P(t) = K_1 V_1(t + \tau) + K_2 V_2(t)$$
$$\tau = t_1 - t_2$$

The mutual coherence function

The intensity in P will be:

$$
\begin{aligned}
I_P &= \langle V_P(t) V_P^*(t) \rangle \\
&= |K_1|^2 \langle V_1(t+\tau) V_1^*(t+\tau) \rangle + |K_2|^2 \langle V_2(t) V_2^*(t) \rangle \\
&\quad + K_1 K_2^* \langle V_1(t+\tau) V_2^*(t) \rangle + K_1^* K_2 \langle V_1^*(t+\tau) V_2(t) \rangle \\
&= |K_1|^2 I_1 + |K_2|^2 I_2 + 2|K_1 K_2| \mathrm{Re}\{\Gamma_{12}(\tau)\},
\end{aligned}
$$

I_1 and I_2: intensities at A_1 and A_2

Mutual coherence function

$$\Gamma_{12}(\tau) = \langle V_1(t+\tau) V_2^*(t) \rangle$$

Visibility of the interference fringes

$$I_P = |K_1|^2 I_1 + |K_2|^2 I_2 + 2|K_1 K_2| \mathrm{Re}\{\Gamma_{12}(\tau)\},$$

We can write the equations as:

$$I_P = I_{P_1} + I_{P_2} + 2\sqrt{I_{P_1} I_{P_2}} Re\{\gamma_{12}(\tau)\}$$

Where $I_{P_i} = |K_i|^2 I_i$ are the intensities due to the two pinholes acting separately, and $\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}}$ is called the **complex degree of coherence of the wave fields at A$_1$ and A$_2$**.

When $I_1 = I_2$ then the visibility of the fringes is:

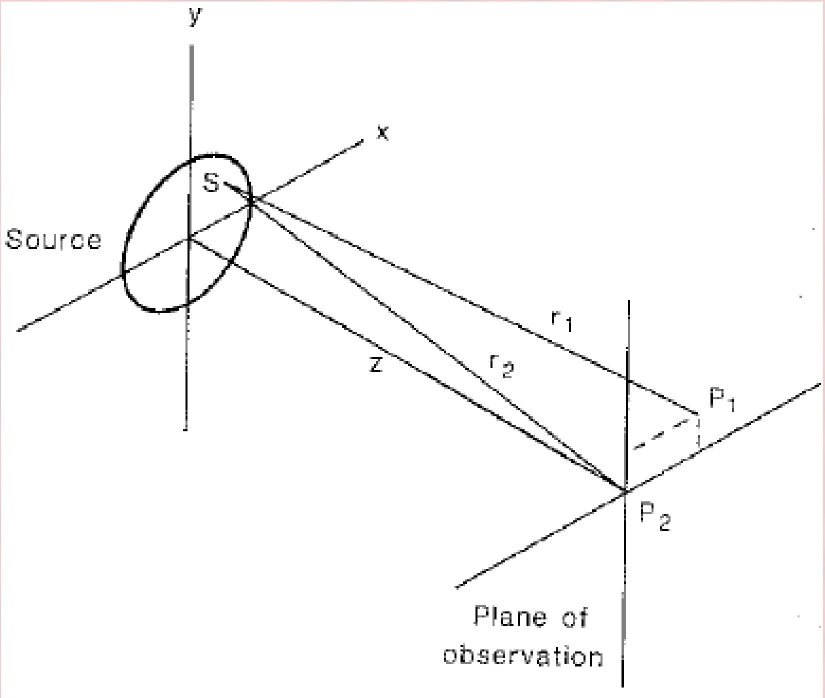
$$\nu = Re\{\gamma_{12}(\tau)\}$$

Spatial coherence (extended sources)

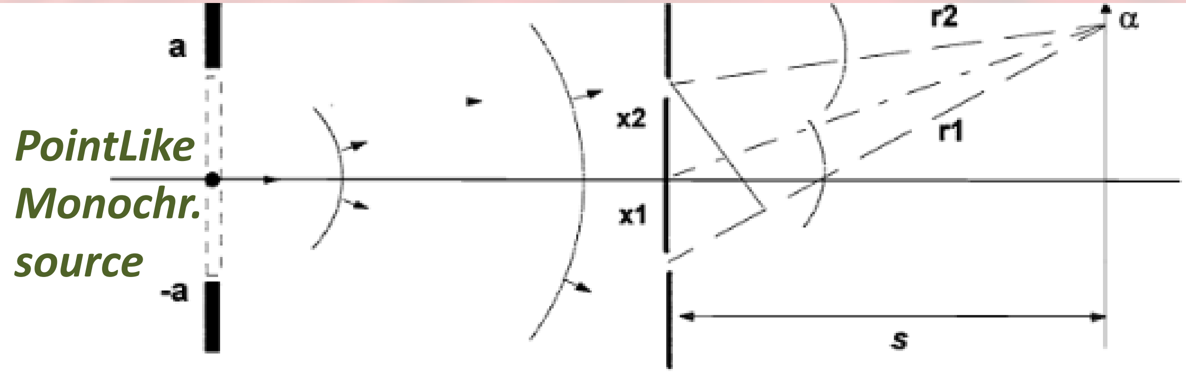
Neglect the time delay τ

When the **difference in the optical paths is small,** the **visibility of the fringes depends only on the spatial coeherence** of the fields. We can evaluate the degree of coherence between the fields at points P_1 and P_2 as follows:

- We **first** obtain an expression for the **mutual coherence function** of the fields **at these two points due to a very small element** on the source.
- We then **integrate** this expression **over the whole area of the source**



Spatial coherence (extended sources)



Interference Equation:

The (ensemble or time) averge intensity, $I(\alpha)$, at α is due to the superposition of complex amplitudes $V(x_1)$ and $V(x_2)$ arriving at x_1 and x_2.

(propagation constant K is omitted in the following derivation)

$$I(\alpha) = \left\langle \left|U(\alpha)\right|^2 \right\rangle = \left\langle \left|V(x_1) + V(x_2)\right|^2 \right\rangle$$

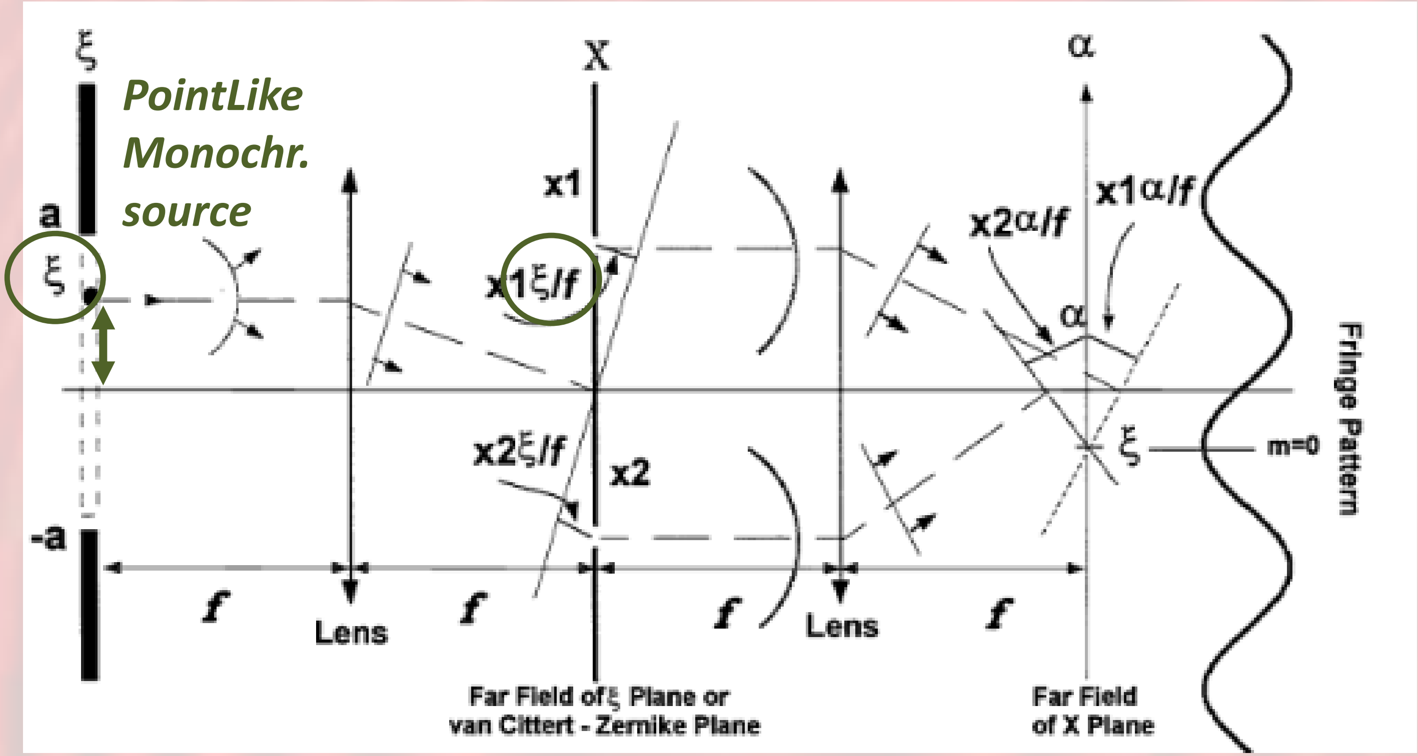
$$= \left\langle \left|V(x_1)\right|^2 \right\rangle + \left\langle \left|V(x_2)\right|^2 \right\rangle + 2\,Re\left\langle V^*(x_1)V(x_2)\right\rangle$$

$$= I_1(\alpha) + I_2(\alpha) + 2\left|G_{12}\right|\cos\varphi_{12}$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,g_{12}\cos\varphi_{12}, \text{ where } g_{12} \equiv \frac{\left\langle V_1^* V_2 \right\rangle}{2\sqrt{I_1 I_2}}$$

$$= \left(1 + \beta g_{12}\cos\varphi_{12}\right)\left(I_1 + I_2\right), \text{ where } \beta \equiv \frac{2\sqrt{I_1 I_2}}{\left(I_1 + I_2\right)}$$

Spatial coherence (extended sources)

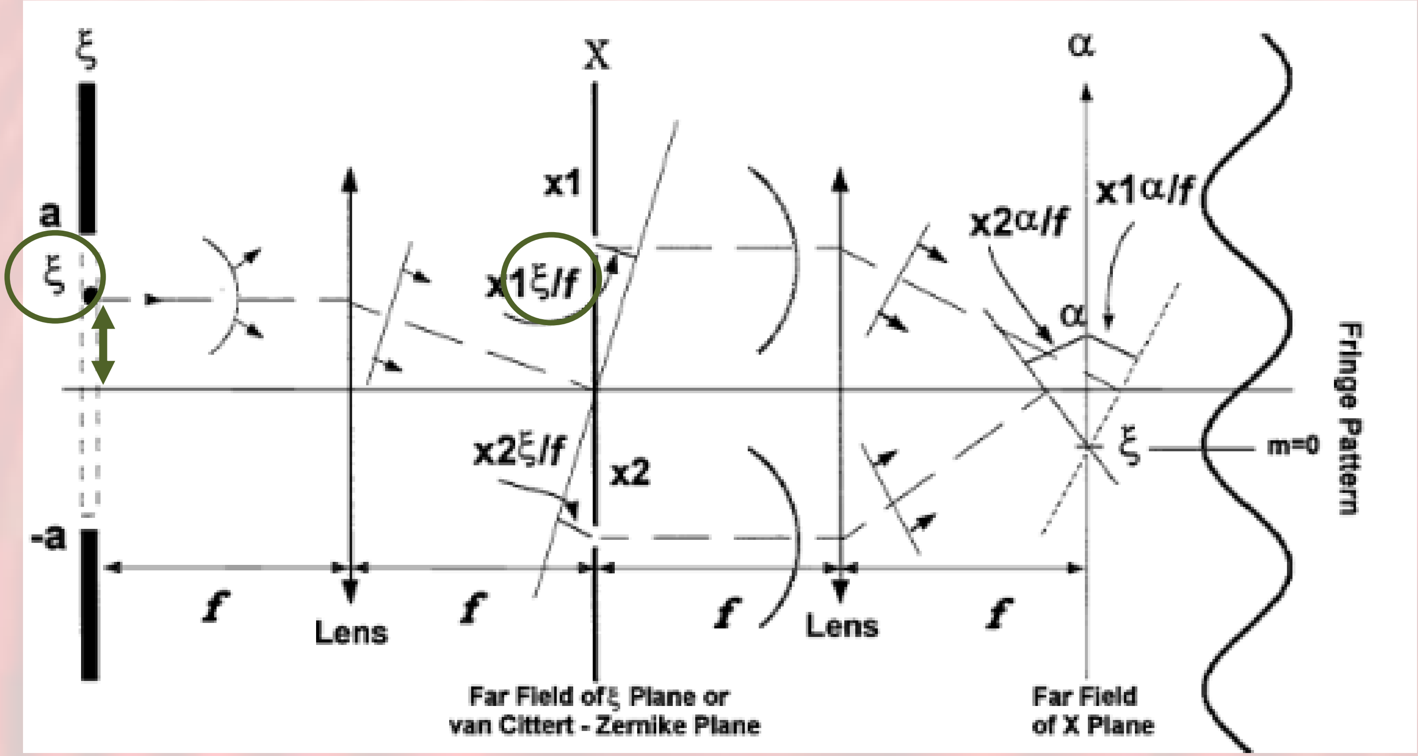


Assume the source is divided into elenments $d\xi_1, d\xi_2$, ... etc. at ξ_1, ξ_2, ... etc. If $V_{m1}(x_1)$and $V_{m2}(x_2)$ ıre elemental complex amplitudes at x_1 and x_2 due to the element $d\xi_m$, then the total disturbances, neglecting the propagation constant, are:

$$V(x_1) = \Sigma_m V_{m1}(x_1) \quad \text{and} \quad V(x_2) = \Sigma_m V_{m2}(x_2)$$

$$V(x_1) = \Sigma_m U(\xi_m)\exp(-ik\, \xi_m x_1/f) \quad V(x_2) = \Sigma_m U(\xi_m)\exp(-ik\, \xi_m x_2/f)$$

Spatial coherence (extended sources)



Then the correlation function becomes:

$$G(x_1,x_2) = \langle V^*(x_1)V(x_2)\rangle$$
$$= \langle \textstyle\sum_m \{V_{m1}(x_1)\}^* \sum_m V_{m2}(x_2)\rangle$$
$$= \textstyle\sum_m \langle \{V_{m1}(x_1)\}^* V_{m2}(x_2)\rangle + \sum_{m\neq n} \sum \langle\{V_{m1}(x_1)\}^* V_{n2}(x_2)\rangle$$

Spatial coherence (extended sources)

For incoherent source points, when m ≠ n, the correlation is zero, or

$$
\begin{aligned}
G(x_1,x_2) &= \Sigma_m \langle \{V_{m1}(x_1)\}^* V_{m2}(x_2) \rangle \\
&= \Sigma_m \langle U^*(\xi_m)\exp(ik\,\xi_m x_1/f)\, U(\xi_m)\exp(-ik\,\xi_m x_2/f) \rangle \\
&= \Sigma_m \langle U^*(\xi_m)\, U(\xi_m) \rangle \exp(ik\,\xi_m(x_1-x_2)/f) \\
&= \Sigma_m I(\xi_m)\exp(ik\,\xi_m(x_1-x_2)/f).
\end{aligned}
$$

For a continuous source we replace the summation by an integral,

$$G(x_1,x_2) = \int I(\xi_m)\exp(ik\,\xi_m(x_1-x_2)/f)\, d\xi.$$

This is the Fourier Transform of the intensity function and is functionally simular to the Fraunhofer diffraction pattern. For a one-dimensional rectangular source of uniform intensity A and of width 2a, the correlation function is integrated over the source yielding:

$$G(x_1,x_2) = 2Aa \operatorname{sinc}[\, ka(x_2-x_1)/f\,].$$

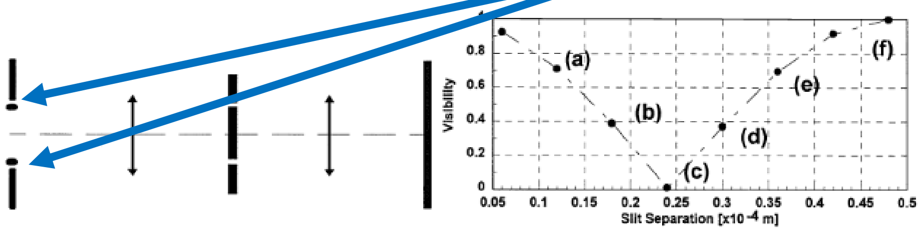
Spatial coherence (extended sources)

The **resulting expression** is similar to the **Fresnel-Kirchhoff diffraction integral** and leads to the **van Cittert-Zernike theorem**, which can be stated as follows:

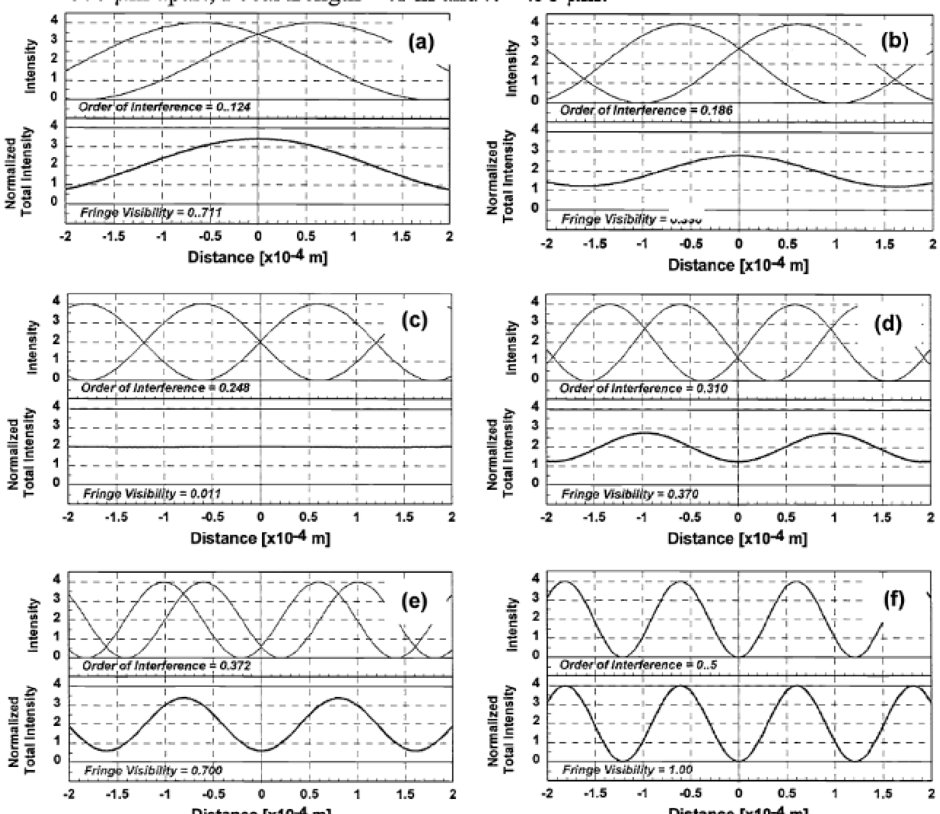
- Imagine that the **source is replaced by an aperture** with an amplitude transmittance at any point proportional to the intensity at this point in the source.
- Imagine that this aperture is **illuminated by a spherical wave converging to a fixed point** in the plane of observation (say P_2) **and we view the diffraction pattern** formed by this wave **in the plane of observation**.
- **The complex degree of coherence** between the wave fields at P_2 and some other points P_1 **is then proportional to the complex amplitude at P_1 in the diffraction pattern**.

Spatial coherence (extended sources)

5. 1 OSCILLATIONS OF VISIBILITY DUE TO A DOUBLE STAR
A Graphical Presentation

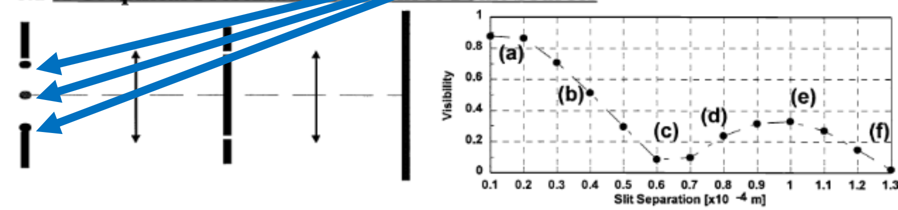


Individual intensities and normalized total intensities for: Two point sources 600 µm apart; Focal Length = .1 m and λ = .58 µm.

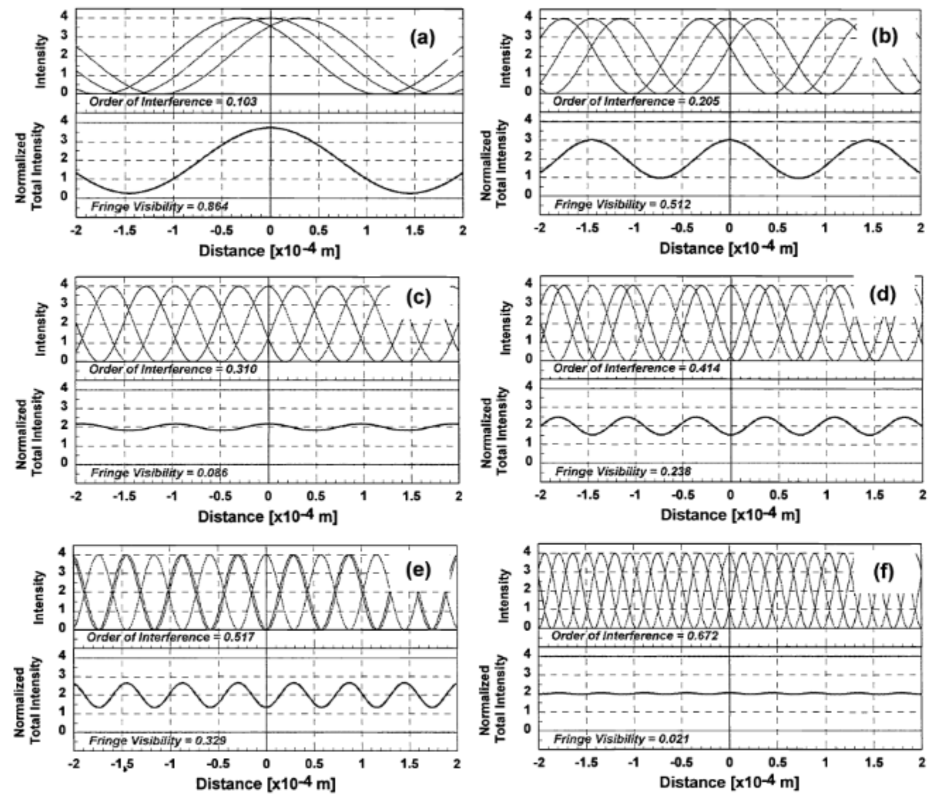


6. VARIATION OF CORRELATION OR VISIBILITY BETWEEN TWO POINT
OF INCREASING SEPARATION FOR THREE EXTENDED SOURCE.

6.1 A Graphical Presentation F ree Point Sources



Individual intensities and normalized total intensities for: Three point sources 300 µm apart; Focal Length = .1 m and λ = .58 µm.



Temporal coherence (polychromatic sources)

For a **point source** radiating over a range of wavelengths, the **complex degree of coherence** between the fields at P_1 and P_2 **depends only on τ, the difference in the transit time**.
The mutual coherence function then reduces to the **autocorrelation function**:

$$\Gamma_{11}(\tau) = \langle V(t+\tau)V^*(t)\rangle$$

The degree of temporal coherence can be written as:

$$\gamma_{11}(\tau) = \frac{\langle V(t+\tau)V^*(t)\rangle}{\langle V(t)V^*(t)\rangle}$$

Coherence length

The frequency spectrum of a source radiating in a range of frequencies $\Delta\upsilon$ can be written as:

$$S(\upsilon) = rect\left[\frac{\upsilon - \bar{\upsilon}}{\Delta\upsilon}\right]$$

The autocorrelation function is given by the Fourier Transform of the frequency spectrum:

$$\gamma_{11}(\tau) = sinc(\tau\Delta\upsilon)$$

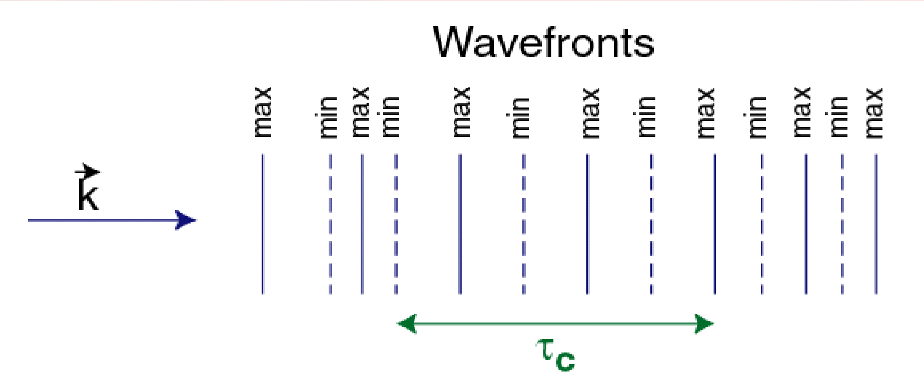
Which drops to zero when $\tau\Delta\upsilon = 1$.

The optical path difference at which fringes disappear is $\Delta p = \frac{c}{\Delta\upsilon}$.

Key Concepts

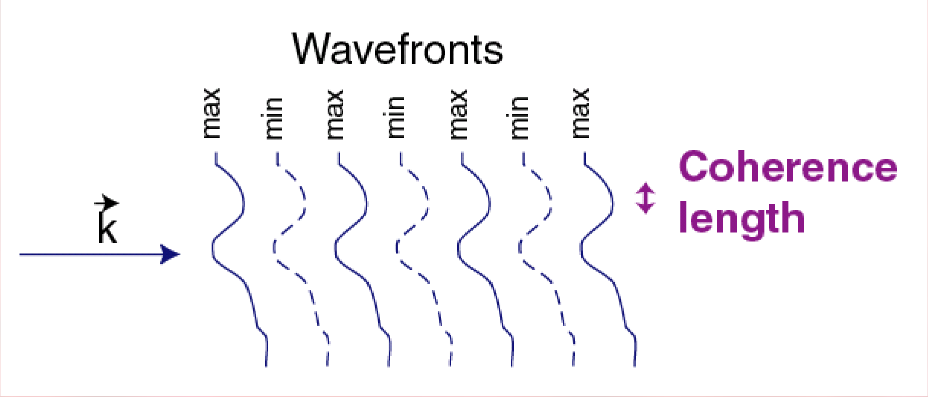
The temporal coherence time is the time the wave-fronts remain equally spaced. That is, the field remains sinusoidal with one wavelength:

Temporal Coherence Time, τ_c



The spatial coherence length is the distance over which the beam wave-fronts remain 'flat':

Spatial Coherence Length



Since there are two transverse dimensions, we can define a coherence area.

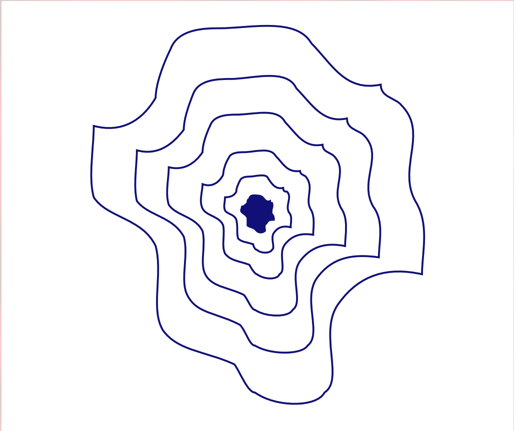
Key Concepts

The van Cittert-Zernike Theorem states that the spatial coherence area A_c is given by:

$$\mathrm{A}_c = \frac{D^2\lambda^2}{\pi d^2}$$

where d is the diameter of the light source and D is the distance away.

Basically, wave-fronts smooth out as they propagate away from the source.



Starlight is spatially very coherent because stars are very far away.

Two-beams interferometers

To make **measurements** using interference, we usually need **two beams** travelling along **different paths**, and an optical setup that makes them interfere.

The two beams, that we call the **reference beam** and **test beam**, must have **the same frequency**.

In order to produce a **stationary interference pattern**, the **phase difference should not change with time**.

The simplest way to meet this requirement is to **derive the two beams from the same source**.

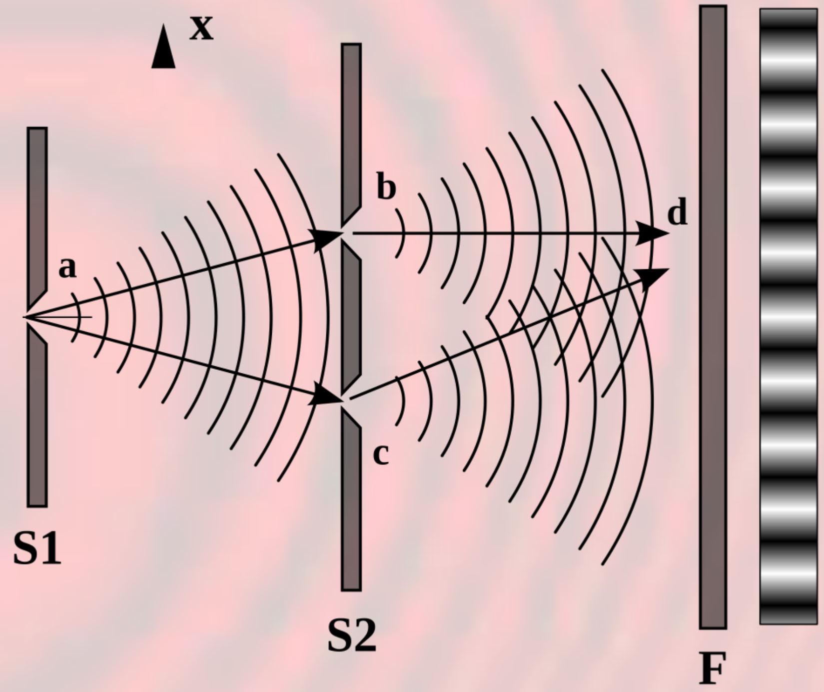
Wavefront division Amplitude division

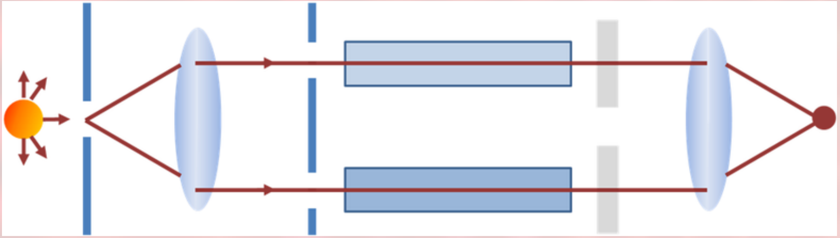
Wavefront division: the Young's experiment

Huygens – Fresnel Principle: Each point on the wavefront acts as a secondary source of a spherical wave.

Small apertures in an opaque screen can be seen as **coherent light sources**

The **Rayleigh interferometer** is based on wavefront division





Wavefront division: localization of the fringes

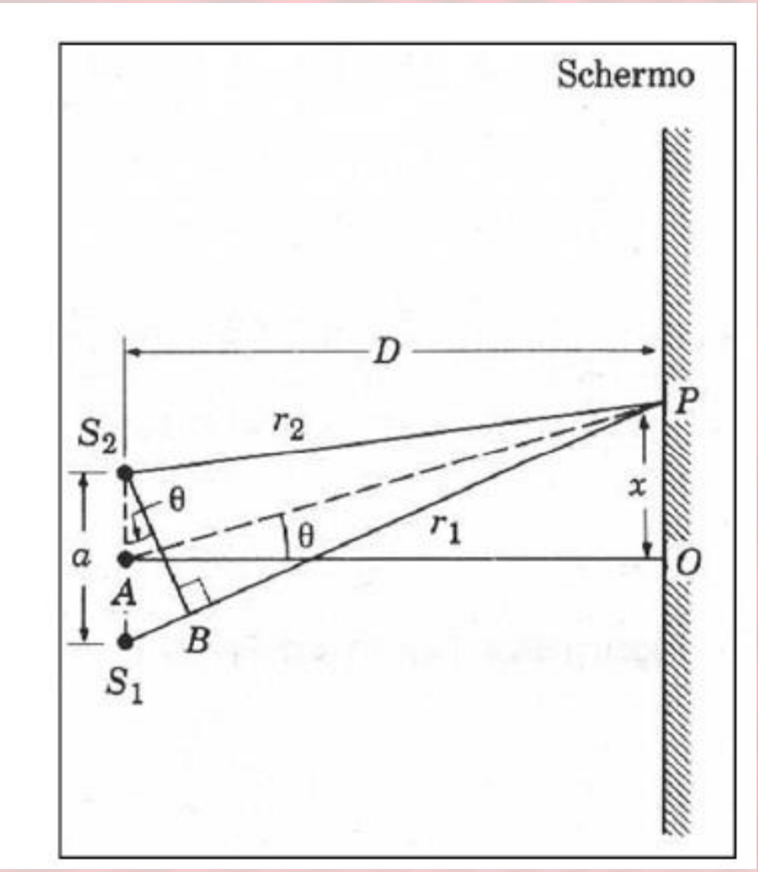
S1 and S2 are two coherent and sincronous sources

$$\boldsymbol{E_1}(\boldsymbol{r}, t) = E_0 cos(\boldsymbol{k} \cdot \boldsymbol{r_1} - \omega t)$$
$$\boldsymbol{E_2}(\boldsymbol{r}, t) = E_0 cos(\boldsymbol{k} \cdot \boldsymbol{r_2} - \omega t)$$

In P the resulting intensity will be:

$$I(p) = 2|E_0|^2 + 2|E_0|\cos(\Delta\varphi)$$

Where $\Delta\varphi = \frac{2\pi}{\lambda}\Delta p = \boldsymbol{k}(\boldsymbol{r_1} - \boldsymbol{r_2})$



Wavefront division: localization of the fringes

When $D \gg a, \rightarrow \boldsymbol{r_1} - \boldsymbol{r_2} \approx a \cdot \sin(\theta)$

$$I(p) = 2|E_0|^2 + 2|E_0|\cos[\frac{2\pi}{\lambda} a \cdot \sin(\theta)]$$

The maximum of intensity occurs when:

$$I_{MAX} \rightarrow \frac{2\pi}{\lambda} a \cdot \sin(\theta) = 2m\pi$$

$$\Downarrow$$

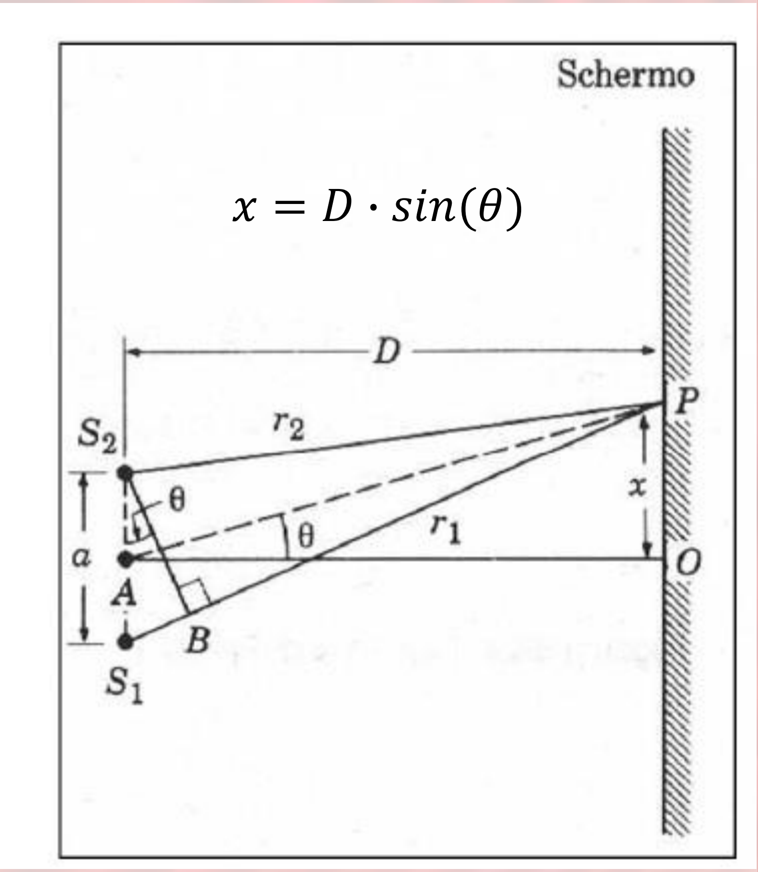
$$\boldsymbol{r_1} - \boldsymbol{r_2} = m\lambda \leftrightarrow x = \frac{m\lambda D}{a}$$

The minimum of intensity occurs when:

$$I_{MIN} \rightarrow \frac{2\pi}{\lambda} a \cdot \sin(\theta) = (2m+1)\pi$$

$$\Downarrow$$

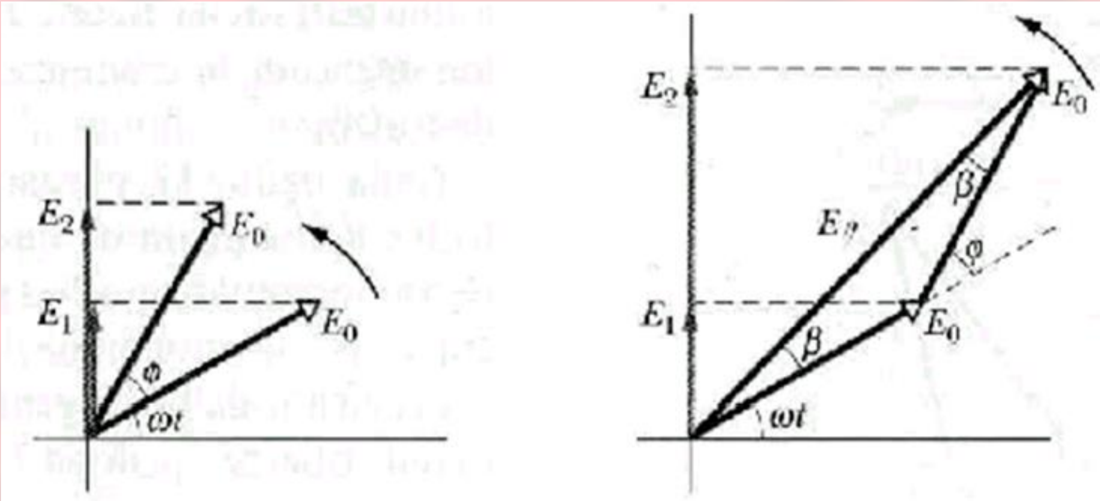
$$\boldsymbol{r_1} - \boldsymbol{r_2} = \frac{(2m+1)\lambda}{2} \leftrightarrow x = (2m+1)\frac{\lambda D}{2a}$$



Superposition of coherent waves: sum of phasors

Two waves are **coherent** if they have the **same frequency** and a **time-invariant phase difference**

When two coherent waves superpose in a point and they have the **electric field in the same plane**, we can use the phasor method to sum them.

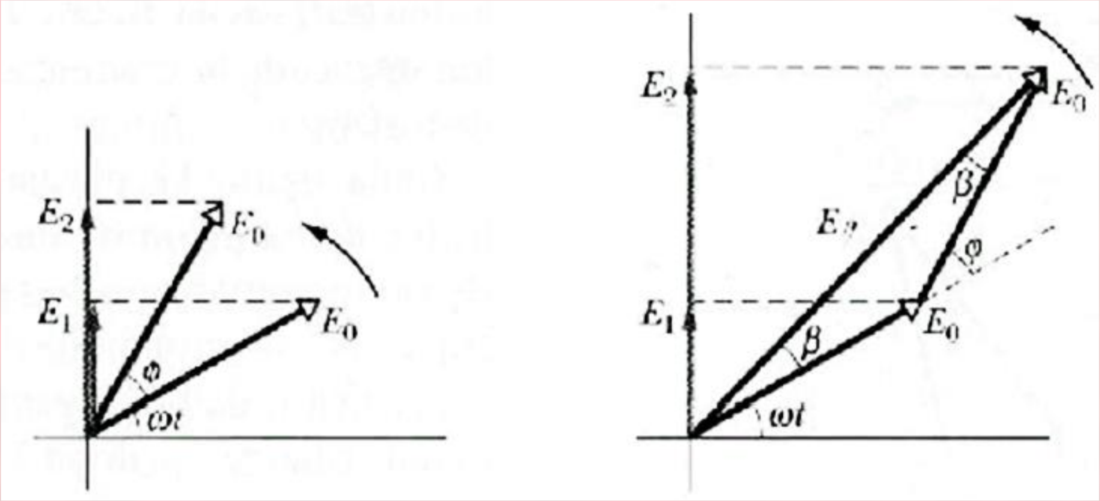


Superposition of coherent waves: sum of phasors

The resulting amplitude is the **sum of the projections of the amplitudes**.

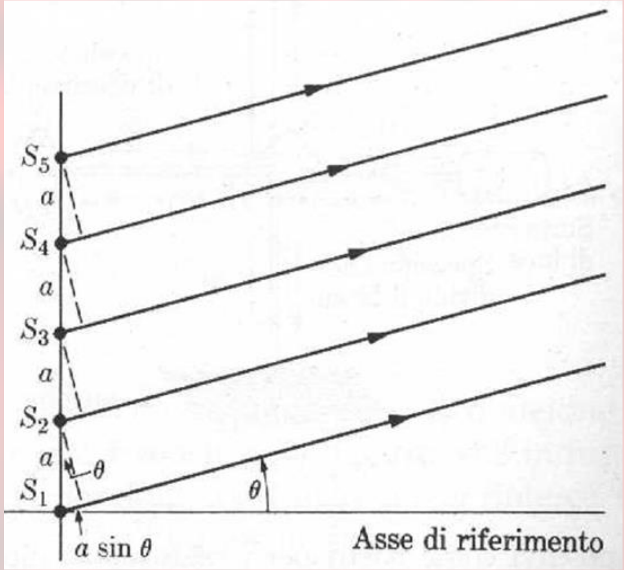
$$\boldsymbol{E_R} = \boldsymbol{E_1} + \boldsymbol{E_2} = E_0 \sin(\omega t) + E_0 \sin(\omega t + \varphi)$$

Where $\boldsymbol{\varphi}$ is the phase difference of the two waves. The maximum intensity will occurr when the two vectors are aligned (**$\boldsymbol{\varphi}$ = 2mπ**)



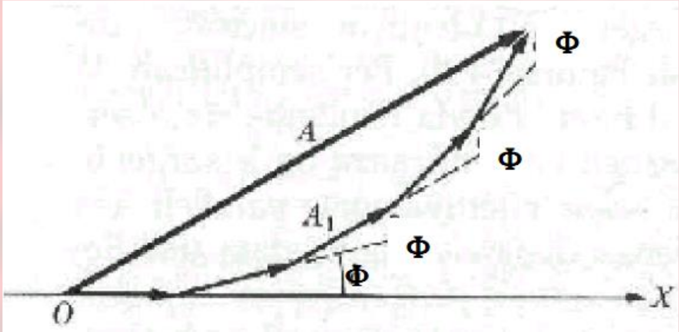
Interference of several coherent sources

We consider several coherent sources equally spaced



The maximum of intensity will occur when all the vectors are aligned, i.e. when

$$\Delta\varphi = 2m\pi$$



$$I_{MAX} \rightarrow \frac{2\pi}{\lambda} a \cdot \sin(\theta) = 2m\pi$$

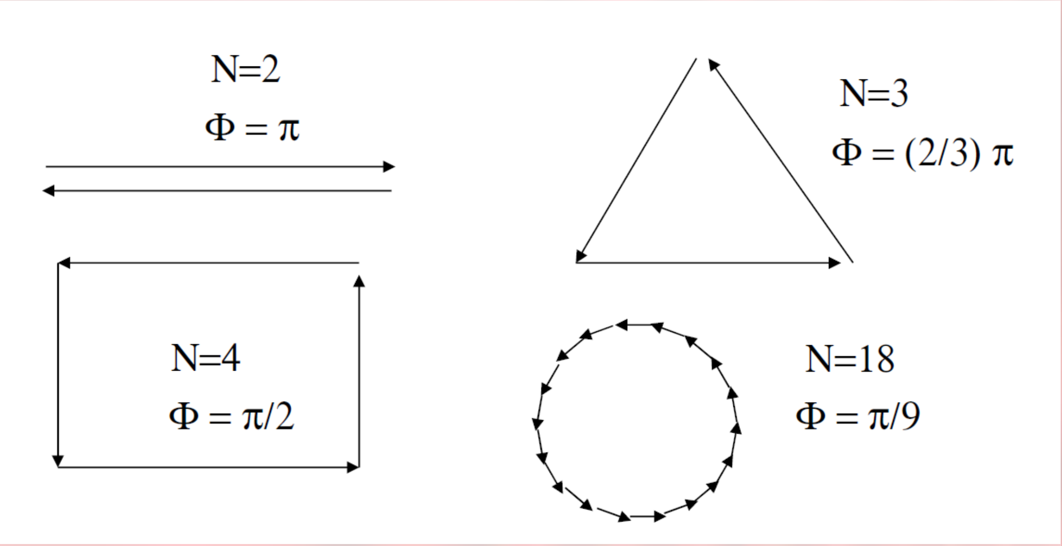
$$I_{MAX} \propto N^2 A^2$$

Interference of several coherent sources

When the vectors form a closed loop, the resulting amplitude is zero.

This condition is satisfied when

$$N\varphi = 2m'\pi \rightarrow \varphi = \frac{2m'\pi}{N}$$



$$m' = 1,2,\ldots(N-1),(N+1),\ldots\ldots(2N-1),(2N+1),\ldots.$$

$$\Phi = \frac{2\pi}{\lambda} a \sin\theta \Longrightarrow \sin\theta = \frac{m'\lambda}{Na}$$

Interference of several coherent sources

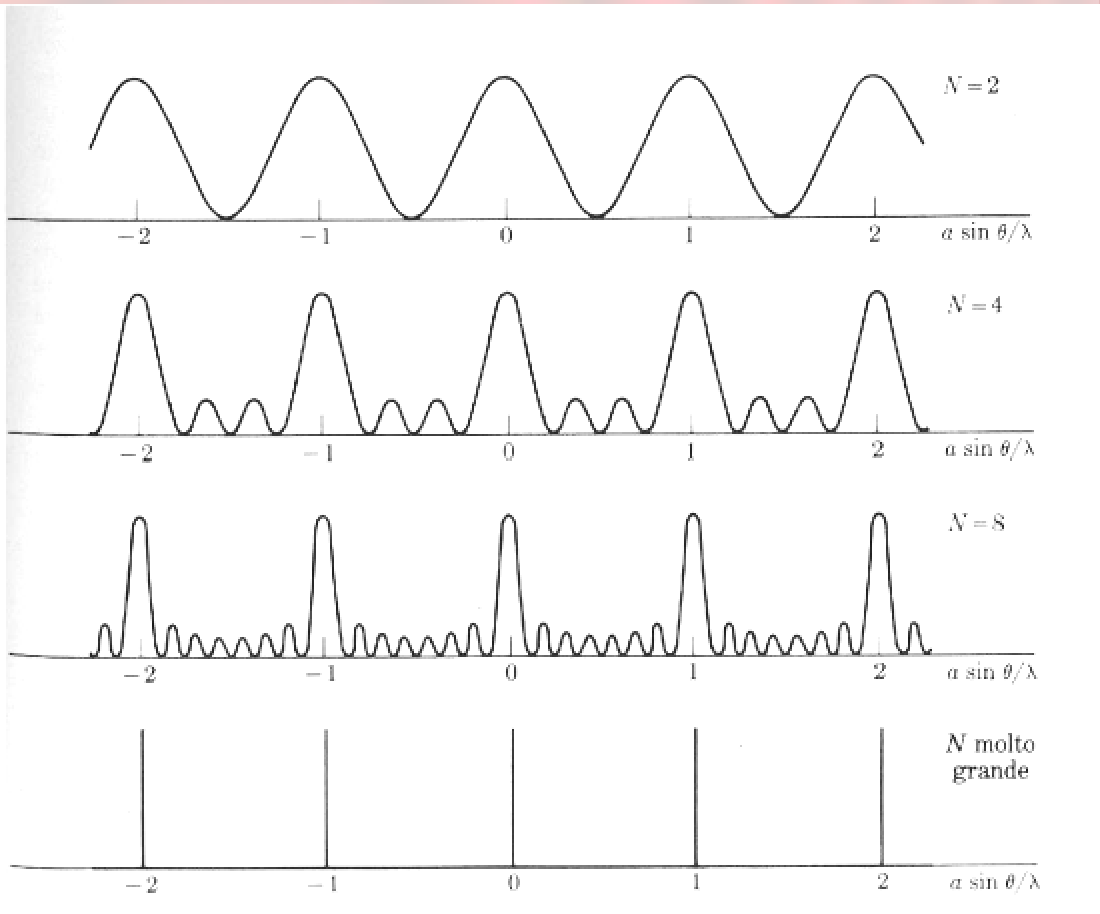
I_{max}

$$\Phi = \frac{2\pi}{\lambda} a \sin\theta \Longrightarrow \sin\theta = \frac{m\lambda}{a}$$

I_{min}

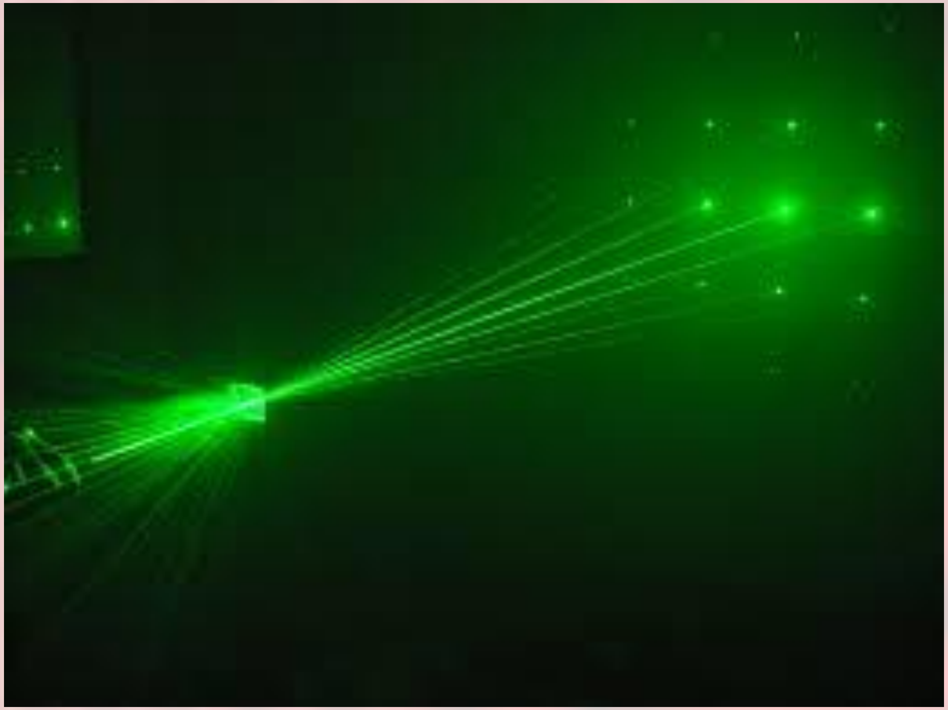
$$\Phi = \frac{2\pi}{\lambda} a \sin\theta \Longrightarrow \sin\theta = \frac{m'\lambda}{Na}$$

Between two principal maxima there are N-1 minima.
Between two minima, there is a local residual maximum

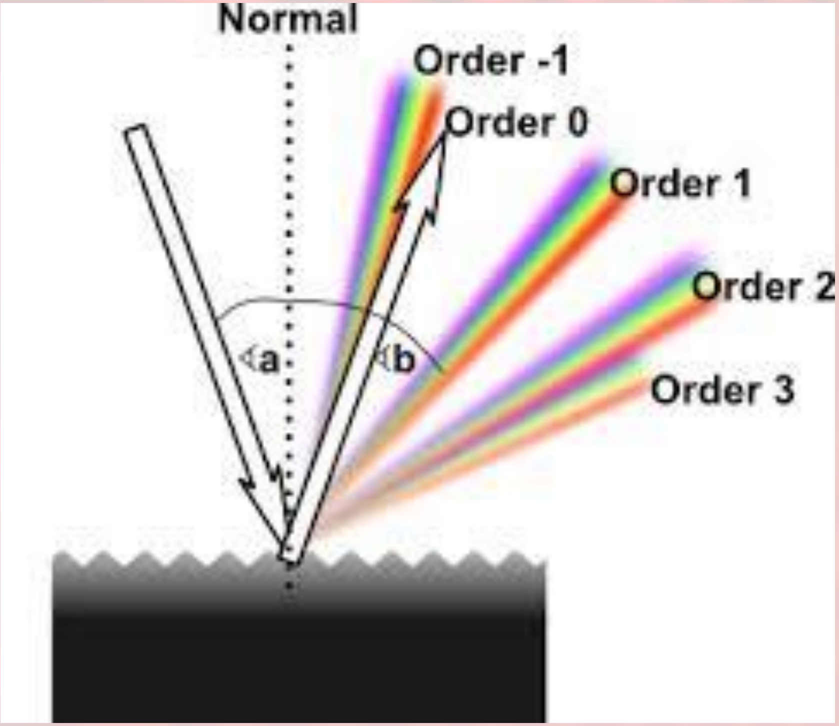


Diffraction gratings

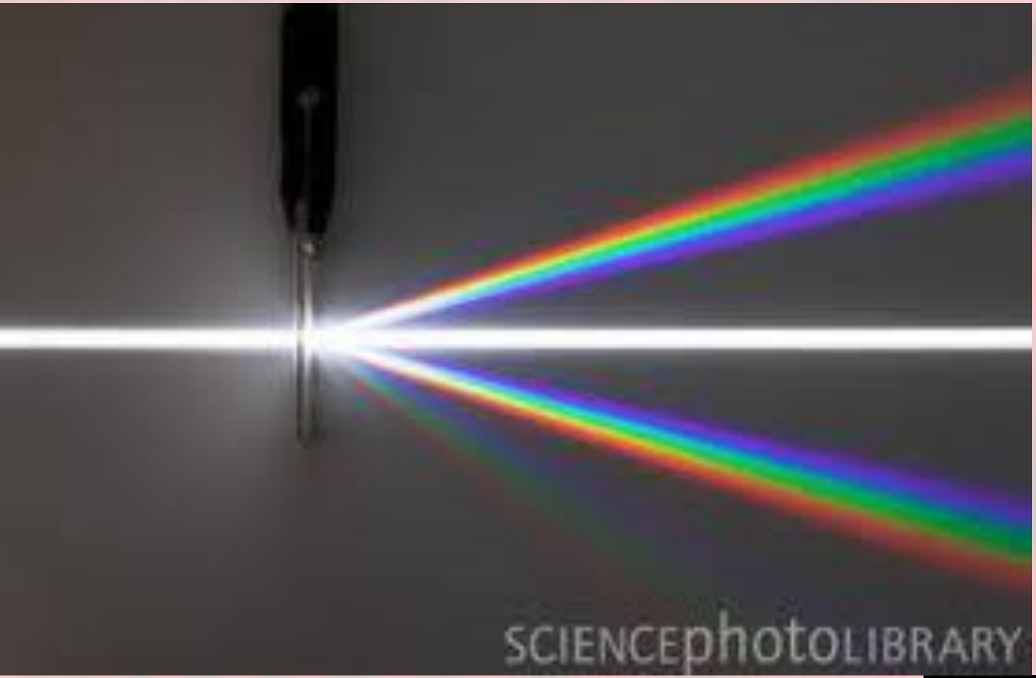
Monochromatic light

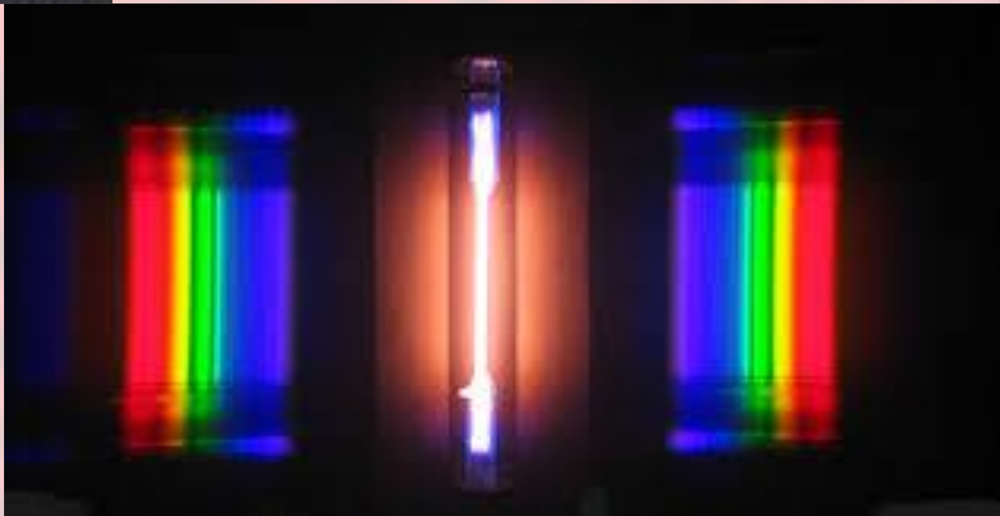


White light



Diffraction gratings





Two-beams interferometers

To make **measurements** using interference, we usually need **two beams** travelling along **different paths**, and an optical setup that makes them interfere.

The two beams, that we call the **reference beam** and **test beam**, must have **the same frequency**.

In order to produce a **stationary interference pattern**, the **phase difference should not change with time**.

The simplest way to meet this requirement is to **derive the two beams from the same source**.

Wavefront division   **Amplitude division**

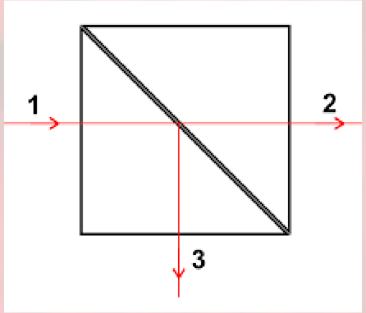
Amplitude division techniques

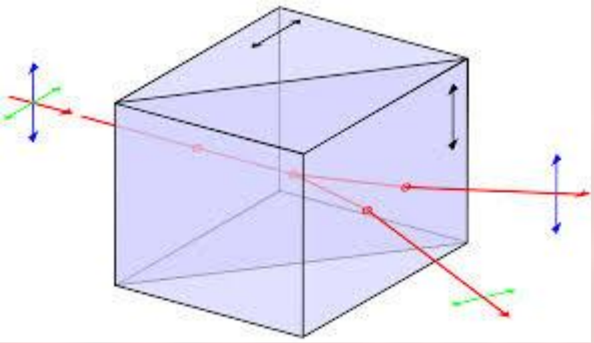
Beam splitters

It is basicaly a **partially transmitting mirror**.

The incident beam (1) is partially transmitted (2) and partially reflected (3)

Usually it is a **dielectric mirror** with 50% of transmitance and 50 % of reflectance



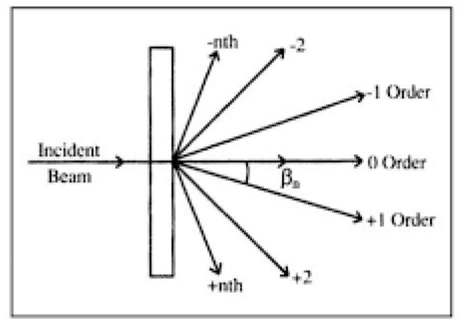


Polarizing prism (wollastone prism)

It separates unpolarized light into two polarized beams

Diffraction gratings

The number of beams and the output angle depends on the periodicity of the grating



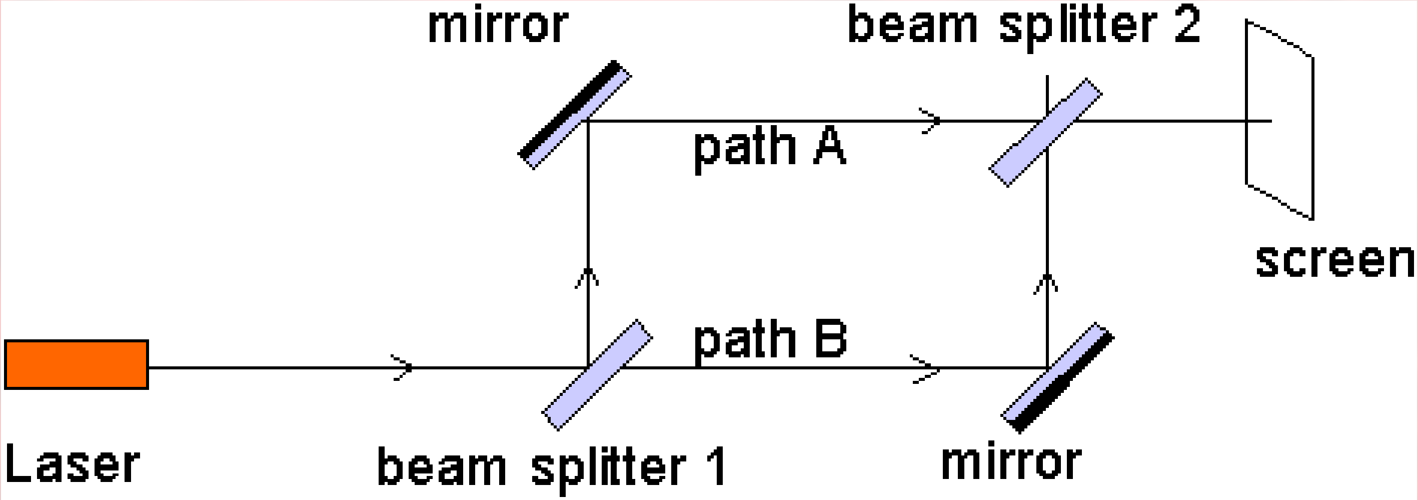
Transmission Grating Diffracted Orders

Mach-Zehnder Interferometer: setup

A Mach Zender interferometer is consituted by **two beam splitters** and **two mirrors**.

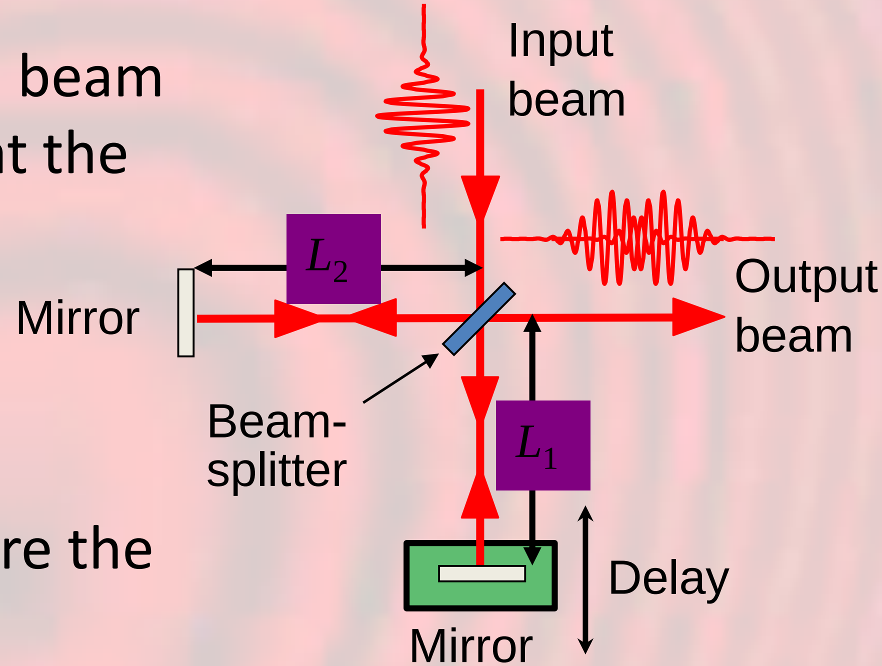
A collimated beam is splitted into a **reference beam** and a **test beam**.

A variation of the optical path in one of the arms produces a phase difference between the two beams



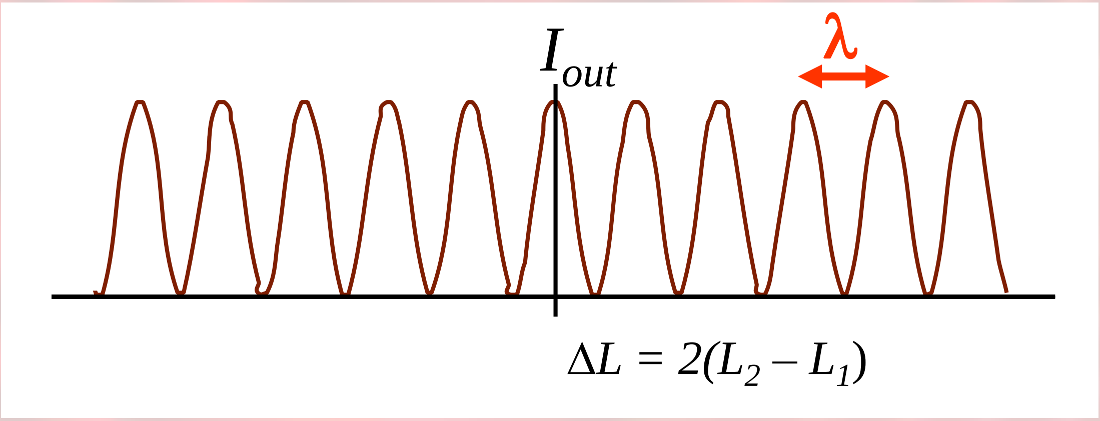
Michelson Interferometer: setup

The Michelson Interferometer splits a beam into two and then recombines them at the same beam splitter.



The most obvious application of the Michelson Interferometer is to measure the wavelength of monochromatic light.

$$I_{out} = 2I\,\{1+\cos(k\Delta L)\} = 2I\,\{1+\cos(2\pi\,\Delta L\,/\,\lambda)\}$$

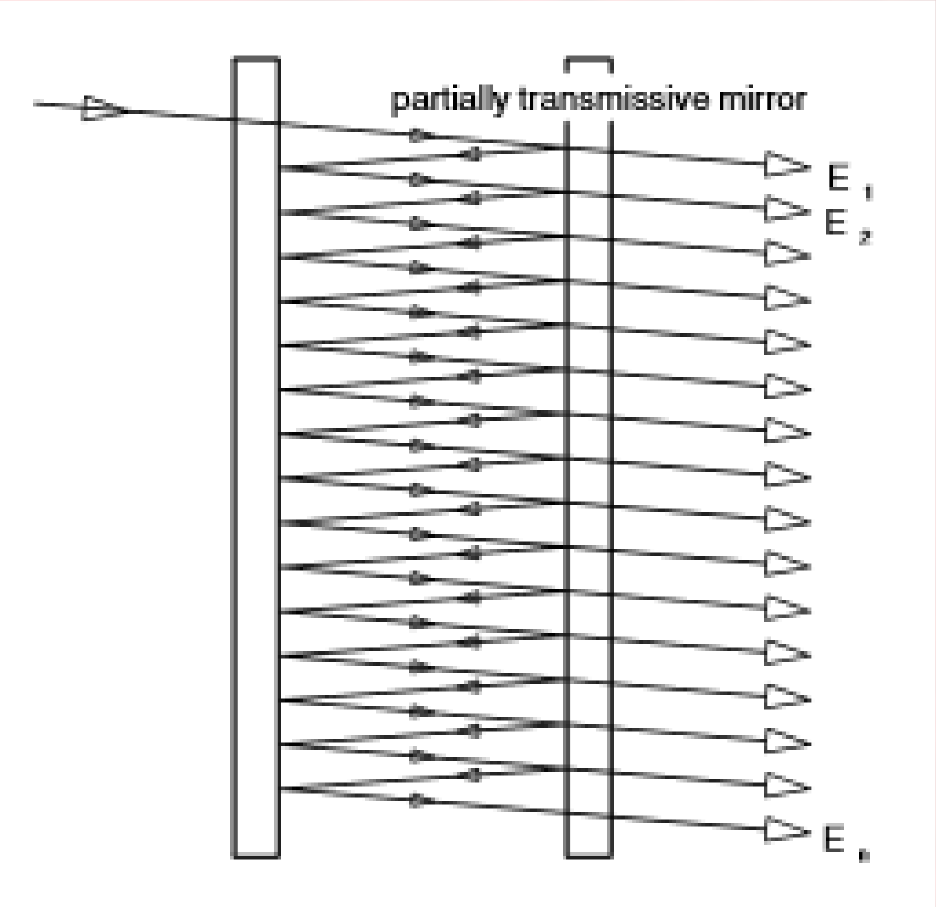


Multiple beam interferometers: Fabry-Perot

A Fabry-Perot cavity is constituted by a pair of **semi-transparent mirrors**

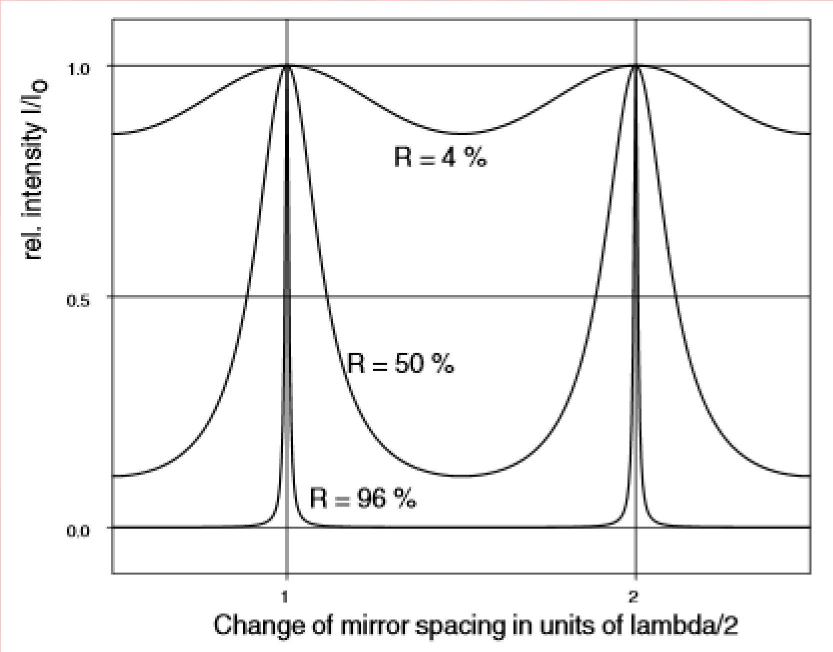
- Multiple reflections between mirrors produce interference between multiple «output» beams

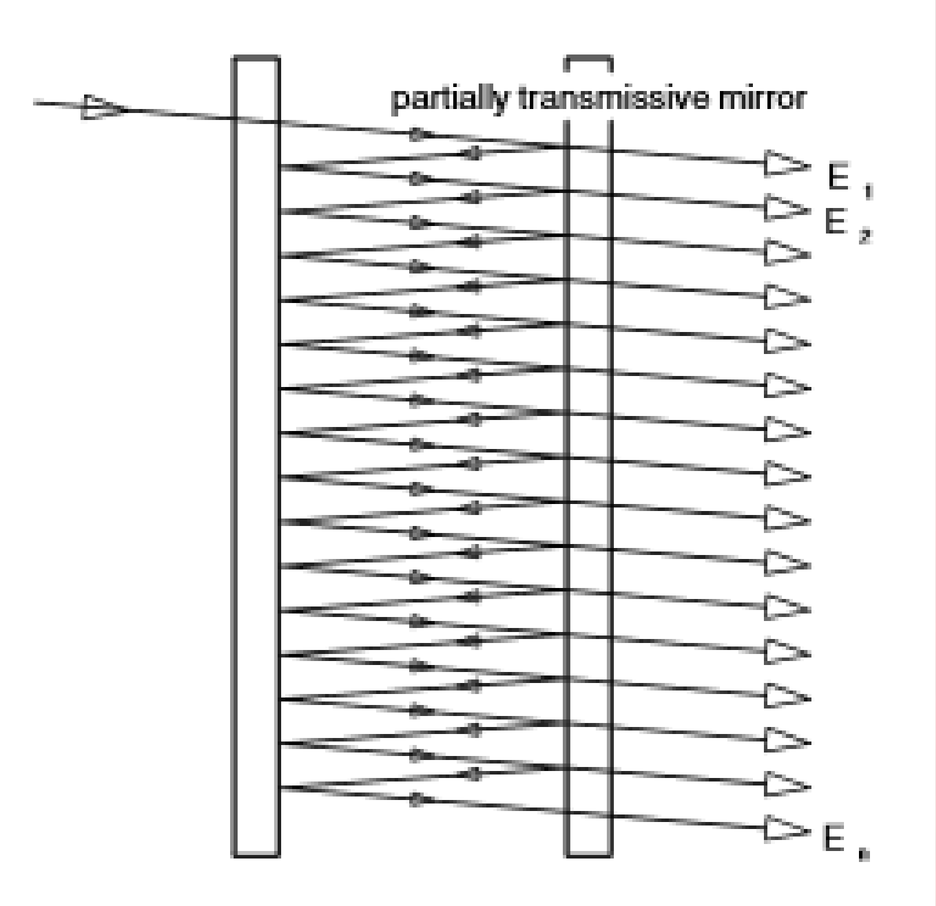
- The final amplitude can be computed by an iterative application of the Fresnel's law of reflection



Multiple beam interferometers: Fabry-Perot

$$I = I_0 \frac{(1-R)^2}{(1-R)^2 + 4 \cdot R \cdot \sin^2\left(\frac{2\pi d}{\lambda}\right)}$$



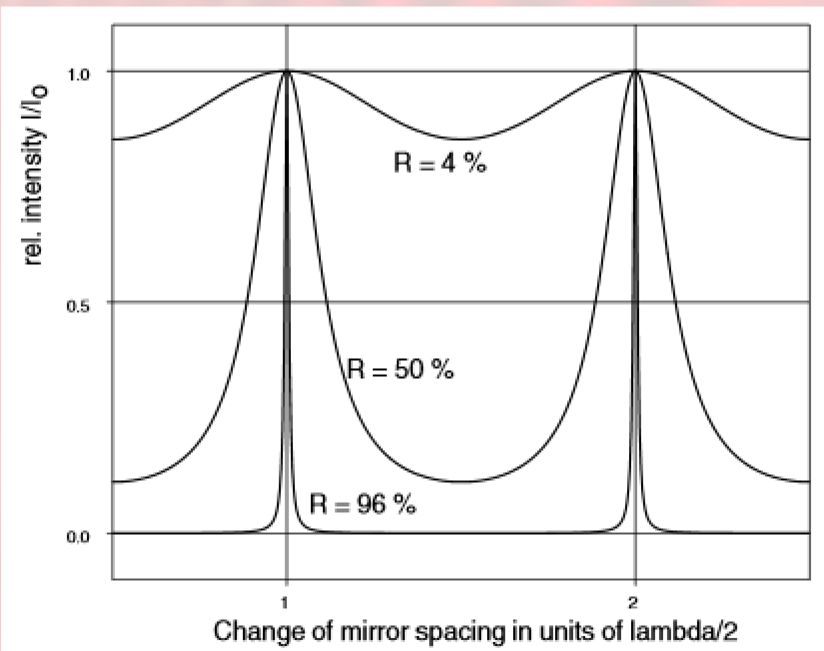


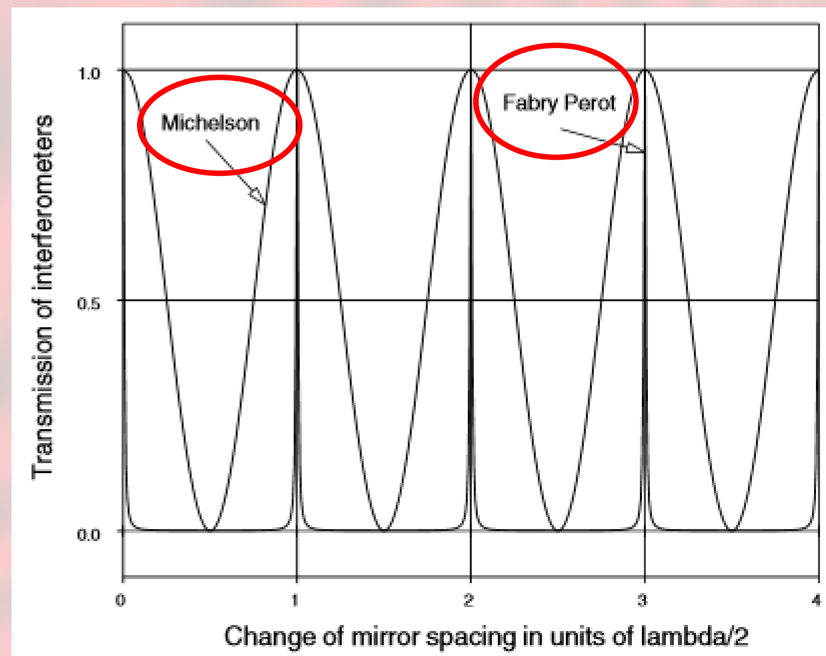
Constructive interference for $d = n\dfrac{\lambda}{2}$

Multiple beam interferometers: Fabry-Perot

$$I = I_0 \frac{(1-R)^2}{(1-R)^2 + 4 \cdot R \cdot \sin^2\left(\frac{2\pi d}{\lambda}\right)}$$

As for N-wave interference, maxima peak are narrower for F-P interferometer as compared to e.g. Michelson or Mach-Zehnder





Constructive interference for $d = n\frac{\lambda}{2}$

Multiple beam interferometers: Fabry-Perot

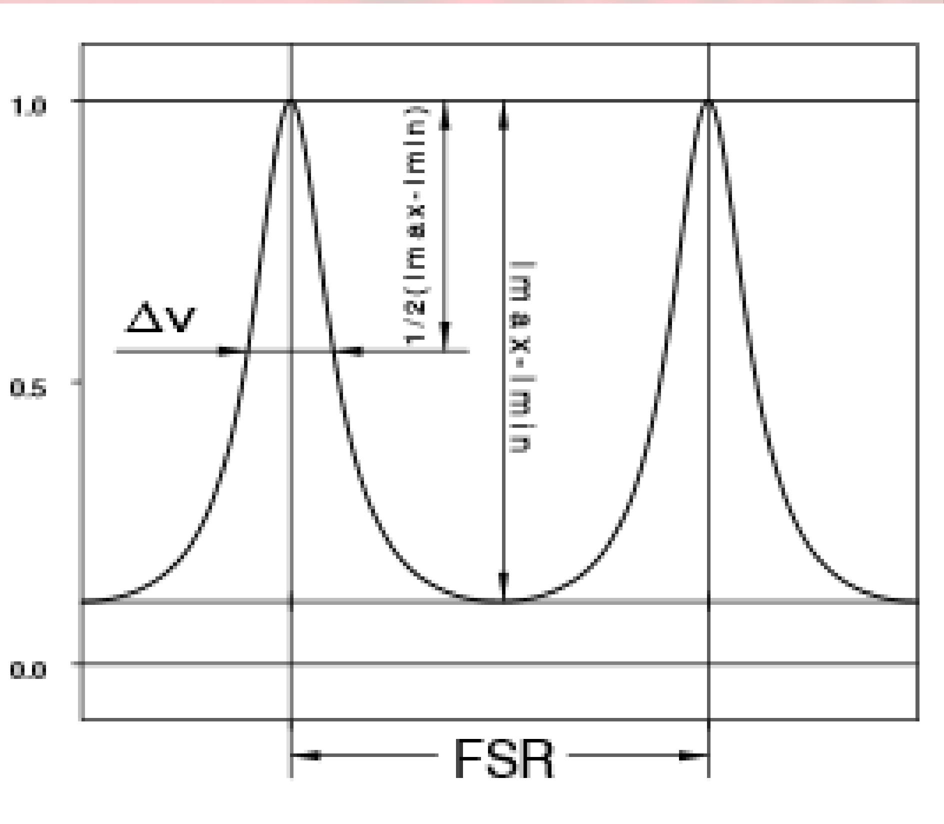
$$d = n \cdot \frac{\lambda_1}{2}$$

Resonance condition

$$\lambda_1 - \lambda_2 = \frac{\lambda_2}{n} = \frac{\lambda_1 \cdot \lambda_2}{2 \cdot d}$$

$$\boxed{\delta v = \frac{c}{2 \cdot d}}$$

Free Spectral Range (FSR)



Finesse

$$F = \frac{FSR}{\Delta v} = \frac{\pi\sqrt{R}}{1-R}$$

i.e. frequency resolution

Classical interference microscopy

Two beam interference microscopes are available using optical setup similar to the macroscopic interferometers.

A very precise measurement of the phase shift can be made by **digital phase shifting.**

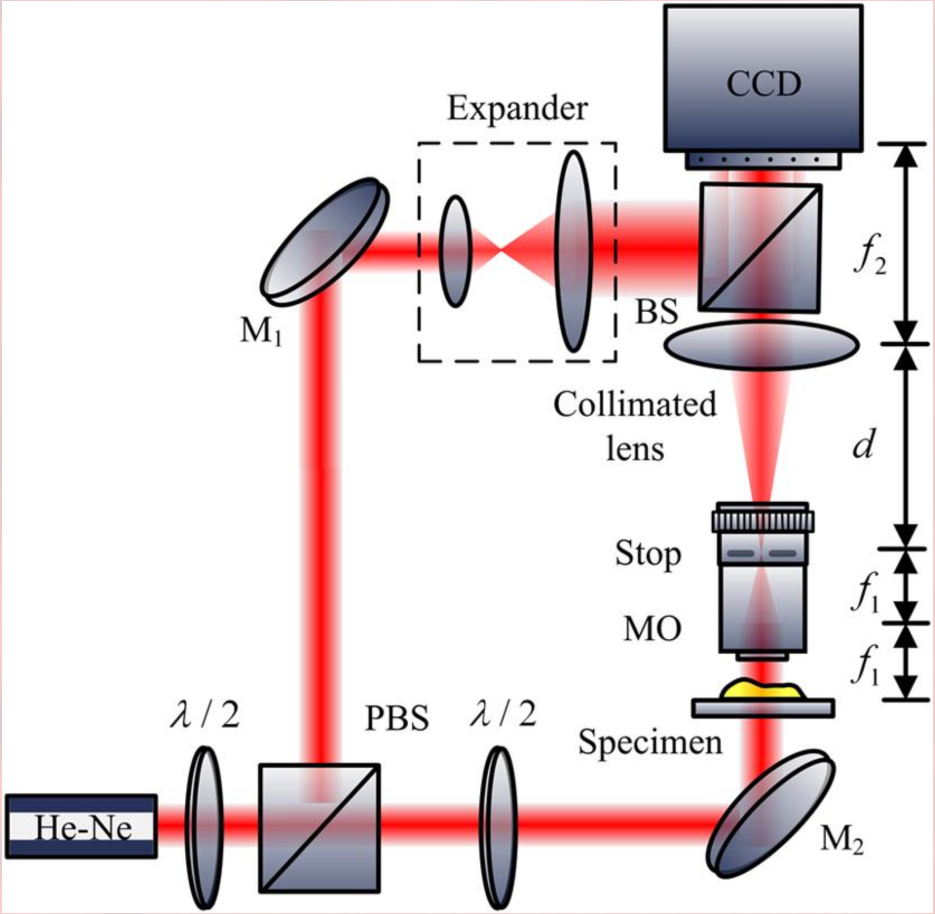
Surface profiles can be measured with very high accuracy (down to 1nm).

A complex post processing of the images is required.

μ-Mach-Zehnder configuration

The interference pattern produced contains mixed information about **the phase difference and the amplitudes of the beams**, but **not** about **the absolute phase** of the test beam

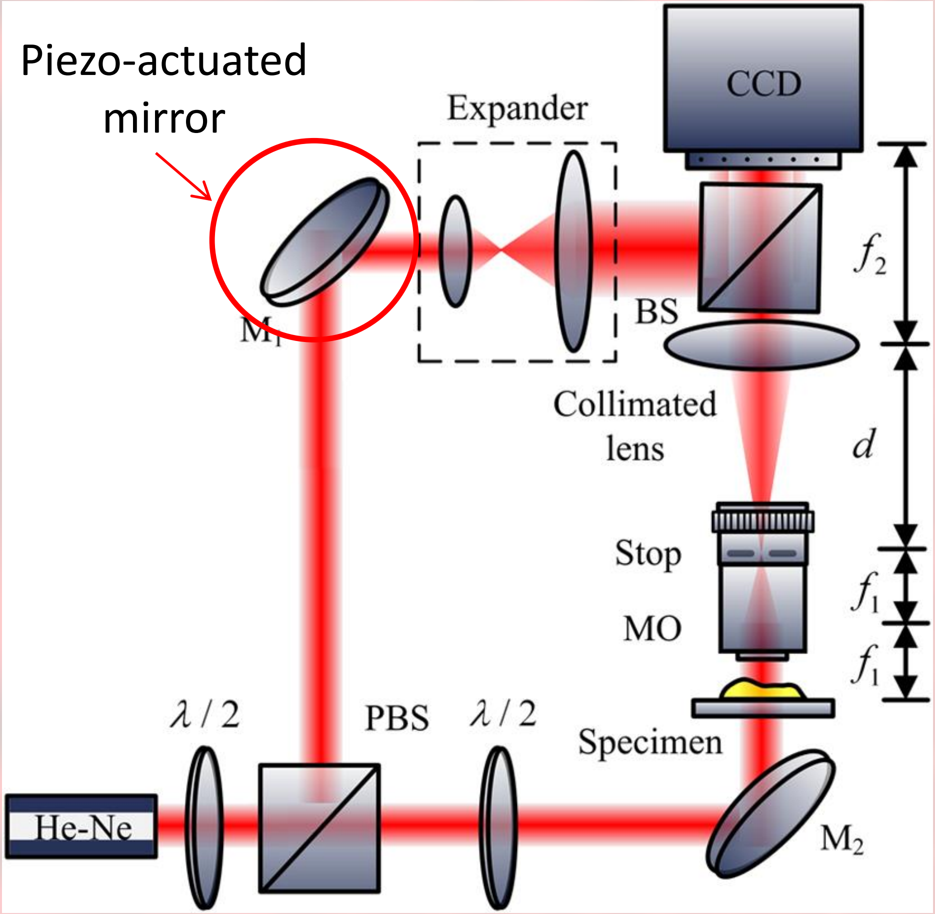
It is possible to **retrieve** the **absolute phase shift** produced by the specimen



Digital Phase Shifting

The movable mirror changes the optical path difference in steps of λ/4

The digital camera records the intensity values in each point



Digital Phase Shifting

Complex amplitude of the test wave:

$$A = ae^{-i\varphi}$$

And reference wave:

$$B = be^{-i\varphi_R}$$

In a given point the intensity will be:

$$I(0°) = a^2 + b^2 + 2ab\cos(\varphi - \varphi_R)$$
$$I(90°) = a^2 + b^2 + 2ab\sin(\varphi - \varphi_R)$$
$$I(180°) = a^2 + b^2 - 2ab\cos(\varphi - \varphi_R)$$
$$I(270°) = a^2 + b^2 - 2ab\sin(\varphi - \varphi_R)$$

The phase difference is then given by:

$$\tan(\varphi - \varphi_R) = \frac{I(90°) - I(270°)}{I(0°) - I(180°)}$$

Digital Phase Shifting

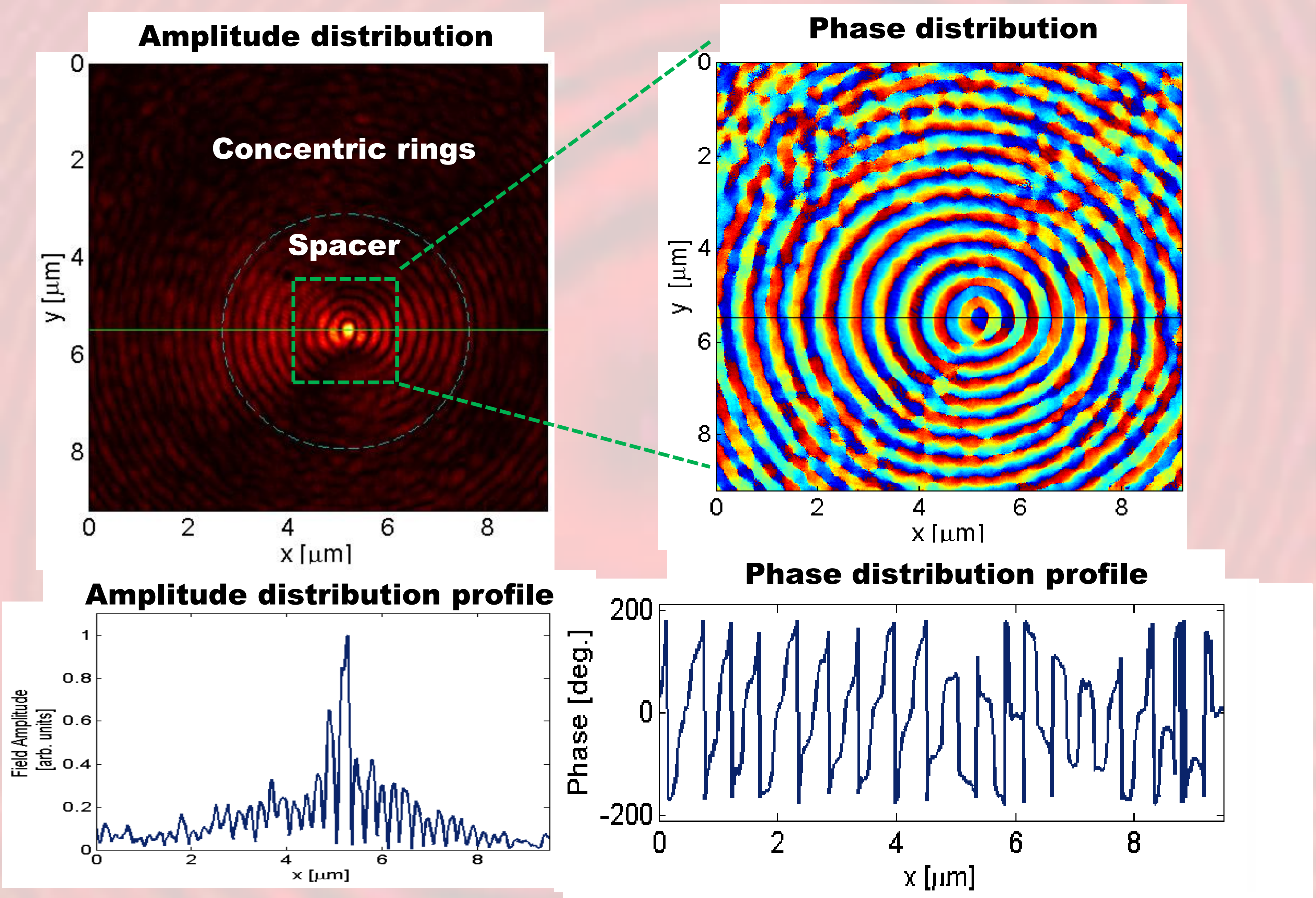
Systematic errors can arise from:

1. Miscalibration of the phase steps
2. Non-linearity of the photodetector
3. Deviations of the intensity distribution in the interference fringes from a sinusoid, due to multiple reflected beams

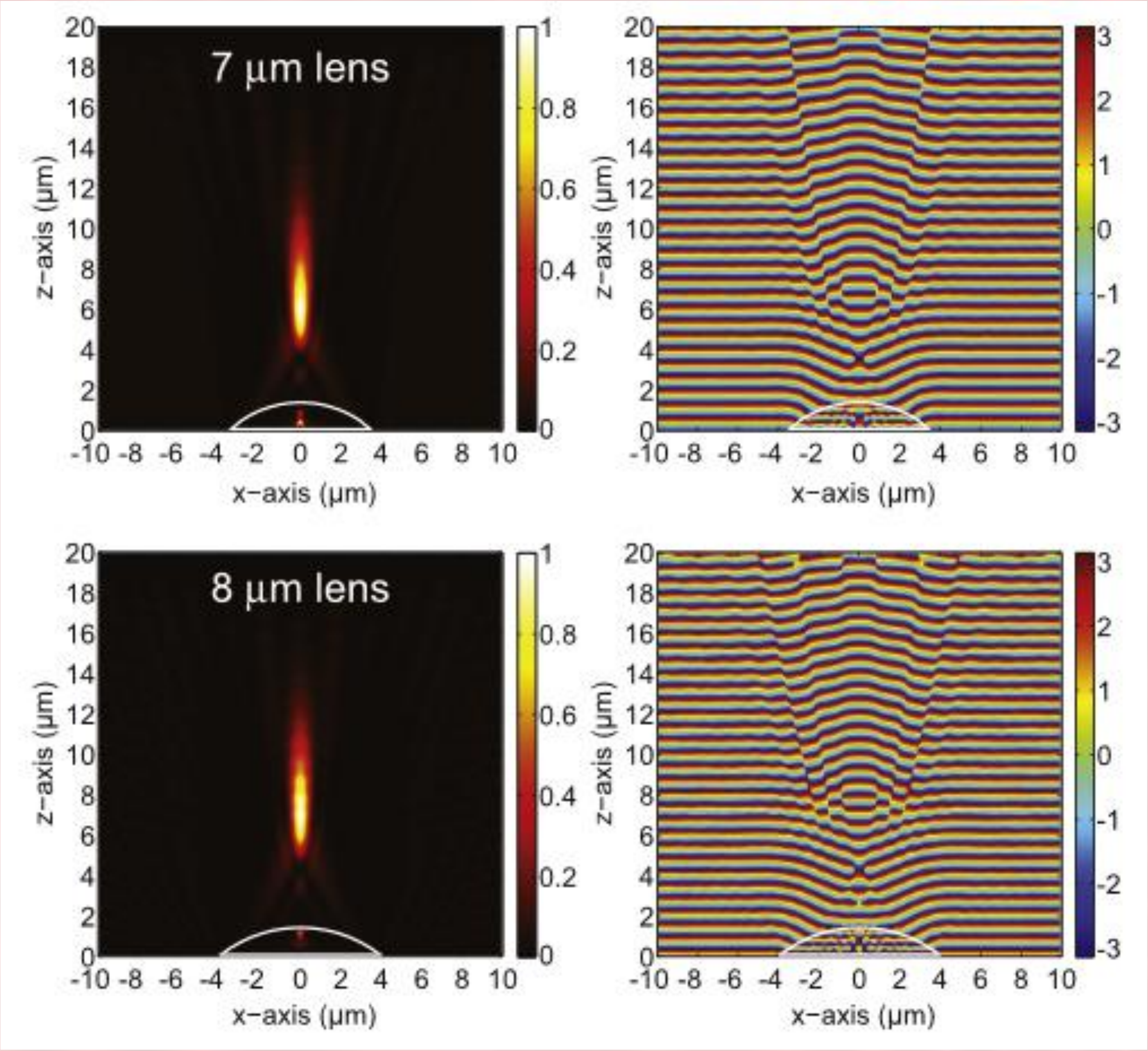
Such errors are unavoidable, but can be minimized adding phase steps in the algorithm.
An example is the 5 steps algorithm:

$$\tan(\varphi - \varphi_R) = \frac{2[I(90°) - I(270°)]}{2I(180°) - I(360°) - I(0°)}$$

Example: In-plane laser focusing



Example: longitudinal focusing by a microlens



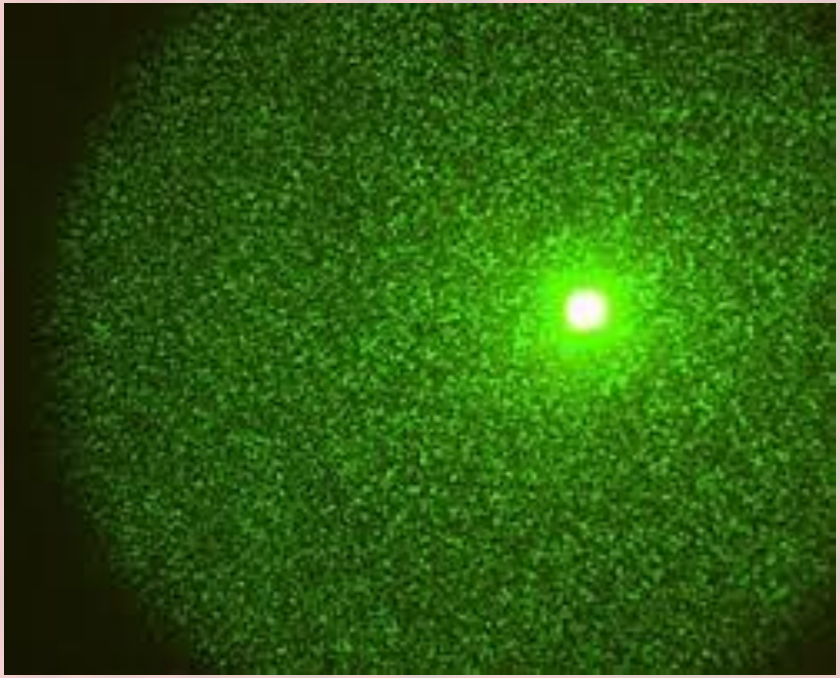
Limits of laser interferometers: Stray light

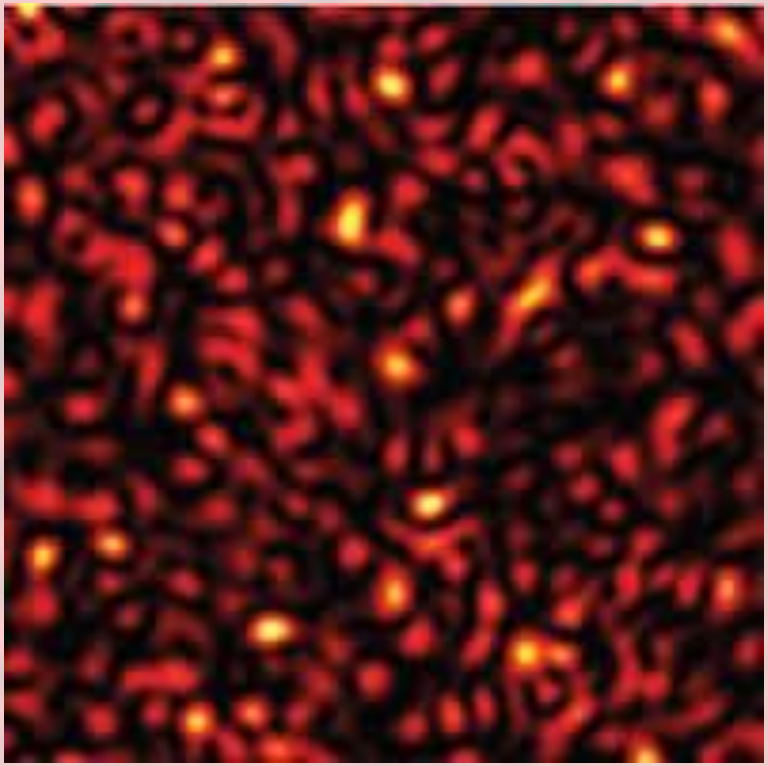
Stray light: Light reflected or scattered from various surfaces in the optical path is coherent with the main beam and adds vectorially resulting in a phase error

Limits of laser interferometers: Speckles

The high degree of coherence of laser light may result in some practical problems:

Speckles: spatial noise due to scattered light that produces random diffraction patterns





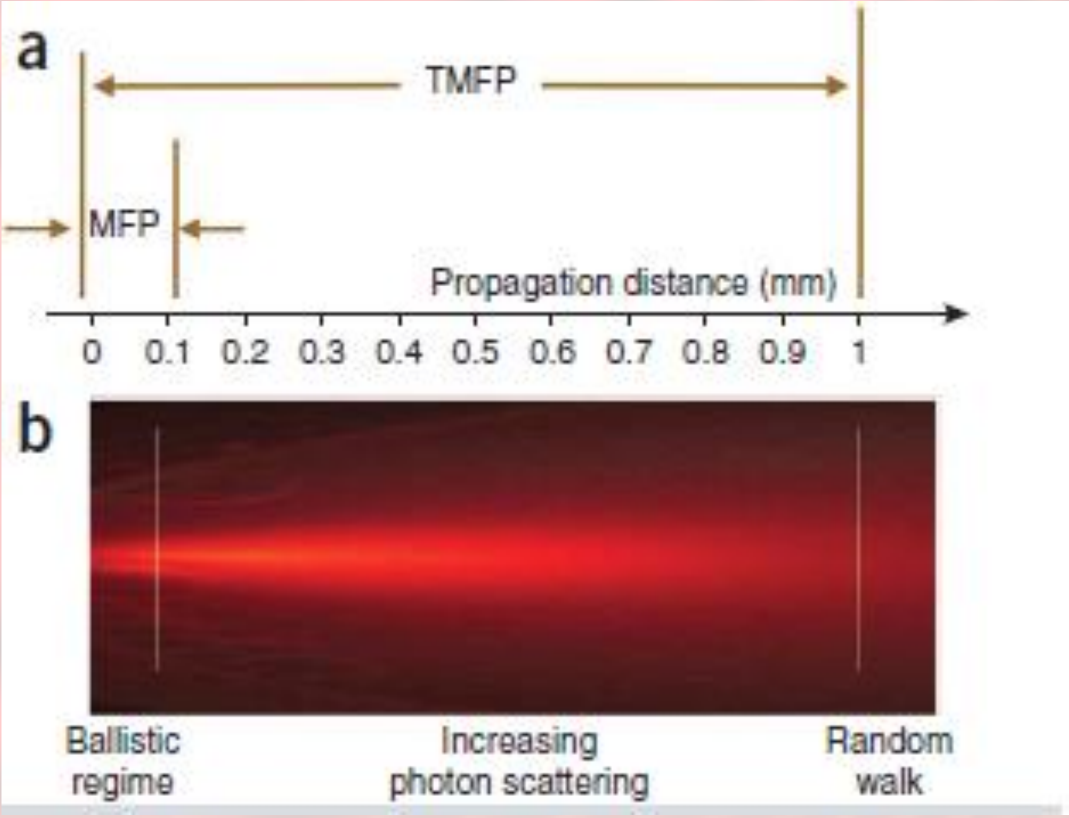
How to deal with coherent light in *complex media*

A complex medium is a transparent medium whose refractive index varies randomly in the space, producing diffusion of light that randomizes the information

Mean free path in turbid media

The Mean Free Path is the average distance between two scattering events

The Transmission Mean Free Path takes into account the mean scattering angle



Imaging in complex media

When coherent light passes through highly scattering media, the information is scrambled into disordered interference patterns called **speckles**

Different techniques have been developed to take advantage from the scattering in order to retrieve information about the inner structure of the complex media

Wave propagation in scattering media

The propagation of light is described by a wave equation describing the wave field evolution in space and time:

$$\nabla^2\Psi(\boldsymbol{r},t) = \frac{n(\boldsymbol{r})^2}{c^2}\frac{\partial^2\Psi(\boldsymbol{r},t)}{\partial^{\mathbf{2}}t}$$

Where Ψ represents the electric field (polarization degree of freedom is neglected for simplicity)

Scattering is caused by local variations of $n(\boldsymbol{r})$ which can occurr due to the presence of small dielectric particles

In homogeneous media, $n(\boldsymbol{r})$ is constant and the solutions are the wave's normal modes, such as the plane waves

Wave propagation in scattering media: spatial degrees of freedom

For a given surface A, only a finite number of indipendent transversal modes can carry energy from the surface to free-space:

$$N_s \approx 2\pi A/\lambda^2$$

Visible light has about 10 million indipendent transversal modes per square millimiter.

Any incident wave can be decomposed into these modes, that therefore correspond to the spatial degree of freedom of the incident light field

The transversal modes represent the basis vectors of the transmission matrix of the sample (spatial degrees of freedom)

Wave propagation in scattering media: frequency degrees of freedom

The ability of focusing thurough scattering media by interference effects is connected to the speckle correlation function

The typical time a photon spends into a medium of thickness L is:

$$\tau_D = L^2/lv_e$$

Where l is the mean free path and v_e is the mean speed of propagation in the medium

In an open medium, solutions can be expanded into quasimodes with frequency width $\delta\omega = \frac{1}{\tau_D}$

Two waves whose spectra are separated by less than $\delta\omega$ will produce a strongly correlated pattern

Wave propagation in scattering media

At the surface, speckle correlation decays at a distance

$$\delta x \approx \frac{\lambda}{2n(\boldsymbol{r})}$$

Which is the typical speckle dimension

Control over the spatial and frequency degrees of freedom of the incident beam allows to control the transmissive properties of the medium

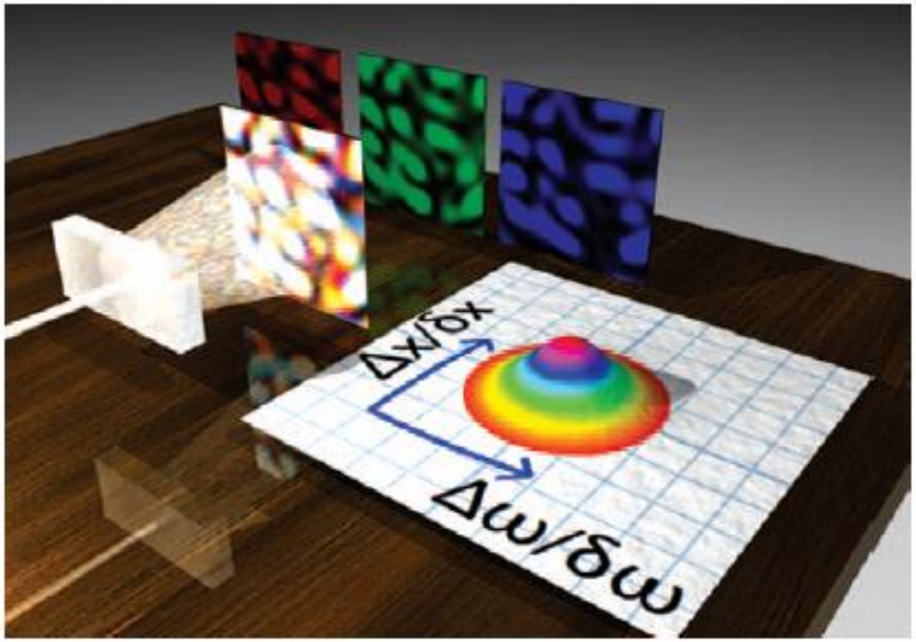


Figure 1 | Speckle correlations in space and frequency. When a white-light beam is incident on a multiply scattering medium, frequency components that are spaced more than the correlation frequency $\delta\omega$ give rise to uncorrelated speckle patterns. The blue and red patterns symbolize frequency components spaced by $\delta\omega$; the green pattern is intermediate. Spatial correlations are lost when the beam is moved by more than one correlation width (the 'speckle size'). The speckle correlation graph shows how speckle correlations are lost as the beam is moved in space or frequency.

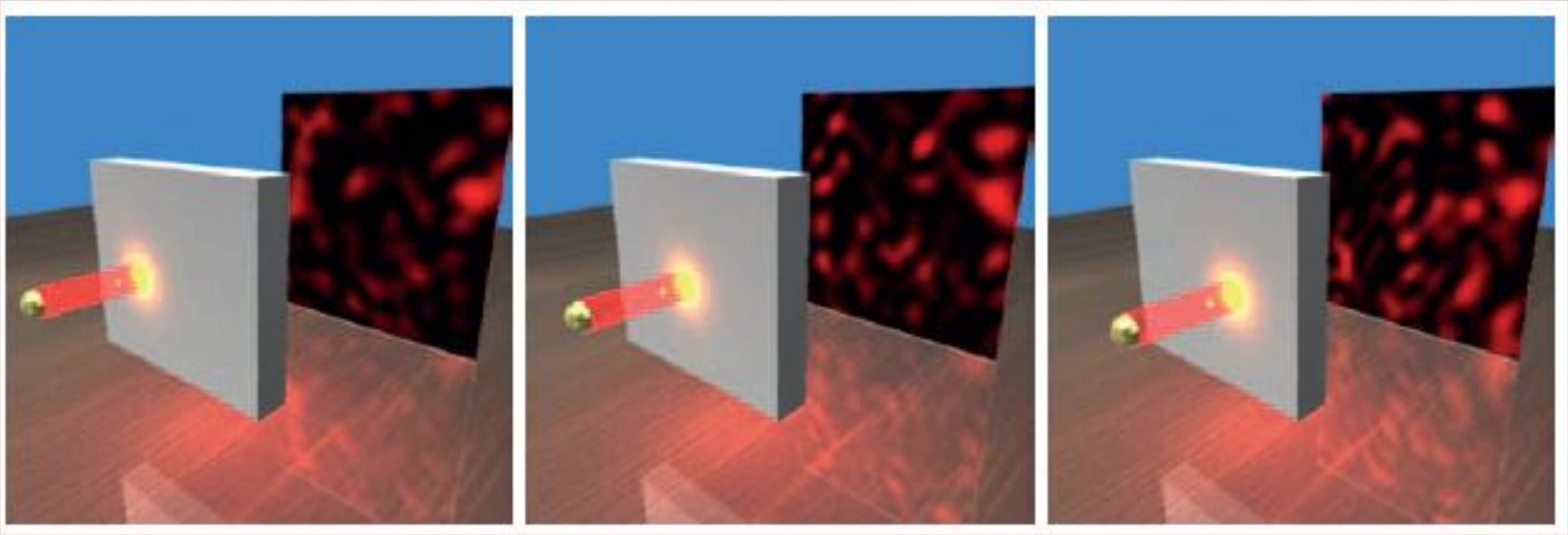
Wave propagation in scattering media

The description is valid for several kinds of waves, from acoustic to microwaves, but there are differences in the hardware that allows to control the degrees of freedom:

- For ultrasound and radiofrequencies, it is possible to reconstruct broadband wavefronts, but with limited spatial resolution
- In the optical regime, CCD and Spatial Light Modulator (SLM) allow for high spatial control but with narrow frequency bandwidth

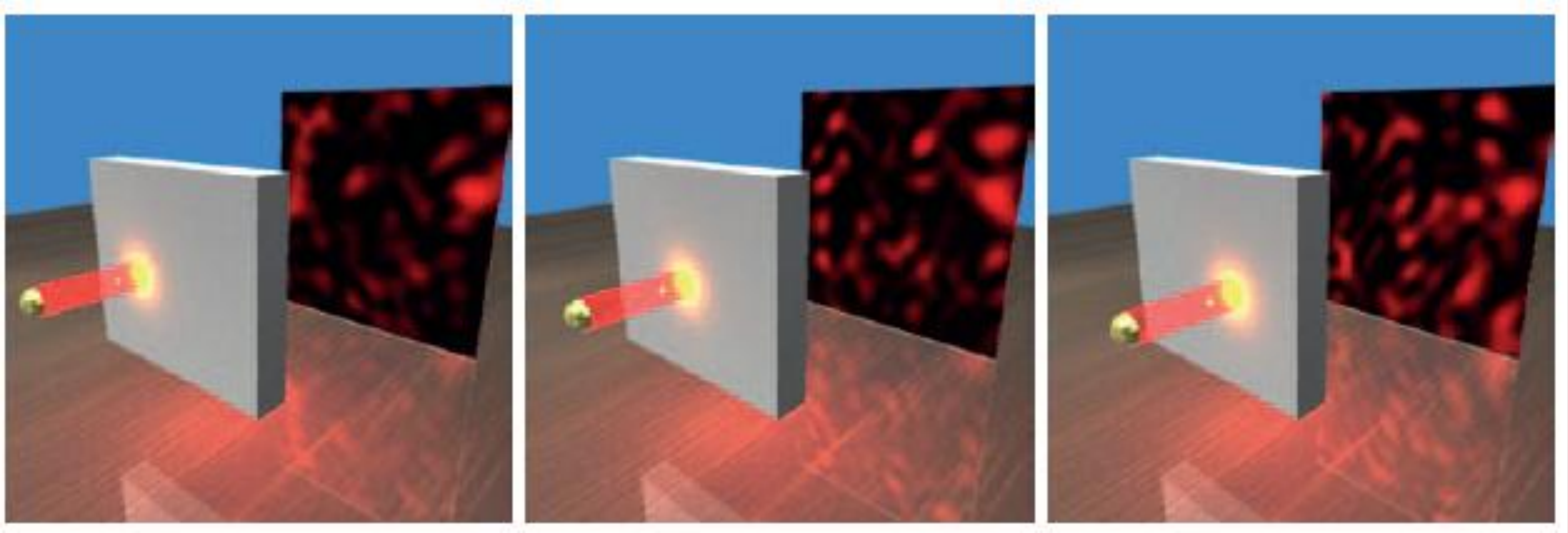
Spatial degrees of freedom to control light

The local transmission function of the scattering sample can be measured by collecting the speckle pattern produced by a known beam in different positions



Spatial degrees of freedom to control light

Each incident mode gives rise to a different interference pattern behind the sample



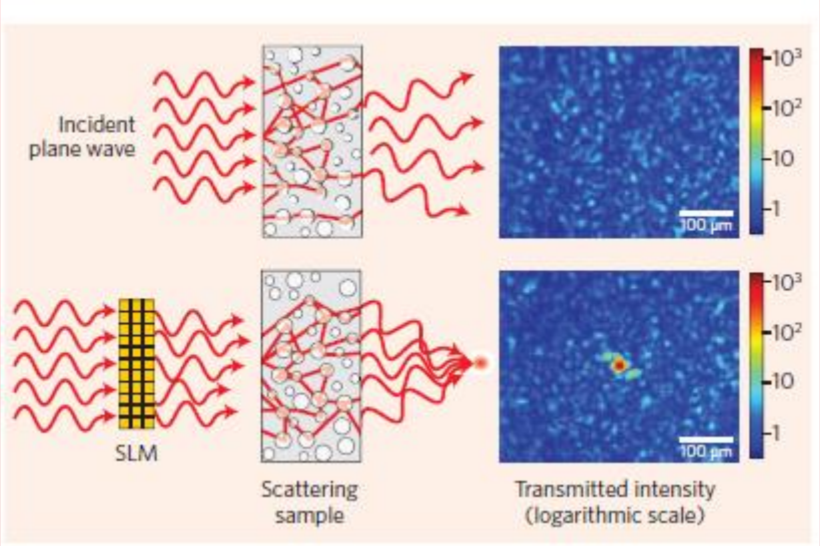
Spatial degrees of freedom to control light

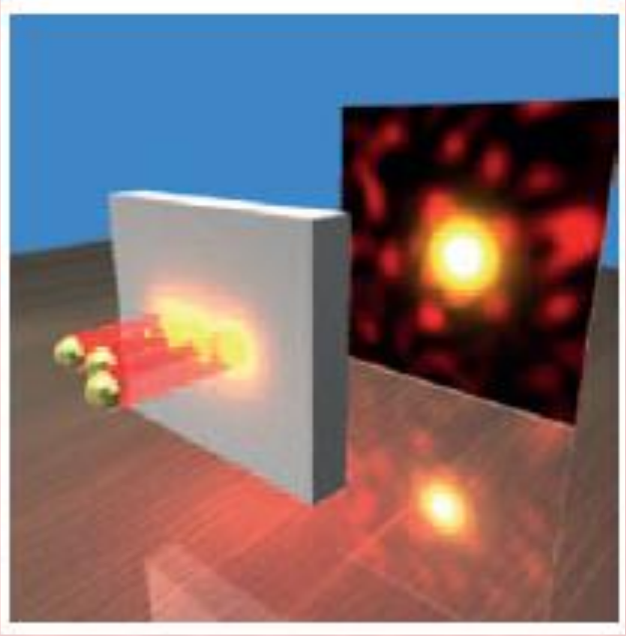
By measuring the intensity on a target it is possible to have a feedback

A plane wave incident on the sample produces a speckle pattern on the target

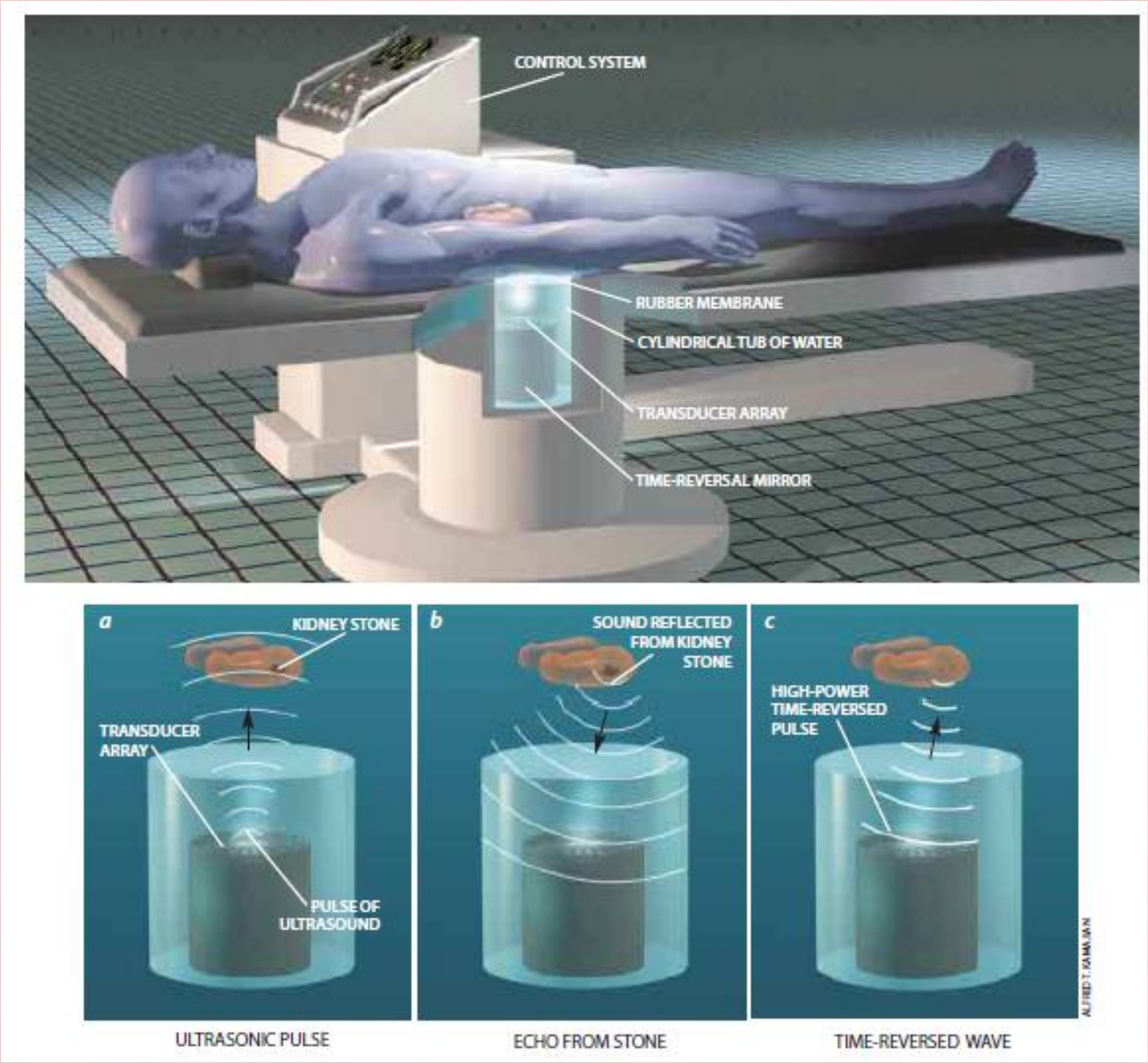
An SLM is used to optimize the phase of thousands of incident modes

The feedback loop allows to optmize the phase modulation in order to achieve constructive interference on the target





Acoustic Phase Conjugation

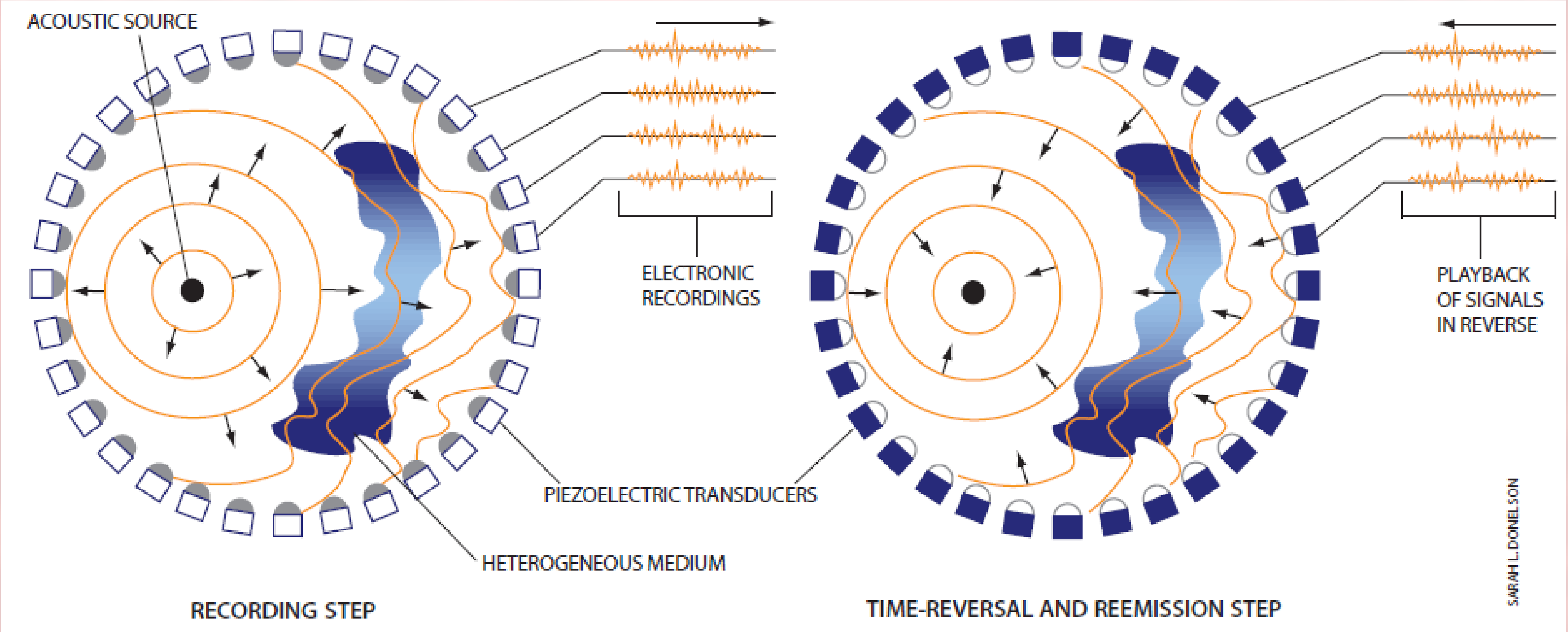


Wave propagation in scattering media

The microscopic wave equation exhibits time-reversal symmetry:

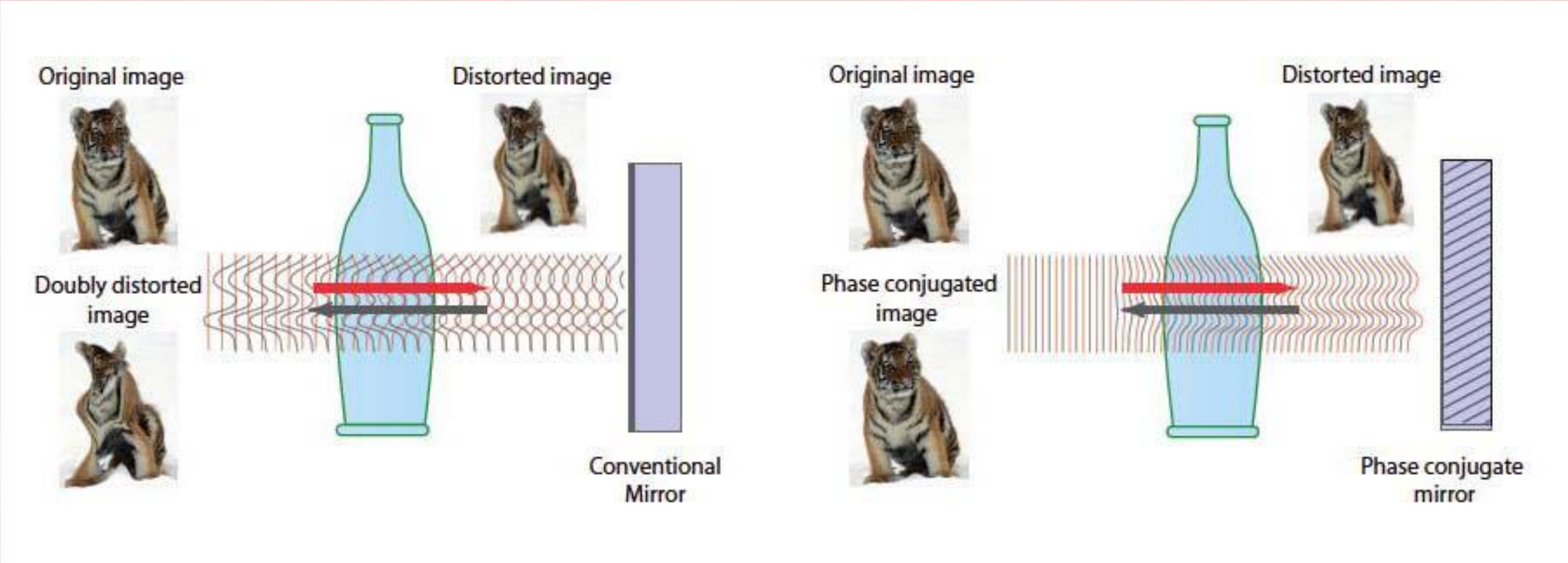
If $\Psi(\boldsymbol{r}, t)$ is solution, then $\Psi(\boldsymbol{r}, -t)$ is solution too.

Scattering from a stationary disorder does not break the time reversal symmetry

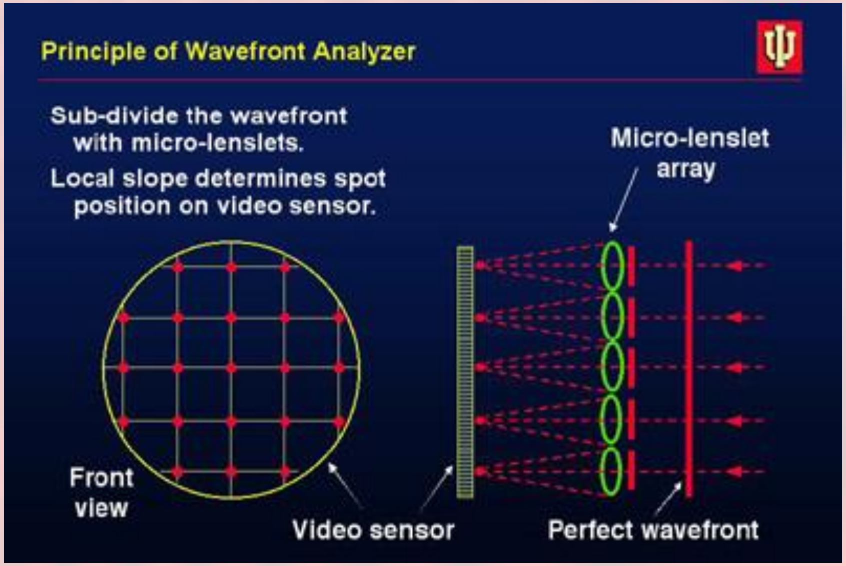


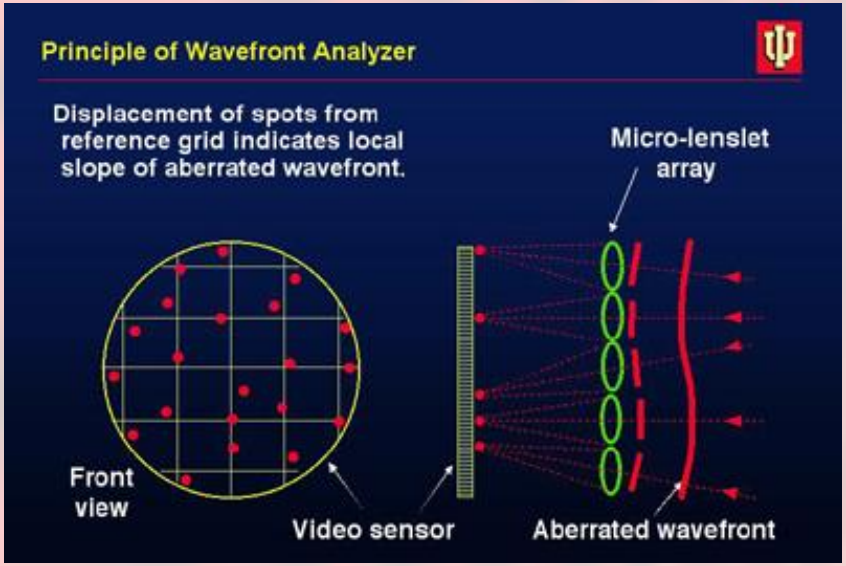
Optical Phase Conjugation

Scattering processes are time-reversable: if we are able to collect phase and amplitude of the scattered field completely and reproduce a backpropagating field with same wavefront, this field should retrace its trajectory across the scattering medium and recover the original input field



Wavefront Analysers





A microlenslet array focuses each section of the incoming beam onto CCD

Dislocations of the focal spots from the ideal position correspond to wavefront distorsions

Frequency degrees of freedom to control light

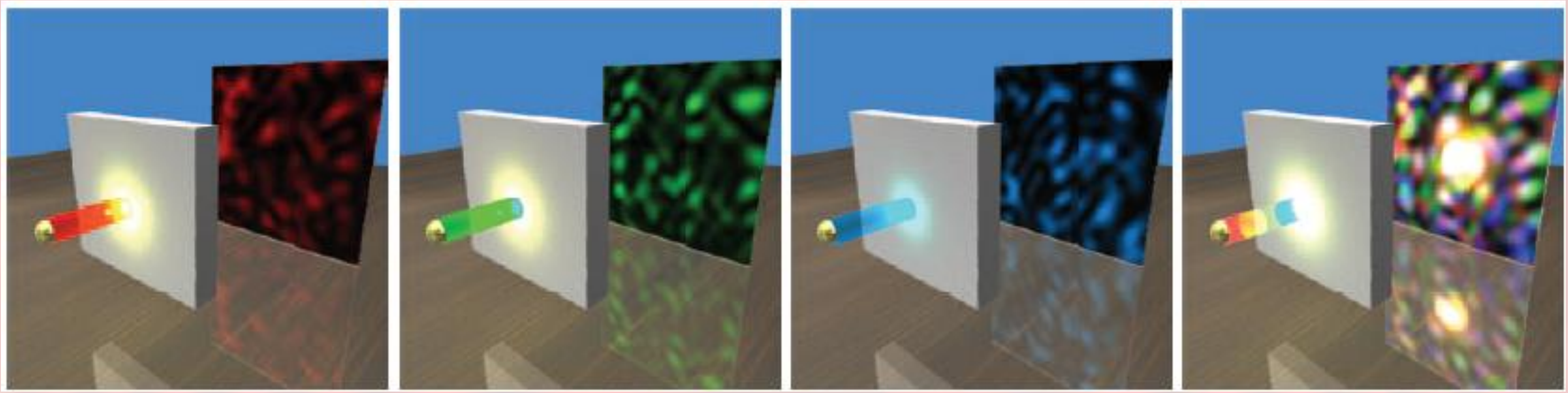
The spatial degrees of freedom provides great flexibility in controlling the propagation through scattering media

The drawback is that wavefront shaping only works for a narrow bandwidth

The effect of optimization is lost when the source is detuned of a frequency larger than the speckle correlation function

Frequency can be seen as an indipendent degree of freedom to control waves in time

Frequency degrees of freedom to control light

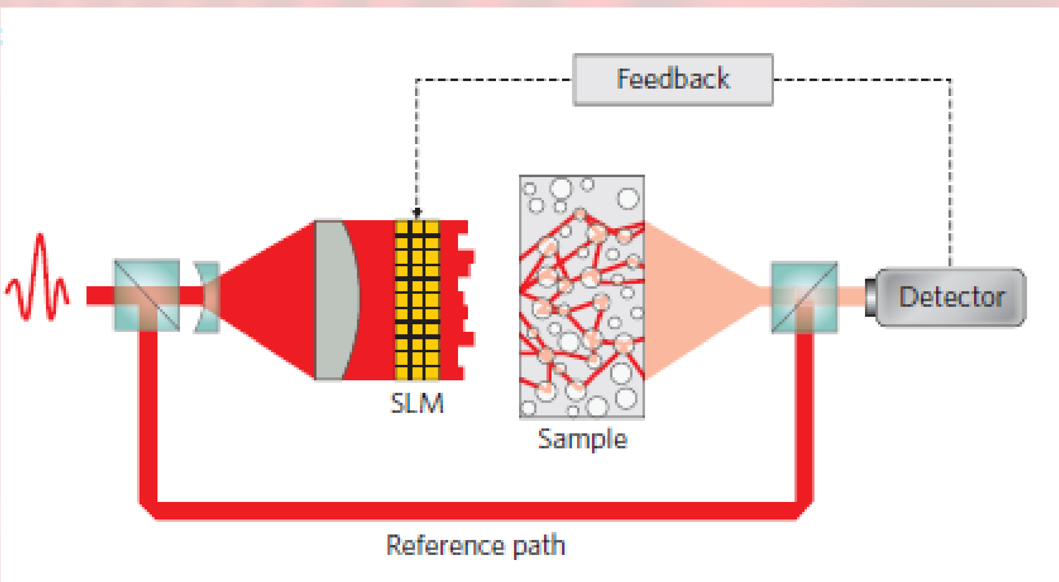


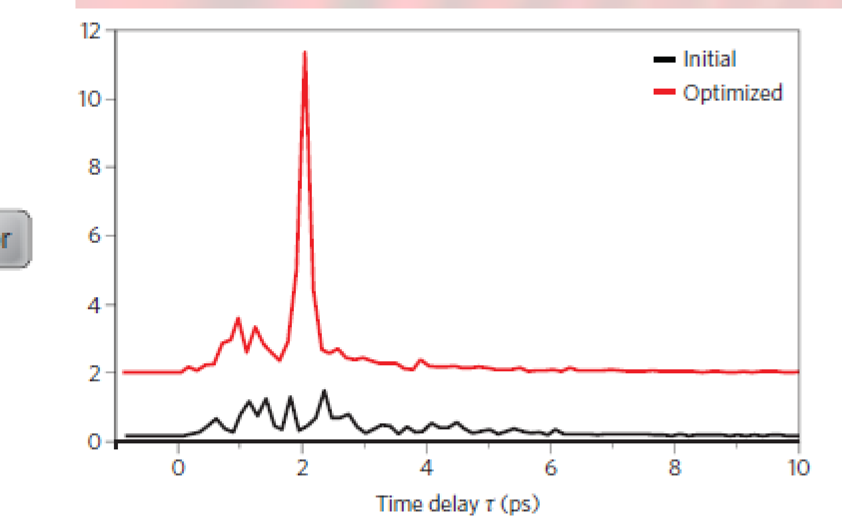
By tuning the source at different frequencies different <u>uncorrelated speckle patterns</u> are obtained

By tuning the phase and amplitudes for the different components makes it possible to control the relative phases and amplitudes of the frequency components

Frequency degrees of freedom to control light

If a short pulse is transmitted through a scattering sample, the phase of the pulse can be optimized to produce constructive interference in a give point and at a given time



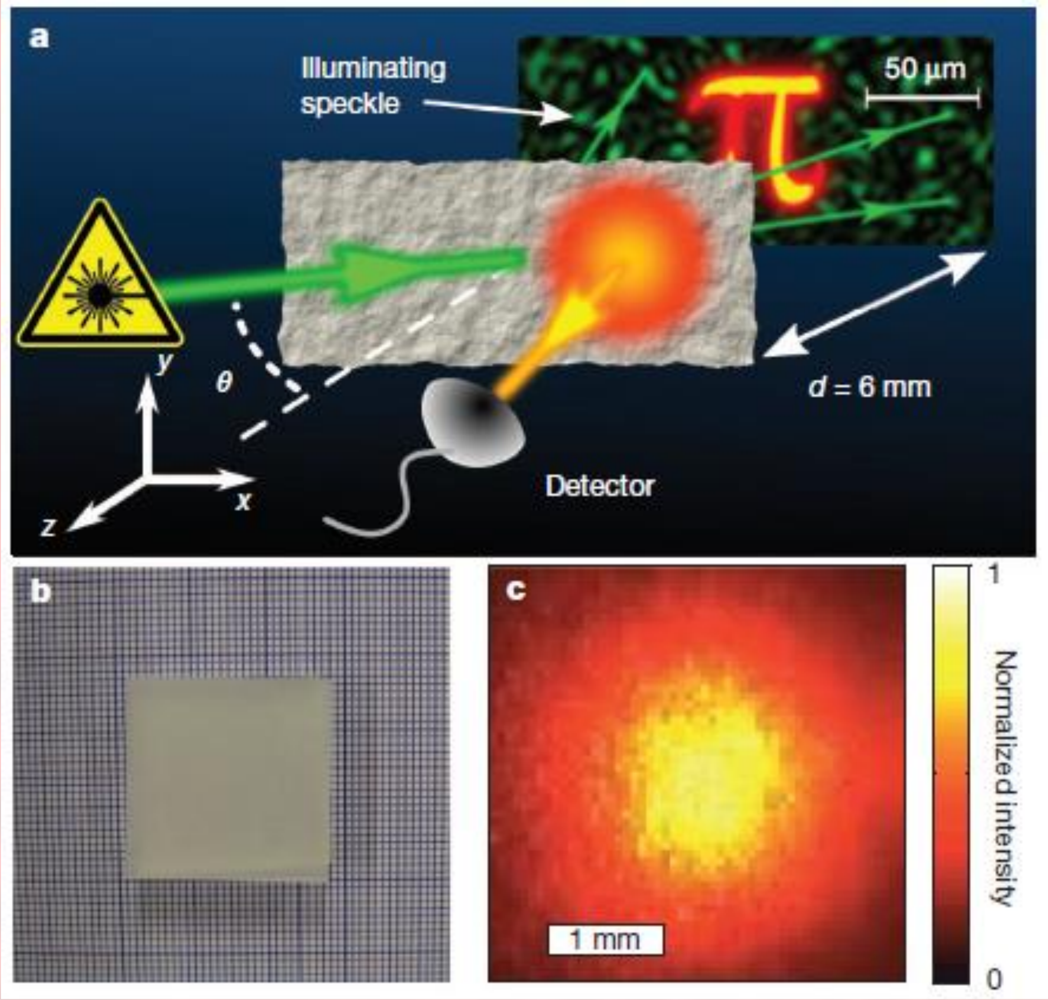


Non-invasive imaging through opaque scattering layers

A 50 μm fluorescent object is hidden behind a thick (d = 6mm) opaque scattering layer that completely hides it

A laser shines the scattering layer producing a speckle pattern that illuminates the object

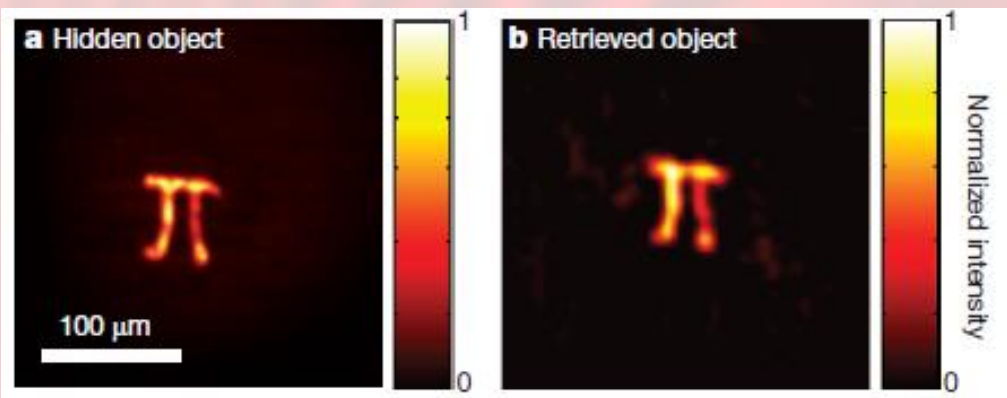
The fluorescent signal is collected

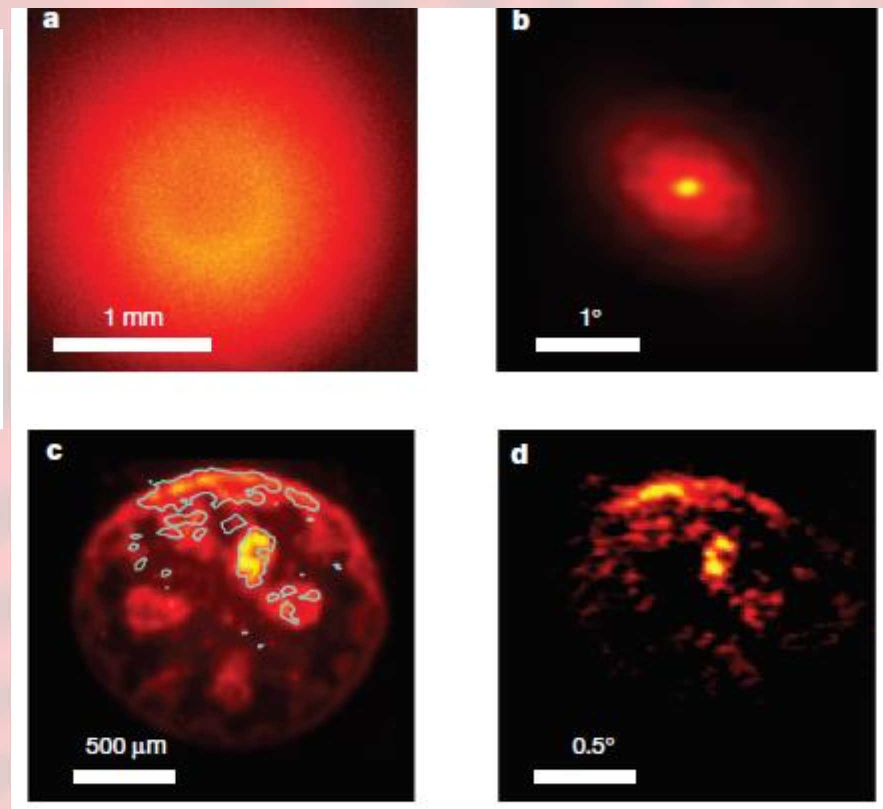


J. Bertolotti et al., Nature 491, 232-4 (2012)

Non-invasive imaging through opaque scattering layers

The original image can be retrieved by inverting the autocorrelation function, for example by using a Gerchberg-Saxton iterative algorithm





J. Bertolotti et al., Nature 491, 232-4 (2012)

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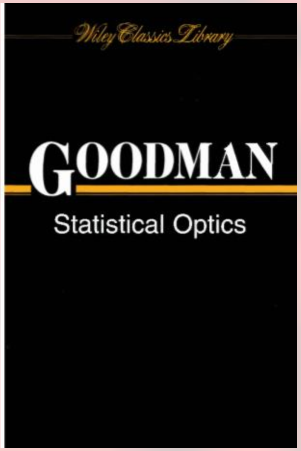

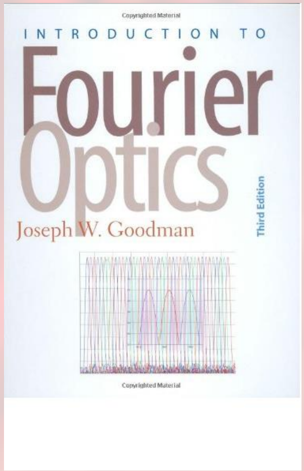

- J. W. Goodman «Introduction to Fourier Optics»

- P. Jacquot, J.-M. Fournier
  «Interferometry in speckle light»