

# Superfluid effects in nuclear collective motion: applications to fission and other processes

**Denis Lacroix**

**IPN Orsay**

## Outline:

- Generalities on time-dependent approaches with pairing
- Highlights of recent applications
- Application to fission
- Collective aspects of Large Amplitude Collective motion
- Stochastic Mean-Field Theories for Large Amplitude Motion

Coll: S. Ayik, B. Yilmaz, C. Simenel,  
G. Scamps, Y. Tanimura

Ultimate Goals : give a unified description of nuclear structure and reactions  
provide predictive theory in explored and unexplored region of nuclear chart

## Status of TD-EDF

→ Symmetry unrestricted simulations

→ State of the art functional consistent with nuclear structure

Kim, Otsuka, Bonche, J. Phys.G23, (1997).

Nakatsukasa, Yabana, PRC71, (2005).

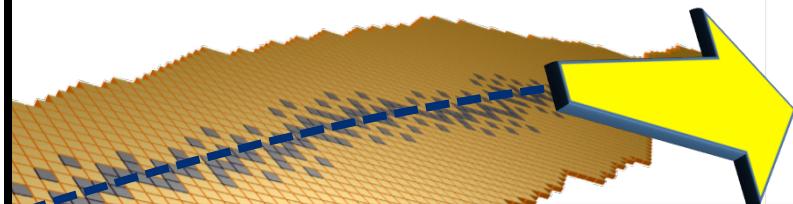
Maruhn, Reinhard, Stevenson, Stone, Strayer, PRC71 (2005).

Umar and Oberacker, PRC71, (2005).

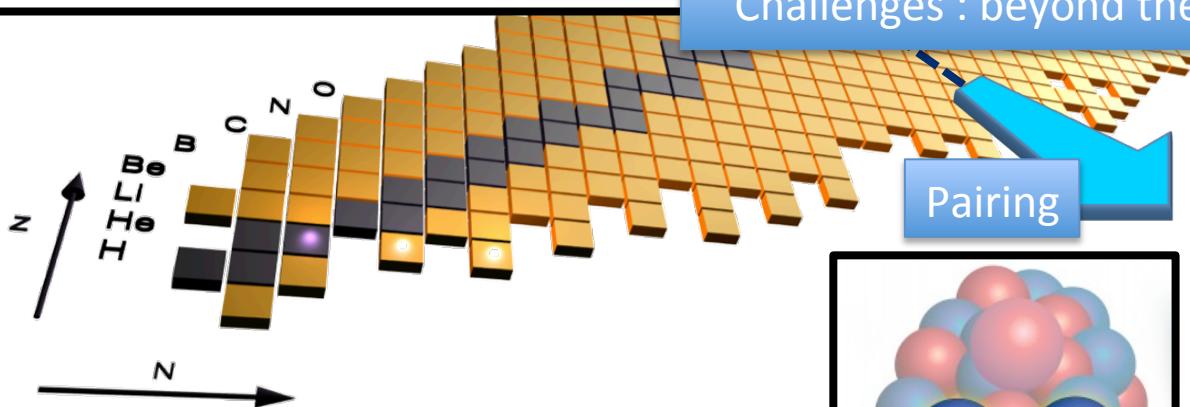
Simenel, Avez, Int. J. Mod. Phys. E17, (2008).

Washiyama, Lacroix PRC78, (2008).

Gao-Feng et al PRCC90, (2014).

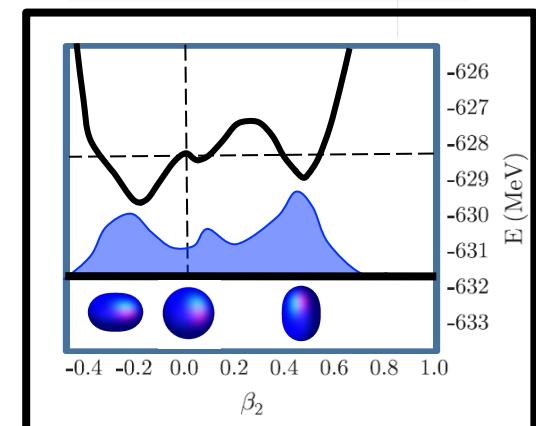


Challenges : beyond the independent particle picture

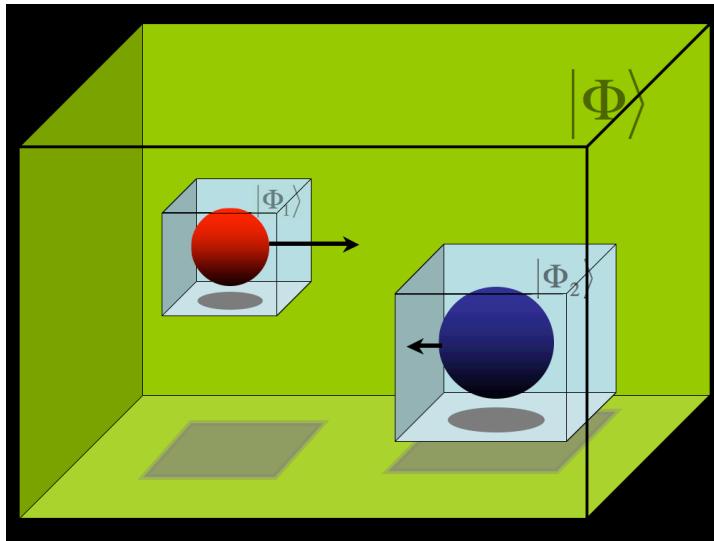


Pairing

Shape coexistence



## Nuclear reaction on a mesh



TDHF is a standard tool  $|\Phi_i\rangle$ : Slater

$$i\hbar \frac{d\rho}{dt} = [h(\rho), \rho] \rightarrow \text{Single-particle evolution}$$

Simenel, Lacroix, Avez, arXiv:0806.2714v2

Introduction of pairing: TDHFB

$$i\hbar \frac{d}{dt} \mathcal{R} = [\mathcal{H}(\mathcal{R}), \mathcal{R}] \quad \mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1 - \rho \end{pmatrix}$$

$\rightarrow$  Quasi-particle evolution

(Active Groups: France, US, Japan...)

BCS limit of TDHFB (also called Canonical basis TDHFB)

**TDHFB = 1000 \* (TDHF)**

Neglect  $\Delta_{ij}$

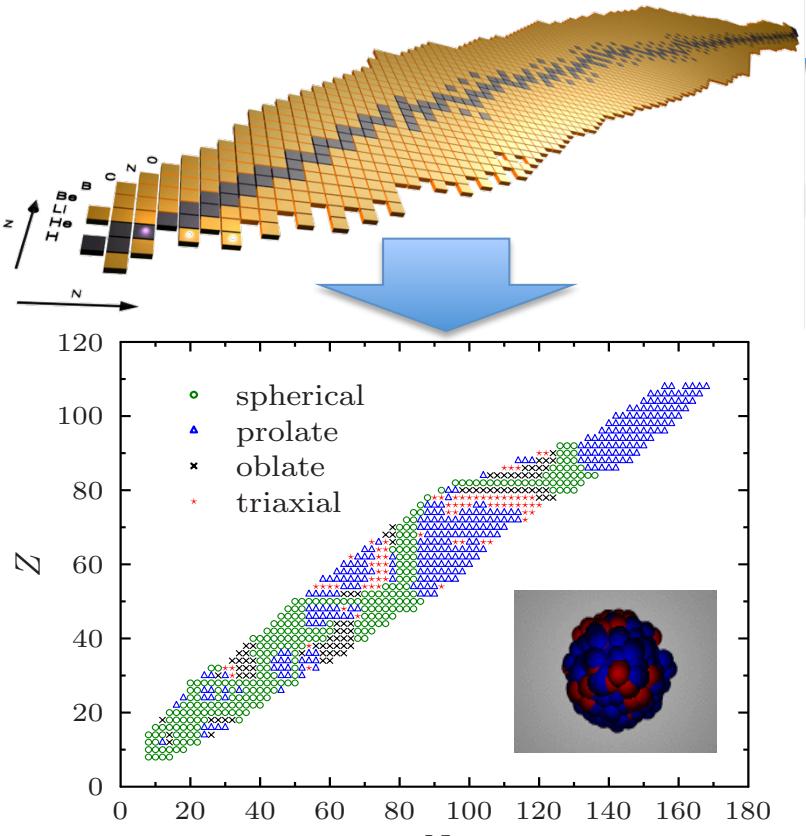
$$|\Phi(t)\rangle = \prod_{k>0} \left( u_k(t) + v_k(t) a_k^\dagger(t) a_{\bar{k}}^\dagger(t) \right) |-\rangle.$$

$\rightarrow$  TDHFB is very demanding Stetcu, Bulgac, Magierski, and Roche, PRC 84 (2011)  
Hashimoto, Scamps, arXiv:1604.07494 (2016)

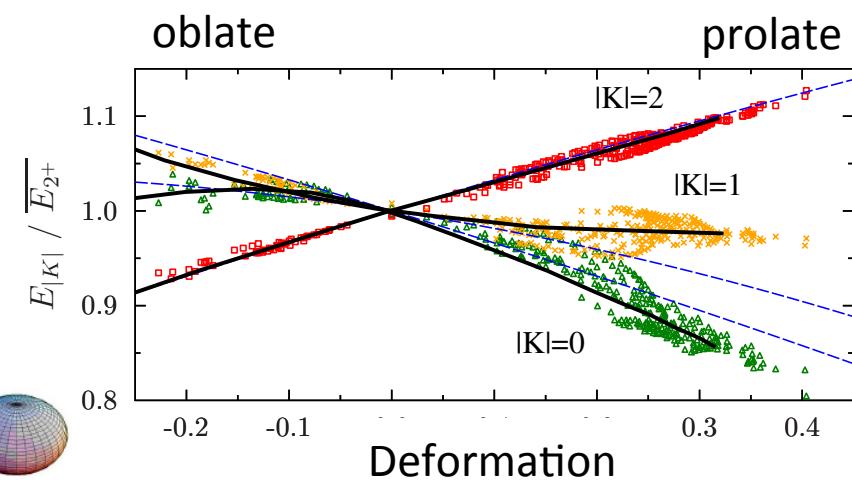
$\rightarrow$  Reasonable results for collective motion Ebata, Nakatsukasa et al, PRC82 (2010)

$\rightarrow$  Sometimes more predictive than TDHFB Scamps, Lacroix, Bertsch, Washiyama, PRC85 (2012)

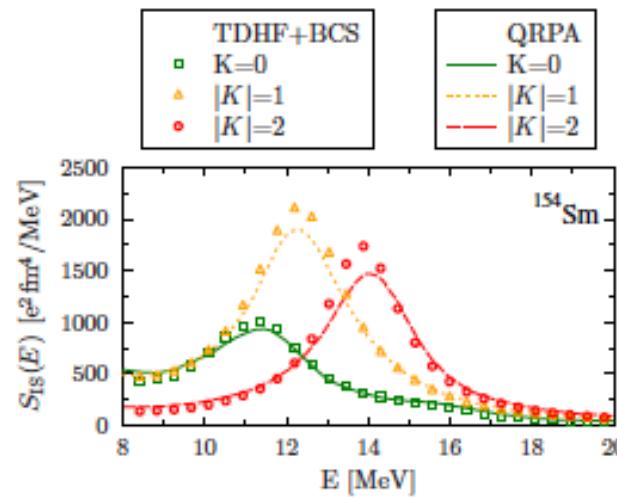
# Large scale study of giant quadrupole resonances



## Energy splitting:



## Example

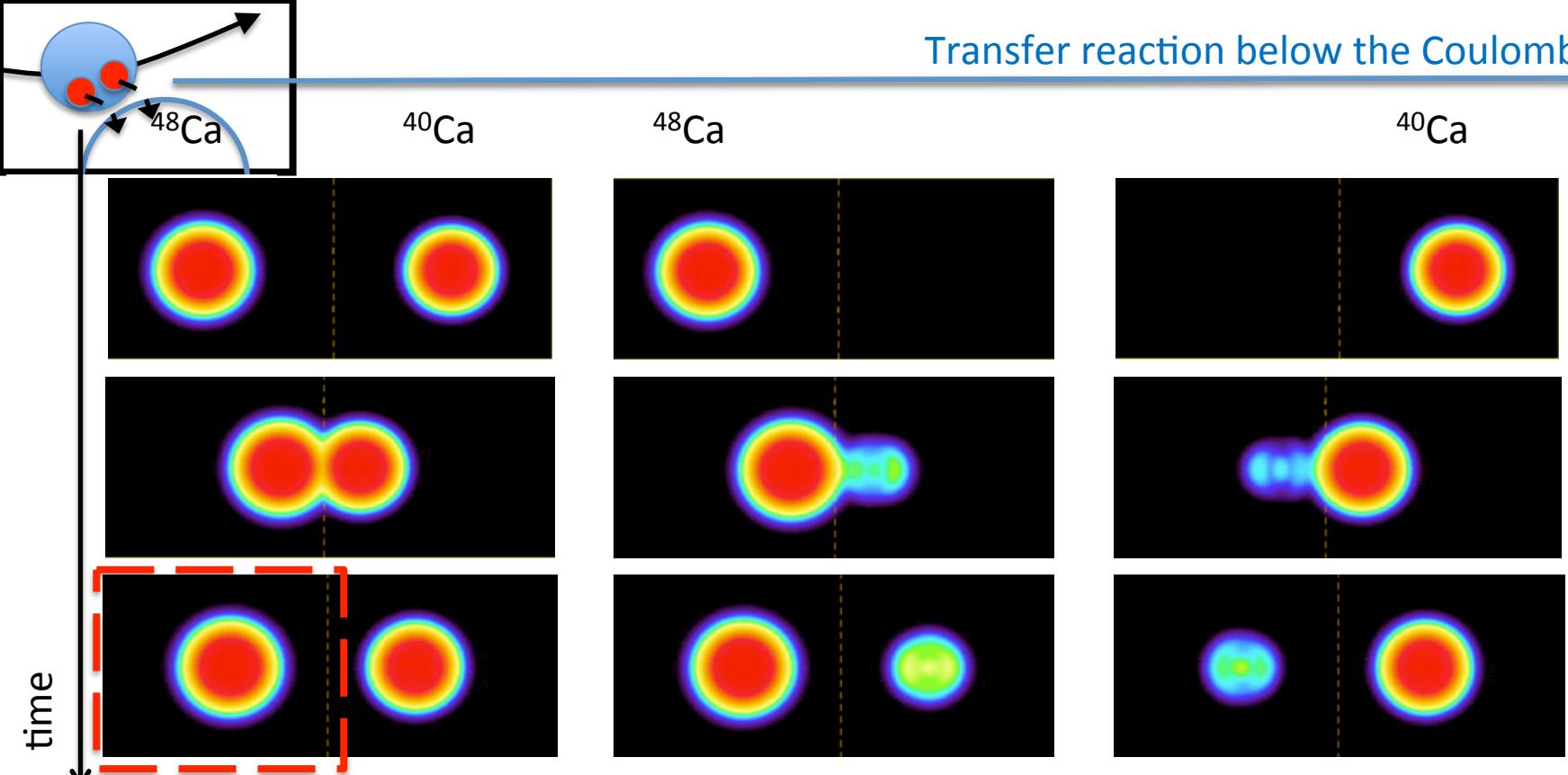


## Good reproduction of average energy

 Damping (fluctuations) is still  
Severely estimated but improves  
In deformed nuclei

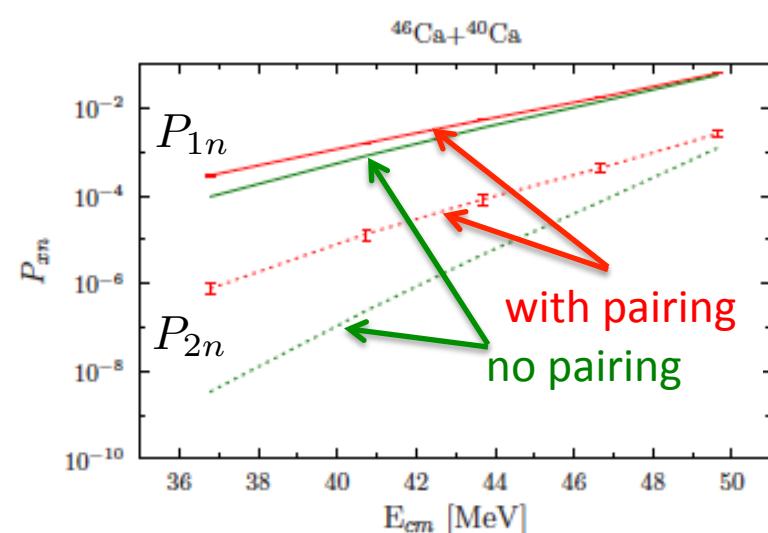
Scamps, Lacroix, PRC88 (2013)  
Scamps, Lacroix, PRC89 (2014)

## Transfer reaction below the Coulomb barrier

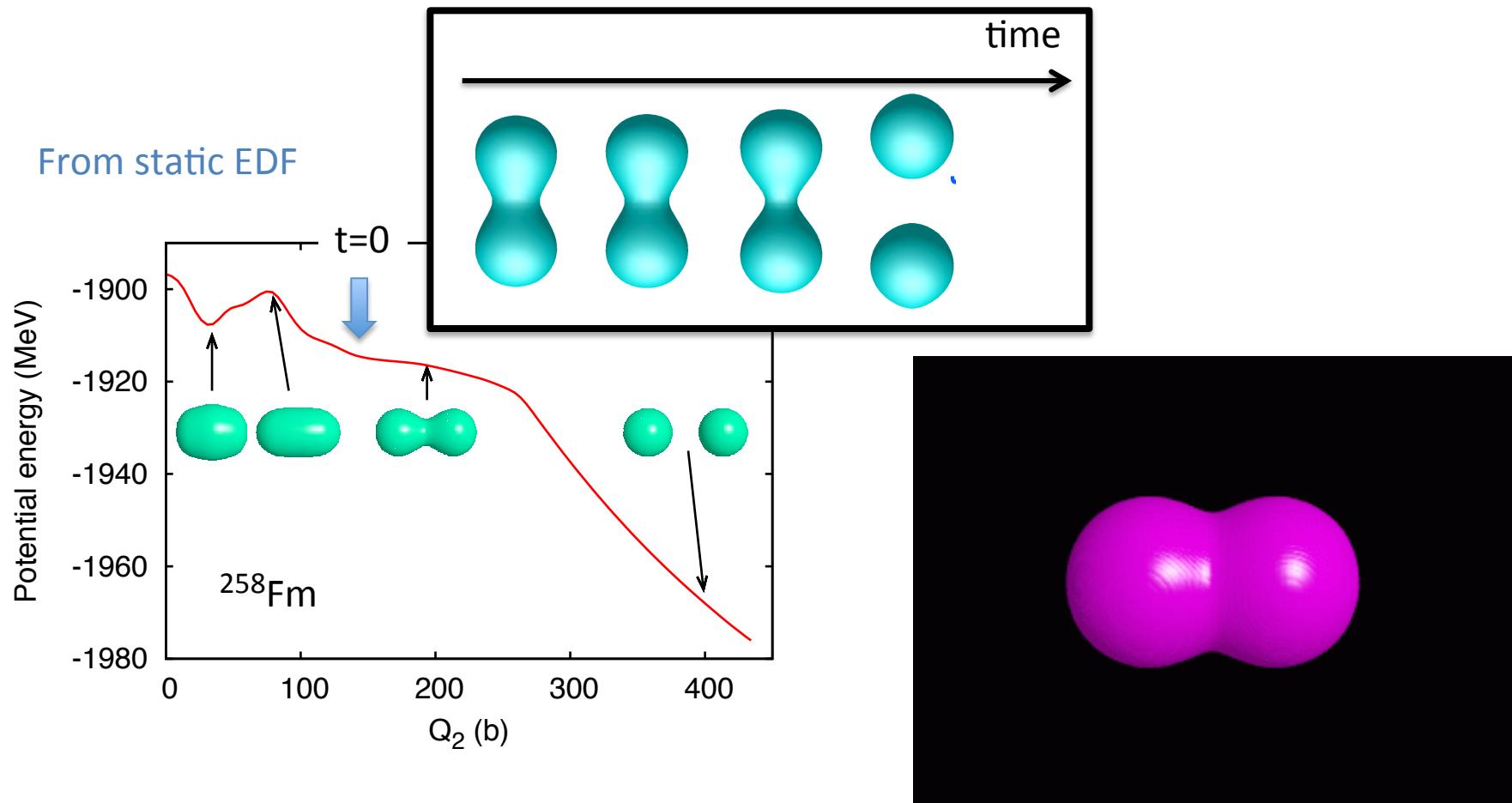


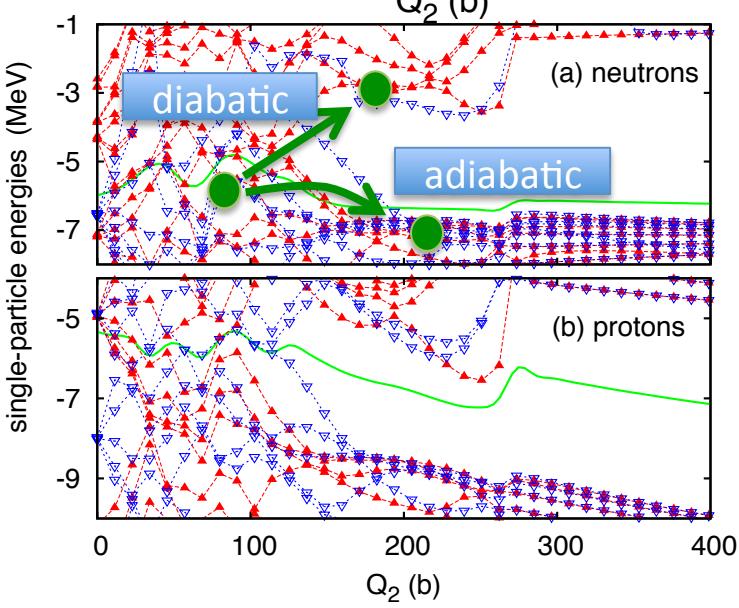
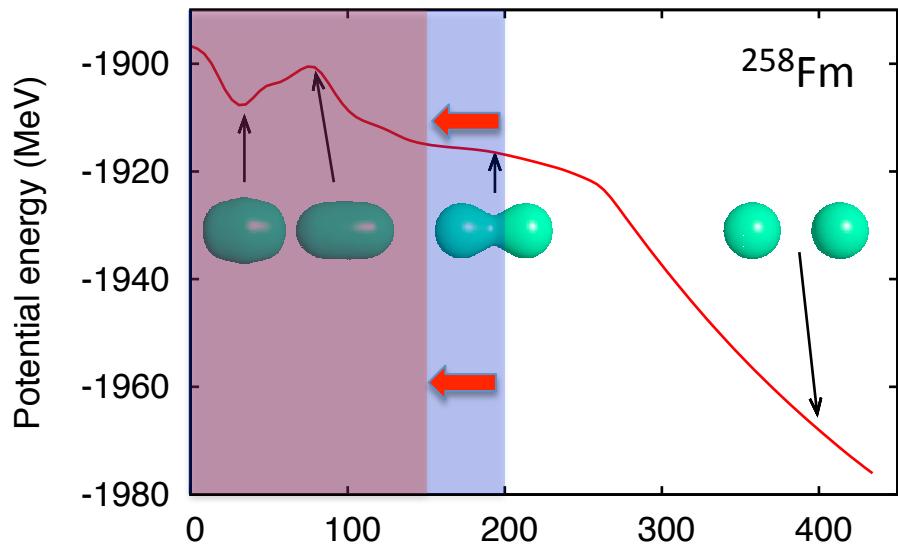
Extract one, two, ...  
nucleons transfer probabilities  
 $P_{1n}, P_{2n}, \dots$

Scamps, Lacroix, PRC 87 (2013).



## Fission with TD-EDF with pairing





Scamps Simenel, Lacroix, PRC 92 (2015)  
Tanimura, Lacroix, Scamps, PRC 92 (2015)

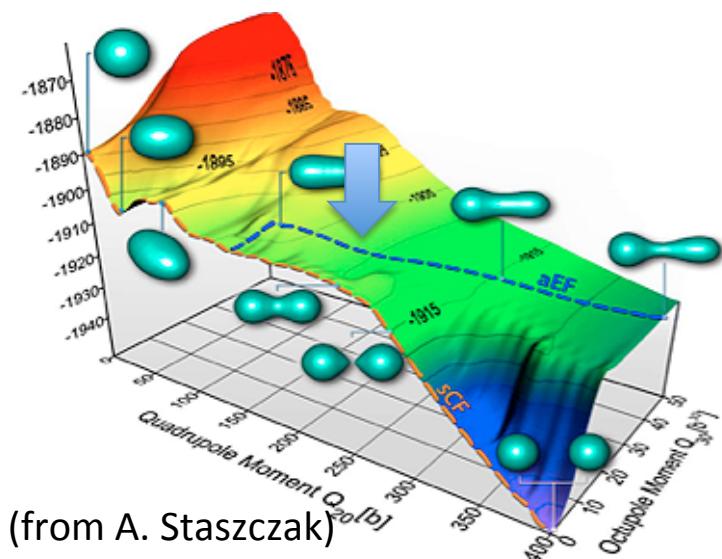
### Fission with TD-EDF without pairing

Simenel, Umar, PRC C89 (2014).  
Goddard, Stevenson, Rios, PRC 92 (2015), 93 (2016)

- Strong interplay between structure and dynamic
- TD-EDF does not follow the adiabatic path
- Existence of a spontaneous fission threshold at larger deformation than the fission barrier
- Still, information on fission can be obtained

### Fission with TD-EDF with pairing

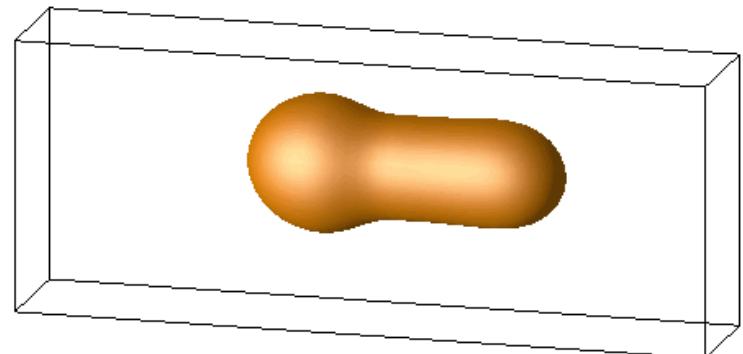
- Improve the threshold anomaly
- Allows to consider the fission of superfluid nuclei



(from A. Staszczak)

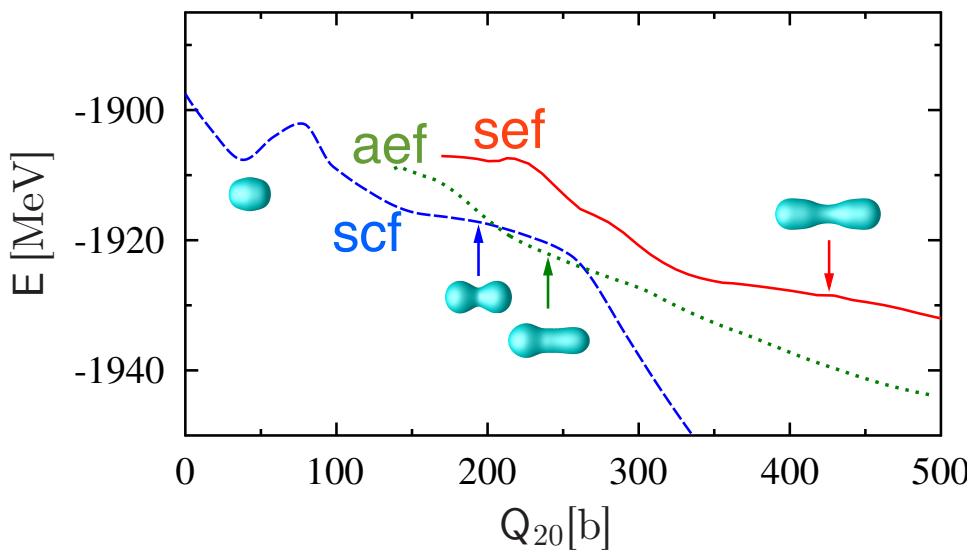
### Time-dependent picture of fission

Scamps Simenel, Lacroix, PRC92 (2015)



(courtesy G. Scamps/C. Simenel)

### Fission along different paths

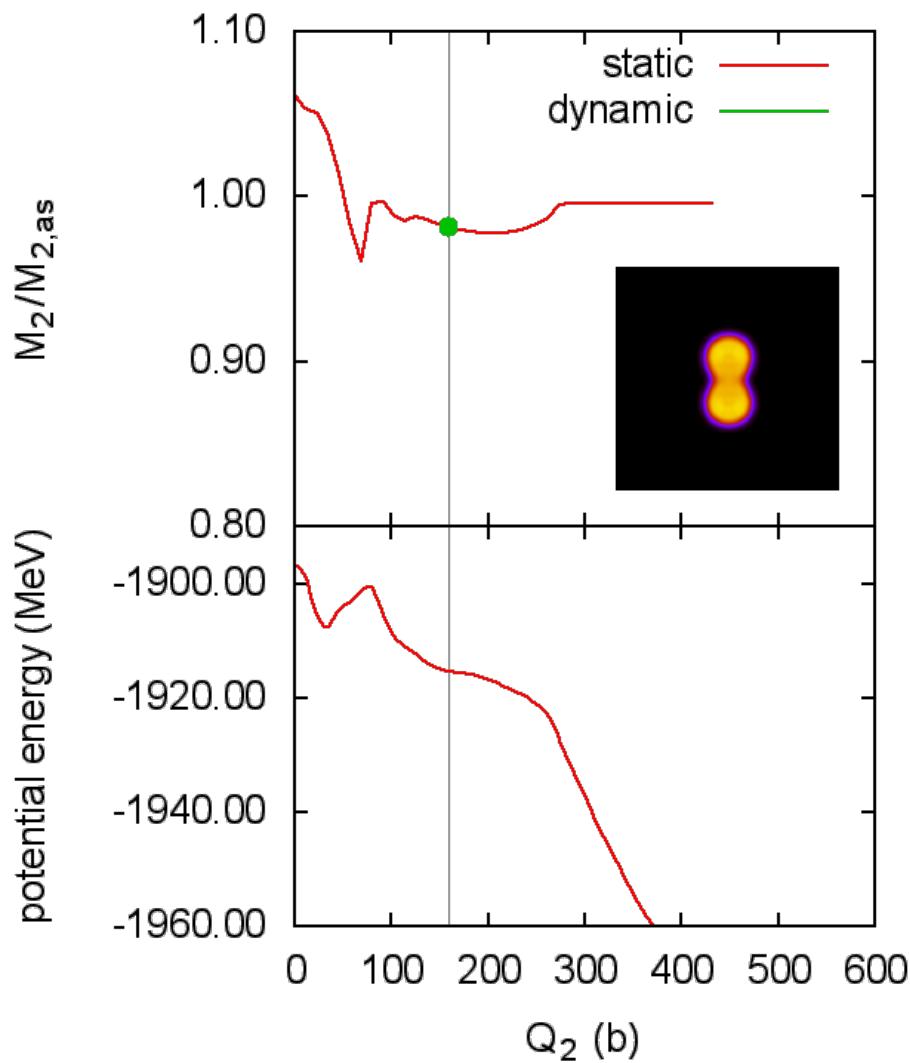


scf: symmetric compact fission

sef: symmetric elongated fission

aef: asymmetric elongated fission

$^{258}\text{Fm}$   $E_x = 0.0 \text{ MeV}$ ,  $t = 0.00 \text{ fm/c}$



Tanimura, Lacroix, Scamps, PRC 92 (2015)

Microscopic dynamic



Collective mass, collective momentum

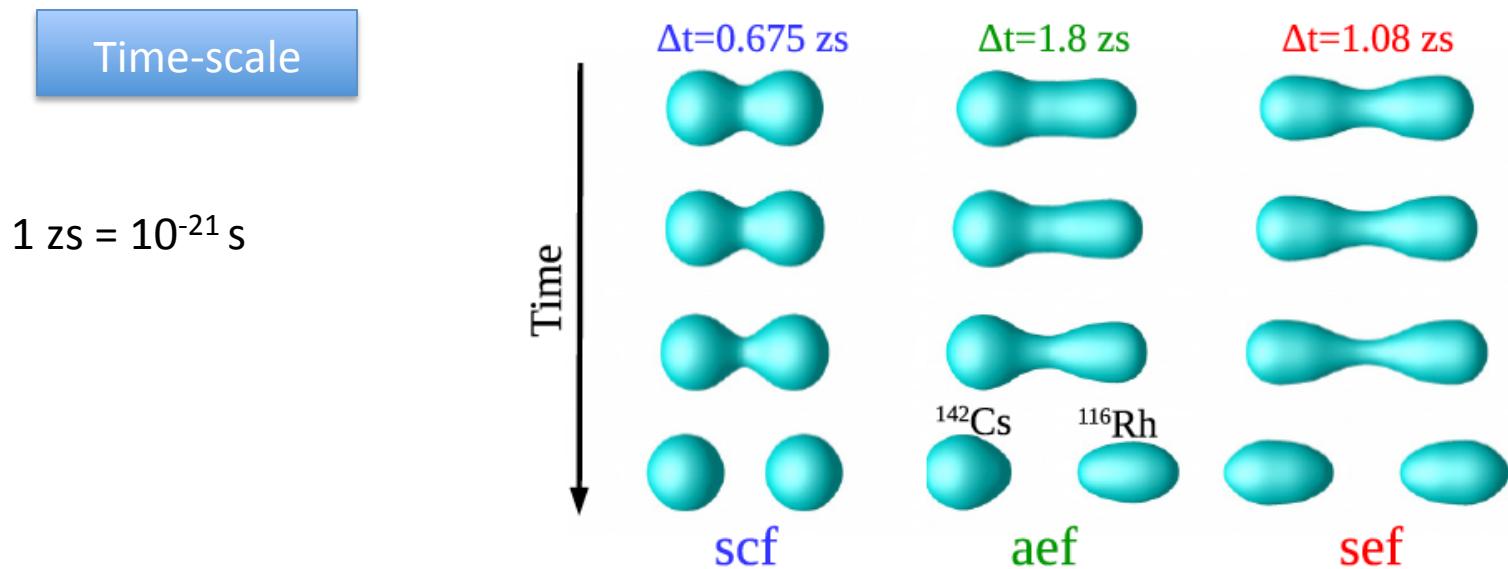
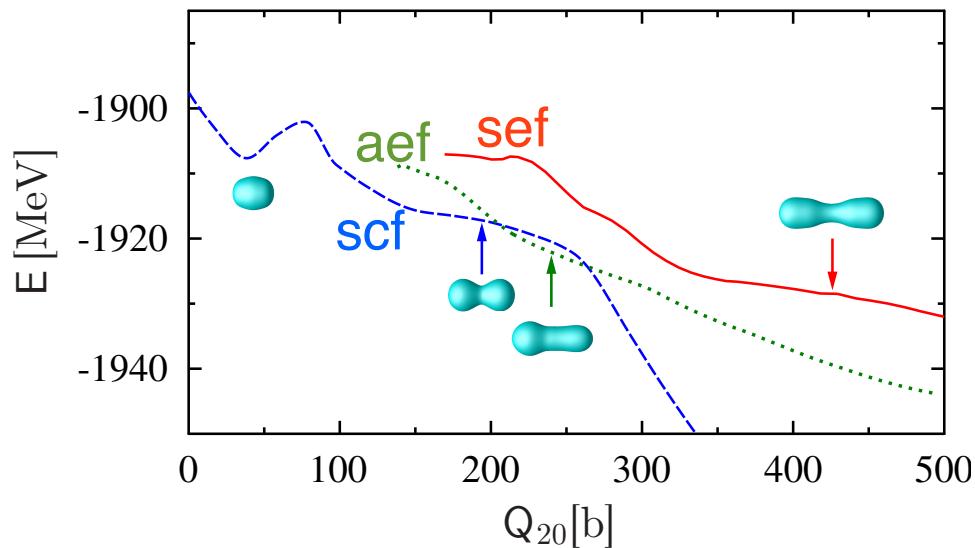


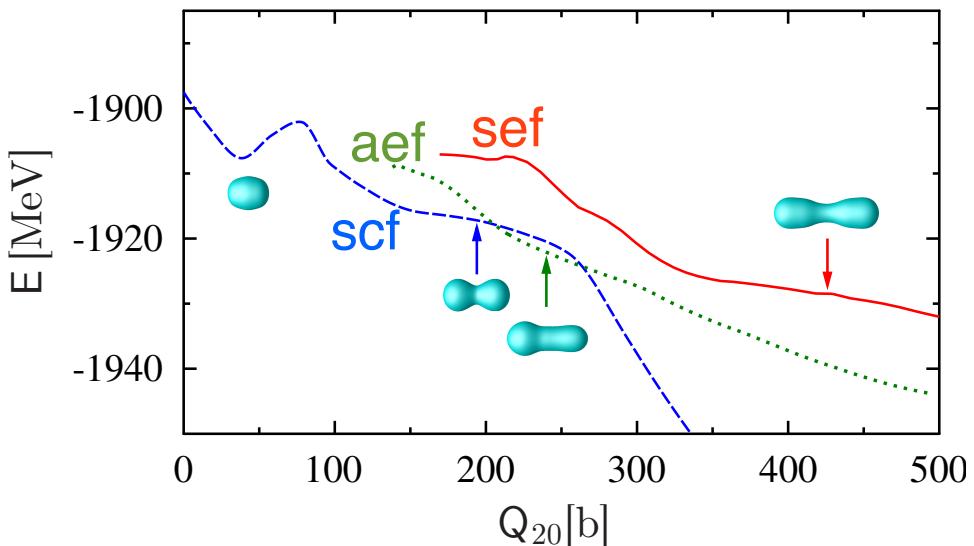
Macroscopic evolution:  
Dissipation, non-adiabatic effects...

→ The system first follows the adiabatic limit

→ Around scission, dynamic is faster and Becomes non-adiabatic

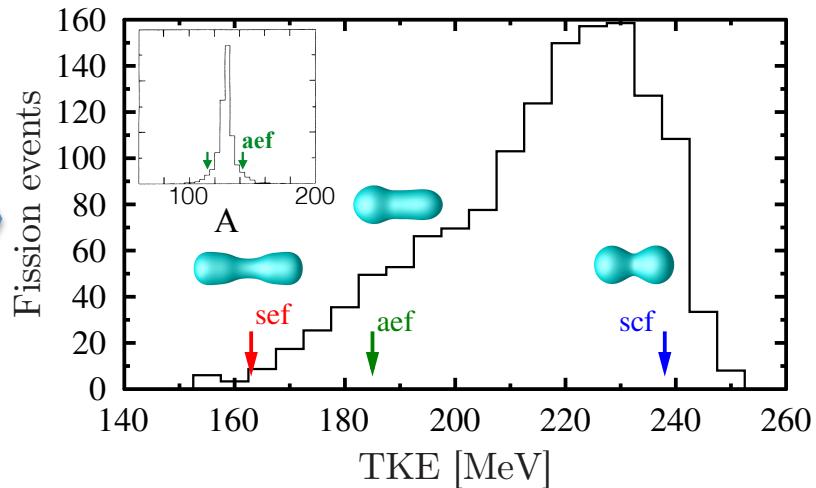
$$E_{\text{diss}} \simeq 20 \text{ MeV} \quad \text{TKE} \simeq 250 \text{ MeV}$$





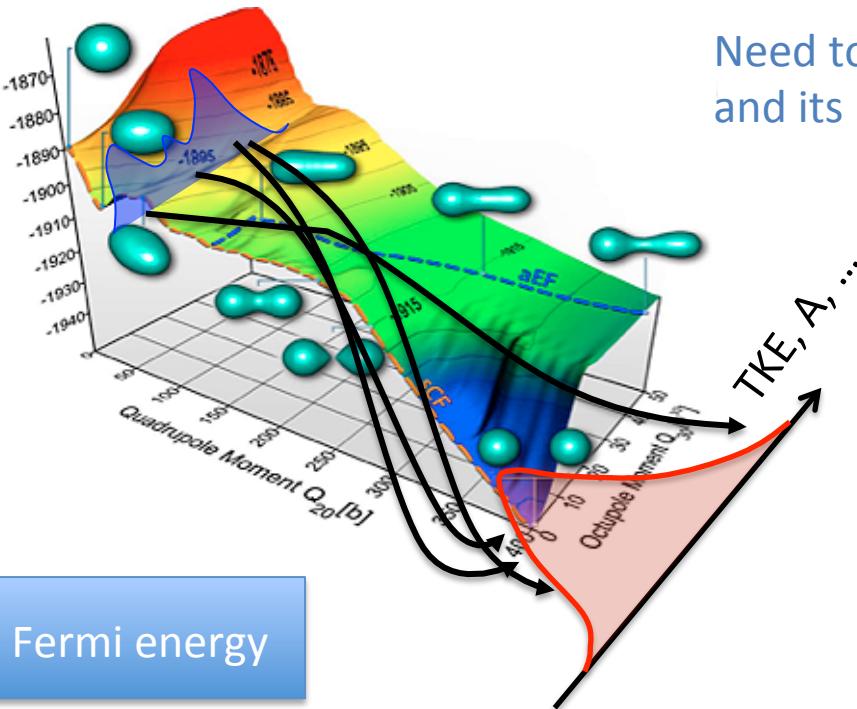
Total Kinetic Energy

Some conclusions



Remaining problem

- Fluctuations are underestimated
- Weight of each paths?

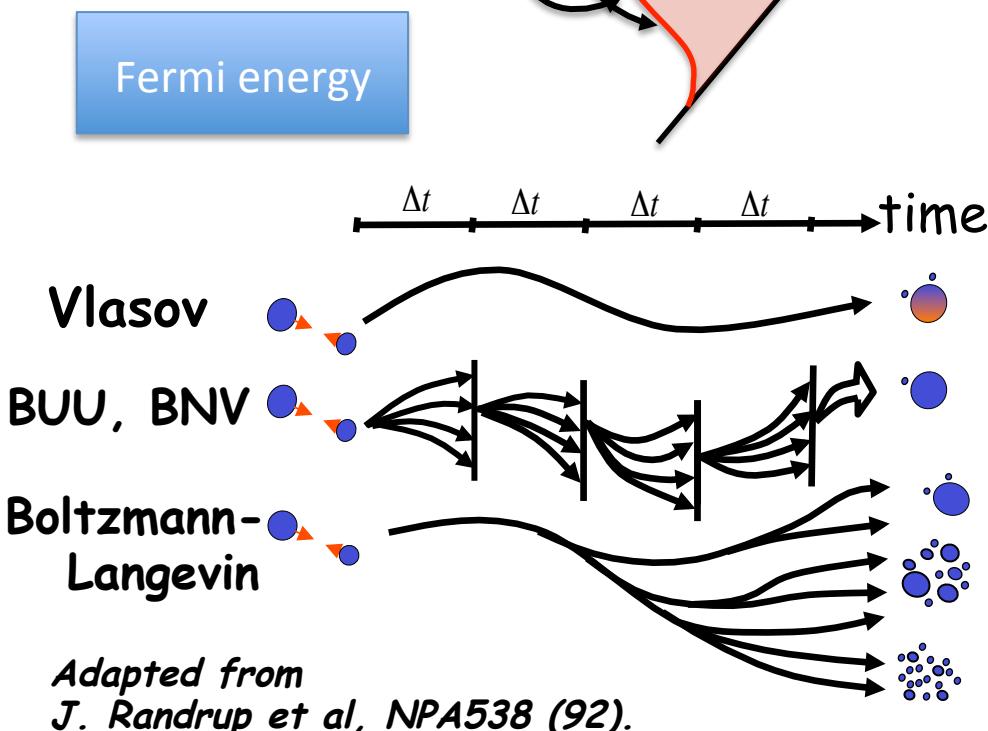


Need to describe configuration mixing  
and its propagation

One possibility is to use Time-Dependent  
Generator Coordinate method  
(beyond adiabatic, number of DOFs, ...)

Our objective: use the stochastic  
mean-field approach to describe fission

Lacroix, Ayik, EPJA (Review) 50 (2014)



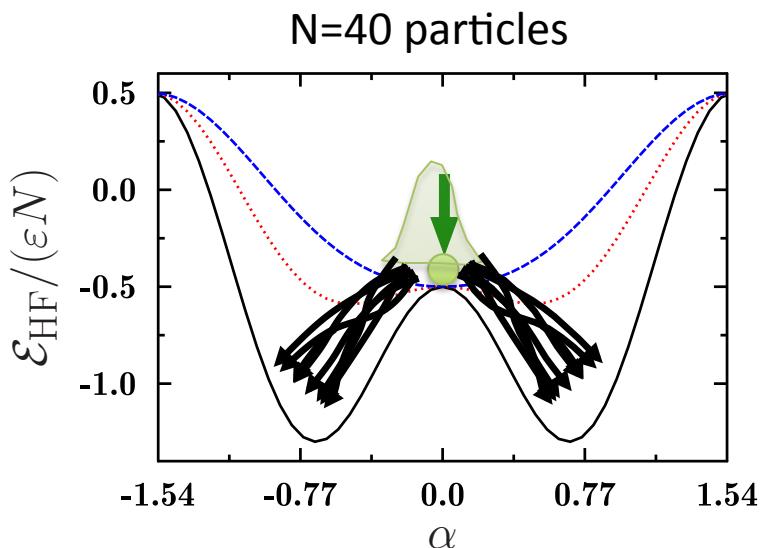
Coulomb  
barrier energy

Stochastic Mean-Field

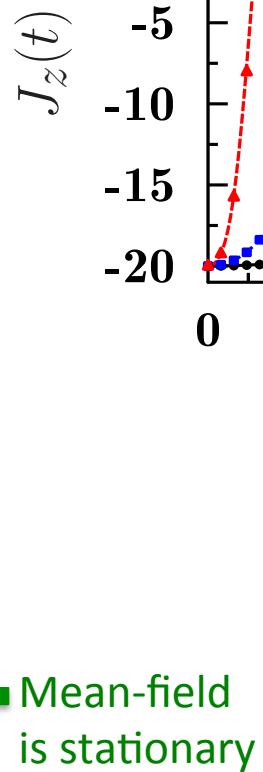
$$\rho_{ij}(t_0) \rightarrow \rho_{ij}(t)$$

Stochastic TDHF

$$|\Phi\rangle\langle\Phi| \rightarrow |\Phi\rangle\langle\Phi|$$

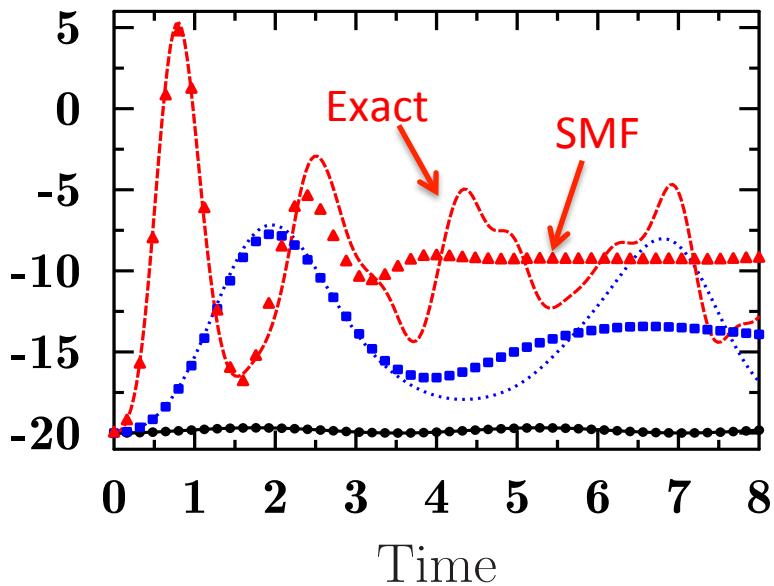
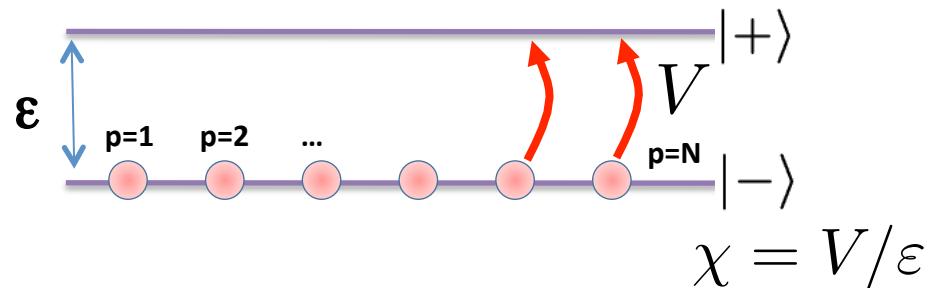


Exact dynamics

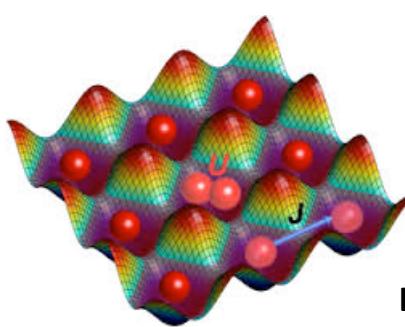


Mean-field  
is stationary

## Two-Level Lipkin Model



Lacroix, Ayik, Yilmaz, PRC 85 (2012)

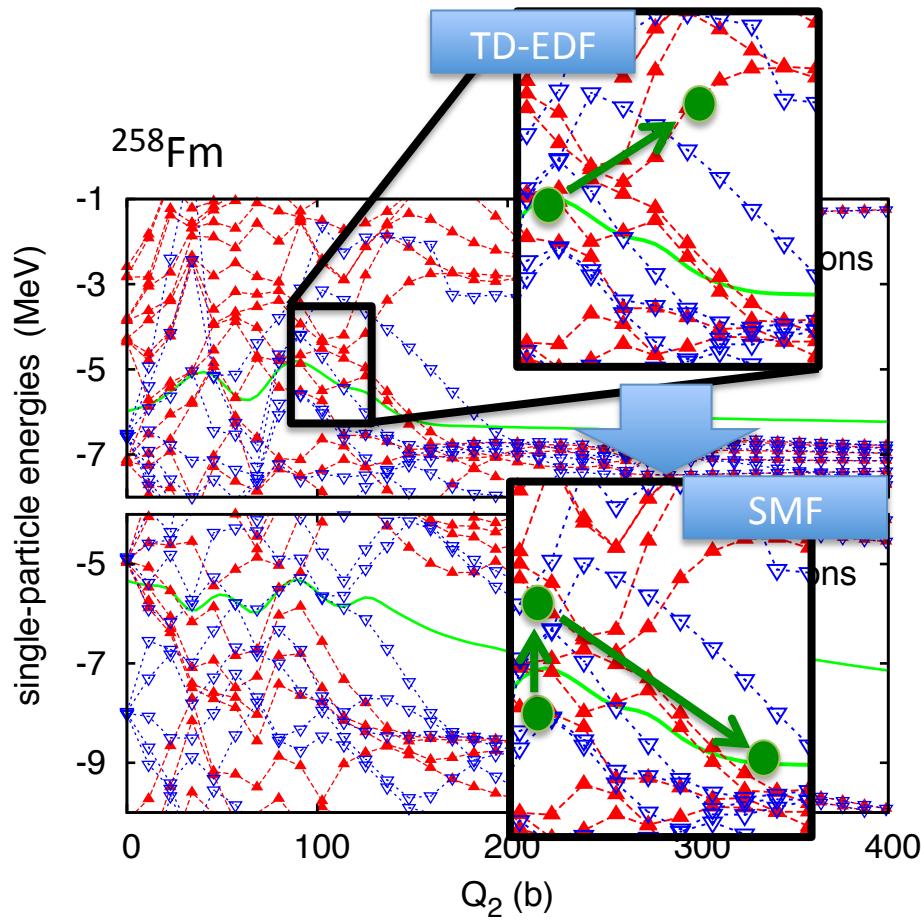


Lacroix et al, PRB90 (2014)

## SMF in density matrix space

$$\rho(\mathbf{r}, \mathbf{r}', t_0) = \sum_i \Phi_i^*(\mathbf{r}, t_0) n_i \Phi_i(\mathbf{r}', t_0)$$

$$\rho^\lambda(\mathbf{r}, \mathbf{r}', t_0) = \sum_{ij} \Phi_i^*(\mathbf{r}, t_0) \rho_{ij}^\lambda \Phi_j(\mathbf{r}', t_0)$$



$$\overline{\rho_{ij}^\lambda} = \delta_{ij} n_i$$

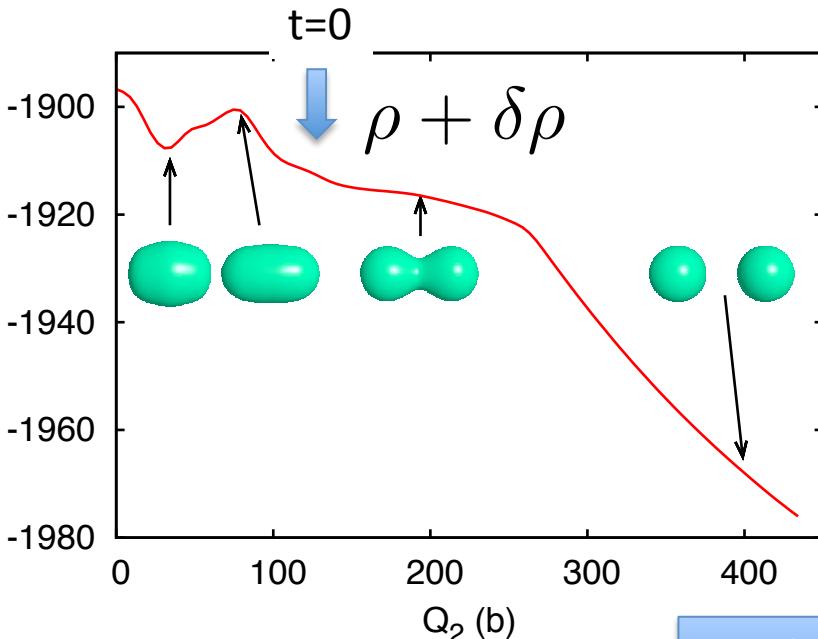
$$\overline{\delta \rho_{ij}^\lambda \delta \rho_{j'i'}^\lambda} = \frac{1}{2} \delta_{jj'} \delta_{ii'} [n_i(1 - n_j) + n_j(1 - n_i)].$$

$$\rho = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \text{holes} & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{Part.}$$

$$\rho = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \neq 0 & & & & & & & & \\ \neq 0 & & & & & & & & \end{pmatrix}$$

## Density evolution with initial fluctuations

Potential energy (MeV)



t =

0.00 fm/c

20

15

10

5

0

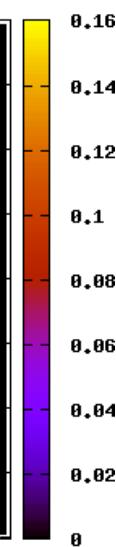
-5

-10

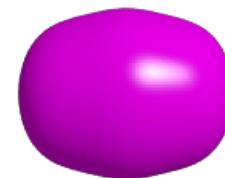
-15

-20

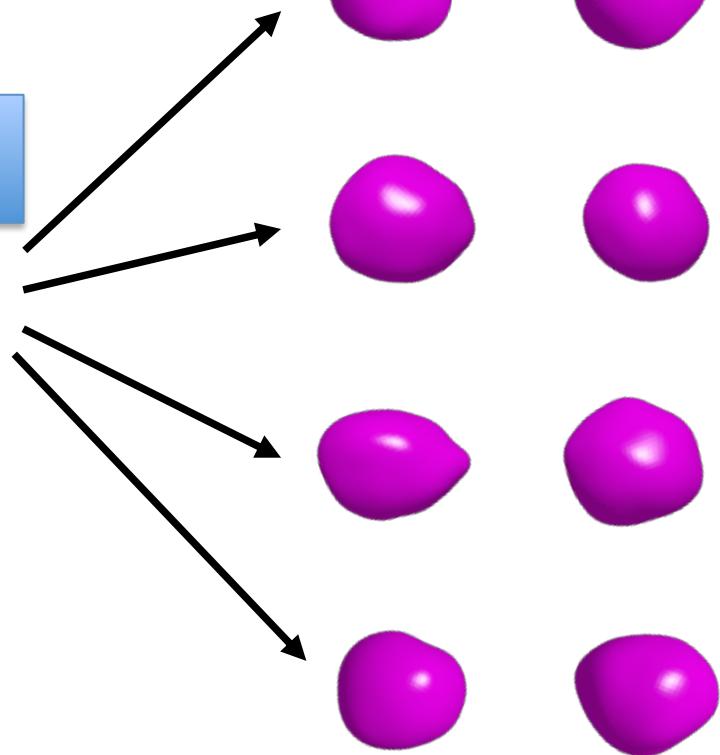
-20 -15 -10 -5 0 5 10 15 20



Fluctuating  
Initial condition

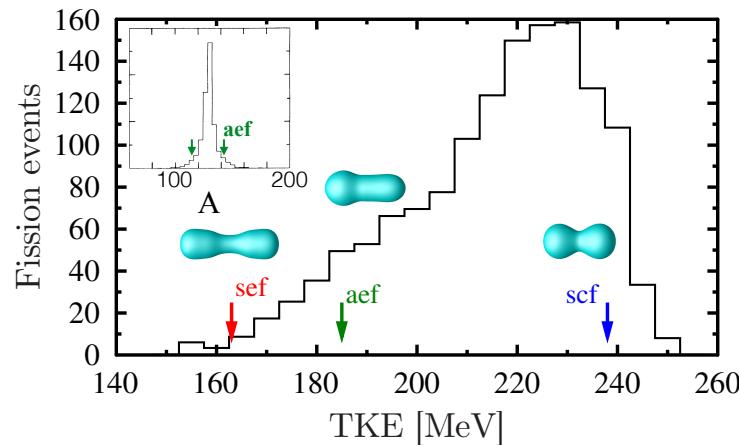


Final  
configurations

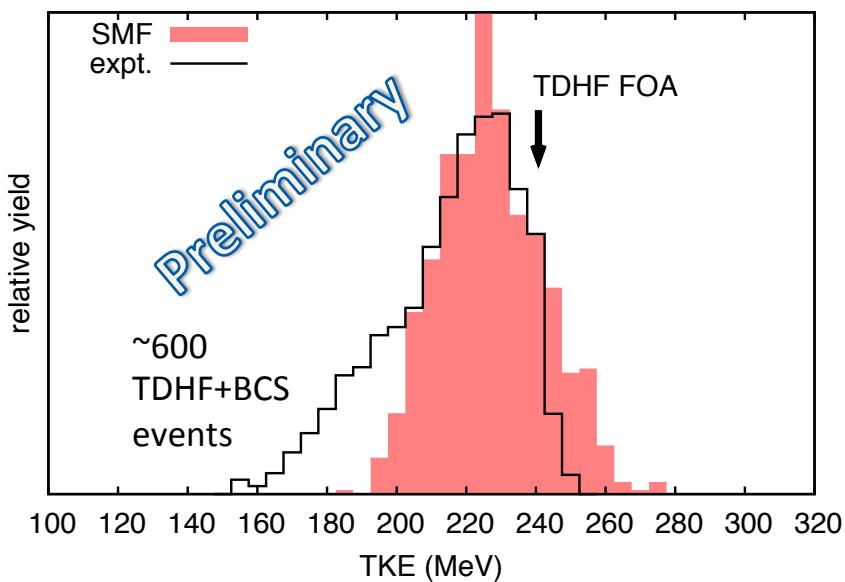


## Total Kinetic energy

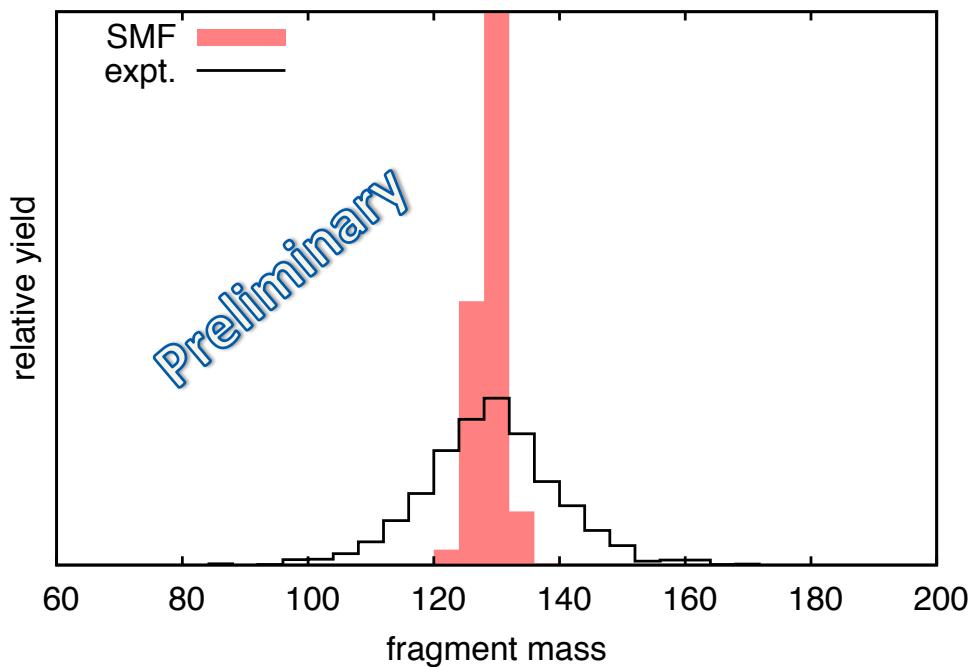
## TDHF+BCS results

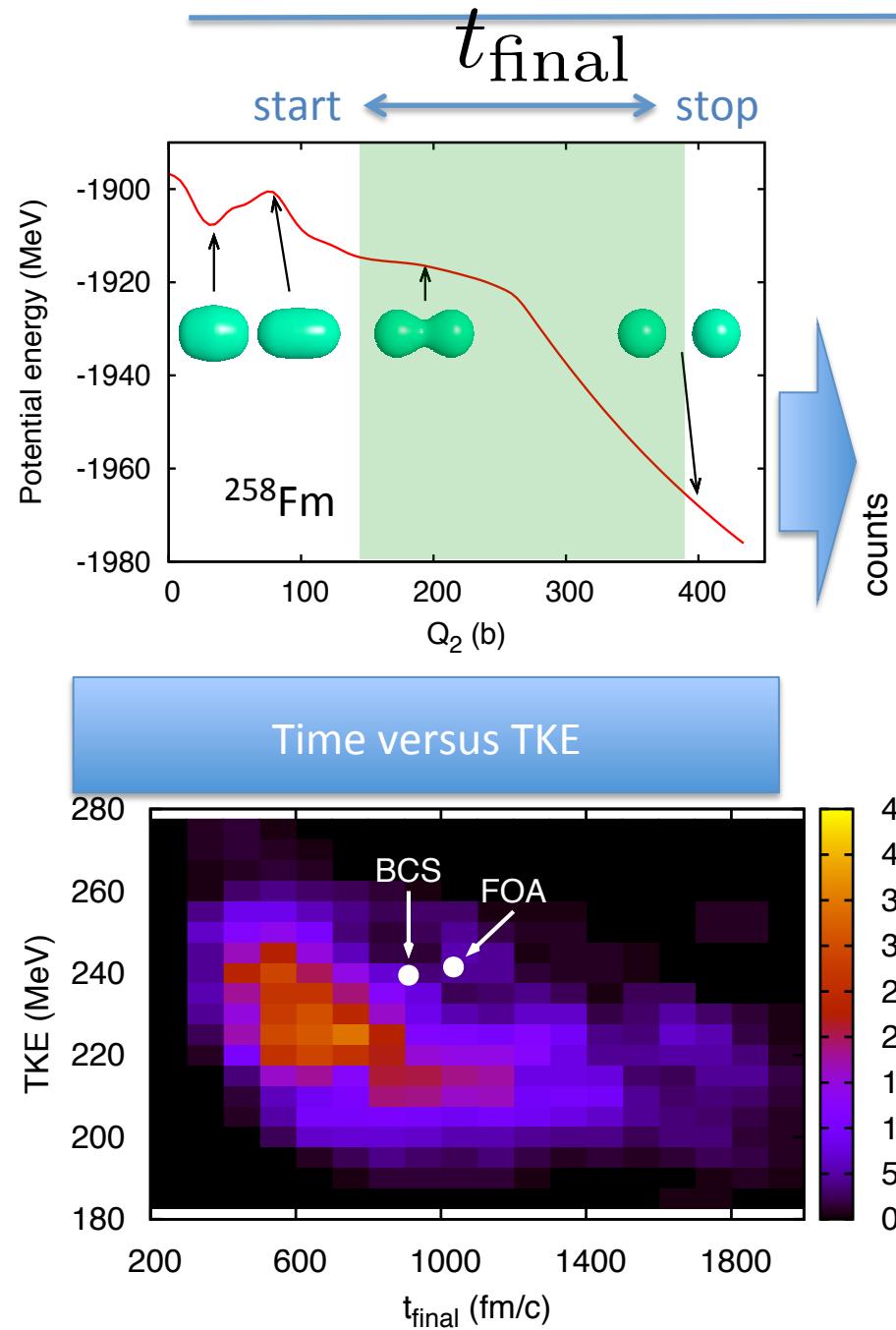


## SMF results

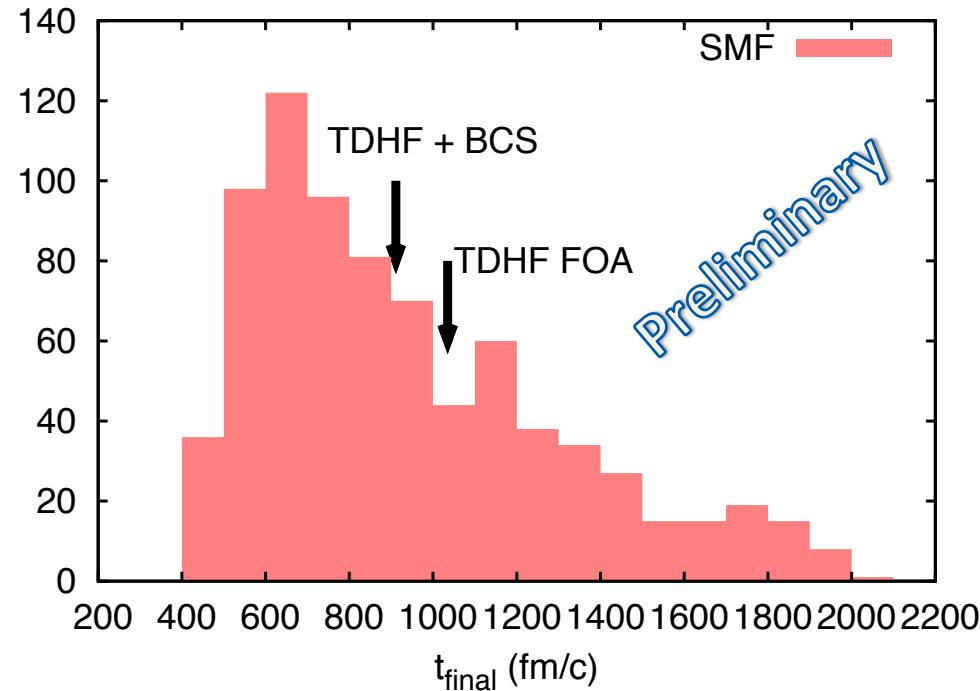


## Mass distribution

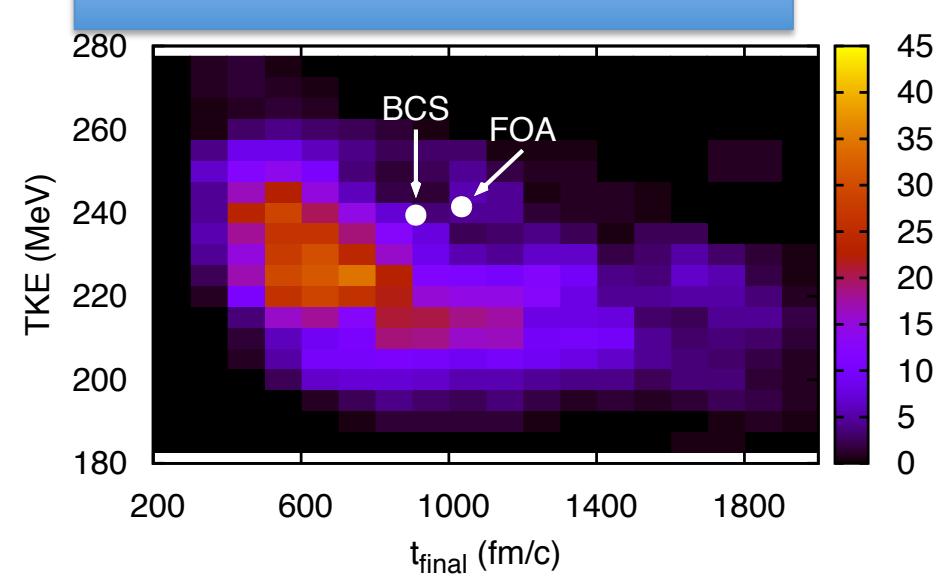




Fission time distribution



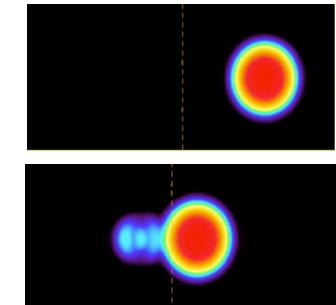
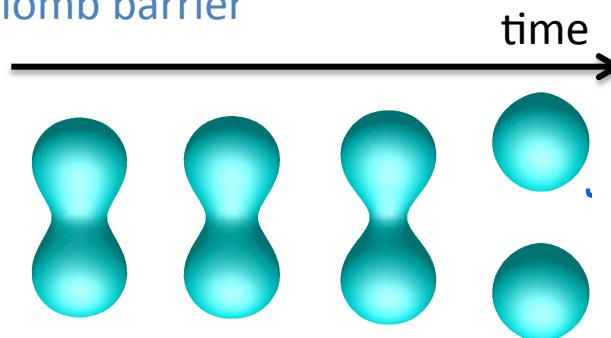
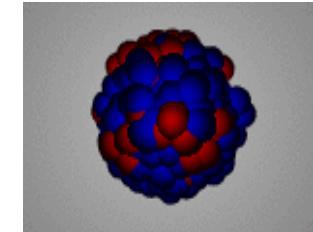
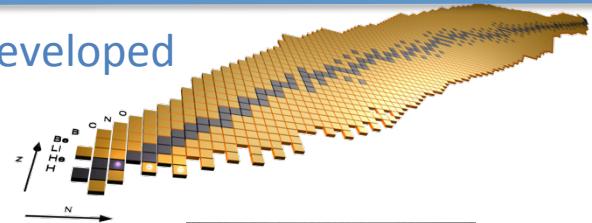
Time versus TKE



- New versatile TD-EDF codes including pairing are now developed
- This open new applications perspectives

### Recent applications

- Systematic of giant resonances in nuclei with various shapes
- Particle transfer below the Coulomb barrier
- Fission of superfluid nuclei



### Stochastic Mean-Field application

- First application with sampling of initial phase-space in TD-EDF
- Preliminary results are encouraging

Emergent collaboration (LNS Catane-IPN Orsay)



**ENSAR<sub>2</sub>**

