Non-Dipolarity of Channeling Radiation at GeV Beam Energies

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Outline

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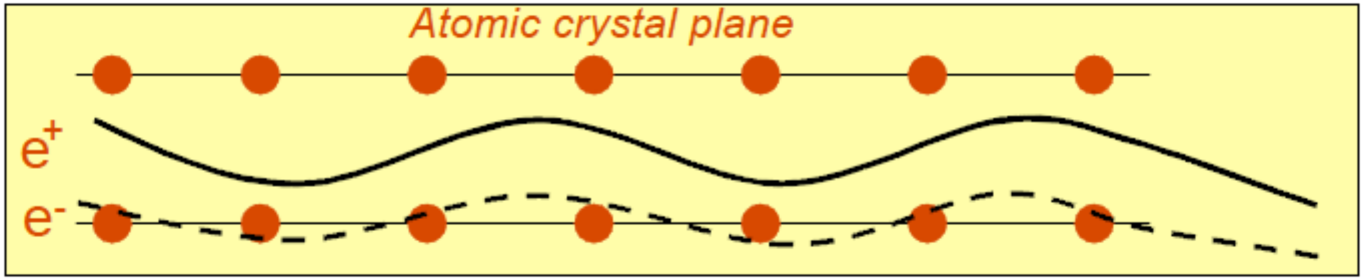
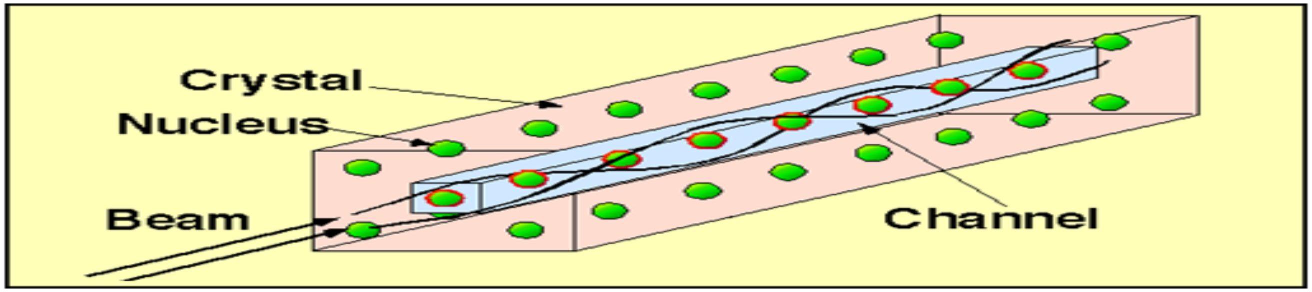
2. Continuum potential
3. Theory of planar channeling radiation (Quantum)
4. Theory of planar channeling radiation (classical) dipole approximation
5. Non-dipole approximation
6. Comparison of non-dipole with dipole approximation
7. Summary

Planar channeling: one-dimensional problem



Axial channeling: two-dimensional problem



2. Continuum potential

Planar channeling

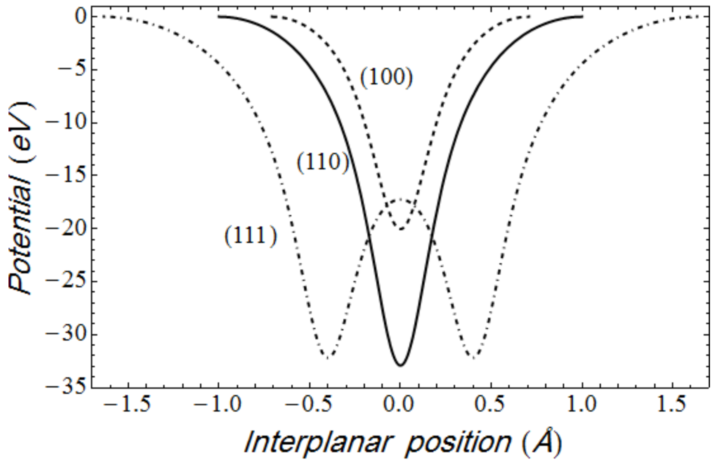
$$V(x) = \sum_n v_n e^{ingx}$$

$$v_n = -\frac{2\pi}{V_c} a_0^2 (e^2/a_0) \sum_j e^{-M_j(\vec{g})} e^{-i\vec{g}.\vec{r}_j} \sum_{i=1}^{4} a_i e^{\left(-\frac{1}{4}\left(\frac{b_i}{4\pi^2}\right)(ng)^2\right)}$$

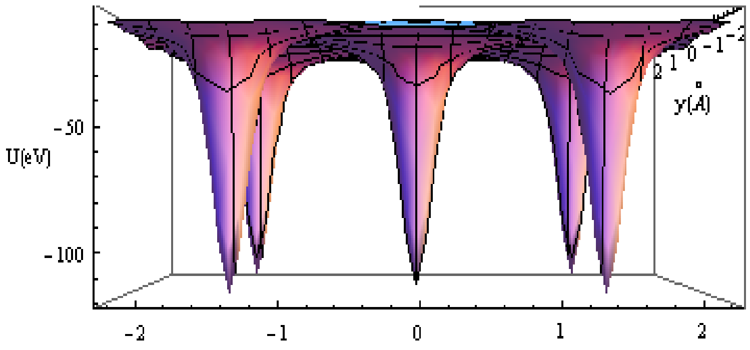
Axial channeling

$$V(x,y) = \sum_{\vec{g}_m} v_{\vec{g}_m} e^{i\vec{g}_m.\vec{r}_\perp}$$

$$v_{\vec{g}_m} = -\frac{2\pi}{V_c} a_0^2 (e^2/a_0) \sum_j e^{-i\vec{g}.\vec{r}_j} \sum_{i=1}^{4} a_i e^{\left(-\frac{1}{4}\left(\frac{b_i}{4\pi^2} + 2\langle u_j^2 \rangle\right)|\vec{g}_m|^2\right)}$$



The planar continuum potentials of diamond for electrons



The <100> axial continuum potential of germanium for electrons

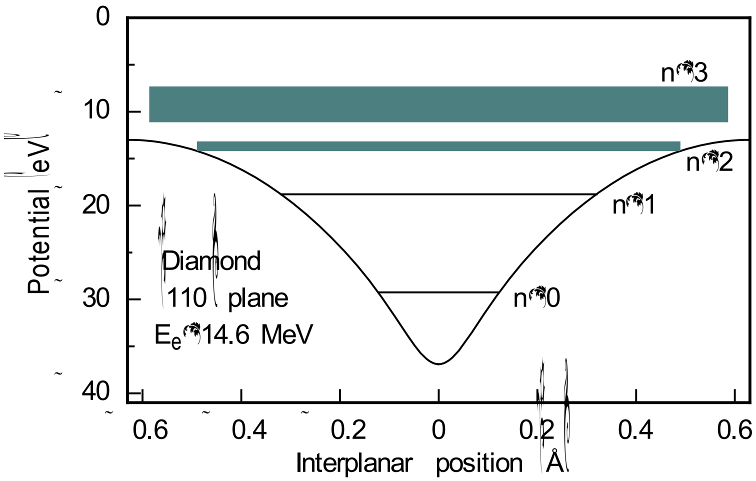
3. Theory of planar channeling radiation (Quantum)

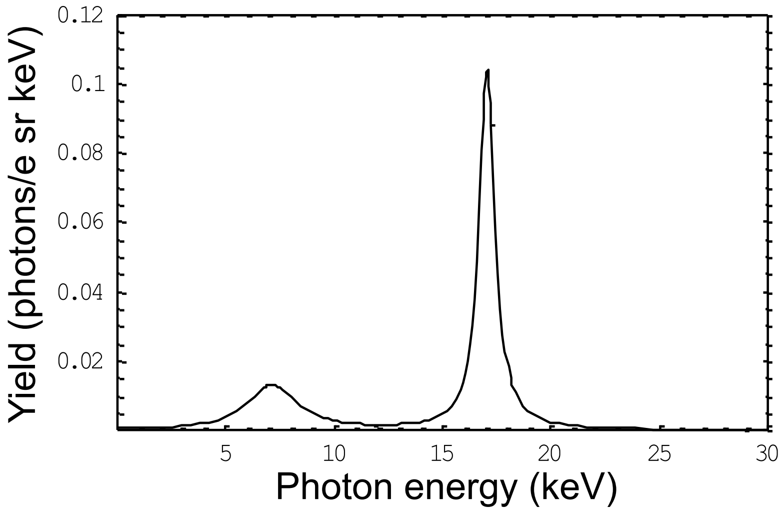
Quantum mechanical model $E_e < 100 MeV$

$$-\frac{\hbar^2}{2m_e\gamma}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$ ⟹ Wave functions $\psi_i(x)$ and eigenvalues E_i

$$E_0 = 2\gamma^2(E_i - E_f)$$

$$\frac{d^2N_{CR}(i\rightarrow f)}{d\Omega_\gamma dE_\gamma}=\frac{\alpha\lambda_c^2}{\pi\hbar c}2\gamma^2(E_i-E_f)\left|\left\langle\psi_f(x)\left|\frac{d}{dx}\right|\psi_i(x)\right\rangle\right|^2\int_0^z dzP_i(z)\times\frac{\Gamma_{tot}/2}{(E_\gamma-E_0)^2+0.25\Gamma_{tot}^2}$$





A Mathematica package for calculation of planar channeling radiation spectra of relativistic electrons channeled in a diamond-structure single crystal (quantum approach)

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4. Theory of planar channeling radiation (classical) dipole approximation

Classical model $E_e > 100 MeV$

Planar : $\gamma m\ddot{x}(t) = F = -\frac{\partial V(x)}{\partial x}$

Angular-energy distribution: $$\frac{d^2E}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_0^\tau e^{i(\omega t - \vec{k}.\vec{r})}\frac{\vec{n}\times((\vec{n}-\vec{\beta})\times\dot{\vec{\beta}})}{(1-\vec{\beta}.\vec{n})^2}dt\right|^2$$

$$c\vec{\beta} = \dot{\vec{r}}(t) \qquad \vec{k} = \omega\vec{n}/c$$

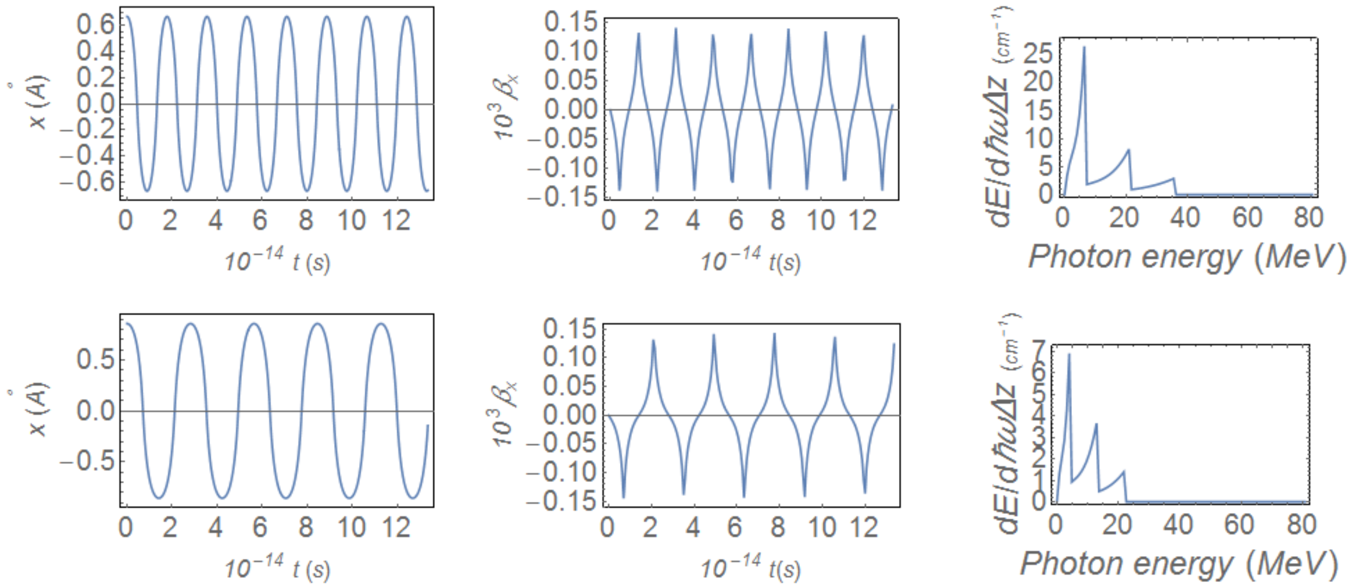
$$\vec{r}(t) = x(t)\vec{e}_x + ct\vec{e}_z$$

Total radiated energy in thin crystal: $$\frac{dE}{d\omega\Delta z} = \frac{e^2}{c^4T^2}\sum_{n=1}^{\infty}\Theta[1-\eta_n](\eta_n^2 - \eta_n + \frac{1}{2})\cdot\left|\dot{x}_{\tilde{\omega}}\right|^2$$

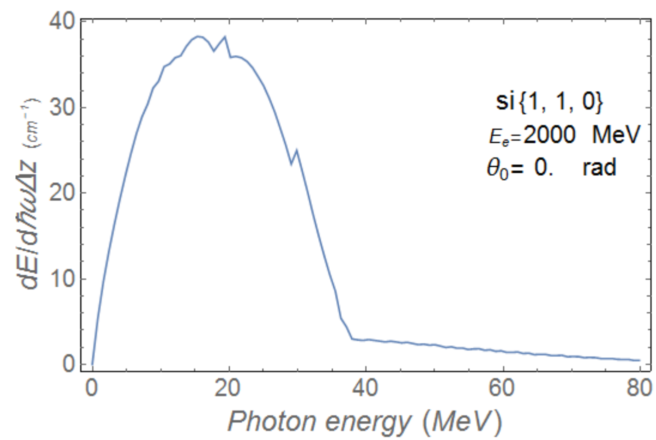
$$\eta_n = \frac{T\omega}{4\pi\gamma^2 n};\ \tilde{\omega} = \frac{2\pi n}{T};\ \dot{x}_{\tilde{\omega}} = \int_0^T \dot{x}e^{i\tilde{\omega}t}$$

4. Dipol approximation

Trajectories, velocities and CR spectra for two different incidence points of 2 GeV electrons to (110) plane of a Si crystal.



Radiation spectrum of 2 GeV electrons channeled along (110) plane of Si in dipol approximation.



5. Non-dipole approximation

$$m\gamma_0\ddot{x} = F_x = -\frac{\partial U(x)}{\partial x} \qquad \gamma_0 = 1/\sqrt{1 - v^2/c^2}$$

At relativistic energies the longitudinal velocity component is coupled with the transverse component through ⟹ $\gamma = \sqrt{1 - (v_x^2 + v_z^2)/c^2}$

conservation law for the longitudinal momentum component $\quad d/dt(\gamma m v_z) = 0$

⇓

$$v_z(t) = v_z(0)\sqrt{\frac{1 - v_x^2(t)/c^2}{1 - v_x^2(0)/c^2}} \approx v_z(0)\left[1 - \frac{1}{2c^2}\left(v_x^2(t) - v_x^2(0)\right)\right].$$

longitudinal component:

$$z(t) = \int_0^t v_z(t')\,dt' = v_z(0)\left(1 + \frac{1}{2}\frac{v_x^2(0)}{c^2}\right)t - \frac{1}{2}\frac{v_z(0)}{c^2}\int_0^t v_x^2(t')dt'$$

5. Non-dipole approximation

$$\langle \dot{z} \rangle = \overline{\beta}_z c = \frac{1}{T}\int_0^T \dot{z}\,(t)dt$$

$$\vec{r}(t) = x(t)\vec{e}_x + \left(\overline{\beta}_z ct + z(t)\right)\vec{e}_z$$

$x(t)$ and z(t) are periodic with period T and $T/2$, respectively

$$\frac{d^2E}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_0^\tau \vec{n}\times\vec{\beta}\, e^{i(\omega t - \vec{k}.\vec{r})}dt\right|^2$$

$$\frac{d^2\overline{E}}{d\omega d\Omega} = \frac{e^2\omega^2}{2\pi c^2}\sum_{n=1}^{\infty}\frac{I_n}{\left(1-\overline{\beta}_z\cos(\vartheta)\right)}\delta\left(\omega - \frac{\omega_n}{\left(1-\overline{\beta}_z\cos(\vartheta)\right)}\right)$$

$$\omega_n = 2\pi n/T$$

$$I_n = |a_n|^2 - |\vec{n}.\vec{a}_n|$$

$$\vec{a}_n = \frac{1}{T}\int_0^T \vec{\beta}\,(t)exp\left(-i\frac{\omega}{c}\left[x(t)cos(\varphi)sin(\vartheta) + z_p(t)cos(\vartheta)\right]\right)e^{i\omega_n t}dt$$

5. Non-dipole approximation

The frequency spectrum is obtained by integration over all emission angles ϑ and φ. Integration over angle φ can be taken easily.

$$\vec{a}_n = \frac{1}{T}\int_0^T \vec{\beta}(t) e^{i\frac{2\pi n t}{T}} \int_0^{2\pi} exp\left(-i\frac{\omega}{c}\left[x(t)cos(\varphi)sin(\vartheta) + z_p(t)cos(\vartheta)\right]\right) d\varphi\, dt$$

$$= \frac{2\pi}{T}\int_0^T \vec{\beta}(t) J_0\left(\frac{\omega x(t) sin(\vartheta)}{c}\right) exp\left[i\left(\omega_n t - \frac{\omega z(t) cos(\theta)}{c}\right)\right] dt$$

$J_0(x)$ is the zero order Bessel function and integration over time must be done numerically.

Due to δ-function under the integral over angle ϑ, the spectrum is restricted by two limits $\omega_{min}=\omega_n/(1+\beta)$ and $\omega_{max}=\omega_n/(1-\beta)$,

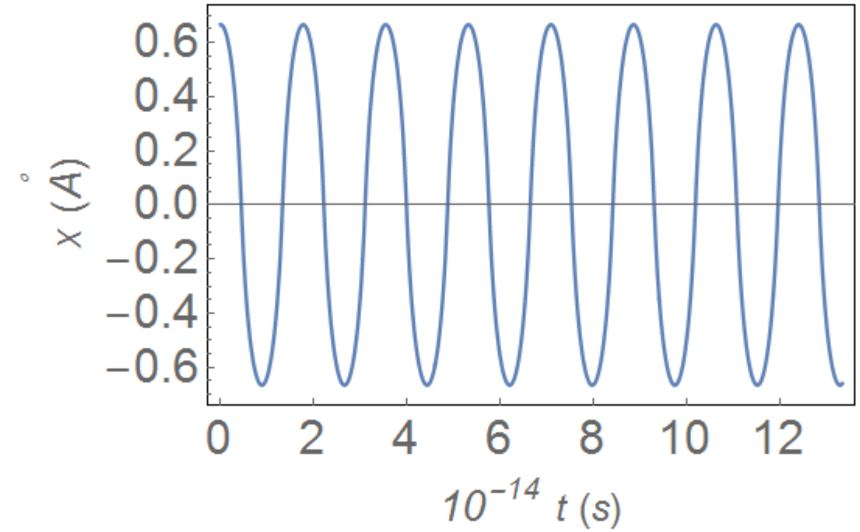
$$\frac{d^2\overline{E}}{d\omega} = \frac{e^2\omega^2}{2\pi c^2}\sum_{n=1}^{\infty}\int_{-1}^{1}\frac{I_n(\vartheta)}{\left(1-\overline{\beta}_z\cos(\vartheta)\right)}\delta\left(\omega - \frac{\omega_n}{\left(1-\overline{\beta}_z\cos(\vartheta)\right)}\right)d(\cos(\vartheta))$$

$$= \frac{e^2\omega}{2\pi c^2}\sum_{n=1}^{\infty} I_n(\theta_m) \qquad \theta_m = \cos^{-1}\left(\frac{1-\omega/\omega_n}{\overline{\beta}_z}\right) \qquad \omega_{min} \le \omega \le \omega_{max}$$

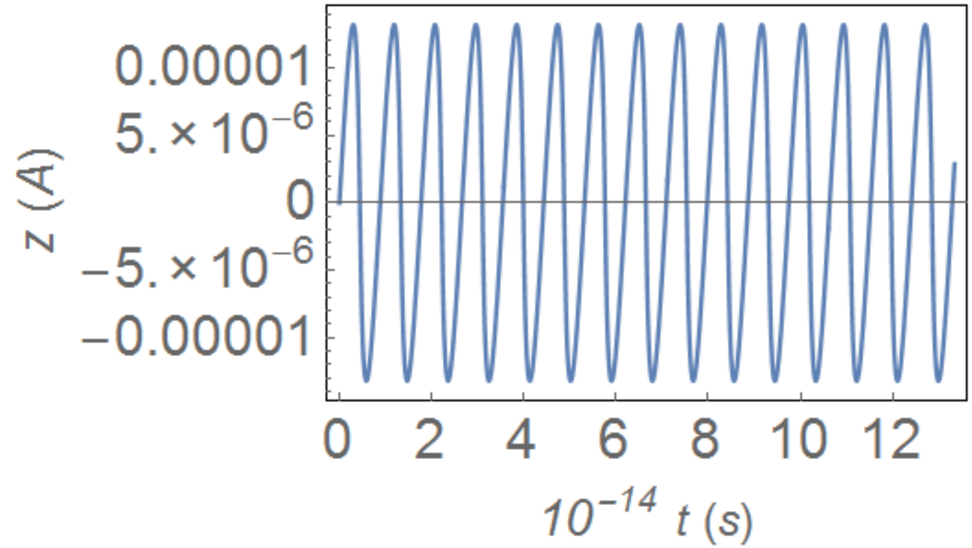
5. Non-dipole approximation

2 GeV electron
(110) plane of Si

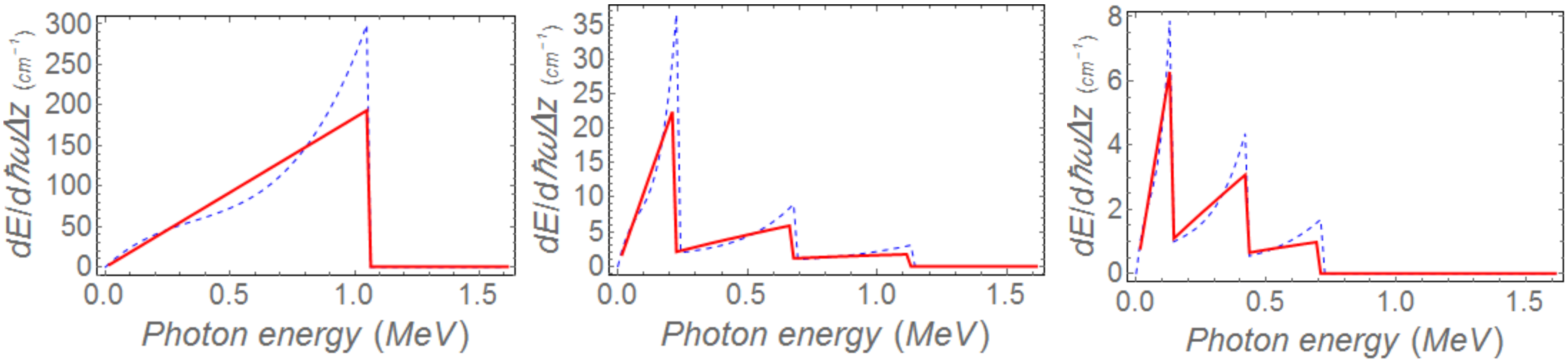
Transversal component ⟹



Longitudinal component ⟹

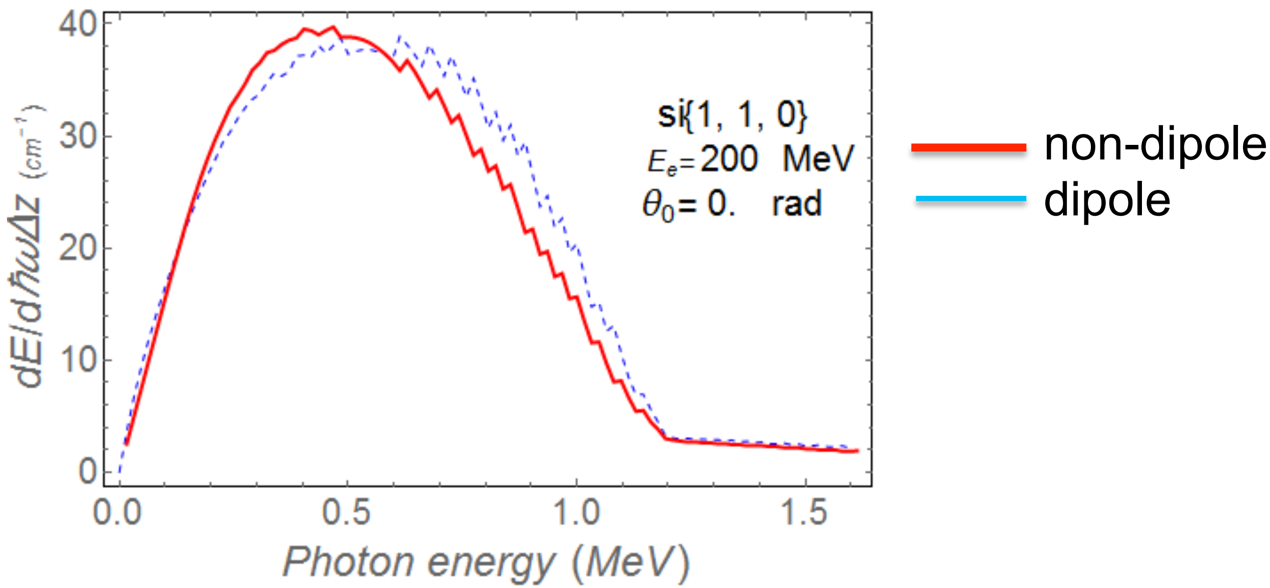


5. Comparison of non-dipole with dipol approximation E_e=200 MeV

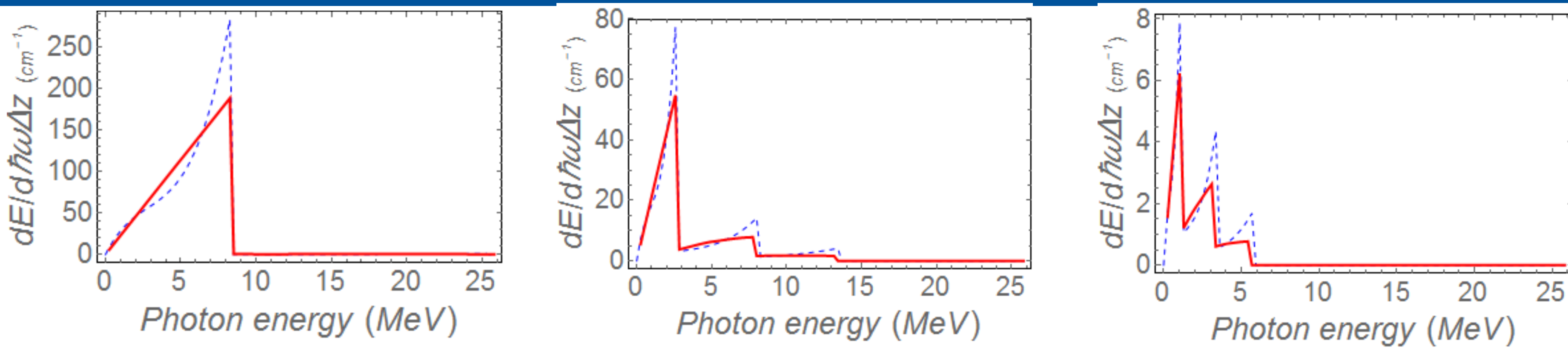


Radiation spectra for different incidence points

Total radiation spectra for 200 MeV electrons Si (110) plane

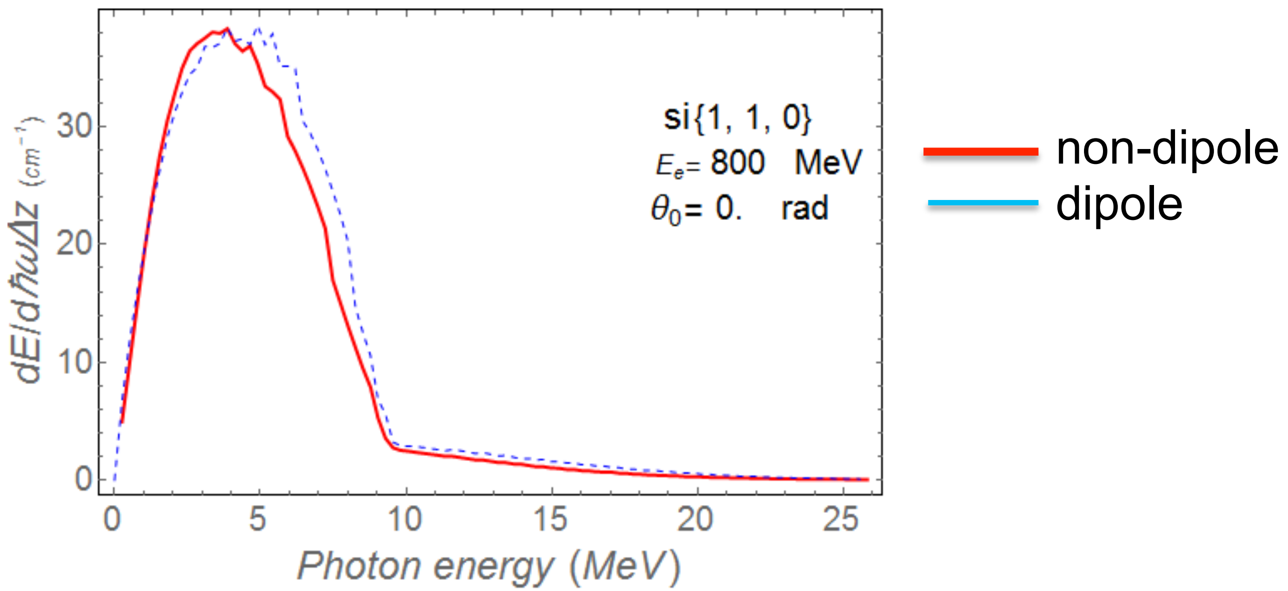


5. Comparison of non-dipole with dipol approximation
Ee=800 MeV

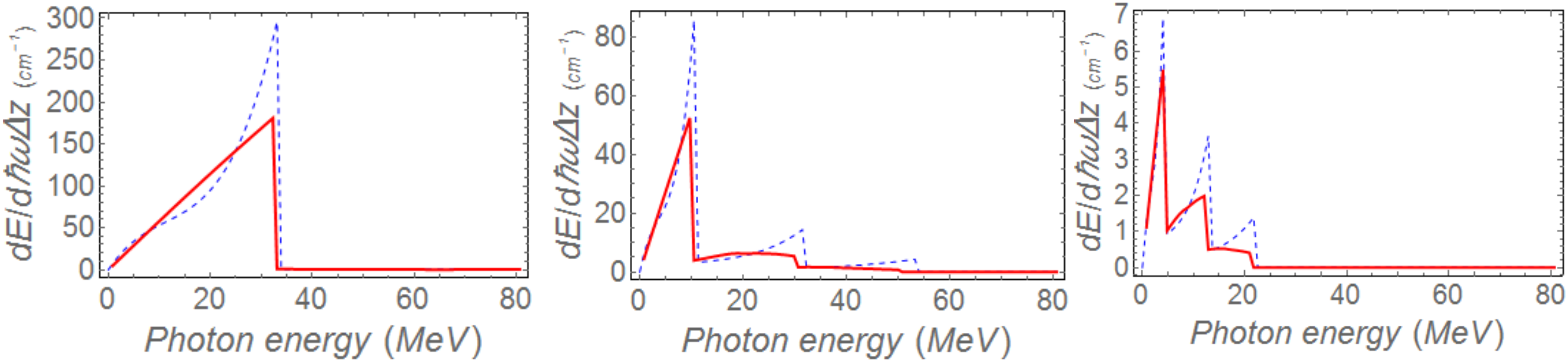


Radiation spectra for different incidence points

Total radiation spectra for 800 MeV electrons Si (110) plane

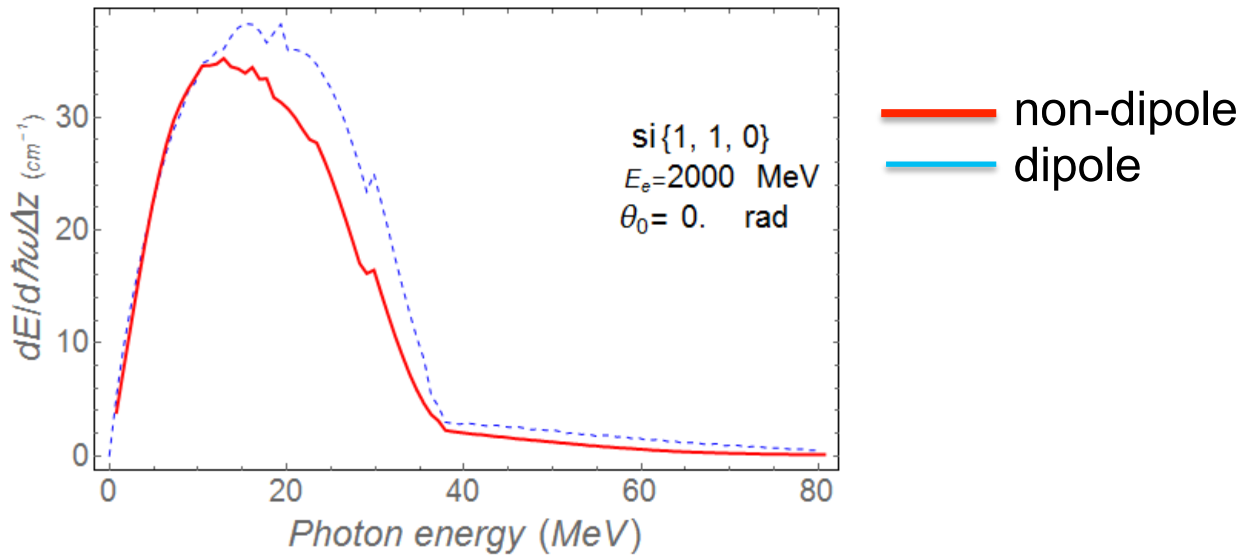


6. Comparison of non-dipole with dipol approximation
Ee=2 GeV

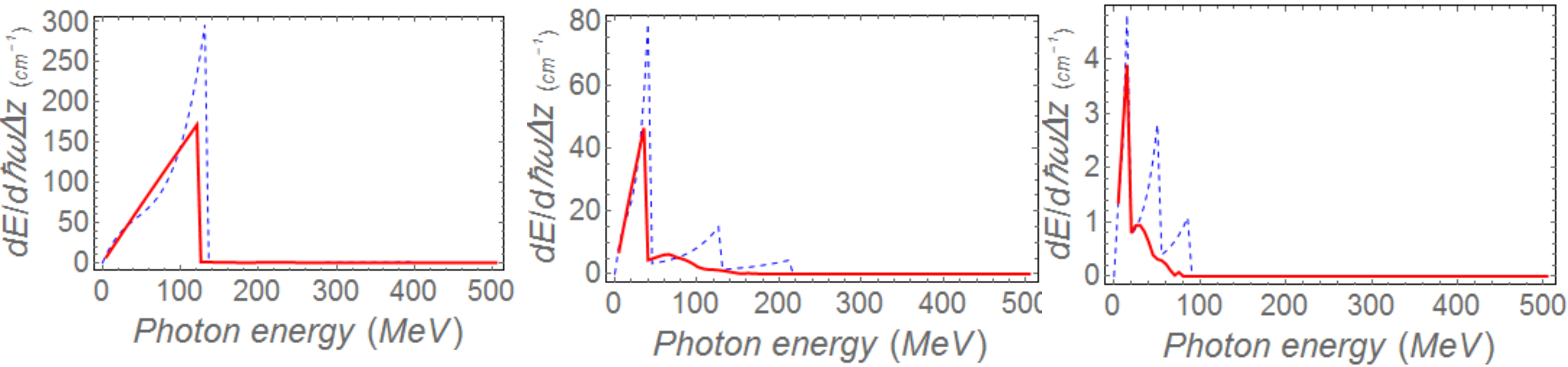


Radiation spectra for different incidence points

Total radiation spectra
for 2 GeV electrons
Si (110) plane

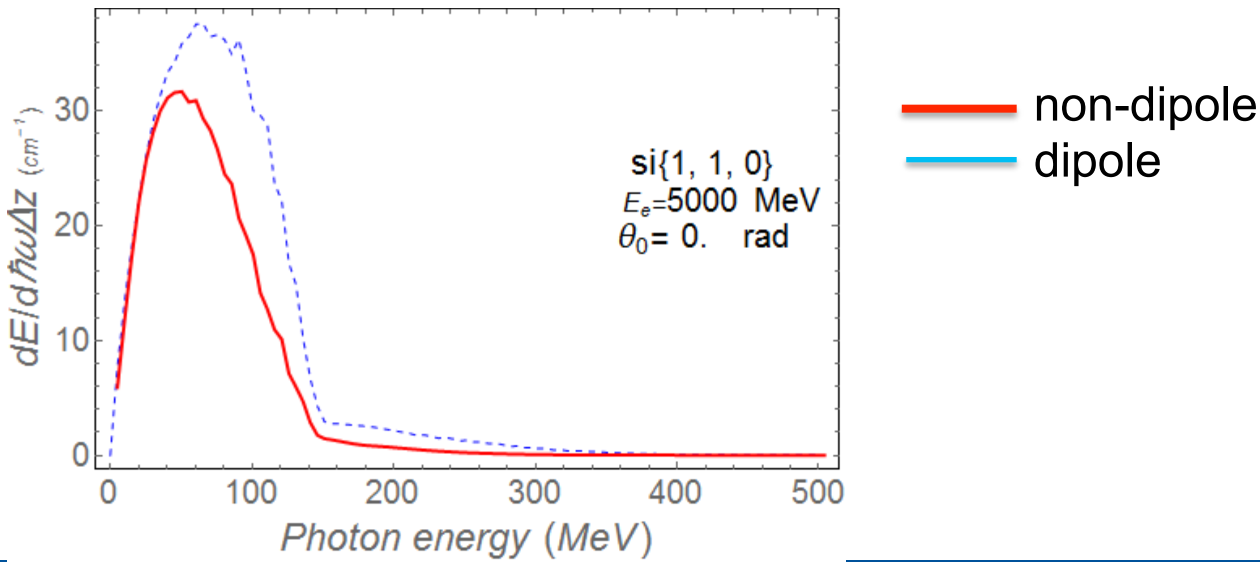


6. Comparison of non-dipole with dipol approximation
Ee=5 GeV



Radiation spectra for different incidence points

Total radiation spectra
for 5 GeV electrons
Si (110) plane



7. Summary

- We treated planar as well as axial channeling radiation at different energies and developed several software codes (*Mathematica*) appropriate for quantum as well as for classical calculations. Users can download the codes from the internet.
- We investigated the influence of non-dipolarity of channeling radiation. This effect can not be neglected at beam energies larger than about 1 GeV.
- This effect is also important for the simulation of positron production by means of channeling radiation.

Thank you