Coherent Radiation Characteristics of Microbunched Positron Bunch Formed in CU

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ASSUME THAT:

- LCLS positron bunch
- Modulated (microbunched) in SASE FEL (process)
- Falls in Crystalline Undulator (CU)
- At a zero angle (to the planes of monocrystal).

THE PROBLEM:

THE GENERATION OF THE COHERENT UNDULATOR RADIATION

- At a resonant frequency, equal to
- Minimal possible frequency formed in CU,
- Due to initial modulation of bunch.

- Why Modulated Bunch ?
- Why CU and CU Radiation ?
- Why Positron Bunch ?
- Why Radiation at a Resonant Frequency ?
- Why Equal to Minimal Possible Frequency ?

We Will Touch
This Questions
in
The End
of Presentation,

When
We Will Know
The Essence
of
The Presentation!

BUNCH RADIATION

- Photon Number Frequency-Angular Average Distribution is:

$$N_{tot}(\omega,\vartheta) = N_{sp}(\omega,\vartheta) + N_{coh}(\omega,\vartheta)$$

$N_s(\omega,\vartheta)$ ← For any source of radiation

F ← For freeform distribution of bunch

$$N_{sp}(\omega,\vartheta) = N_s(\omega,\vartheta)\cdot N_b(1-F)$$

$$N_{coh}(\omega,\vartheta) = N_s(\omega,\vartheta)\cdot {N_b}^2 F$$

- Where N_b is the number of bunch particles radiated

The Single Particle Radiation Part

$$N_{tot}(\omega,\vartheta) = \langle\frac{d^2 N_{ph}}{d\omega dO}\rangle$$

$$=$$

$$\frac{\alpha\omega\sqrt{\varepsilon}}{4\pi^2}\left|\int_0^T [\vec{n}\times\vec{\beta}(\tau)]e^{i(\omega\tau-\vec{k}\vec{r}(\tau))}\,d\tau\right|^2$$

$$\cdot$$

$$\langle\left|\sum_{j=1}^{N_b} e^{-\frac{\omega}{v_\parallel}z_j} e^{-ik_x x_j} e^{-ik_y y_j}\right|^2\rangle$$

$$=$$

$$N_s(\omega,\vartheta)\cdot\left(N_b^2 F + N_b(1-F)\right)$$

Form Factor, Longitudinal and Transverse Form Factors of Bunch

$$F = F_Z(\omega) \cdot F_R(\omega, \vartheta)$$

$$F_Z(\omega) = \left| < e^{(-ik_{\parallel}Z)} > \right|^2 = \left| \int f(Z)\, e^{(-ik_{\parallel}Z)} dZ \right|^2$$

$$F_R(\omega) = \left| < e^{(-ik_{\parallel}R)} > \right|^2 = \left| \int f(R)\, e^{(-ik_{\perp}R)}\, dR \right|^2$$

- There are probability density functions (bunch particle density distribution functions)
- And probability functions (form factors of bunch)
- On longitudinal Z (movement direction) and transverse R directions.

STEPS OF SOLVING THE PROBLEMS OF COHERENT RADIATION

- Is necessary the availability of radiation (at least a single particle radiation):

$N_s \neq 0 \quad \text{or} \quad N_{sp} \neq 0$

- Is provided the coherence condition:

$N_b F \gg 1 \quad \text{or} \quad F \to 1 \quad \Leftarrow \quad N_b \gg 1$

- Due to bunch distribution, takes place spontaneous radiation amplification:

$N_{tot} = N_s \cdot N_b (1 + (N_b - 1)F) \cong N_{sp} \cdot (1 + K)$

$K = N_b F = N_b \quad \Leftarrow \quad F \to 1$

Modulated Bunch

- Is used the model of bunch
- With the initial Gaussian density distributions:

$$f_0(Z) = \frac{1}{\sqrt{2\pi}\sigma_Z} e^{-\frac{1}{2}\cdot\left(\frac{Z}{\sigma_Z}\right)^2}$$

$$f(R) = \frac{1}{\sqrt{2\pi}\sigma_R} e^{-\frac{1}{2}\cdot\left(\frac{R}{\sigma_R}\right)^2}$$

- And microbunched by harmonic law:

$$f_M(Z) = b\cos\left(\frac{\omega_r}{V} Z\right)$$

Modulated Bunch Form Factor

$$f(Z) = f_0(Z) \cdot f_M(Z)$$

$$f(Z) = \frac{1}{\sqrt{2\pi}\sigma_Z} e^{-\frac{1}{2}\cdot\left(\frac{Z}{\sigma_Z}\right)^2} \cdot b \cos\left(\frac{\omega_r}{V} Z\right)$$

- b is the modulation depth
- ω_r is the resonant frequency (frequency of modulation)

$$F = F_Z \cdot F_R = e^{-\left(\frac{(\omega-\omega_r)\sigma_Z}{c}\right)^2} \cdot e^{-\left(\frac{\omega\sigma_R\vartheta}{c}\right)^2}$$

It is taken into account, that

- The radiation angle ϑ is small, and
- Is ignored the summand containing the expression $\omega + \omega_r$ at exponent.

CU Radiation

- In CU, except to the channeling radiation takes place CU radiation due to the positron oscillation by trajectory of "periodic curve" of crystal.
- Channeling preservation condition in CU

$$\Theta_{max} = \frac{2\pi A}{l} < \Theta_L = \sqrt{\frac{2\nu}{\gamma}}$$

- Takes place due to the choice of CU parameters
  - A , l are the amplitude and space period of CU
  - ν , γ are potential depth of planar channel and bunch energy (Lorentz factor) in terms of positron rest energy
  - Θ_{max} is the maximal bending angle
  - Θ_L is the Lenhardt's angle

CU Radiation Characteristics

- Photon Number Frequency-Angular Distribution for Single Particle, Due to the Medium Polarization, is:

$$N_s(\omega,\vartheta) = \pi\alpha\left(\frac{qn}{4\gamma}\right)^2 \frac{\omega}{\omega_{min}}\phi(\omega,\vartheta)\cdot$$
$$\cdot\left(1+\left(1-(\vartheta\gamma_p)^2\frac{\omega}{2\omega_{min}}\right)^2\right)$$

- $\alpha = 1/137$ is the fine structure constant
- $q = \gamma\Theta_{max}$ is the CU parameter
- $\gamma = E/mc^2$ is the Lorentz factor, $\gamma_p = \omega_p/\Omega = l/\lambda_p$
- $n = L/l$ is the CU period number
- l is the CU space period, L is the CU length
- $\Omega = 2\pi V/l$: CU frequency; $\omega_p(\lambda_p)$: plasma frequency.

Frequency Interval & Threshold Energy

$$\phi(\omega,\vartheta) = \frac{\sin^2 Z}{Z^2}$$

$$Z(\omega,\vartheta) = \pi n \frac{\omega}{\Omega} \cdot \left(\vartheta^2 - \frac{\Omega^2}{\omega_p^2} \left(1 - \frac{\omega_{min}}{\omega}\right) \left(\frac{\omega_{max}}{\omega} - 1\right) \right)$$

- At $Z = 0 \quad \phi = 1$
- $\omega_{min} \leq \omega \leq \omega_{max}$

- $Q = 1 + q^2/2$
- $\gamma_{th} = \gamma_p \sqrt{Q}$

$$\omega_{\substack{min \\ max}} = \frac{\Omega \gamma^2}{Q} \left(1 \mp \sqrt{1 - \left(\frac{\gamma_{th}}{\gamma}\right)^2} \right)$$

Photon Distribution at Minimal Frequency

- When $\gamma \gg \gamma_{th}$

  $\omega_{min} = {\omega_p}^2/(2\Omega)$ or $\lambda_{max} = 2{\lambda_p}^2/l$

  $\omega_{max} = 2\Omega\gamma^2/Q$

- Photons with boundary frequencies is radiated at a zero angle. ⟸ $Z = 0$
- X-ray radiation is generated at a zero angle with minimal possible frequency, due to CU parameters.
- For minimal possible frequency we have:

$$N_s(\omega_{min}, 0) = 2\pi\alpha\left(\frac{qn}{4\gamma}\right)^2$$

Coherent Radiation Linewidth

- If choose the spatial period CU:

$$l = 2{\lambda_p}^2/\lambda_r$$

- Then : $\omega_{min} = \omega_r$ ⇐ $\lambda_{max} = 2{\lambda_p}^2/l$
- And :

$$F_R = 1 \qquad F_Z = e^{-\left(\frac{\omega_r \sigma_Z}{c}\right)^2 \left(\frac{\omega}{\omega_r} - 1\right)^2}$$

- Coherent radiation distribution is defined by distribution of F_Z .
- Linewidth of coherent radiation :

$$\frac{\Delta\omega}{\omega} = \frac{\lambda_r}{2\pi\sigma_Z}$$

Coherent Radiation

- Total number and gain of coherent radiation:

$$N_{tot} = \sqrt{\pi}\alpha\frac{\lambda_r}{\sigma_z}\left(\frac{qnN_b}{8\gamma}\right)^2 b^2$$

$$K = \frac{N_{tot}}{N_s} = \frac{N_b\lambda_r}{8\sqrt{\pi}\sigma_z}b^2 = \left(\frac{b}{b_c}\right)^2$$

- Critical value of modulation depth for generation of coherent radiation:

$$b \gg b_c = \sqrt{\frac{8\sqrt{\pi}\sigma_z}{N_b\lambda_r}} \quad \Rightarrow \quad K \gg 1$$

Parameters

Parameters of CU

$(1,1,0)$ *the channeling planes*

of diamond monocrystal

$d = 1.26$ Å

$\nu = 4.87 \cdot 10^{-5} \quad (U_0 = 25\ eV)$

$\lambda_p = 3.26 \cdot 10^{-6} cm \quad (\hbar\omega_p = 38\ eV)$

Parameters

Parameters of LCLS Bunch

$N_b = 1.56 \cdot 10^9$

$\sigma_z = 9 \cdot 10^{-4} cm$

$E = 13.6\ GeV$ $(\gamma = 2.66 \cdot 10^4)$

Choice

Selected parameters of CU

$$q = 1.5 \qquad (A = 1.27\ \text{Å})$$

$$L = 1\ cm$$

$$(n = 704)$$

$$l = 14.2 \cdot 10^{-4} cm$$

$$q_{ch} = \sqrt{2\nu\gamma} = 1.6$$

Result

Generated Coherent Radiation Characteristics

$$N_{tot} = 2.43 \cdot 10^{12} b^2$$

$$K = 1.84 \cdot 10^{3} b^2$$

$$\frac{\Delta\omega}{\omega} = 5.3 \cdot 10^{-6}$$

$$b_c = 2.3 \cdot 10^{-2}$$

EXPERIMENT

IS PROPOSED THE EXPERIMENT ACCORDING TO AFOREMENTIONED PARAMETERS

- If the gain of radiation will be found, then we can determine an important parameter b_c .
- O therwise, its upper limit b_c .

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This All Questions

Have

The Same Answer:

For Generation of

➲X-ray Coherent Radiation

➲!

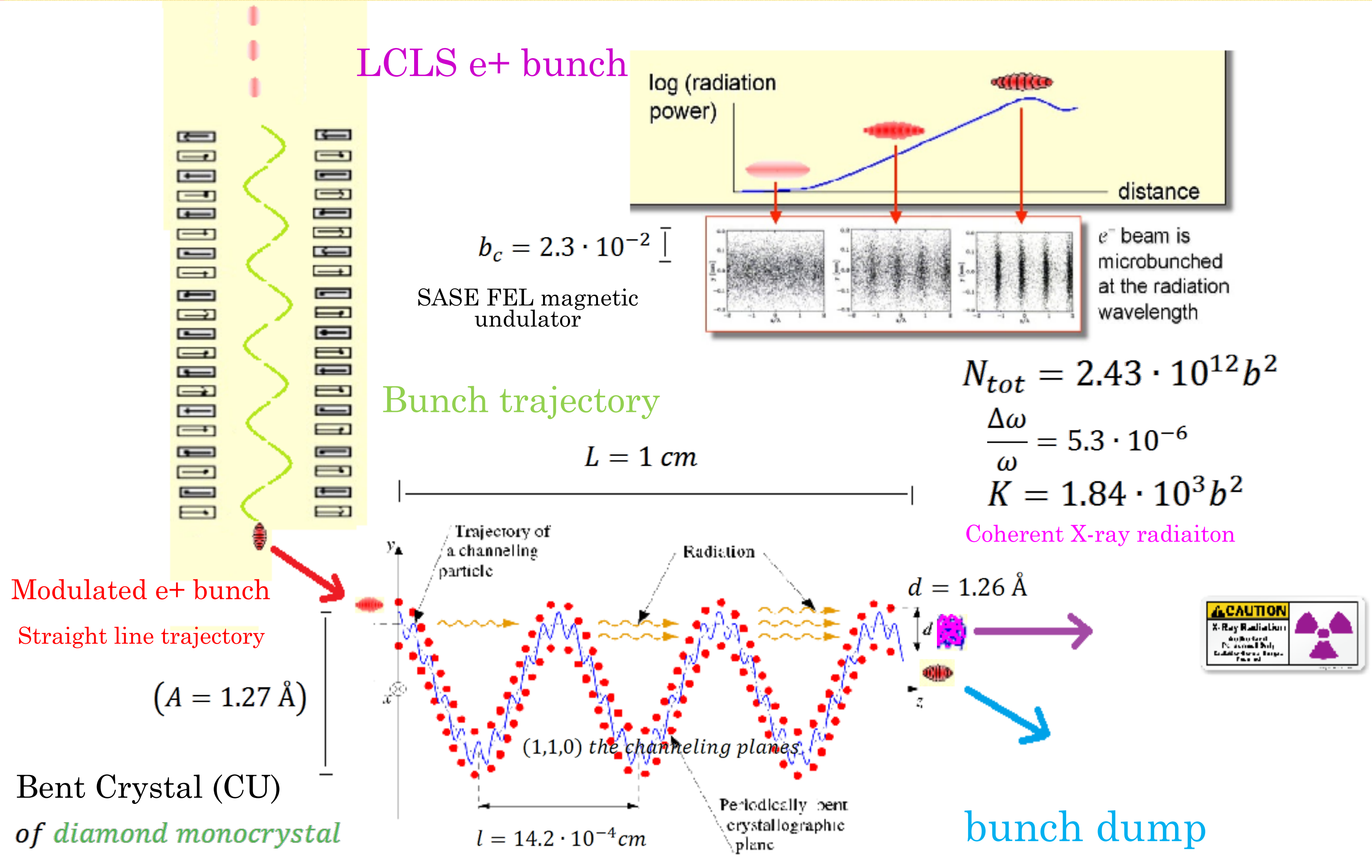

Illustration of Experiment

THANK YOU

∞

FOR YOUR

∞

ATTENTION !

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I'm Free for Questions !

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