Diffracted transition radiation of a beam of relativistic electrons in a thin single-crystal plate

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In the present work, diffracted transition radiation (DTR) of the beam of relativistic electrons crossing a thin single-crystal plate has been considered. The expression for the DTR angular density has been derived for the case when the path of the electron in the target is considerably less than extinction length. The kinematic character of DTR of the beam of ultra-relativistic electrons crossing a thin single-crystal plate has been proved. The numerical calculation carried out shows the considerable influence of divergence of the beam on the angular density of DTR at high and super high energies of the electrons.

1. Geometries of the radiation processes

Let us consider a beam of relativistic electrons crossing a monocrystalline plate in Bragg (figure 1) scattering geometries.

$$\mathbf{V} = \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\psi^2\right)\mathbf{e}_1 + \boldsymbol{\psi},$$

$$\mathbf{e}_1\boldsymbol{\psi} = 0, \quad \mathbf{n} = \left(1 - \frac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \boldsymbol{\theta}_0, \tag{1}$$

$$\mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B, \quad \mathbf{n}_\mathbf{g} = \left(1 - \frac{1}{2}\theta^2\right)\mathbf{e}_2 + \boldsymbol{\theta}, \quad \mathbf{e}_2\boldsymbol{\theta} = 0,$$

$$\boldsymbol{\theta} = \boldsymbol{\theta}_\parallel + \boldsymbol{\theta}_\perp, \quad \boldsymbol{\theta}_0 = \boldsymbol{\theta}_{0\parallel} + \boldsymbol{\theta}_{0\perp}, \quad \boldsymbol{\psi} = \boldsymbol{\psi}_\parallel + \boldsymbol{\psi}_\perp .$$

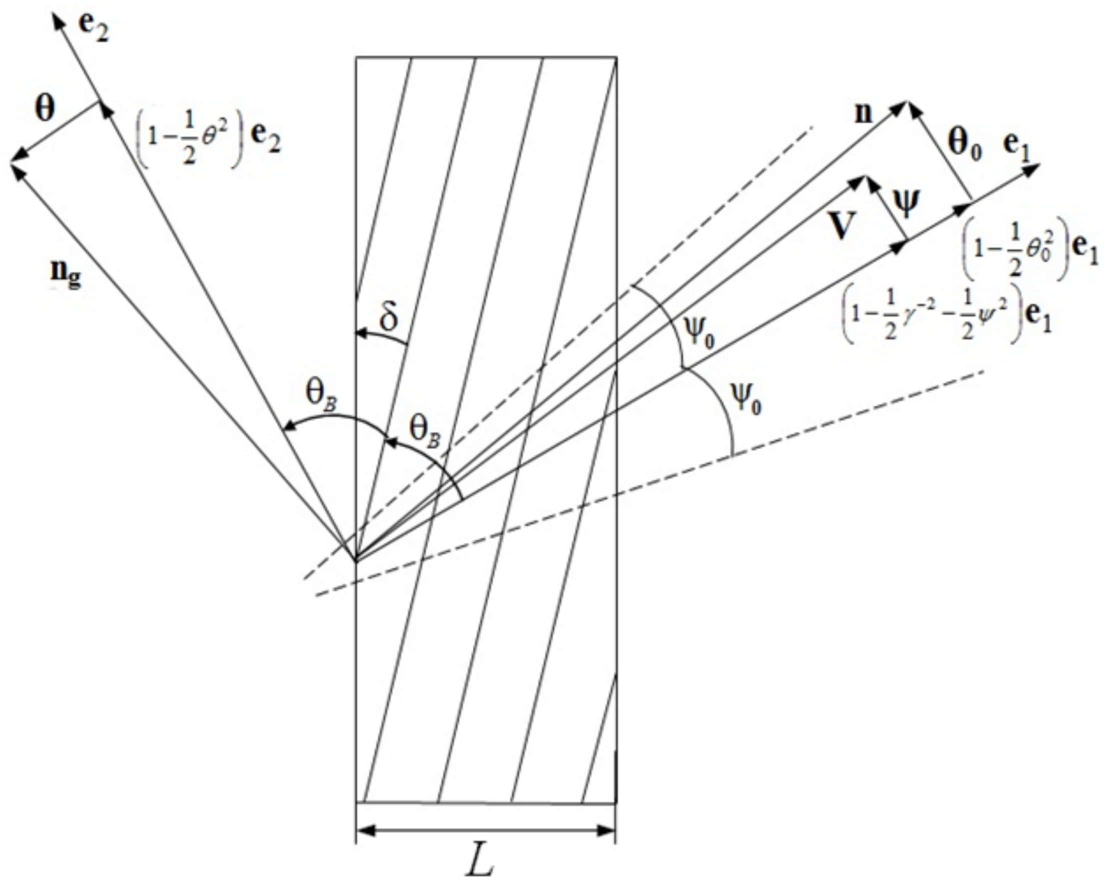

Figure 1. Bragg scattering geometry.

Let us consider the electromagnetic processes in crystalline medium, which are characterized by a complex permittivity

$$\varepsilon(\omega, \mathbf{r}) = 1 + \chi(\omega, \mathbf{r}), \quad (2)$$

where $\chi(\omega, \mathbf{r}) = \chi_0(\omega) + \sum_{\mathbf{g}}{}' \chi_{\mathbf{g}}(\omega) \exp(i\mathbf{gr})$, $\chi(\omega, \mathbf{r})$ is the dielectric susceptibility, $\chi_{\mathbf{g}}(\omega) = \chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega)$ is the Fourier coefficient of the expansion of the dielectric susceptibility of the crystal into series over the reciprocal lattice vectors $\mathbf{g}$, and $\chi_0(\omega)$ is the average dielectric susceptibility.

In the present work the two-wave approach of dynamic diffraction theory are used, in which both the incident and diffracted waves are considered as equitable in process of self repumping one into another in crystalline target.

2. Spectral-angular density DTR in a thin crystal

Let us consider the relativistic electron coherent X-radiation in Bragg scattering geometry (see Fig.1). If we perform the analytical procedures similar to those used in
[S.V. Blazhevich, A.V. Noskov. Coherent X-radiation of relativistic electron in a single crystal under asymmetric reflection conditions // *Nucl. Instr. and Meth. B.* 266 (2008) 3770.]
we will obtain the expressions for the spectral-angular density DTR for the propagation direction of the emitted photon $\mathbf{k}_{\mathbf{g}} = k_{\mathbf{g}}\mathbf{n}_{\mathbf{g}}$ (see Fig.1) taking into account the direction deviation of the electron velocity $\mathbf{V}$ relative to the electron beam axis $\mathbf{e}_1$:

$$\omega \frac{d^2 N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \Omega^{(s)^2} \times$$

$$\left(\frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2} - \frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi'_0} \right)^2 \times \tag{3}$$

$$\times \frac{\varepsilon^2}{\xi^{(s)^2} - \left(\xi^{(s)^2} - \varepsilon\right) \coth^2\left(\dfrac{b^{(s)} \sqrt{\varepsilon - \xi^{(s)^2}}}{\varepsilon} \right)},$$

where the notations are used:

$$\Omega^{(1)} = \theta_{\perp} - \psi_{\perp}, \quad \Omega^{(2)} = \theta_{//} + \psi_{//}, \quad \varepsilon = \frac{\sin(\theta_B - \delta)}{\sin(\theta_B + \delta)}, \; b^{(s)} = \frac{1}{2\sin(\theta_B + \delta)} \frac{L}{L_{ext}^{(s)}},$$

$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1+\varepsilon}{2\nu^{(s)}}, \quad \eta^{(s)}(\omega) = \frac{2\sin^2\theta_B}{V^2\left|\chi'_{\mathbf{g}}\right|C^{(s)}}\left(1 - \frac{\omega(1-\theta_{//}\cot\theta_B)}{\omega_B}\right), \quad \nu^{(s)} = \frac{\chi'_{\mathbf{g}}C^{(s)}}{\chi'_0},$$

$$L_{ext}^{(s)} = 1/\omega\left|\chi'_{\mathbf{g}}\right|C^{(s)}. \tag{4}$$

The parameter ε is an important parameter which determines the degree of asymmetry of the reflection of the radiation field in a crystal plate with respect to the target surface.

Parameter $b^{(s)}$ characterizing the thickness of the crystal plate is the ratio of half path of the electron in the target $L_e = L/(\theta_B + \delta)$ to the extinction length $L_{ext}^{(s)}$.

To find the angular density of DTR let us integrate the expression (3) over the frequency function $\xi^{(s)}(\omega)$ using the relation $\frac{d\omega}{\omega}=-\frac{\left|\chi'_{\mathbf{g}}\right|C^{(s)}}{2\sin^2\theta_B}d\xi^{(s)}$. As the result we will obtain the expression describing the angular densities of DTR in Bragg scattering geometries:

$$\frac{dN_{\mathrm{DTR}}^{(s)}}{d\Omega}=\frac{e^2\left|\chi'_{\mathbf{g}}\right|C^{(s)}}{2\pi^2\sin^2\theta_B}\Omega^{(s)2}\times$$
$$\left(\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2}-\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi'_0}\right)^2\times \tag{5}$$
$$\int_{-\infty}^{\infty}\frac{\varepsilon^2}{\xi^{(s)^2}-\left(\xi^{(s)^2}-\varepsilon\right)\coth^2\left(\frac{b^{(s)}\sqrt{\varepsilon-\xi^{(s)^2}}}{\varepsilon}\right)}d\xi^{(s)}(\omega),$$

Let us consider the extreme cases when the electron path in the target expressed in extinction length is $b^{(s)} << \sqrt{\varepsilon}$ or $b^{(s)} >> \sqrt{\varepsilon}$, which can be written as $\frac{L_e}{L_{ext}^{(s)}} << 2\sqrt{\varepsilon}$, $\frac{L_e}{L_{ext}^{(s)}} >> 2\sqrt{\varepsilon}$.

Because the parameter ε in the real experiments possesses the value $0.5 < \varepsilon < 3$ the pointed inequalities practically correspond to inequalities $L_e << L_{ext}^{(s)}$, $L_e >> L_{ext}^{(s)}$

The approximations of integral of DTR spectra for this extreme cases we will obtain in the following form

$$\int_{-\infty}^{\infty} \frac{\varepsilon^2}{\xi^{(s)^2} - \left(\xi^{(s)^2} - \varepsilon\right)\coth^2\left(\frac{b^{(s)}\sqrt{\varepsilon - \xi^{(s)^2}}}{\varepsilon}\right)} d\xi^{(s)}(\omega) = \\ = \varepsilon\sqrt{\varepsilon}\pi \cdot \tanh\left(\frac{b^{(s)}}{\sqrt{\varepsilon}}\right) \approx \begin{cases} \varepsilon\sqrt{\varepsilon}\pi, & b^{(s)} >> \sqrt{\varepsilon} \\ \pi\varepsilon b^{(s)}, & b^{(s)} << \sqrt{\varepsilon} \end{cases} \tag{6}$$

Using the obtained approximations, we will derive the angular densities of DTR in Bragg scattering geometries for conditions $b^{(s)} << \sqrt{\varepsilon}$:

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{b^{(s)}<<\sqrt{\varepsilon}} = \frac{e^2\omega_B\chi_{\mathbf{g}}'^{\,2}C^{(s)\,2}}{4\pi\sin^2\theta_B}\,\Omega^{(s)2}\times$$

$$\left(\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2}-\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi_0'}\right)^2\varepsilon\frac{L}{\sin(\theta_B+\delta)}. \tag{7}$$

In the case when $b^{(s)} >> \sqrt{\varepsilon}$ the expression (5) will take the form

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{b^{(s)}>>\sqrt{\varepsilon}} = \frac{e^2\left|\chi_{\mathbf{g}}'\right|C^{(s)}}{2\pi\sin^2\theta_B}\Omega^{(s)2}\times$$

$$\left(\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2}-\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi_0'}\right)^2\varepsilon\sqrt{\varepsilon}\,. \tag{8}$$

In case of enough high energy of electrons ($\gamma >> \frac{1}{\sqrt{|\chi_0'|}}$) for the beams where the magnitude of deviation angle of electron in the beam $\boldsymbol{\psi}(\psi_\perp, \psi_{//})$ is less or close to characteristic value of angle of DTR angular density maximum γ^{-1}, i.e. when the condition $\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 << -\chi_0'$ is fulfilled, the expressions (7) and (8), describing the angular density of DTR go correspondently over

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{\substack{b^{(s)} << \sqrt{\varepsilon} \\ \gamma >> \frac{1}{\sqrt{|\chi_0'|}}}} = \frac{e^2 \omega_B \chi_{\mathbf{g}}'^{\,2} C^{(s)\,2}}{4\pi \sin^2 \theta_B} \frac{\Omega^{(s)\,2}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2\right)^2} \varepsilon \frac{L}{\sin(\theta_B + \delta)} \qquad (9)$$

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{\substack{b^{(s)} >> \sqrt{\varepsilon} \\ \gamma >> \frac{1}{\sqrt{|\chi_0'|}}}} = \frac{e^2 \left|\chi_{\mathbf{g}}'\right| C^{(s)}}{2\pi \sin^2 \theta_B} \frac{\Omega^{(s)\,2}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2\right)^2} \varepsilon\sqrt{\varepsilon}\,. \qquad (10)$$

Let us note that the condition $b^{(s)} << \sqrt{\varepsilon}$ meaning that the length of electron path in the target is considerably less than the extinction length of x-ray waves in the crystal $L_e << L_{ext}^{(s)}$ completely excludes the repumping of the incident and diffracted waves in each other. We can see also that angular density in formula (9) is proportional to target thicknesses L that is typical of kinematic approaching in radiation process description.

In fig. 2 the curves plotted according to (5) and (9) for a relativistic electron crossing a thin ($L = 0.2\ \mu m$) monocrystalline plate of silicon Si (111), under conditions $\gamma >> \frac{1}{\sqrt{|\chi_0'|}}$ and $b^{(s)} << \sqrt{\varepsilon}$, notably under $\gamma \approx 10000$ and $\frac{1}{\sqrt{|\chi_0'|}} \approx 258$ are shown. One can see that the angular densities of DTR calculated by formulas (5) and (9) practically coincide. The calculations were carried out for σ-polarized ($s = 1$) waves under condition $\theta_{//} = 0$. The angular density of the radiation was calculated for a separated electron moving along the axis of the electron $\mathbf{e}_1$ ($\psi = 0$) (see in fig.1).

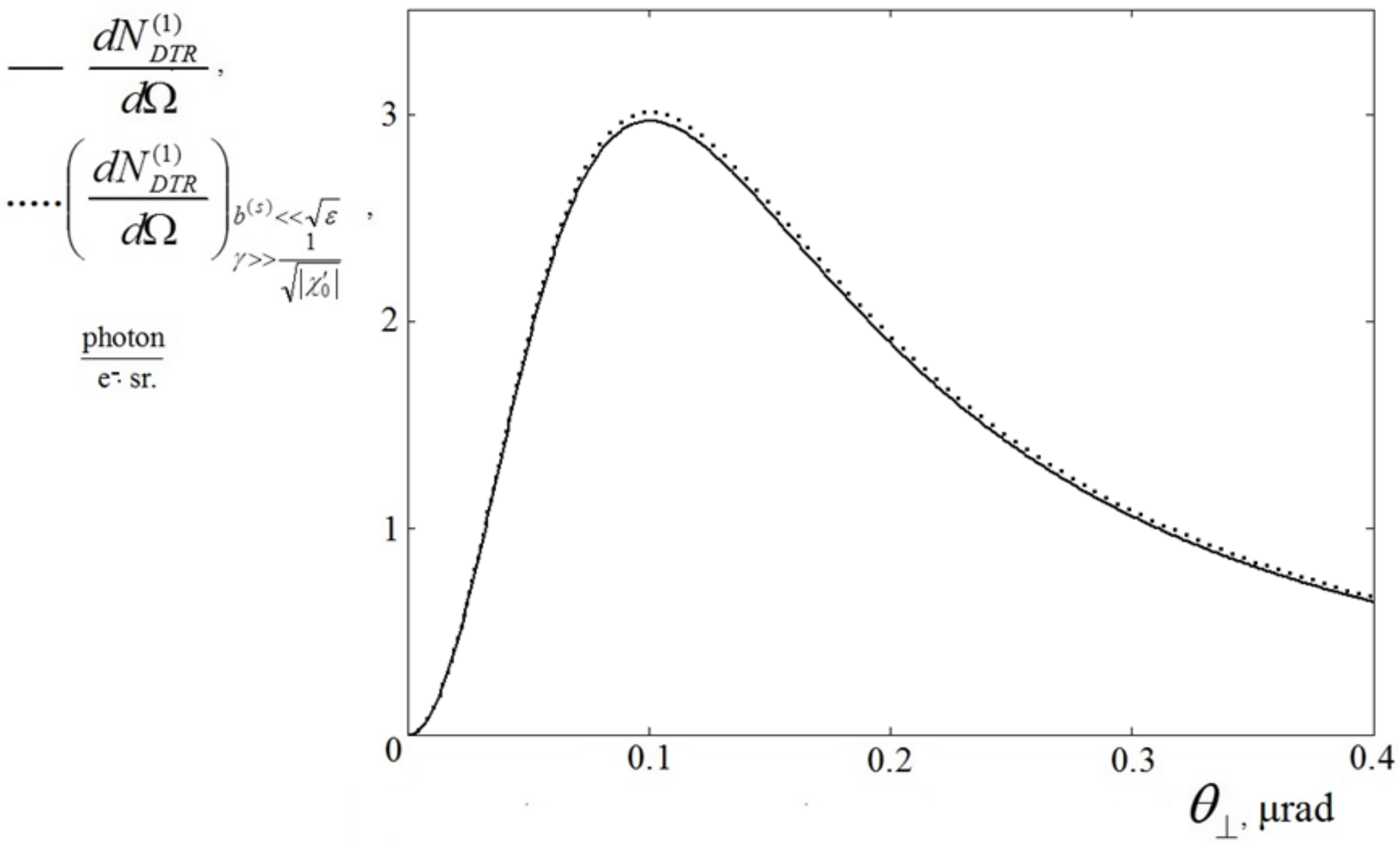

Fig. 2. The angular density of the DTR generated by a relativistic electron in the beam incident in direction of the beam axis. The curves are plotted by dynamical formula (5) (solid) and by its asymptotic kinematic formula (9) (dashed). $Si(111)$, $L = 0.2\,\mu m$, $\theta_B = 14.5^o$, $\delta = -7^o$, $\varepsilon = 3$ $\omega_B = 8$ кэВ, $E_e = 5$ ГеВ, $\psi = 0$, $\theta_{//} = 0$; $b^{(s)} \approx 0.264$, $\frac{1}{\sqrt{|\chi'_0|}} \approx 258$.

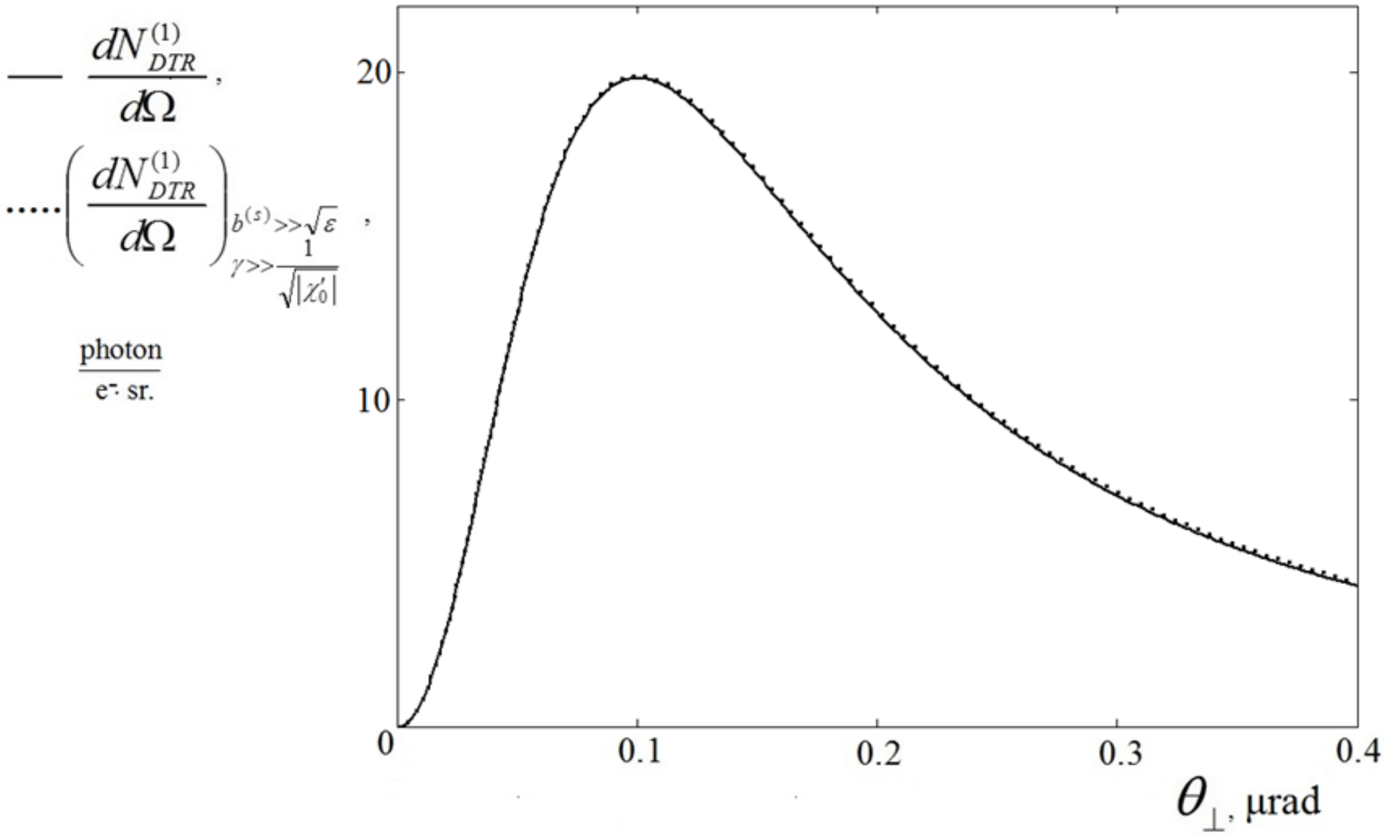

Fig.3. The angular density of the DTR generated by a relativistic electron in the beam incident in direction of the beam axis. The curves are plotted by dynamical formula (5) (solid) and by its asymptotic kinematic formula (10) (dashed). $Si(111)$, $L = 5\,\mu m$, $\theta_B = 14.5^o$, $\delta = -7^o$, $\varepsilon = 3$ $\omega_B = 8$ кэВ, $E_e = 5$ ГеВ, $\psi = 0$, $\theta_{//} = 0$; $b^{(s)} \approx 6.59$, $\frac{1}{\sqrt{|\chi'_0|}} \approx 258$.

4. Influence of divergence of electron beam on DTR angular density

Let us consider the influence of the divergence of the beam of relativistic electrons crossing a thin monocrystalline target on the angular density of DTR. For this purpose, we will average the expressions for angular density of the radiation generated by one of electrons over all its possible straight trajectories in the beam. As an example, let us carry out the averaging of the DTR density (9) using the Gauss distribution function

$$f(\psi) = \frac{1}{\pi\psi_0^2} e^{-\frac{\psi^2}{\psi_0^2}}, \qquad (11)$$

where ψ_0 parameter would mean a divergence of the radiating electron beam (see in fig. 1). The angle ψ_0 defines the cone limiting the part of the electron beam outside of which the density is reduced by more than three times in comparison with the density on the beam axis.

For this distribution function the expression for averaged angular density of DTR generated by the electron beam normalized per number of electrons in the beam will take the form

$$\left\langle \frac{dN_{DTR}^{(s)}}{d\Omega} \right\rangle_{\substack{b^{(s)} \ll \sqrt{\varepsilon} \\ \gamma \gg \frac{1}{\sqrt{|\chi'_0|}}}} = \frac{e^2 \omega_B \chi'^2_{\mathbf{g}} C^{(s)^2}}{4\pi \sin^2 \theta_B} \frac{\varepsilon L}{\sin(\theta_B + \delta)} \frac{1}{\pi \psi_0^2} \times$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\Omega^{(s)^2} e^{-\frac{\psi^2}{\psi_0^2}}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2\right)^2} d\psi_\perp d\psi_{//}. \tag{12}$$

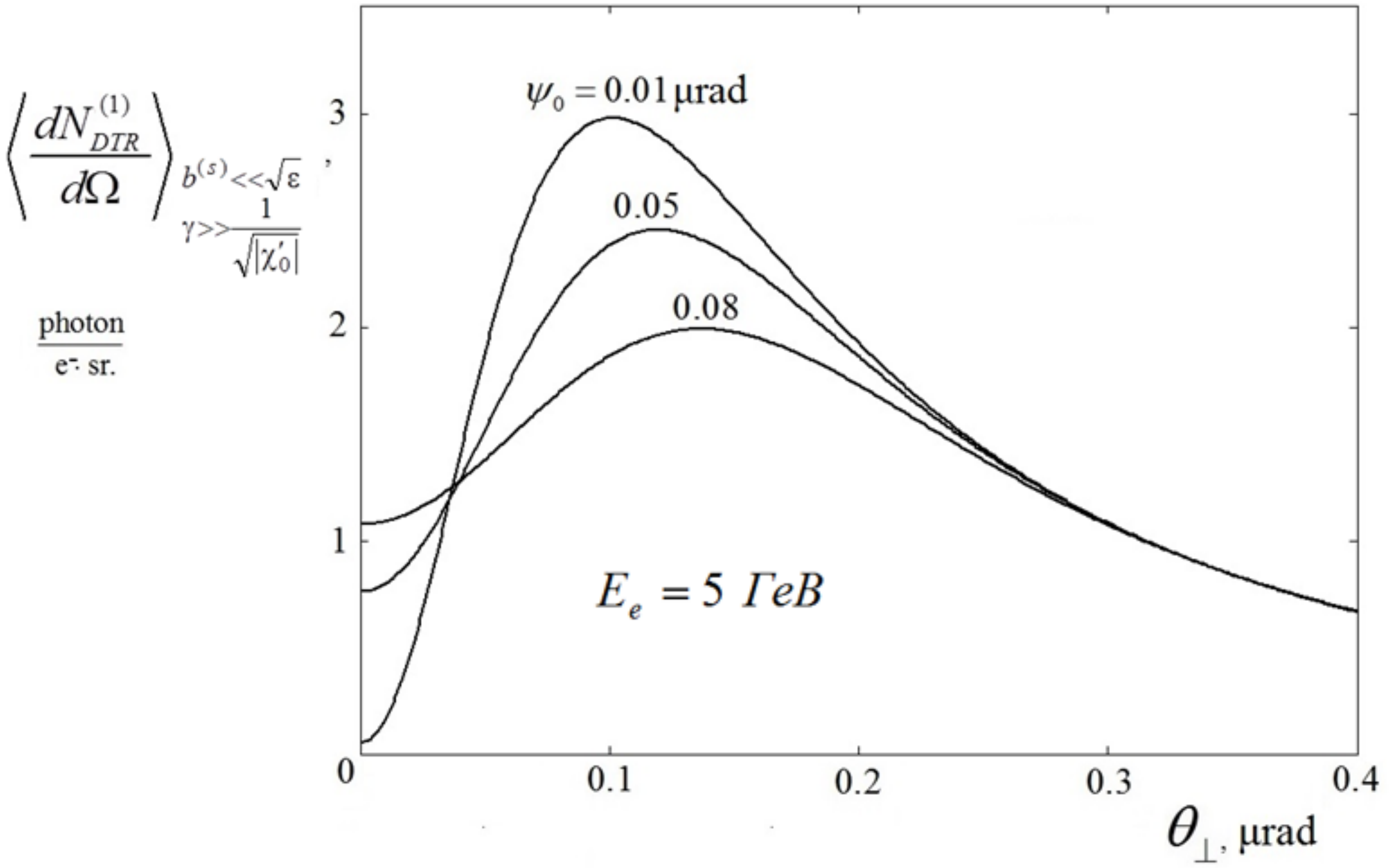

Fig. 4. Influence of the DTR angular density on the relativistic electron beam divergence (all the parameters are the same as in fig. 2).

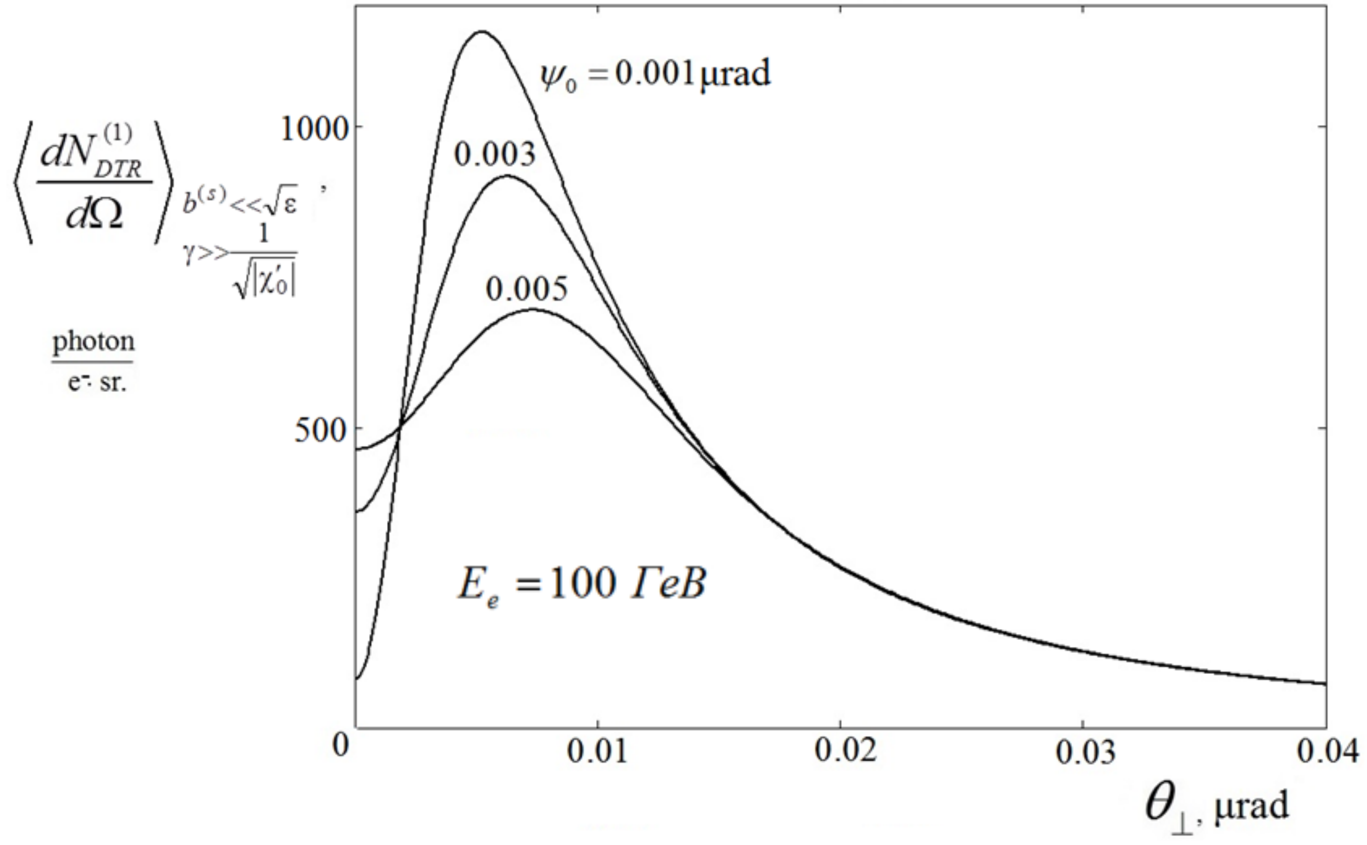

Fig. 5. Influence of the relativistic electron beam divergence on the DTR angular density (all the parameters are the same as in fig. 2, with the exception of the electron energy $E_e = 100$ $ГeB$)

5. Conclusion

In the present work the diffracted transition radiation (DTR) of the beam of super relativistic electrons crossing a thin single-crystal plate in scattering geometry of Bragg has been considered. In the first time the expression describing the DTR angular density was derived for case when the path of relativistic electron in the target is considerably less than length of X-ray extinction in crystal. The kinematic character of DTR by the beam of ultra-relativistic electrons crossing a thin single-crystal target has been proved. The numerical calculation carried out shown the effect of considerable influence of the beam divergence on the DTR angular density. The derived expression can be effectively used under working out the new methods of the beam divergence measurements on the new electron colliders on super high energy.

- Thank you for your attention