Manifestation of the band structure in the photon emission spectrum of the fast above-barrier oriented particle

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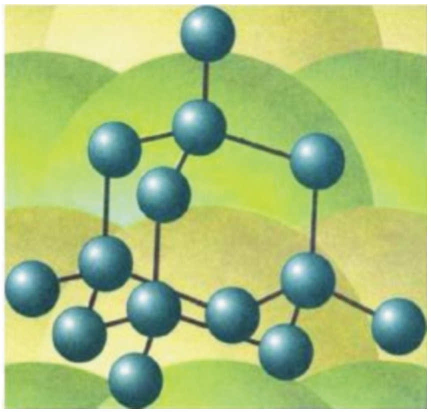
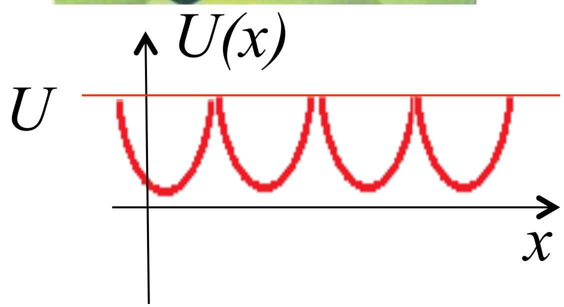
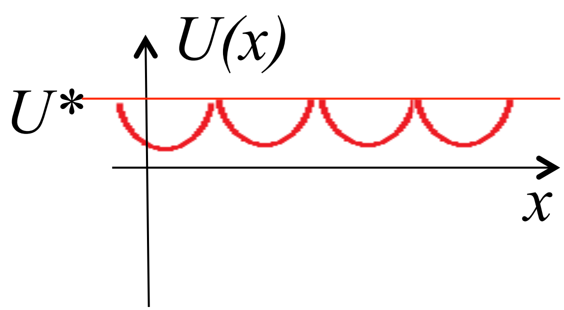






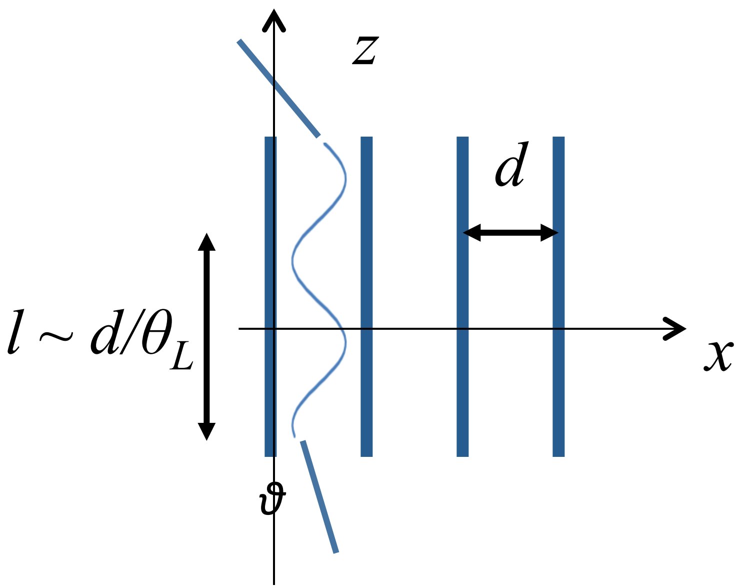
Standard approach:

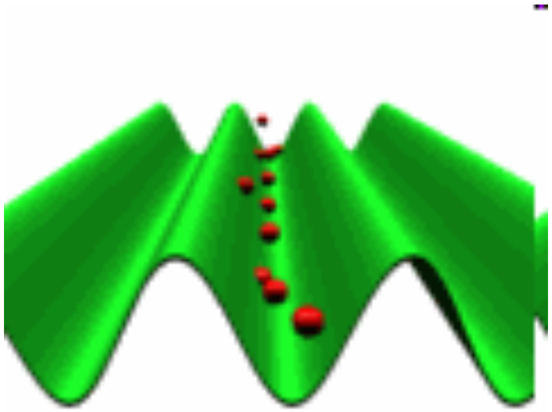
$$U(x) = \int dydz(Ze/r)exp(-r/R_{atom})$$

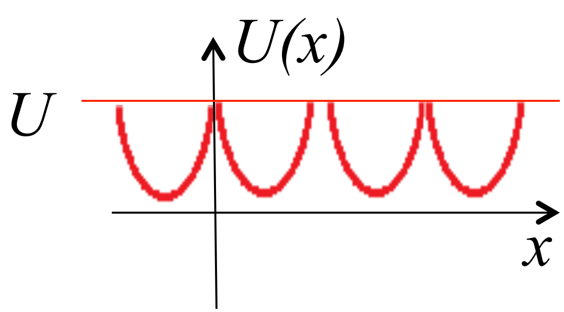
In covalence crystals (C, Si) 4 electrons are far away from nucleus and evenly distributed

$$U^*(x) = \int dydz(Z-4)e/r)exp(-r/R_{ion}) < U(x)$$

Planar channeling - thick crystal







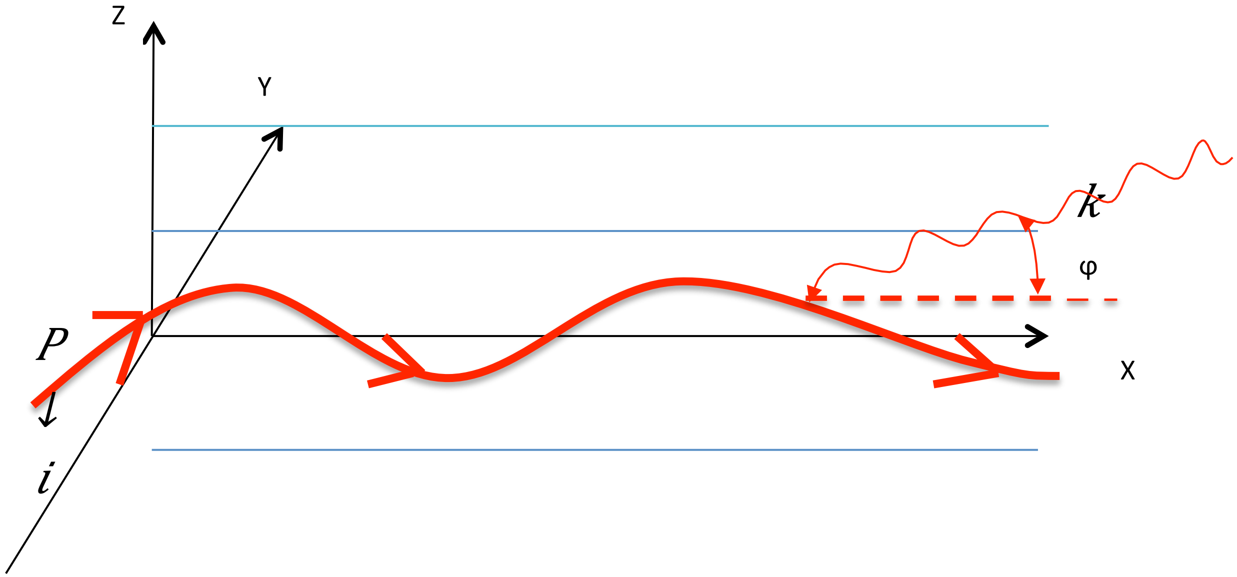
$\vartheta < \vartheta_L = (2U/E)^{1/2}$ Lindhard angle

$E \gg mc^2$; $U \sim 20 - 40$ eV

d – interplane distance;

l – period of trajectory

$$P_i = P_i(x) + P_{i\perp}(\theta) = P_0\left(1-\theta_0^2/2\right) i + P_0\,\theta_0\, j$$



$$P_{i\perp}^2/2m \lessapprox \langle U\rangle, \quad P_0^2\,\theta_0^2/2m \lessapprox \langle U\rangle, \qquad \theta_0 \le 2m\langle U\rangle/P_o^2 = \theta_L^2$$

In "accompanying" system

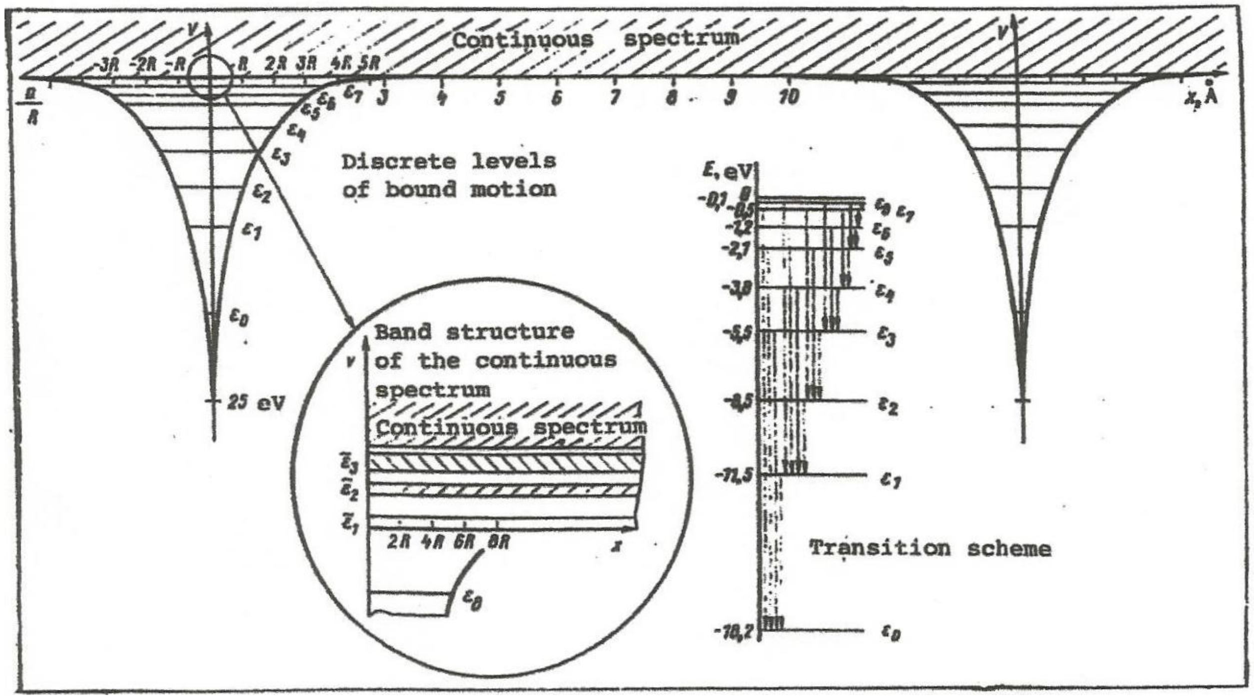
$$\omega=\omega_0\sqrt{1-V^2/C^2}\,/\left(1+Vk/\omega_0\right)=\omega_0\sqrt{1-V^2/C^2}\,/\left(1+V/C\cos\psi\right)$$

$\psi=\pi-\varepsilon$

$$\omega=\omega_0\sqrt{1-V^2/C^2}\,/\left(1-V/C+V/C\;\varepsilon^2/2\right)$$

$$\Rightarrow\omega=\omega_0\,1/\left(\gamma^{-1}+\varepsilon^2/2\gamma\right)$$

•Structure of energy bands and radiative transitions of 56-MeV electrons channeled along the (110) plane in Si



$$\overline{\Delta E} \sim \frac{U_{acc}}{N} \sim \frac{Uo\,\gamma}{\sqrt{\gamma}} \sim U_0\sqrt{\gamma}$$

$$U_0(Si) \approx 25eV \quad\quad R \approx 0{,}2\ \dot{A}$$

$$\overline{\omega_{12}}(lab) \sim \gamma^{3/2}$$

$$\omega_{max}(lab) \sim U_0\gamma^2$$

EQUATION FOR THE WAVE FUNCTION OF THE ORIENTED PARTICLE

$$\left[E_{n,p}^2/c^2 - 2eE_{n,\bar{p}}U(\vec{r})/c^2 + e^2U^2(\vec{r})/c^2 + \hbar^2\Delta_{\vec{r}} - m^2c^2 + ie\hbar\vec{\alpha}\nabla_{\vec{r}}U(\vec{r})/c\right]\psi_{n,\bar{p}}(\vec{r}) = 0.$$

$$\left(\frac{\hbar^2}{2m}\Delta_{\vec{r}} + \frac{E_{n,\vec{p}}^2 - m^2c^4}{2mc^2}\right)\psi_{n,\vec{p}}(\vec{r}) = \left(eE_{n,\vec{p}}U(\vec{r})/mc^2\right)\psi_{n,\vec{p}}(\vec{r}).$$

$$\left(-\frac{\hbar^2}{2m}\Delta_x + \left(eU(x)E_{n,\vec{p}}/mc^2\right)\right)\psi_{n,\kappa}(\vec{r}) = \varepsilon_{n,\kappa}\psi_{n,\kappa}(\vec{r}).$$

$$\left(-\frac{\hbar^2}{2m}\Delta_x + \frac{E_{n,\vec{p}}}{mc^2}V(x)\right)\psi_{n,\kappa}(\vec{r}) = \varepsilon_{n,\kappa}\psi_{n,\kappa}(\vec{r}).$$

$$V(x) = \sum_{G_x} V_{G_x}\exp(iG_x x) = V_0 + 2\sum_{G_x>0} V_{G_x}\cos(G_x x).$$

$$V_0 = -2V_G$$

$$2S = G_x x$$

$$\left(-\frac{\hbar^2 G_x^{\,2}}{4\cdot 2m}\Delta_S + \frac{E_{n,\vec{p}}}{mc^2}V(2S)\right)\psi_{n,\kappa}(2S) = \varepsilon_{n,\kappa}\psi_{n,\kappa}(2S).$$

$$\psi_{n,\kappa}(S) \equiv U(S)$$

DIMENSIONLESS EQUATION FOR THE WAVE FUNCTION OF THE FAST PARTICLE

$$\frac{\partial^2 U}{\partial S^2} + \left(d\!\!/\!\!\theta - 2q\cos 2S\right)U\left(S\right) = 0, \qquad V_0 = -2V_{G_x}, \text{ n=4}$$

$$d\!\!/\!\!\theta = \frac{8m\varepsilon_{n,\kappa}}{G_x{}^2\text{h}^2} - \frac{8E\,V_0}{\text{h}^2c^2G_x{}^2};\; q = \frac{8EV_{G_x}}{\text{h}^2c^2G_x{}^2};\quad S = G_x x/2;\quad G_x \equiv nG_{\text{ì èí}} = nG = n2\pi/d;\; E_\perp \equiv \varepsilon_{n,\kappa}$$

$$d\!\!/\!\!\theta = \frac{8m\varepsilon_{n,\kappa}}{n^2G^2\text{h}^2} + \frac{2\cdot 8EV_{G_x}}{\text{h}^2n^2c^2G^2} = \frac{8m\varepsilon_{n,\kappa}}{n^2G^2\text{h}^2} + \frac{16EV_{G_x}}{\text{h}^2n^2c^2G^2};\; q = \frac{8EV_{G_x}}{\text{h}^2c^2n^2G^2};$$

n=4

$$d\!\!/\!\!\theta = \frac{m\varepsilon_{n,\kappa}}{2G^2\text{h}^2} + \frac{EV_{G_x}}{\text{h}^2c^2G^2} = \frac{m\varepsilon_{n,\kappa}}{2G^2\text{h}^2} + \frac{EV_{G_x}}{\text{h}^2c^2G^2};\; q = \frac{EV_{G_x}}{2\text{h}^2c^2G^2};$$

$$2S = G_x x;\quad G_x \equiv nG_{\text{ì èí}} = nG = n2\pi/d;\; E_\perp \equiv \varepsilon_{n,\kappa}$$

ESTIMATION OF THE DIMENSIONLESS CONSTANTS FOR THE PARTICLE IN Si

$$\frac{2\text{h}^2 G^2}{m} = \frac{2\cdot 10^{-68}10^{-20}}{10^{-30}} = 2\cdot 10^{-18} J = \frac{2\cdot 10^{-18}}{1.6\cdot 10^{-19}} J = 12.5eV$$

$$\frac{EV_G}{\text{h}^2 c^2 G^2} \to \frac{EV_G}{\text{h}^2 c^2 G^2} = \frac{mEV_G}{\text{h}^2 mc^2 G^2} = \gamma \frac{2mV_G}{2\text{h}^2 G^2} = 2\gamma V_G \,/\, \frac{2\text{h}^2 G^2}{m} \approx \gamma 30eV \,/\, 12.5eV \approx 2.4\gamma$$

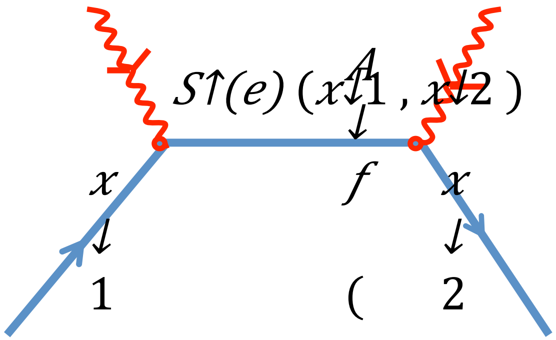
$$\partial/\theta = \frac{\varepsilon_{n,\kappa}}{12.5eV}\frac{16}{n^2} - 2.4\gamma\frac{16}{n^2};\; q = 2.4\gamma\frac{8}{n^2};\;\; S = G_x x;\;\; G_x \equiv nG_{\text{ì èí}} = nG = n2\pi / d;\; E_\perp \equiv \varepsilon_{n,\kappa}$$

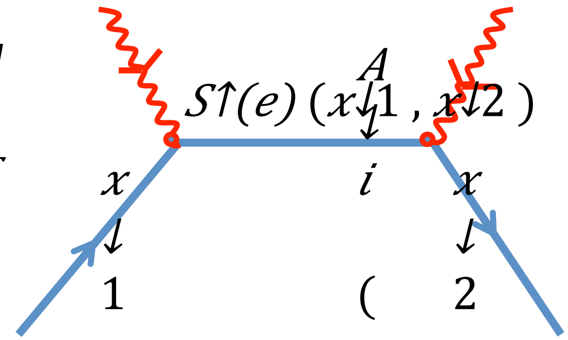
$$\partial/\theta = \varepsilon_{n,\kappa} / 12.5eV + 2.4\gamma;\; q = 1.2\gamma;\;\; 2S = G_x x;\;\; G_x \equiv 4G_{\text{ì èí}} = 4G = 4\cdot 2\pi / d;\; E_\perp \equiv \varepsilon_{n,\kappa}$$

$$\varepsilon_{n,\kappa} = 12.5eV\left(\partial/\theta - 2.4\gamma\right)$$
$$q = 1.2\gamma;$$

$$\varepsilon_{n,\kappa} = \left(\partial/\theta - 2.4\gamma\right), \quad q = 1.2\gamma;$$

Generation of a photon by a "bound" electron





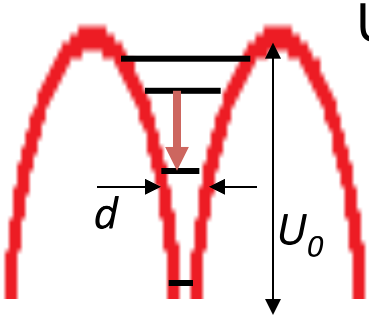
$$A_i(x) = A_i(r) e^{-i\omega_i t} = \frac{e_i}{\sqrt{2\omega_i}} e^{i(k_i r - \omega_i t)}$$

$$A_f(x) = A_f(r) e^{i\omega_f t} = \frac{e_f}{\sqrt{2\omega_f}} e^{-i(k_f r - \omega_f t)}$$

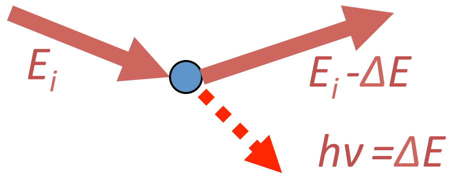
$$\psi_i(x) = \psi_i(r) e^{-iE_i t}$$

$$\psi_f(x) = \psi_f(r) e^{-iE_f t}$$

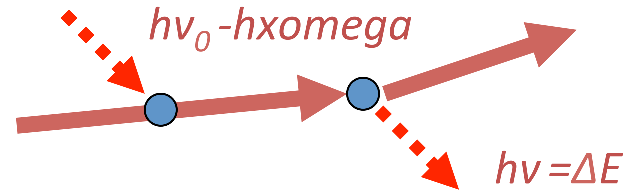
Channeling photon radiation conditions



Usual spontaneous radiation



Plasmon "wings" in radiation



1. Potential in lab. system: $U_0 \sim 20eV;\ d \sim 0{,}2\text{-}0{,}3\ A\ (Si);$
   in accomponying system (v_x = 0): $U = U_0\,(E/mc^2)$

2. Number of levels: $N \sim P_{z\,max} d\,/\,h \sim (EU_0)^{1/2} d\,/\,hc$
3. Distance between levels in acc. system: $\Delta E \sim U/N \sim (EU_0)^{1/2}(h/mcd)$

4. Plasmon energy in acc. system:

$$hxomega = 2hxomega_0/(mc^2/E + E\psi^2/mc^2) < 2h\nu_0 E/mc^2$$

5. In resonance conditions radiation "wings" can be very effective:

$$\Delta E \sim (EU_0)^{1/2}(h/mcd) = h\nu < 2h\nu_0 E/mc^2 = 2hE/mc\lambda_0$$

6. Resonance can be reached by correctly orienting laser beam, if:

$$E/U_0 > (\lambda_0/2d)^2 \sim 10^{7\text{-}8}$$

FORMALISM

$$\frac{d^2w}{d\omega d\Omega}=\frac{e^2\omega}{2\pi}\sum_f |M_{if}|^2\delta[\omega-\omega_{if}-(E_p^{\parallel}-E_{p-k}^{\parallel})].$$

$$E_p^{\parallel}-E_{p-k}^{\parallel}\approx\omega-\frac{\omega}{2(E_i-\omega)}[(\theta^2+E_i^{-2})E_i-\omega\theta^2\cos^2\varphi].$$

$$\delta\left(\frac{\omega}{2(E-\omega)}\left[\left(\theta^2+\frac{1}{\gamma^2}\right)E-\omega\theta^2\cos^2(\varphi)\right]-\omega_{if}\right)=$$

$$=\delta\left(\varepsilon_i-\varepsilon_f-\frac{\omega}{2(E-\omega)}\left[\left(\theta^2+\frac{1}{\gamma^2}\right)E-\omega\theta^2\cos^2(\varphi)\right]\right).$$

$$\delta\left(\varepsilon_i-\varepsilon_f-\frac{\omega}{2(E-\omega)}\left[\left(\frac{1}{\gamma^2}\right)E\right]\right)=$$

$$=\delta\left(A_i(q)-A_f(q')-2q+2q'-\frac{\omega}{2(E-\omega)}\frac{1.44E}{q^2}\right)$$

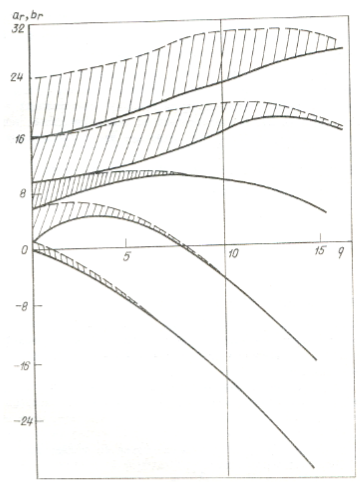
$$\delta\left(12.5A_i(q)-12.5A_f(q(1-x))-25q\cdot x-\frac{x}{(1-x)}\frac{300000}{q}\right)=$$

$$=\frac{1}{12.5}\delta\left(A_i(q)-A_f(q(1-x))-2q\cdot x-\frac{x}{(1-x)}\frac{24000}{q}\right)$$

$$0.72\frac{E}{1.2\gamma}=0.6\frac{E}{\gamma}=0.6\cdot 0.5MeV=300000eV$$

Orientation dependence of the band structure of relativistic positrons in single crystal

The transverse motion band spectrum of a oriented fast particle in the approximation of a sinusoidal crystal potential (band spectrum allocated to 25MeV energy particles). The lower band border and the upper band border vs pulse (in dimensionless units) for the crystallographic plane (110) in silicon Si.

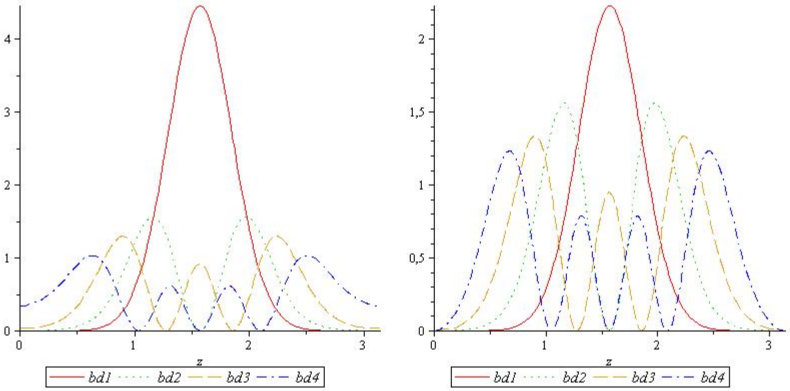


$$V(x)=\bar{V}+2\sum_{n=1,2}V_{4n,0,0}\cos(4Gx).$$

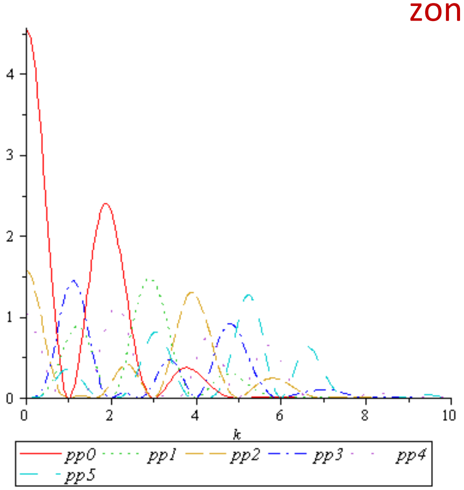
$$\frac{\partial^2 U}{\partial S^2}+(a-2q\cos 2S)U(S)=0,$$

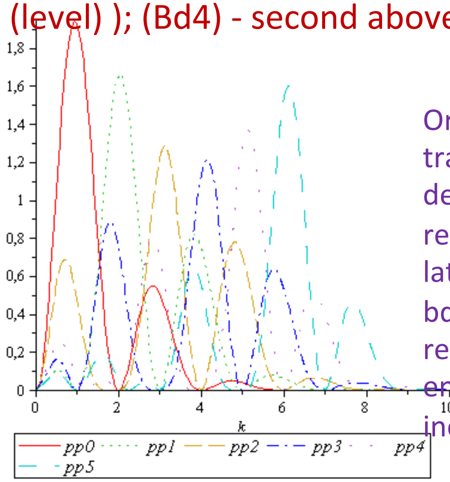
$$a=E_{\perp}^2/4G^2\hbar^2c^2-\frac{8EV_0}{2\hbar^2c^2G^2};$$

$$q=\frac{EV_G}{2\hbar^2c^2G^2};\quad S=2Gx;\quad G\equiv G_{мин}=2\pi/d;$$



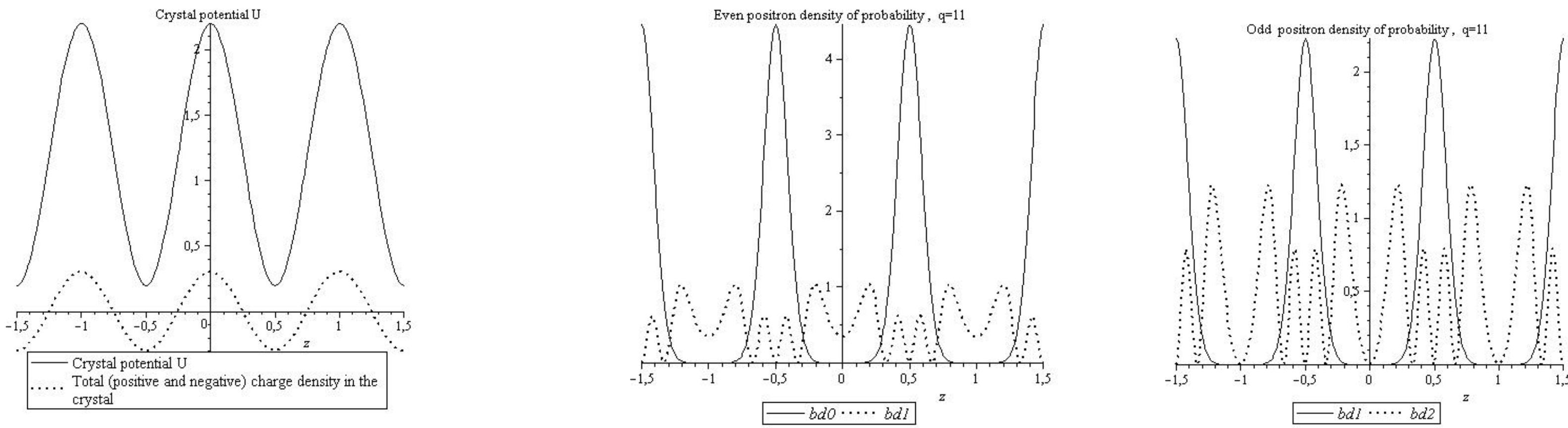
Squares module of the even (a) and odd (b) wave functions of positrons with an energy of 28 MeV in the planar channel (110) in a single crystal Si. (BD1) - first deep sub-barrier level; (Bd2) - the second level in the middle of the channel; (BD3) - the first above- barrier zone (level)); (Bd4) - second above- barrier zone





Orientation dependence of the probability of the population of the transverse motion positron levels for the even and odd levels depending on the angle of incidence of the positron in a crystal relative to the plane (110) in the Si. Angle θ is measured in reciprocal lattice vectors. Levels of oriented particles are numbered by index bd. On the x-axis the wave vector is specified as a fraction of the reciprocal lattice vector, with five of the reciprocal lattice vectors at energy 28 MeV correspond approximately to an Lindhardt angle of incidence of the positron

Along the x-axis of the graph the planar potential U is shown (solid line) and a graph of the distribution of the total density of positive and negative charges in a crystal (a line consisting of points).



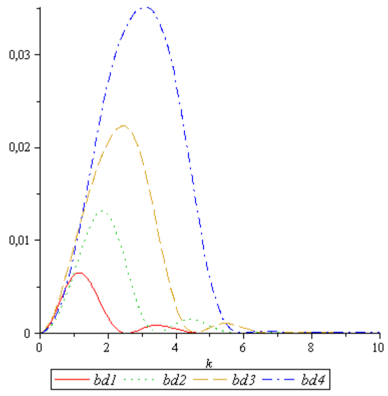
Squares module of the even (a) and odd (b) wave functions of positrons with an energy of 28 MeV in the planar channel (110) in a periodic potential with parameters corresponding to the plane 110 in the silicon single crystal. x-coordinate is listed in dimensionless units as a fraction of the interplanar distance. In Figure A: (bd0) - the first deep sub-barrier zone; (Bd1) - third near-barrier zone; Fig. b: (bd1) - the second zone; (Bd2) - Fourth (above-barrier) area.

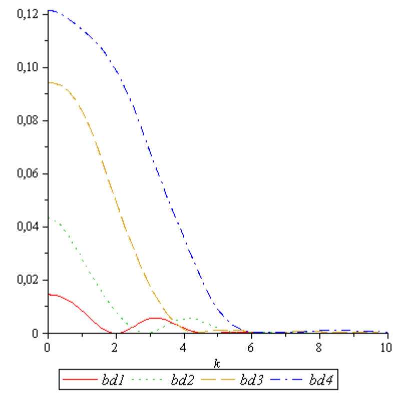
Orientation dependence

<u>Squared modulus of the matrix element of the transition positron tor the even and odd levels depending on the angle of incidence of the positron in a crystal.</u>

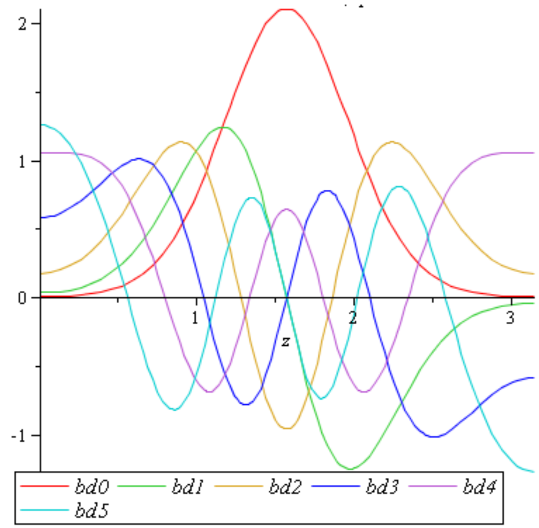
Angle θ is measured in reciprocal lattice vectors.

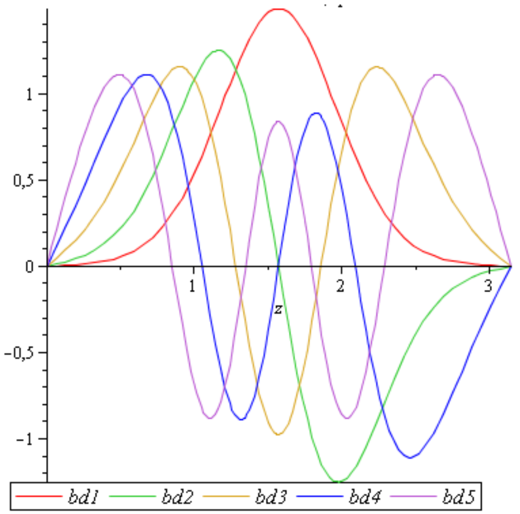
Levels of oriented particles are numbered with index bd. On the x-axis the wave vector is specified as a fraction of the reciprocal lattice vector, with five of the reciprocal lattice vectors at energy 28 MeV correspond approximately to an Lindhard angle of incidence of the positron





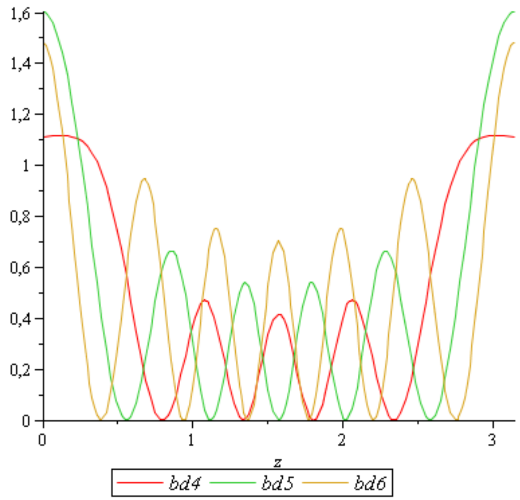
a. First six even and b. first six odd wave functions of the positron with the energy corresponding to the parameter q = 11.3 (about 10 MeV). It shows part of the wave functions corresponding to only the right half of the corresponding channel

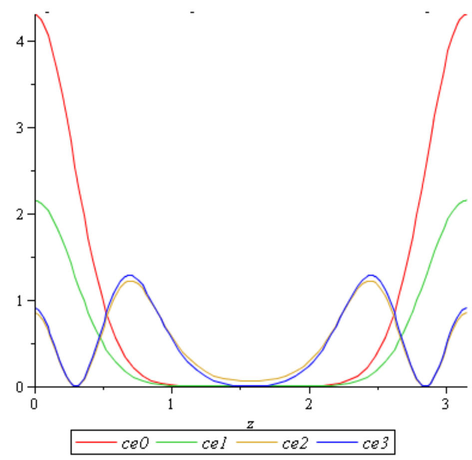




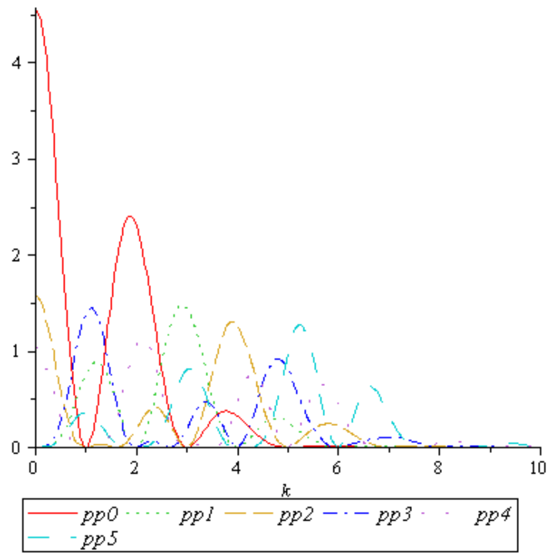
a. - The square modulus of the wave function of the positron over barrier three zones of the transverse motion at q = 11 a and

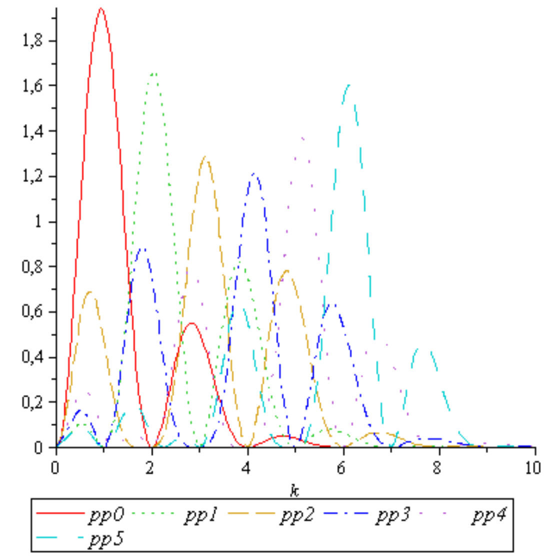
b. - Squared modulus of the electron wave functions for the four zones sub barrier transverse motion at q = -10



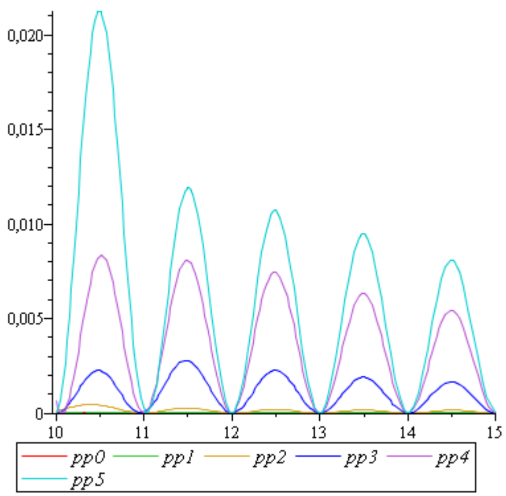


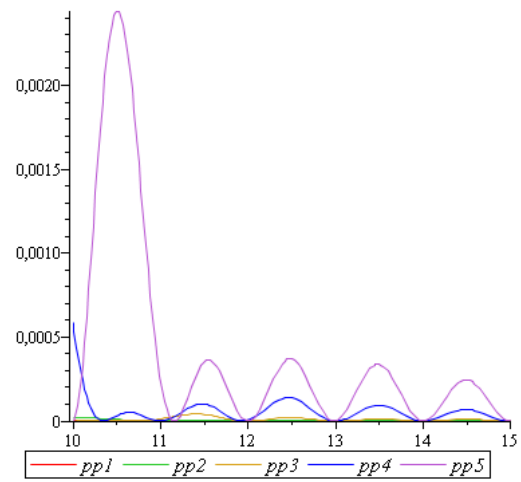
Orientation dependence of the probability of the population of the lower boundary of the zone of the cross-traffic for positron in the even and odd states, respectively, depending on the angle of incidence of the positron in a crystal relative to the plane (110) in the Si. The angle θ is measured in the number of reciprocal lattice vectors corresponding to the projection of the momentum of the positron channeling across the planes, divided by the total momentum of the positron. Levels of oriented particles are numbered with Indeks bd. On the x-axis the wave vector is specified as a fraction of the reciprocal lattice vector. The four vectors of the reciprocal lattice at energy 28 MeV correspond approximately to an Lindhardt angle of incidence of the positron, so that the value of the argument along the x-axis, approximately equal to four, shares sub barrier and over barrier states.



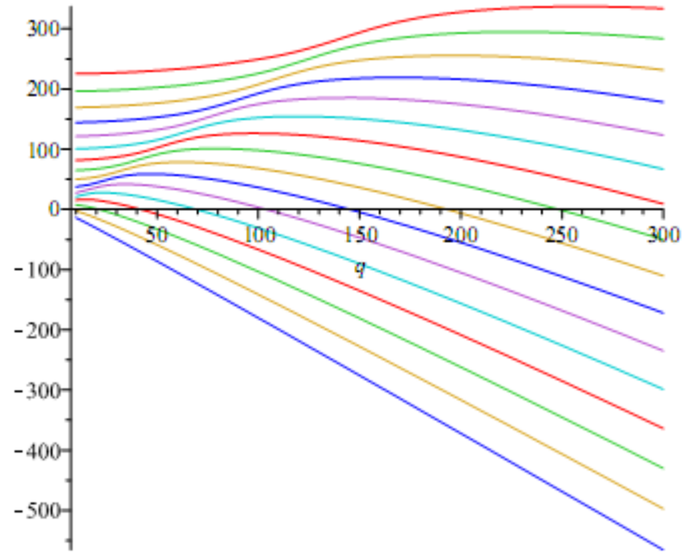


Orientation dependence of the probability of the population of the transverse motion of the positron levels for even and odd levels depending on the angle of incidence of the positron in a crystal relative to the plane (110) in the Si. Angle θ is measured in reciprocal lattice vectors. Levels of oriented particles are numbered with index bd. On the x-axis the wave vector is specified as a fraction of the reciprocal lattice vector. Five of the reciprocal lattice vectors at energy 28 MeV correspond approximately to an Lindhardt angle of incidence of the positron





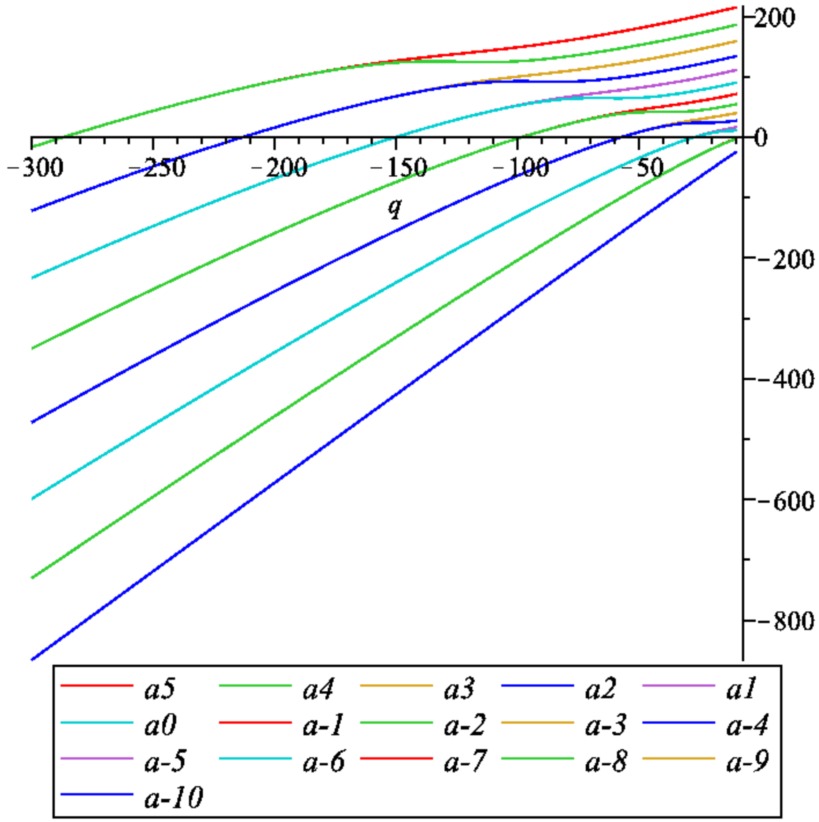
The lower border of 15 bands of low transverse-motion for channeled particles with the energy corresponding to a range of dimensionless parameter q from zero to 300



$$\Delta\left(E_{\perp}^{2}\right)=4\mathrm{h}G^{2}c^{2}\cdot 2^{4r+5}\sqrt{2\pi}q^{r/2+3/4}e^{-4\sqrt{q}}\ /\ r\ /\ .$$

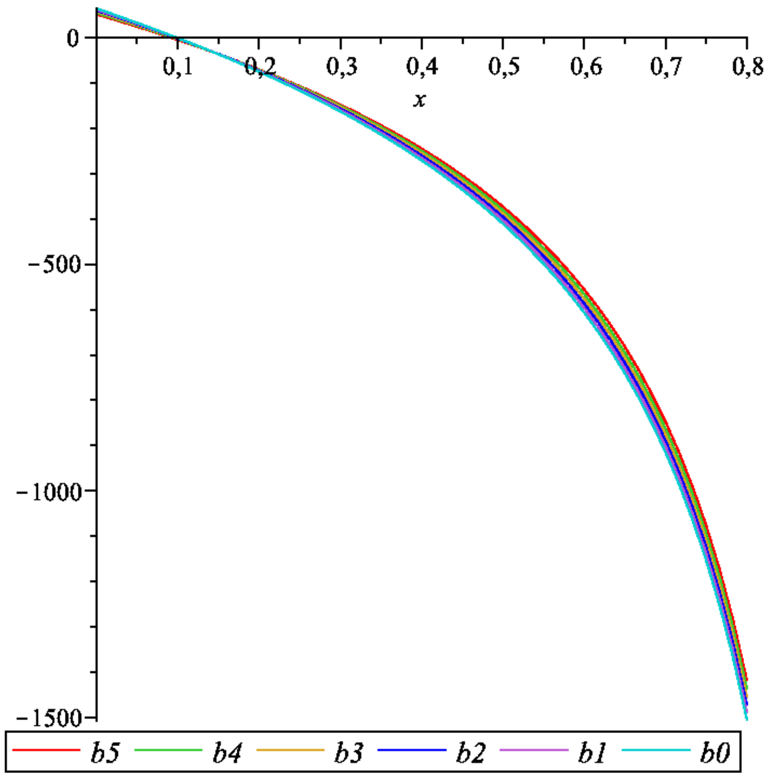
BAND STRUCTURE FOR THE ELECTRON

$$plot(\,[\,(\mathrm{MathieuA}(15-n,\,q)+q)\$(n=0\,..15)\,],\ q=-300\,..-10,\ legend=[\,'a\|(5-n)'\$(n=0\,..15)\,]\,)$$



EMISSION OF THE HIGH ENERGY PHOTONS IN NON DIPOLE TRANSITIONS BETWEEN NON ADJACENT BANDS WITH DIFFERENT BASIS ENERGIES OF THE ABOVE BARRIER PARTICLE

$$plot\left(\left[\left(-2\cdot 71\,x-\frac{24000\,x}{71\,(1-x)}+\text{MathieuA}(6-n+2,\,71)-\text{MathieuA}(6-n,\,71\,(1-x))\right)\$(n=1..6)\right],\,x=0..0.8,\,legend=\left['b\,\|\,(6-n)'\$(n=1..6)\right]\right)$$

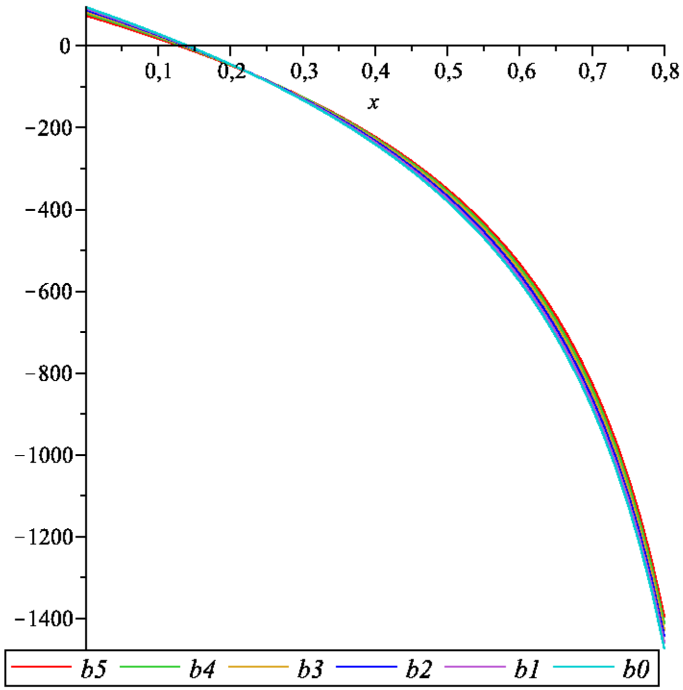


INDEPENDENCE OF THE PHOTON ENERGY ON THE BAND NUMBER

EMISSION OF THE HIGH ENERGY PHOTONS IN NON DIPOLE TRANSITIONS BETWEEN NON ADJACENT BANDS OF THE ABOVE BARRIER PARTICLE

INITIAL SET OF BANDS AND FINAL SET OF BAND CORRESPOND TO DIFFERENT ENERGIES

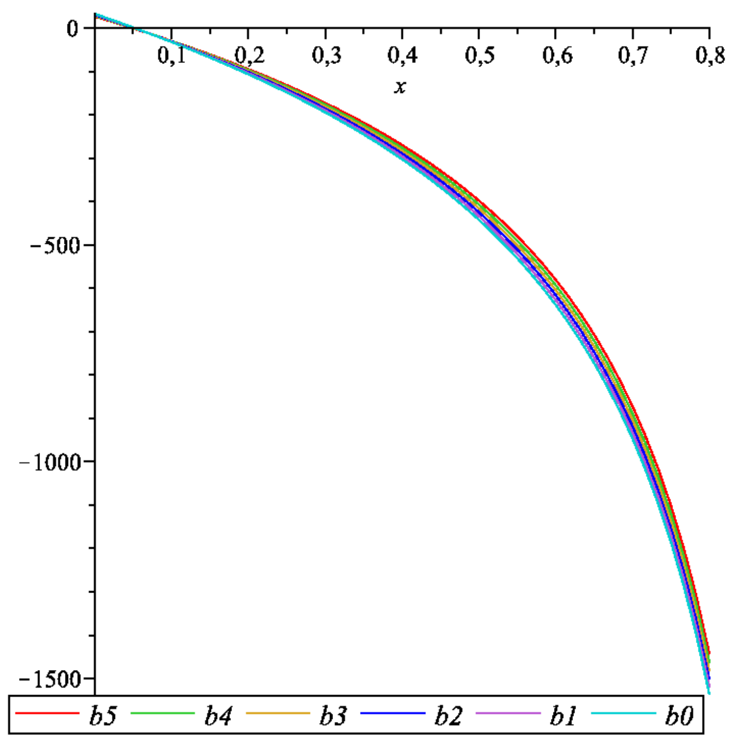
$$plot\left(\left[\left[\left(-2\cdot 71\,x-\frac{24000\,x}{71\,(1-x)}+\mathrm{MathieuA}(6-n+3,\,71)-\mathrm{MathieuA}(6-n,\,71\,(1-x))\right)\right)\$(n=1..6)\right],\,x=0..0.8,\,legend=\left[{}'b\|(6-n)'\$(n=1..6)\right]\right)$$



INDEPENDENCE OF THE PHOTON ENERGY ON THE BAND NUMBER

EMISSION OF THE HIGH ENERGY PHOTONS IN NON DIPOLE TRANSITIONS BETWEEN ADJACENT (NEAREST) BANDS OF THE ABOVE BARRIER PARTICLE

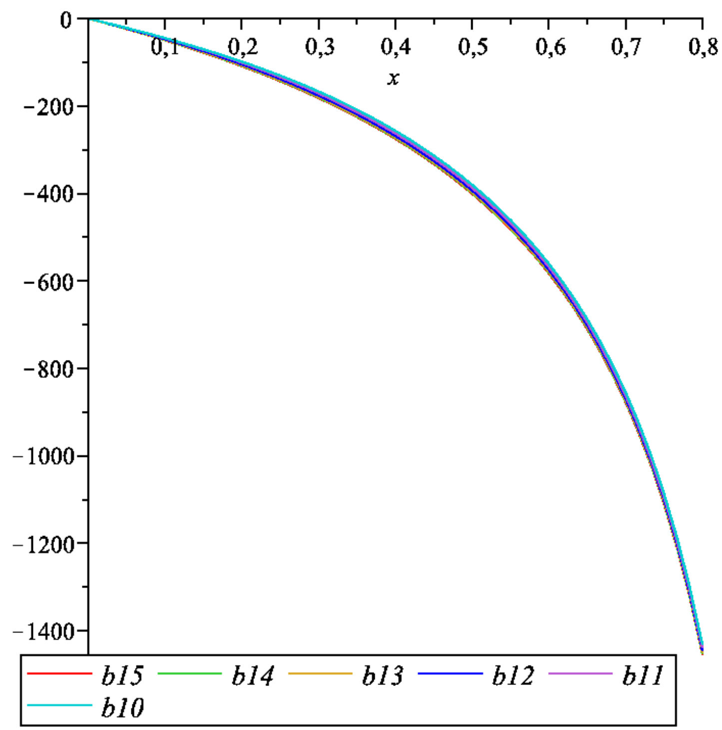
$$plot\left(\left[\left(-2\cdot 71\,x - \frac{24000\,x}{71(1-x)} + \mathrm{MathieuA}(6-n+1,\,71) - \mathrm{MathieuA}(6-n,\,71(1-x))\right)\$(n=1..6)\right],\, x=0..0.8,\, legend=\left['b\,\|\,(6-n)'\$(n=1..6)\right]\right)$$



IMPOSSIBILITY

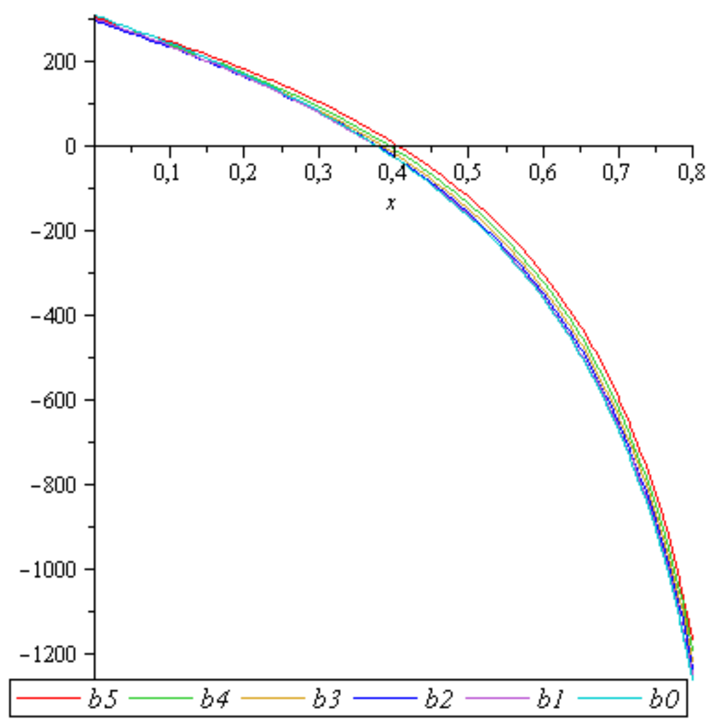
OF THE EMISSION OF THE HIGH ENERGY PHOTONS IN NON DIPOLE TRANSITIONS BETWEEN THE STATES OF THE SAME BAND NUMBER OF THE ABOVE BARRIER PARTICLE

$$plot\left(\left[\left(-2\cdot 71\,x-\frac{24000\,x}{71\,(1-x)}+\mathrm{MathieuA}(16-n,\,71)-\mathrm{MathieuA}(16-n,\,71\,(1-x))\right)\$(n=1\,..6)\right],\,x=0\,..0.8,\,legend=\left['b\,\|\,(16-n)'\$(n=1\,..6)\right]\right)$$



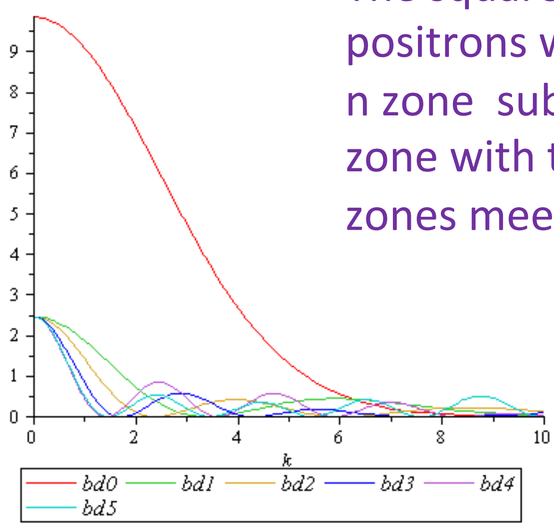
DEPENDENCE OF THE PHOTON ENERGY ON THE BAND NUMBER
EMISSION OF THE HIGH ENERGY PHOTONS IN NON DIPOLE TRANSITIONS BETWEEN ADJACENT (NEAREST) BANDS OF THE ABOVE BARRIER PARTICLE

$$plot\left(\left[\left(-2\cdot 71\,x-\frac{24000\,x}{71\,(1-x)}+\mathrm{MathieuA}(6-n+13,\ 71)-\mathrm{MathieuA}(6-n,\ 71\,(1-x))\right)\$(n=1\,..6)\right],\ x=0\,..0.8,\ legend=\left['b\,\|\,(6-n)'\$(n=1\,..6)\right]\right)$$

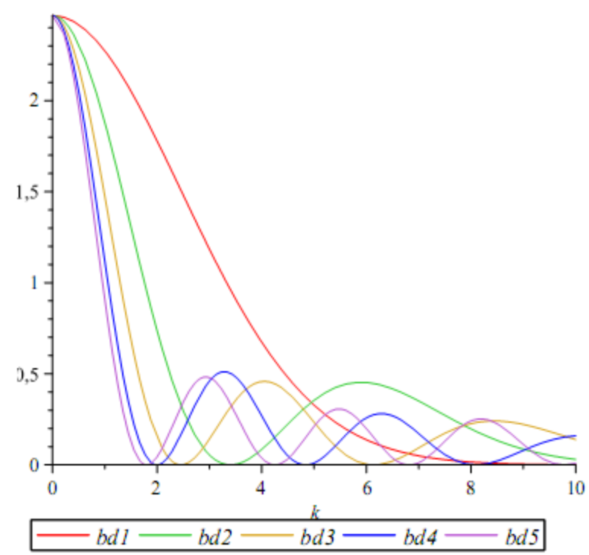


NON_DIPOLE MATRIX ELEMENTS

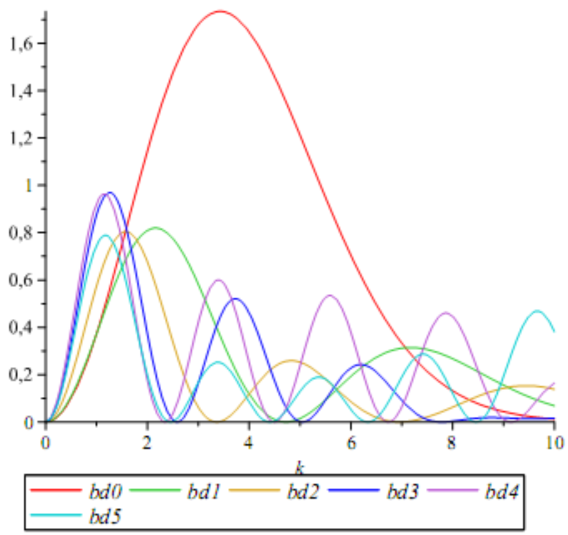
The square modulus of the matrix elements of the transition energy of positrons corresponding to q = 11, from even state n zone sub barrier movement to an even state of sub barrier movement with zone number n. Designation: bdi i = 1, ..., 5 of zones meet the number n = i



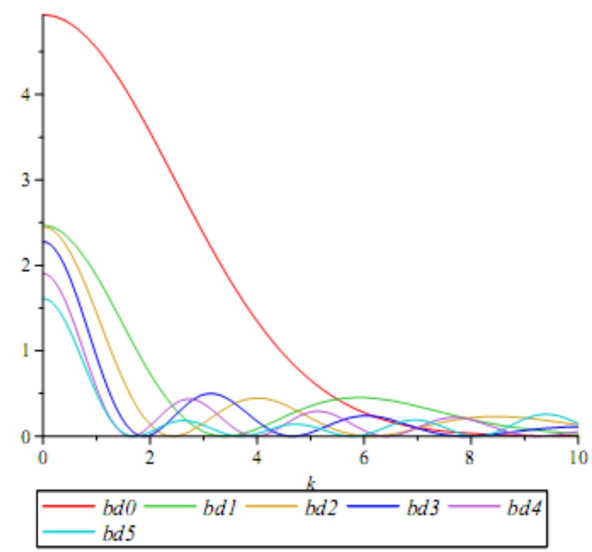
The square modulus of the matrix elements of the transition of positrons with energy corresponding to q = 11, from an odd state n zone sub barrier movement to the odd sub barrier movement zone with the number n +1. Designation bdi with i = 1, ..., 5 of zones meet the number n = i.



The square modulus of the matrix elements of the transition of positrons with energy corresponding to q = 11, from n zone odd state of the sub barrier movement to an odd state of the sub barrier movement with the zone number n +1. Designation bdi with i = 1, ..., 5 corresponds to n = i band number.

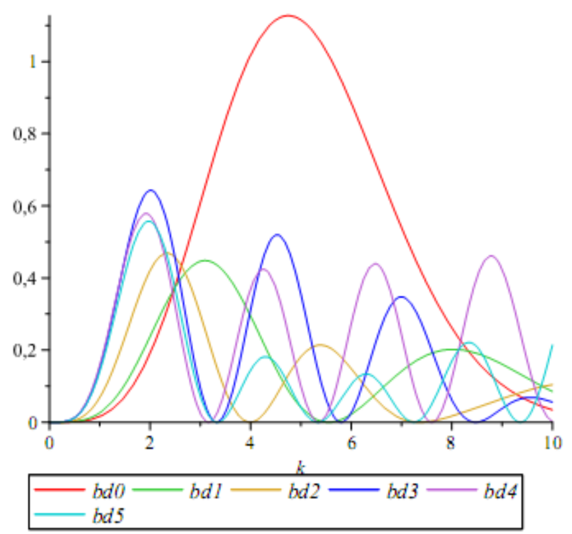


The square modulus of the matrix elements of the positron transition with the energy corresponding to q = 11, from n zone even state of sub barrier movement in an odd state sub barrier movement zone number n +1. Designation bdi with i = 1, ..., 5 answers n = i number of the band.

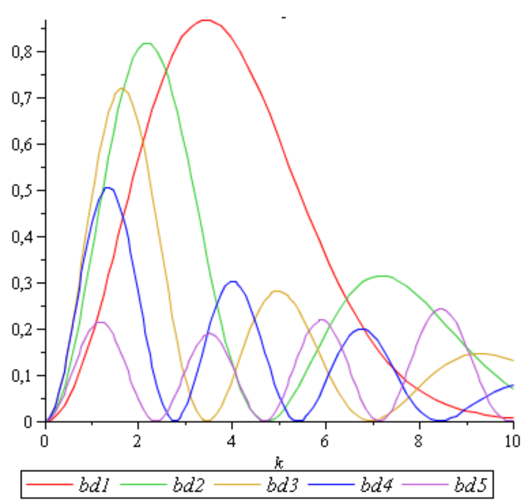


The square modulus of the matrix elements of the positron transition with the energy corresponding to q = 11, from n zone even state of sub barrier movement in an even state sub barrier movement with zone number n +2. Designation bdi c i = 1, ..., 5 answers n = i Zone number.

$$plot\left(\left[\left[\left(\left(\int_0^{\pi} \mathrm{MathieuCE}(n, 11, z)\cos(k\cdot z)\mathrm{MathieuCE}(n+2, 11, z)\,\mathrm{d}z\right)^2 + \left(\int_0^{\pi} \mathrm{MathieuCE}(n, 11, z)\sin(k\cdot z)\mathrm{MathieuCE}(n+2, 11, z)\,\mathrm{d}z\right)^2\right)\$(n = 0..5)\right], k = 0..10, title\right.$$

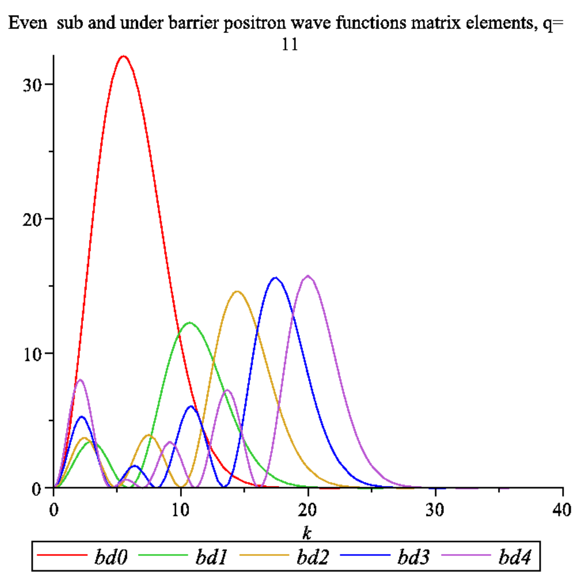


The square modulus of the matrix elements of the positron transition with the energy corresponding to q = 11, from even state of n zone sub barrier movement in an n odd state sub barrier movement zone. Designation bdi c i = 1, ..., 5 answers n = i Zone number.

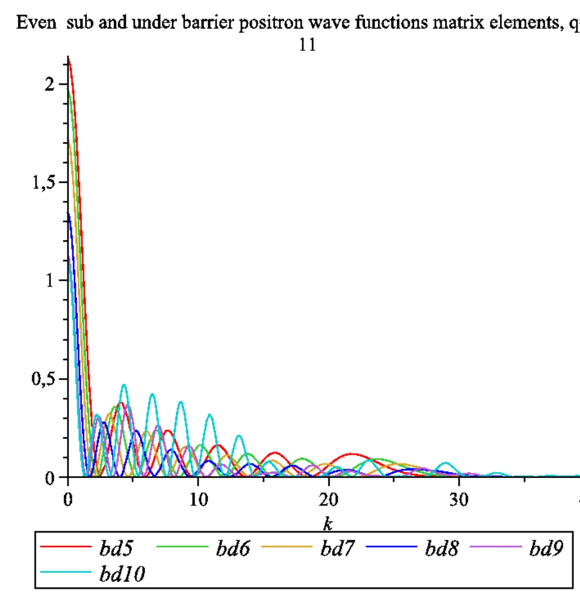


SQUARED MODULO OF THE TWO TYPES OF NON DIPOLE COMPLEX MATRIX ELEMENTS

$$plot\left(\left[\left(\left(\int_0^{\pi} \mathrm{MathieuCE}(n,\,51,\,z)\cos(k\cdot z)\frac{\partial}{\partial z}\mathrm{MathieuCE}(n,\,71,\,z)\,\mathrm{d}z\right)^2 + \left(\int_0^{\pi} \mathrm{MathieuCE}(n,\,51,\,z)\sin(k\cdot z)\frac{\partial}{\partial z}\mathrm{MathieuCE}(n,\,71,\,z)\,\mathrm{d}z\right)^2\right)\$(n = 0\,..4)\right],\, k = 0\,..40,\, title\right.$$



$$plot\left(\left[\left(\left(\int_0^{\pi} \mathrm{MathieuCE}(n,\,51,\,z)\cos(k\cdot z)\mathrm{MathieuCE}(n,\,71,\,z)\,\mathrm{d}z\right)^2 + \left(\int_0^{\pi} \mathrm{MathieuCE}(n,\,51,\,z)\sin(k\cdot z)\mathrm{MathieuCE}(n,\,71,\,z)\,\mathrm{d}z\right)^2\right)\$(n = 5\,..10)\right],\, k = 0\,..40,\, title\right.$$



RESUME

1.The calculation of the quasi-Bloch energy spectrum of the oriented fast charged particle entering the crystal at an angle substantially greater than the Lindhard angle is performed.

2. It is shown that the band structure with the presence of allowed and forbidden bands has been preserved during the passage of fast charged particles high above the crystal potential.

3.The processes of the photon generation by the quantum crystal-oriented particle entering into the crystal at an angle substantially greater than the Lindhard angle are considered.

4.The probability of the photon excitation by the quantum above-barrier channeled particle is calculated. It is proved that all of the essential features of the above-barrier band structure manifest themselves as the components in the emission spectrum of the crystal-oriented fast charged particle.