Coherent processes and channeling at high energy in thin crystals

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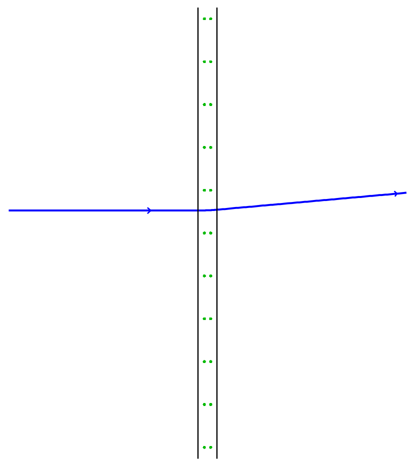
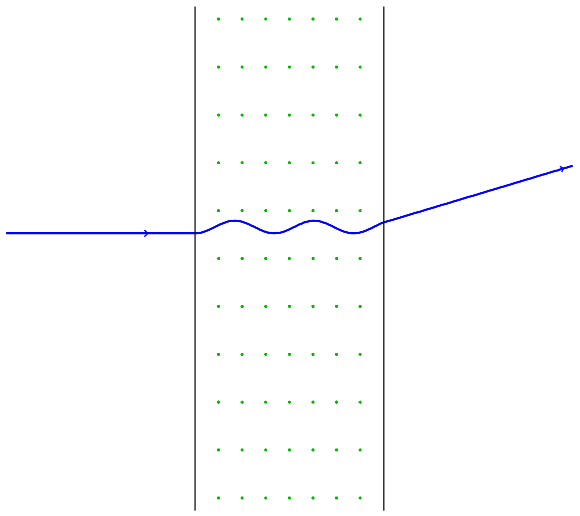
- Coherent processes and channeling
- Transitional region from ultrathin to thin crystals
- Quantum and classical theories of scattering
- Quantum and classical effects at scattering (coherence, interference, rainbow, …)
- How do quantum levels appear at regular motion and dynamical chaos?
- ….

Desenzano del Garda, Italy, 2016

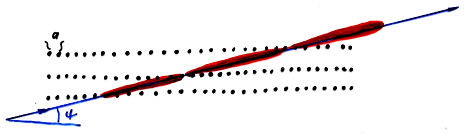
Ultrathin, Thin and Thick Crystals





Coherent effects

B. Ferretti (1950)
M. Ter-Mikaelian (1953)
&
H. Überall (1956)
....
G. Diambrini (1968)
....

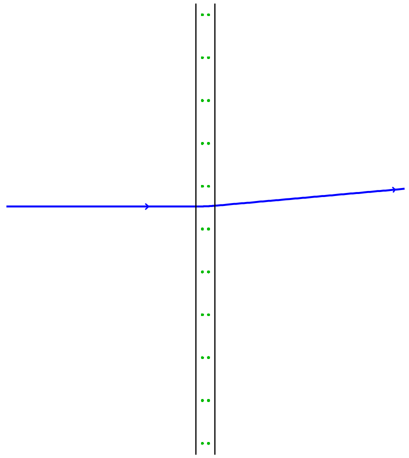


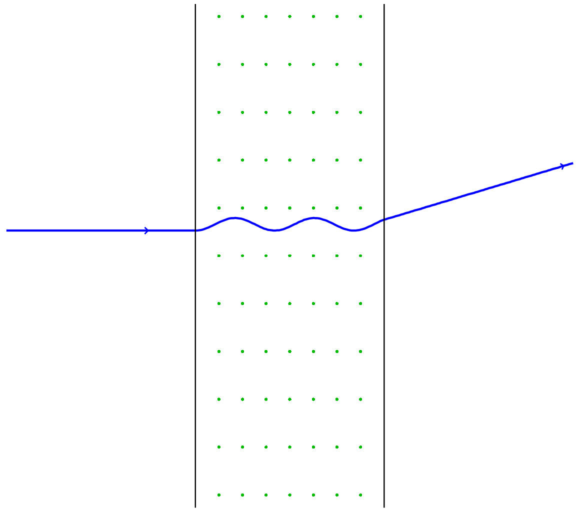
Channeling

J. Lindhard (1965)
....
G. Gemmell (1974)
....

Transitional region from ultrathin to thick crystals?

Methods of description





continuous strings (planes) potential

Born approx.
higher Born, eikonal approx.
semiclassical,
clasical approx.,

close to rectilinear motion

continuous strings (planes) potential

quantum levels for $\varepsilon_\perp$

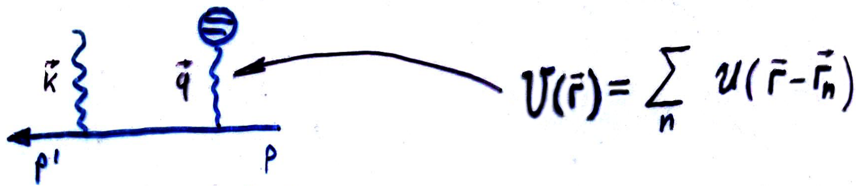
$n_{ax} \sim \varepsilon_{MeV} \qquad n_{pl} \sim \sqrt{\varepsilon_{MeV}}$

classical mechanics
semiclassical (WKB)
undulator type motion

A. Akhiezer, N. Shul'ga (1970-1993)

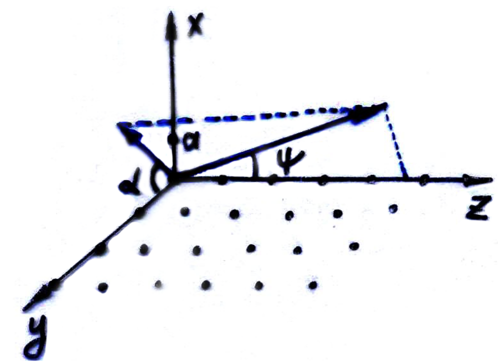
Coherent Bremsstrahlung in Born Approximation

(Ferretti 1950, Ter-Mikaelian 1952, Überall 1960)



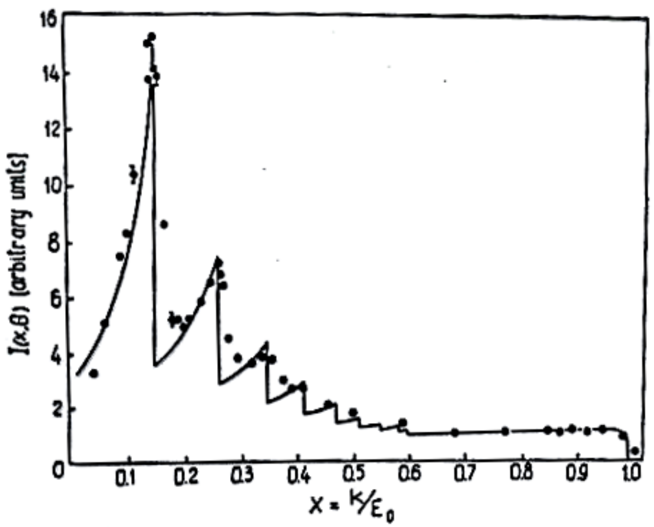
$$\omega\frac{d\sigma}{d\omega} = \frac{2e^2\delta}{m^2\Delta}\frac{\varepsilon'}{\varepsilon}\sum_{\vec{g}}\frac{g_\perp^2}{g_\parallel^2}\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta}{g_\parallel}\left(1-\frac{\delta}{g_\parallel}\right)\right]\left|U_g\right|^2 e^{-g^2\overline{u^2}}$$

$$q_\parallel \geq \delta = \omega m^2/2\varepsilon\varepsilon', \qquad g_\parallel = g_z + \psi\left(g_y\cos\alpha + g_x\sin\alpha\right) \geq \delta$$



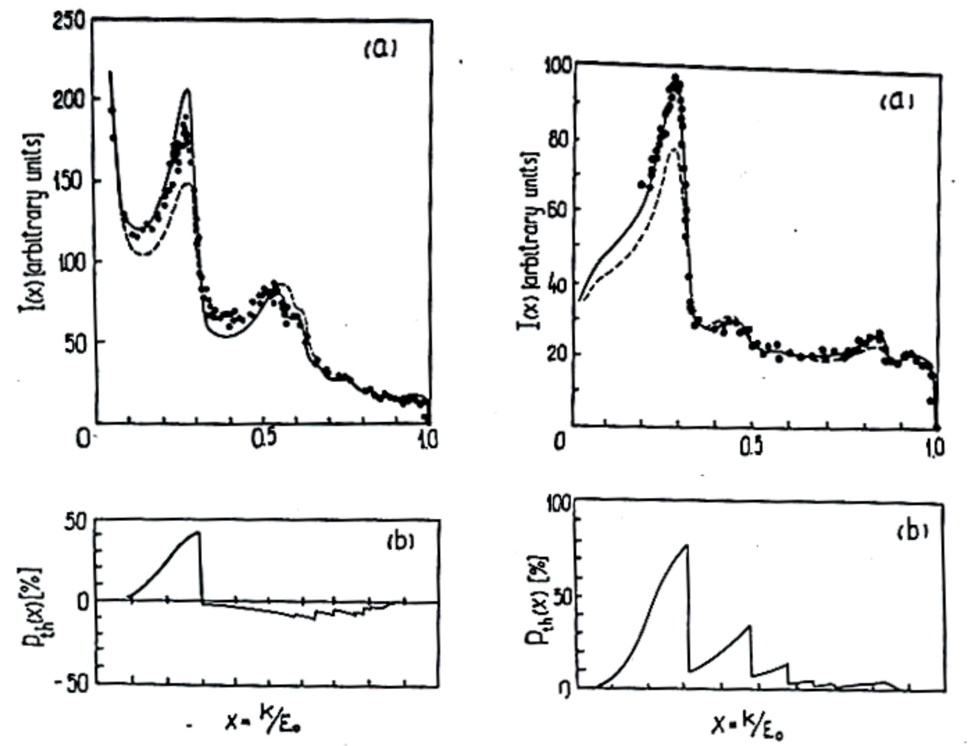
Experiment $\varepsilon \sim 1-5\ \mathrm{GeV}$ (1962 - 1965)

Frascati, DESY,Kharkov, Protvino, Tomsk, Yerevan, SLAC, …



Frascati

ε=1 GeV, θ =4,6 mrad

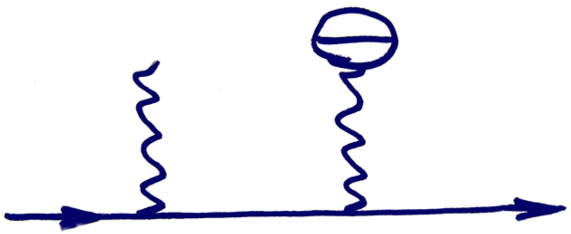


DESY

ε=4,8 GeV, θ =3,4 mrad

Generalization of CB theory

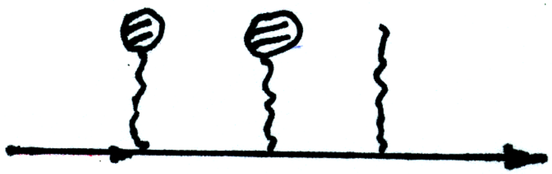
The main idea:

-For  $d\sigma_{coh} >> d\sigma_{atom}$

-The relative contribution of higher Born approximation can be also increased (A.Akhiezer, P.Fomin, N.Shul'ga 1971)

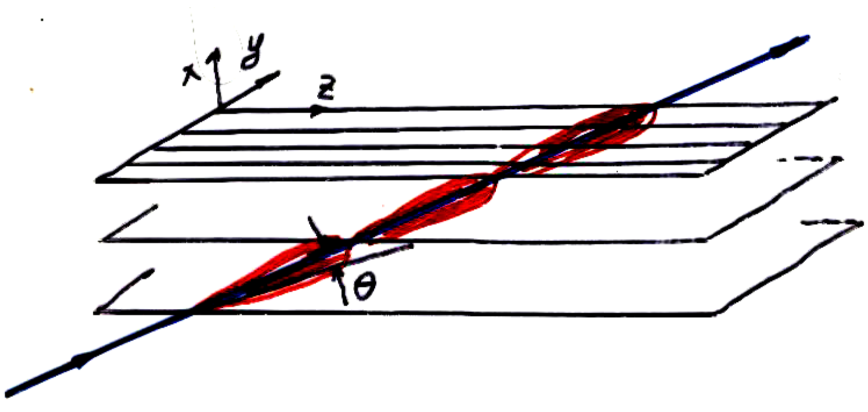
Second Born approximation in CB theory

A.Akhiezer, P.Fomin, N.Shul'ga (1970)



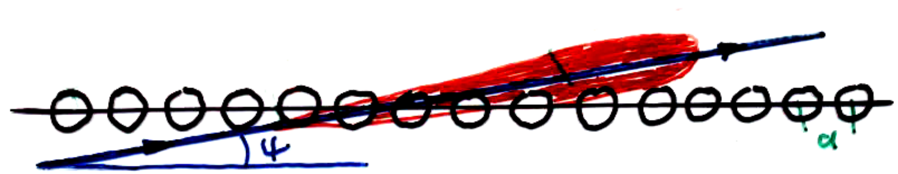
$$d\sigma_c = d\sigma_{coh}^{Born} \cdot \left(1 \pm \eta \frac{\theta_c^{\ 2}}{\theta^2}\right), \qquad \hbar\omega \ll \varepsilon$$

$\eta \sim 1$ $\qquad \theta_c = \sqrt{4Ze^2/\varepsilon a}$ − *critical channeling angle*



Higher Born approximation in the CB theory

A.Akhiezer, N.Shul'ga (1975)



$$N_{coh} \sim \min\left(\frac{l_{coh}}{a}, \quad \frac{R}{\psi a} \right)$$

$$\frac{Ze^2}{hc} << 1 \quad \rightarrow \quad N_{coh} \frac{Ze^2}{hc} \sim \frac{R}{\psi a} \frac{Ze^2}{hc} << 1$$

Quickly destroys for $\psi \rightarrow 0$

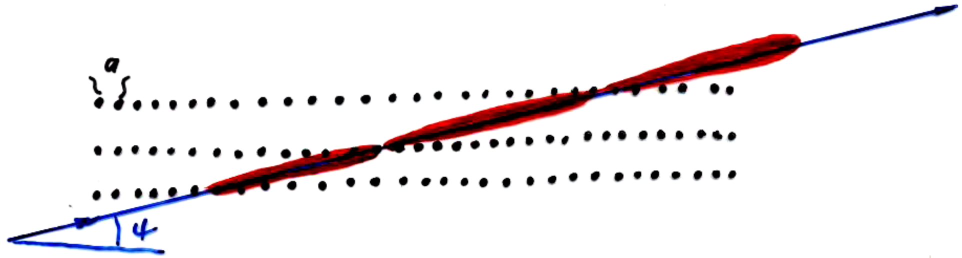
PARADOX

This condition did not fulfill practically for experiments (1960-1970) on verification of F – T – Ü theoretical results.

But the experiments were in good agreement with this theory !!!
Why ???

Eikonal, semiclassical, classical CB theory

A.Akhiezer, V. Boldyshev, N.Shul'ga (1975 - 1979)



Semiclassical approximation

$$\frac{N_c Ze^2}{\hbar c} = \frac{R}{\psi a}\frac{Ze^2}{\hbar c} >> 1$$

!!!

Classical Electrodynamics

$$N_c \frac{Ze^2}{\hbar c} >> 1, \quad \hbar\omega << \varepsilon$$

$$d\sigma^{(WKB)} = d\sigma\{\vec{r}_{cl}(t)\}$$

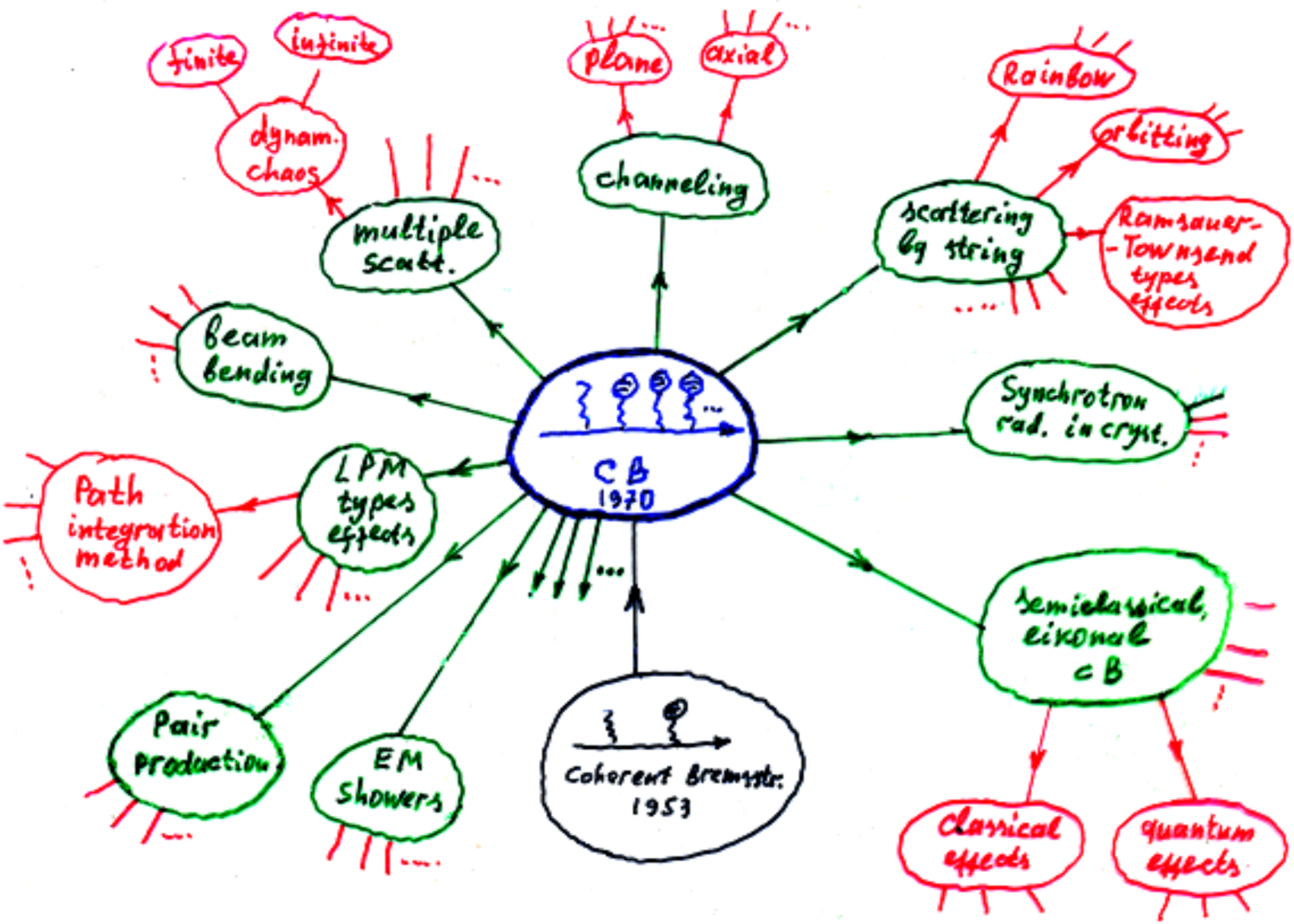
- Radiation is determined by the classical trajectory !!!
- It is necessary to know the types of particles' motion in crystal
- Same methods for description of CB and LPM effects !!!

New area of research

The interaction of high-energy particles with matter in conditions of effectively strong interaction of the particle with atoms of media (semiclassical, classical approximations)

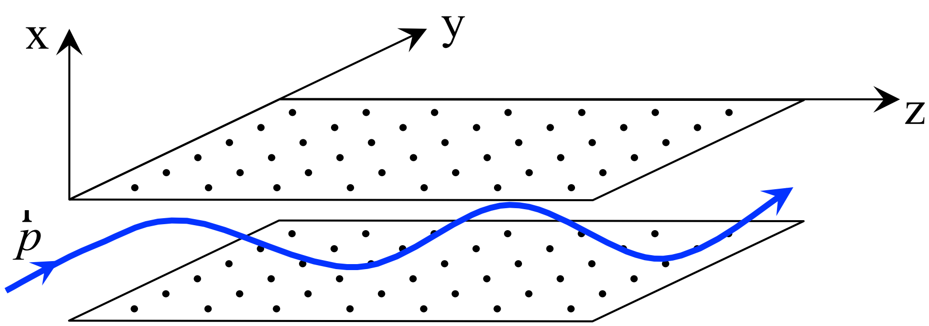
$$N_c \frac{Ze^2}{\hbar c} >> 1$$

Problems generated by the theory of coherent radiation in crystals (situation for 1993)



Phenomenon of Planar Channeling

J.Lindhard (1965)



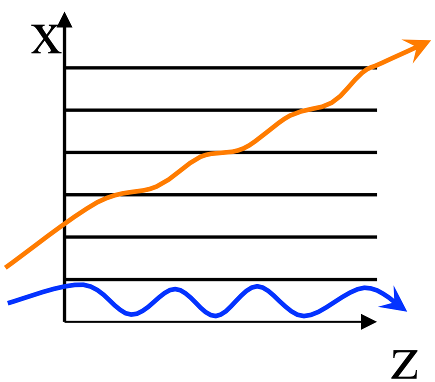
$$p_z = const \approx p$$

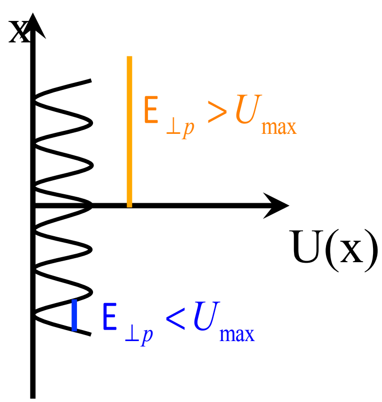
$$p_y = const \approx 0$$

$$\ddot{x} = -\frac{1}{E}\frac{\partial}{\partial x}U(x)$$

$$E_\perp = \frac{E\dot{x}^2}{2} + U(x)$$

$$E_\perp = \frac{E\psi_c^2}{2} = U_{max} \quad \Leftrightarrow \quad \psi_c \sim \sqrt{2U_{max}/E}$$





Quantum consideration

$$\psi = e^{i(pz-\varepsilon t)}\varphi(x,t)$$

$$i\hbar\partial_t\varphi = \left(-\frac{\hbar^2}{2\varepsilon}\frac{\partial^2}{\partial x^2} + U(x)\right)\varphi(x,t)$$

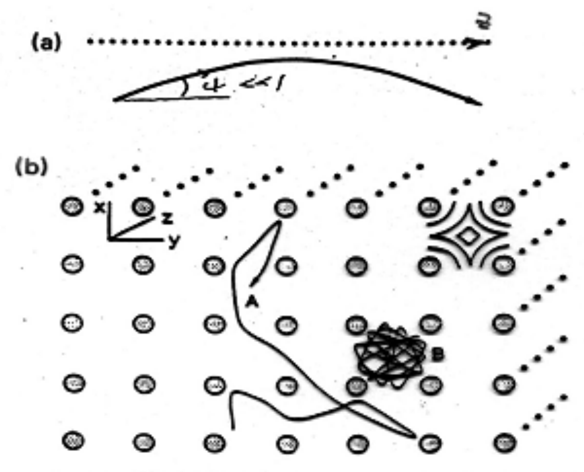
$$n_{levels} \sim \sqrt{E_{MeV}}$$

Phenomenon of Above Barrier Motion: A. Akhiezer, N. Shul'ga (1978)

Axial Channeling and Above-Barrier Motion
(continuous string potential)

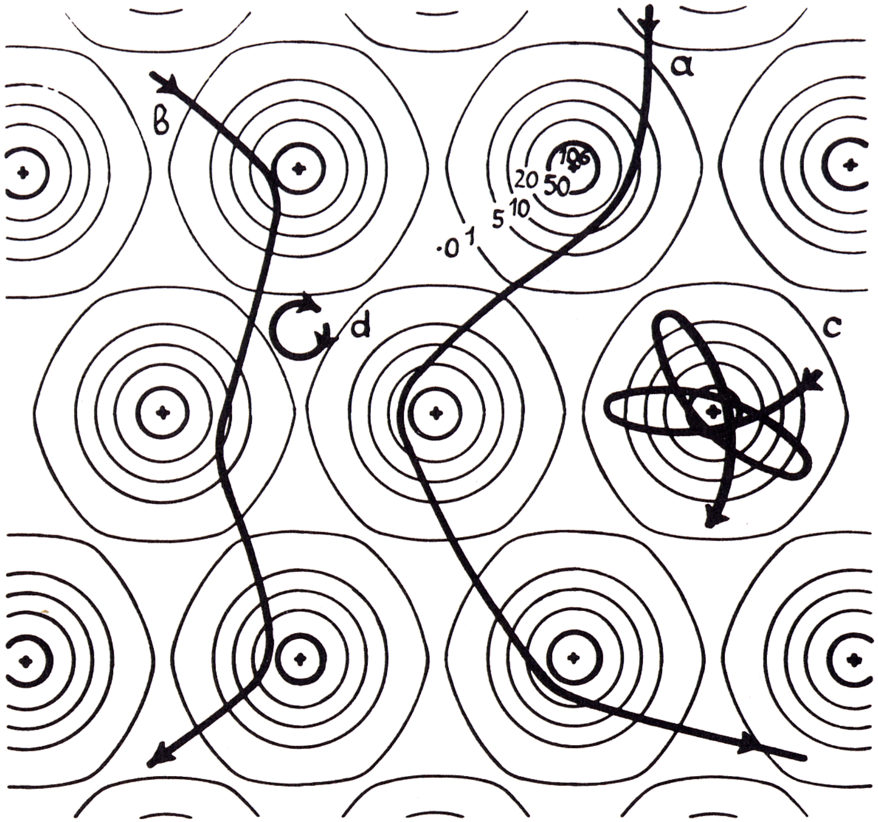
$$\frac{d\mathbf{p}}{dt} = -\nabla U(\mathbf{r})$$

$$U(\mathbf{r}) \to U(x,y) = \frac{1}{L}\int_0^L dz \sum_n u(\mathbf{r}-\mathbf{r}_n)$$



$$\frac{d\mathbf{p}}{dt} = -\nabla U(x,y) \qquad \to \qquad \begin{cases} p_z = const >> p_\perp \\ \dot{\vec{p}} = -\dfrac{1}{\varepsilon}\nabla U_\perp(x,y) \end{cases}$$

Particle motion in periodical field of crystal atomic strings Si <111>



Classical consideration

$$\ddot{\vec{\rho}} = -\frac{c^2}{\varepsilon}\frac{\partial}{\partial\vec{\rho}}U(\vec{\rho})$$

Quantum consideration

$$\psi = e^{i(pz-\varepsilon t)}\varphi(\vec{\rho},t)$$

$$i\hbar\partial_t\varphi = \left(-\frac{\hbar^2}{2\varepsilon}\nabla_\perp^2 + U(\vec{\rho})\right)\varphi(\vec{\rho},t)$$

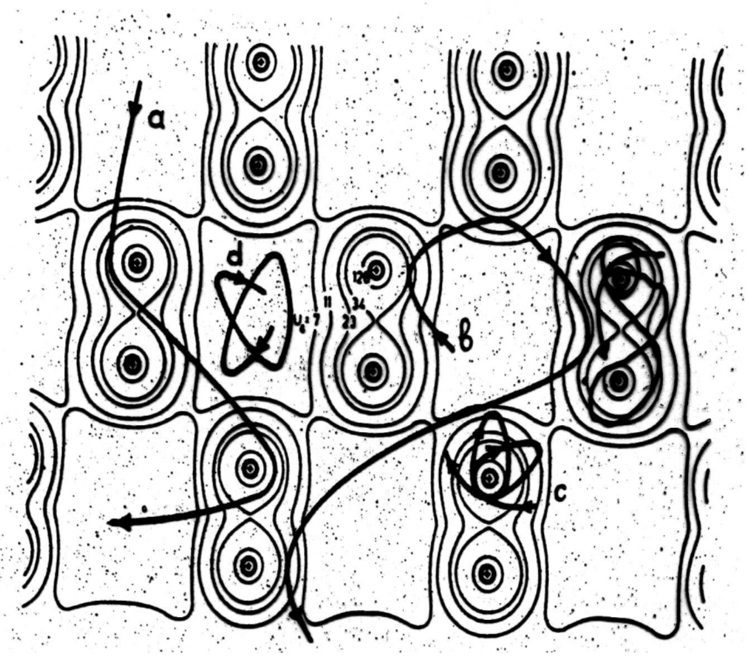
$$n_{levels} \sim \varepsilon_{MeV}$$

- From where appear the bound energy levels at channeling?
- How do the levels appear at dynamical chaos?

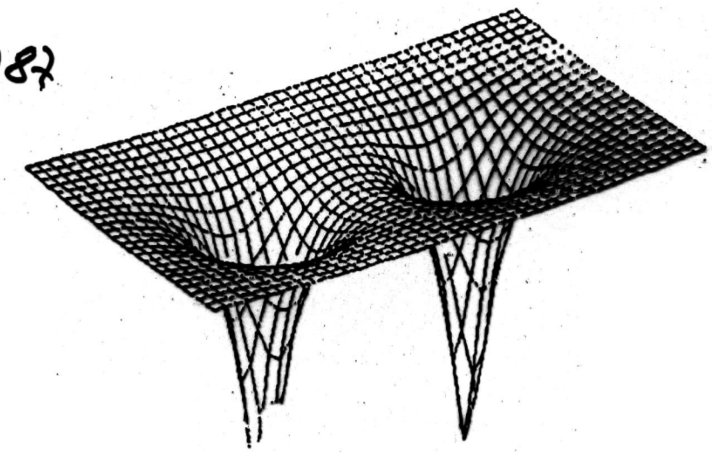
Dynamical chaos at channeling

Yu.Bolotin, V.Gonchar, V.Truten’, N.Shul’ga (1986)

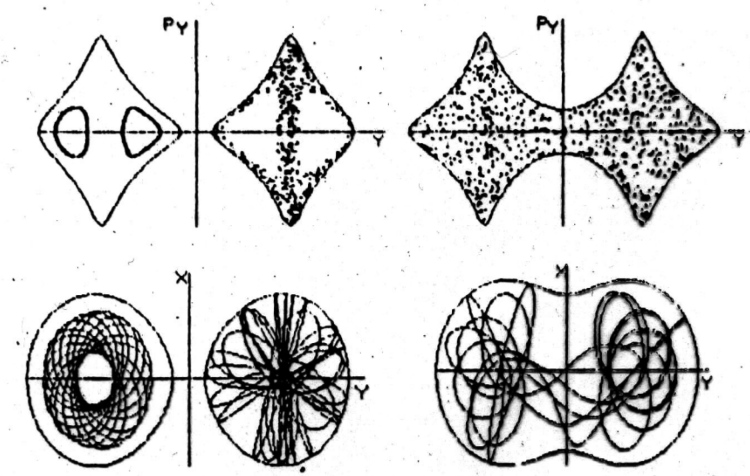
Continuous string potential Ge, <110>.



1987

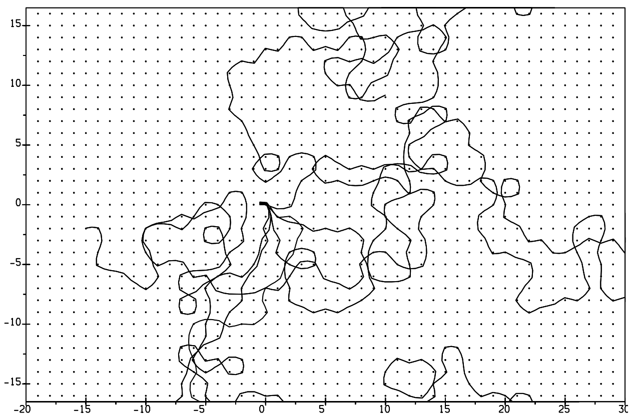


Si, <110>, 9~

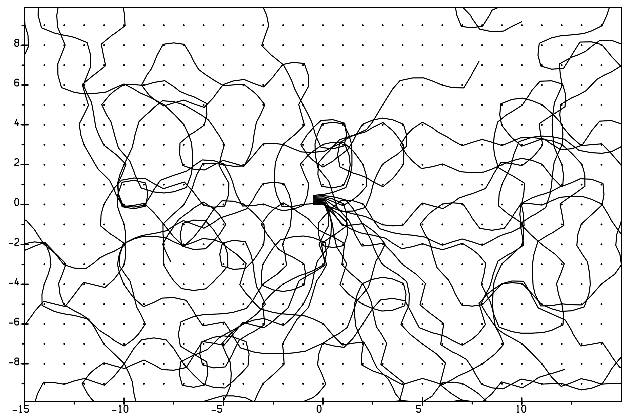


Dynamical Chaos at Multiple Scattering for $e^{\mp}$

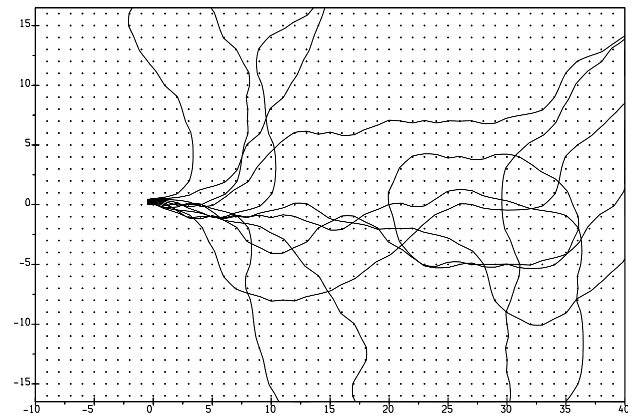
$$z = \psi / \psi_c$$



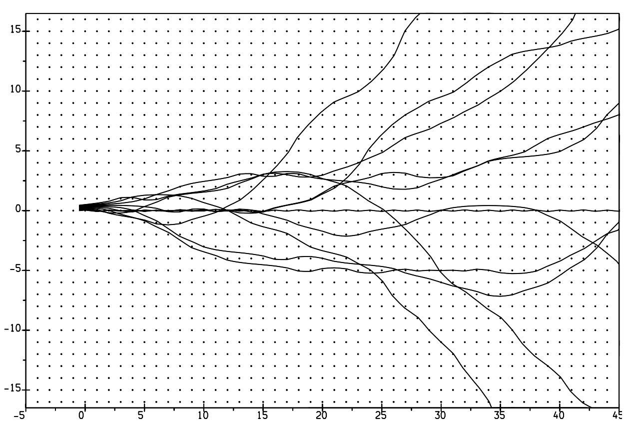
Z=0.5



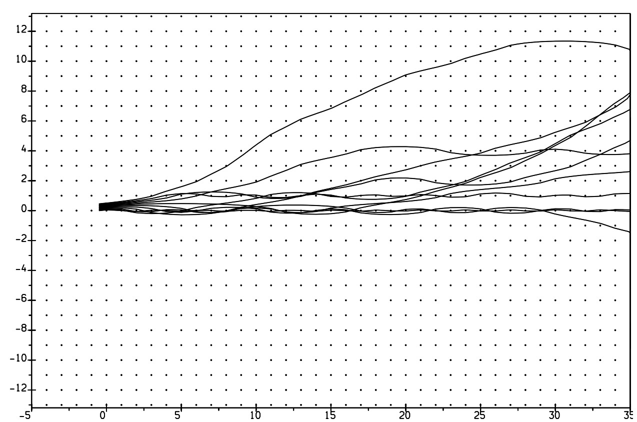
Z=0.7



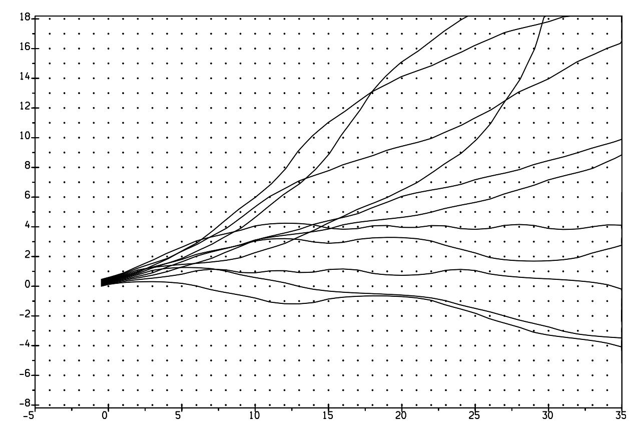
Z=1.5



Z=1



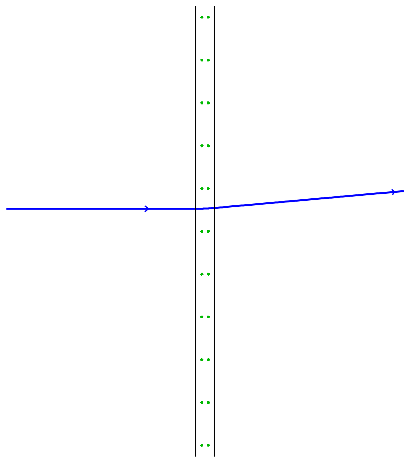
Z=2, $\alpha = 6^0$

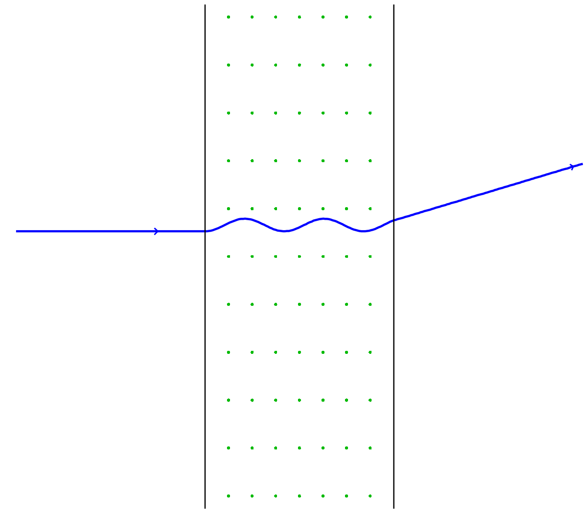


Z=2, $\alpha = 15^0$

A. Akhiezer, N Shul’ga, V. Truten’, Physics Reports, 1991

Ultrathin, Thin and Thick Crystals





Coherent effects

B. Ferretti (1950)
M. Ter-Mikaelian (1953)
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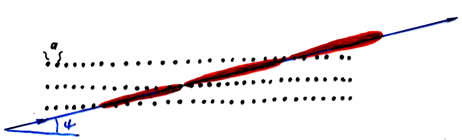


Channeling

J. Lindhard (1965)
....
G. Gemmell (1974)
....

Transitional region from ultrathin to thick crystals?

New direction of work

- **Ultrathin crystals**

Experiments:

J.S. Rosner, Golovchenko et al. Phys. Rev. B18 (1978) 1066.
M. Mothapothula et al. NIM B283 (2012) 29
V. Guidi et al. Phys. Rev. Lett. (2012)

Experiment: 2MeV protons scattering in L=55nm Si

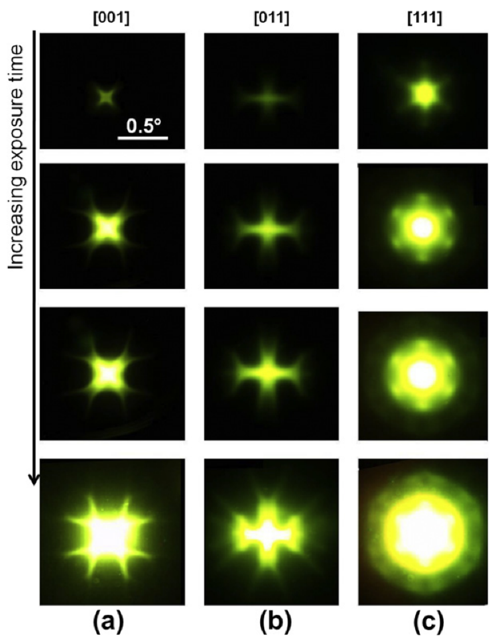
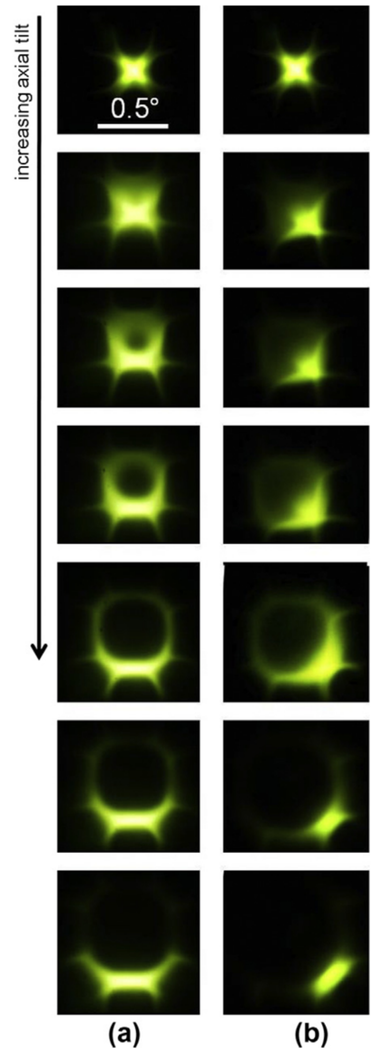


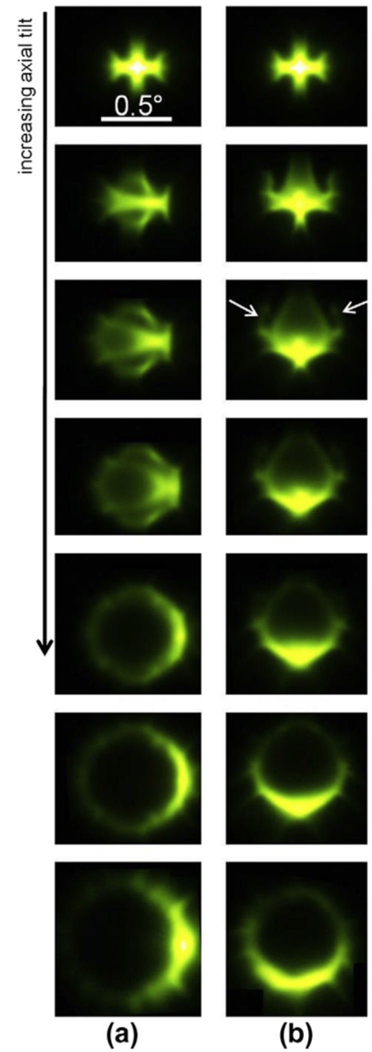
Fig. 2. Experimental channeling patterns for 2 MeV protons from a 55 nm [001] Si membrane at alignment with the (a) [001], (b) [011] and (c) [111] axes. Downwards direction shows the effect of increasing camera exposure.

$$\psi = 0$$

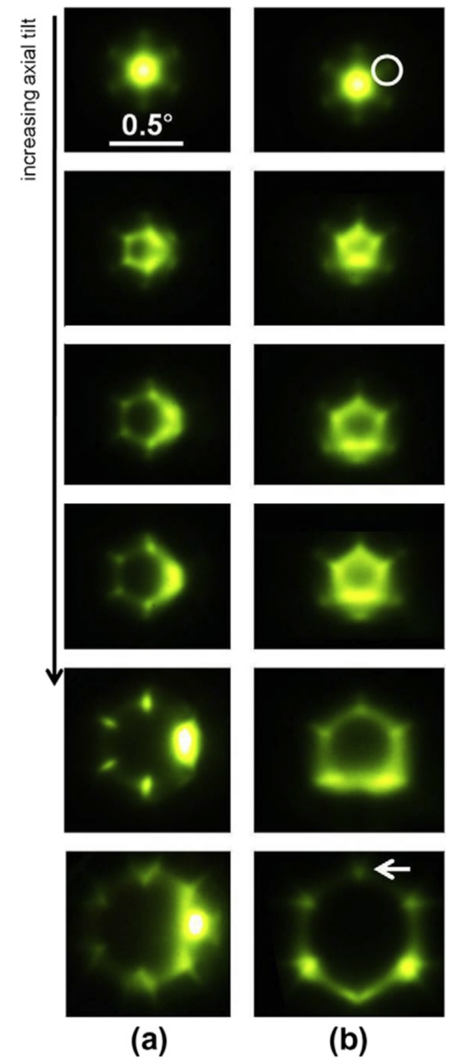
different expositions



$\langle 100 \rangle$



$\langle 110 \rangle$



$\langle 111 \rangle$

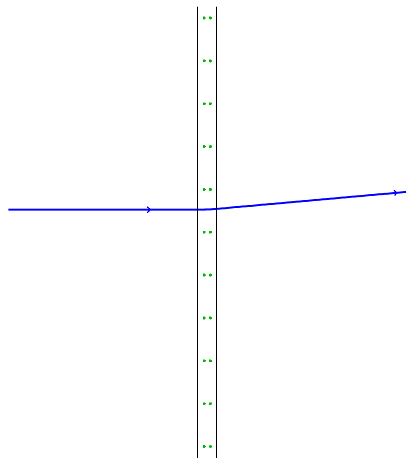
$$\psi_c > \psi > 0$$

M. Mothapothula et al. NIM B283 (2012) 29

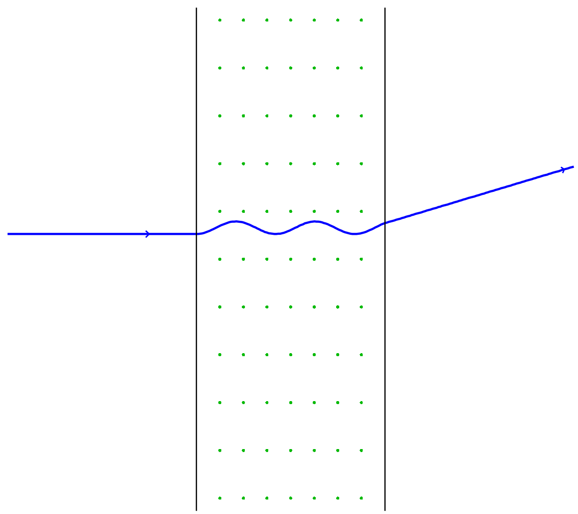
Quantum and Classical theories of high energy electron scattering in ultrathin crystals

N.F. Shul'ga,   S.N.Shul'ga

arxiv: 2016



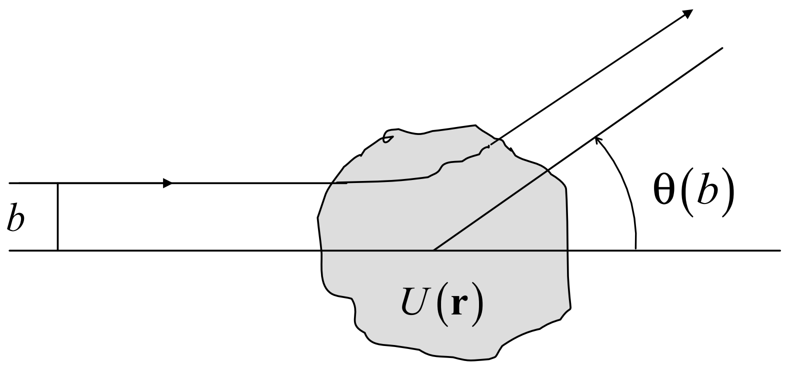
~~Channeling~~



Channeling

- Transitional region from ultrathin to thick crystals
- Scattering (rainbow, bound states levels, interference, coherence, …)
- How do quantum levels and zones appear at regular motion and dynamical chaos?
- Radiation

Classical theory of scattering in thin crystals



$$\boldsymbol{\vartheta} = \boldsymbol{\vartheta}(\mathbf{b})$$

$\downarrow$ inversion

$$\mathbf{b} = \mathbf{b}(\boldsymbol{\vartheta})$$

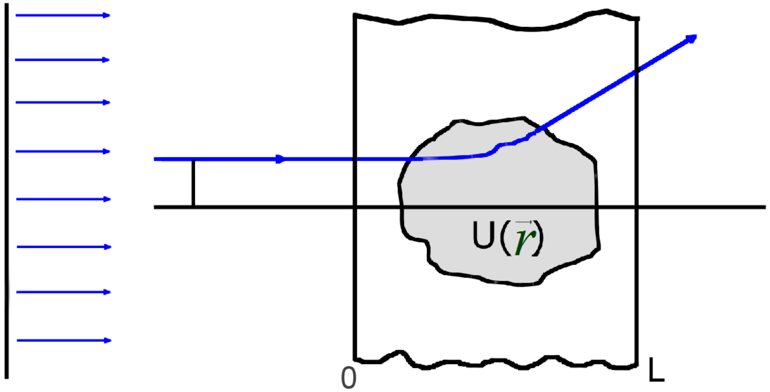
$$d\sigma_{cl}(\boldsymbol{\vartheta}) = d^2 b \quad \Rightarrow \quad d^2\vartheta \sum_n \frac{1}{\left|\partial\boldsymbol{\vartheta}/\partial\mathbf{b}\right|_n}\Bigg|_{\mathbf{b}=\mathbf{b}_n(\boldsymbol{\vartheta})} = d^2\vartheta \int d^2 b \; \delta\big(\boldsymbol{\vartheta} - \boldsymbol{\vartheta}(\mathbf{b})\big)$$

$$\ddot{\boldsymbol{\rho}} = -\frac{c^2}{E}\nabla U(\boldsymbol{\rho})$$

$$\boldsymbol{\rho}(t) = \boldsymbol{\rho}(\mathbf{b}, t) \qquad\qquad \boldsymbol{\vartheta}(b) = \frac{1}{v}\dot{\boldsymbol{\rho}}(\mathbf{b}, T)$$

Gauss Theorem in the Scattering Theory

N. Bondarenco, N. Shul'ga Phys. Lett. B 427 (1998) 114



$$\psi = e^{i\vec{p}\vec{r}}\varphi(\vec{r})$$

$$a(\vartheta) = -\frac{1}{4\pi}\int_V d^3r\, e^{-i\vec{p}'\vec{r}}\,\bar{u}'\gamma_0 U(\vec{r})\psi(\vec{r}) = -\frac{1}{4\pi}\int_V d^3r\ \mathrm{div}\left[\bar{u}'\vec{\boldsymbol{\gamma}}\psi(\vec{r})e^{-i\vec{p}'\vec{r}}\right] =$$

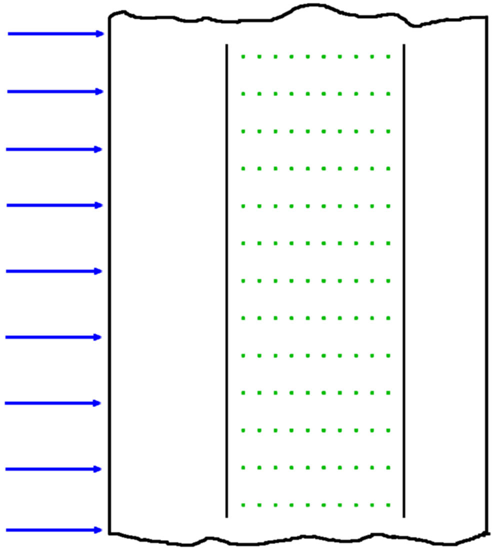
$$= -\frac{i}{4\pi}\oint d\vec{S}\,\bar{u}'\vec{\gamma}\psi(\vec{r})e^{-i\vec{p}'\vec{r}} =$$

$$= -\frac{ip}{2\pi}\oint d^2\rho\, e^{i\vec{q}\vec{r}}\left(\varphi(\vec{r}) - 1\right)\Big|_{z=-L}^{z=L}$$

$$\frac{d\sigma_q}{do} = \left|a(\vartheta)\right|^2$$

$$\vec{q} = \vec{p} - \vec{p}'$$

Spectral Method



$\psi_{in} = e^{i\vec{p}\vec{r}}$ $\psi = e^{i\vec{p}\vec{r}}\varphi(\vec{r})$

$$\psi = e^{i(pz-\varepsilon t)}\varphi(\vec{\rho}, z)$$

$$i\hbar v\partial_z\varphi(\vec{\rho}, z) = \left(\frac{\vec{p}_\perp^{\,2}}{2\varepsilon} + U(\vec{\rho})\right)\varphi(\vec{\rho}, z) =$$

$$= \left(\hat{H}_0 + U(\vec{\rho})\right)\varphi$$

wave function $\varphi(\vec{\rho}, z+\Delta z) = e^{-\frac{i}{\hbar}\left(\hat{H}_0 + U(\vec{\rho})\right)\Delta z}\varphi(\vec{\rho}, z)$

$$\varphi^{WKB}(\vec{\rho}, z) = \sqrt{\int d^2b\ \delta\left(\vec{\rho} - \vec{\rho}\left(\vec{b}, z, \vec{p}\right)\right)}\cdot e^{iS/\hbar}$$

*W.H. Miller, Adv. Chem. Phys. **61** (1974) 1823*
*A. Akhiezer, N. Shul'ga, Phys. Rep. **234** (1993) 297*

Spectral Method

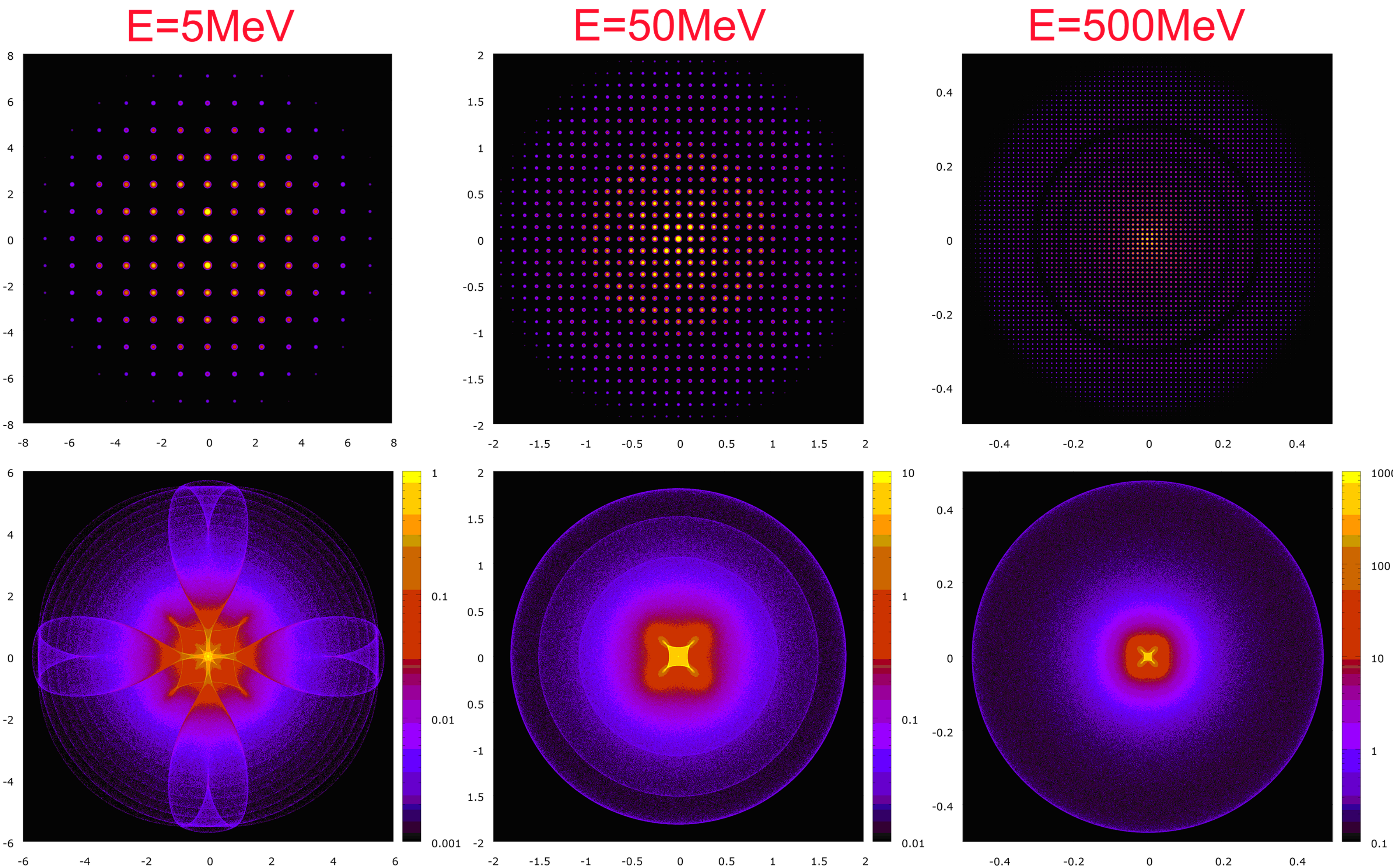
Optics (resonant frequencies in waveguides and optical fibers)
M. Feit et al. J. Comp. Phys. 47 (1982) 412.

Nuclear Physics
Yu. Bolotin et al. Phys. Lett. A 323 (2004) 218.

Channeling
S. Dabagov et al. NIM B30 (1988) 185 (ε~MeV)
A. Kozlov, N. Shul'ga, et al. Phys. Lett. A374 (2010)4690 (levels and zone structure)
N. Shul'ga. V. Syshchenko et al. NIM B309 (2010)153 (levels for dynamical chaos in thick crystals)

Scattering
S. Shul'ga, N. Shul'ga et al. arxiv:1512.04601v1 (2015)

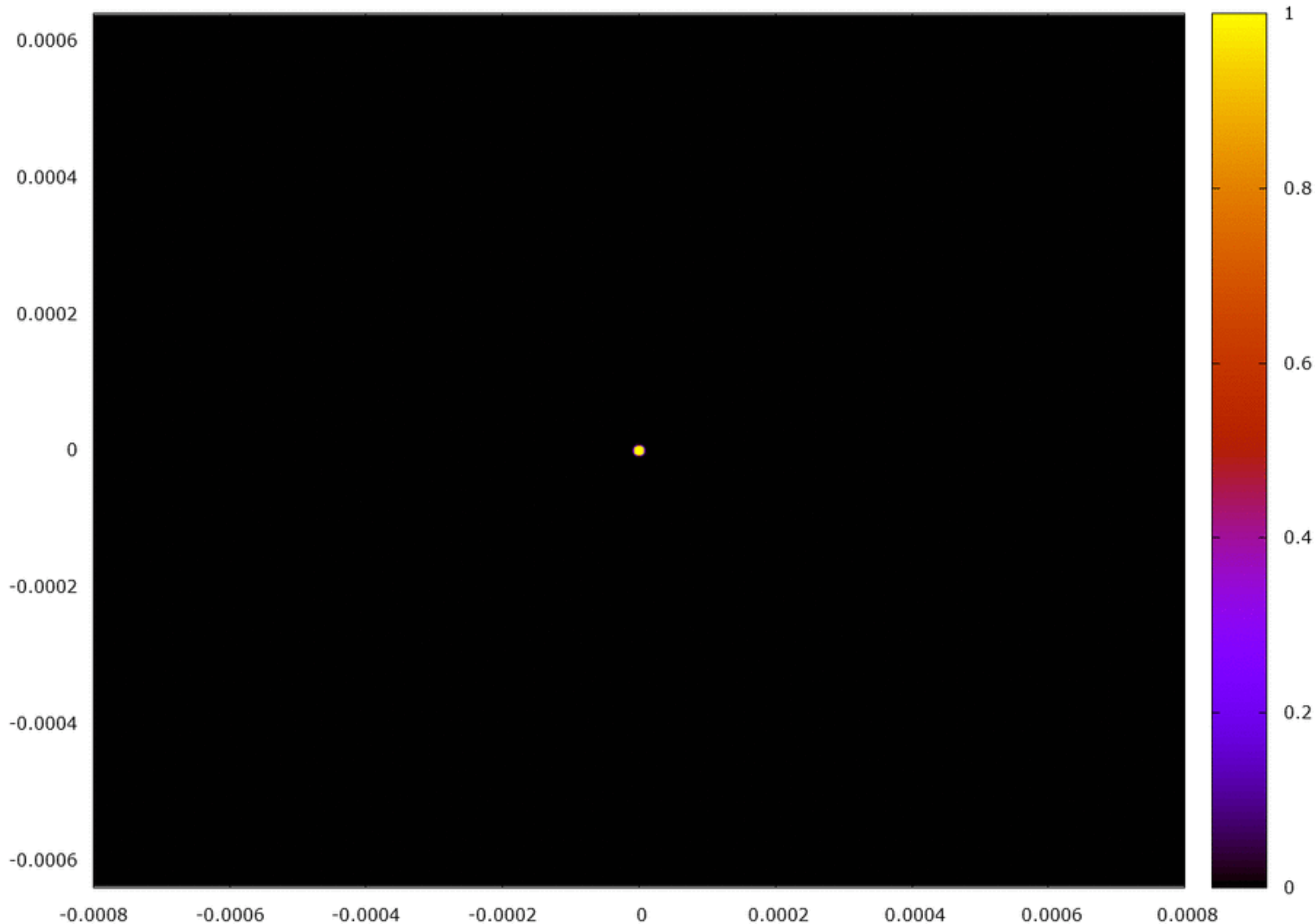
Quantum and classical angular distributions of electrons in 1000Å Si <100>



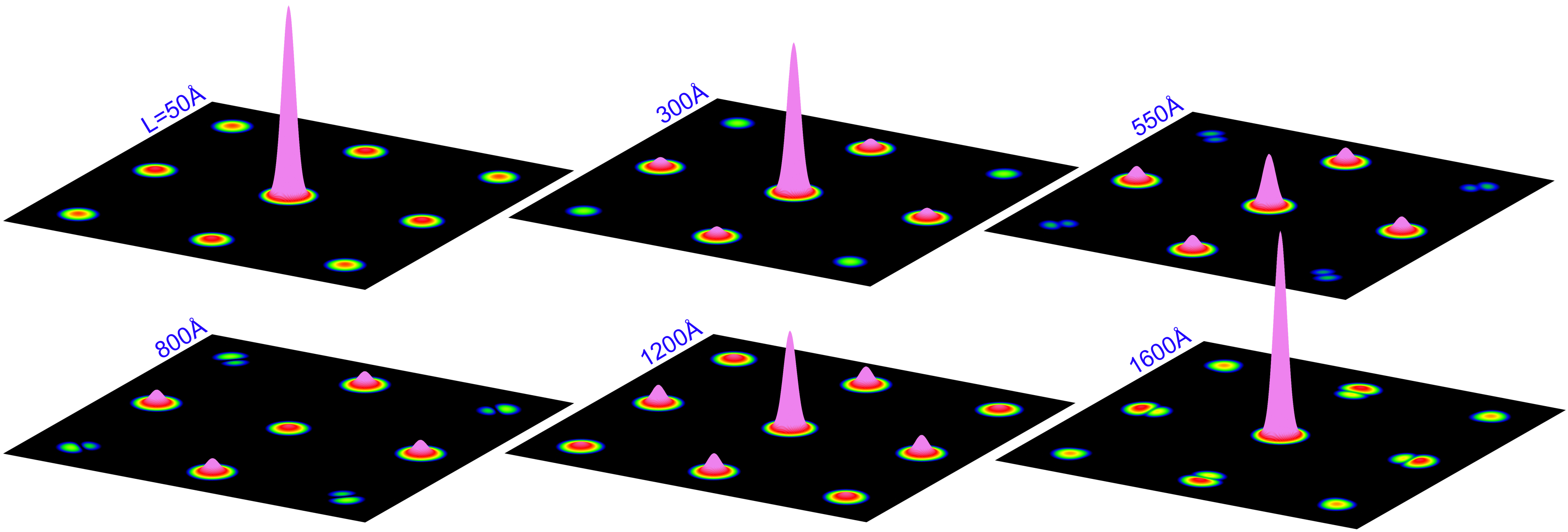
Prediction of rainbow scattering in crystal

S. Fomin, N. Shul'ga Phys. Lett. A73 (1979) 131

Scattering of a 140 MeV electron at (110) Si crystal axis, $\psi_i = 0$, L=0--2000Å



Quantum angular distributions of electrons in ultrathin Si <100> crystal

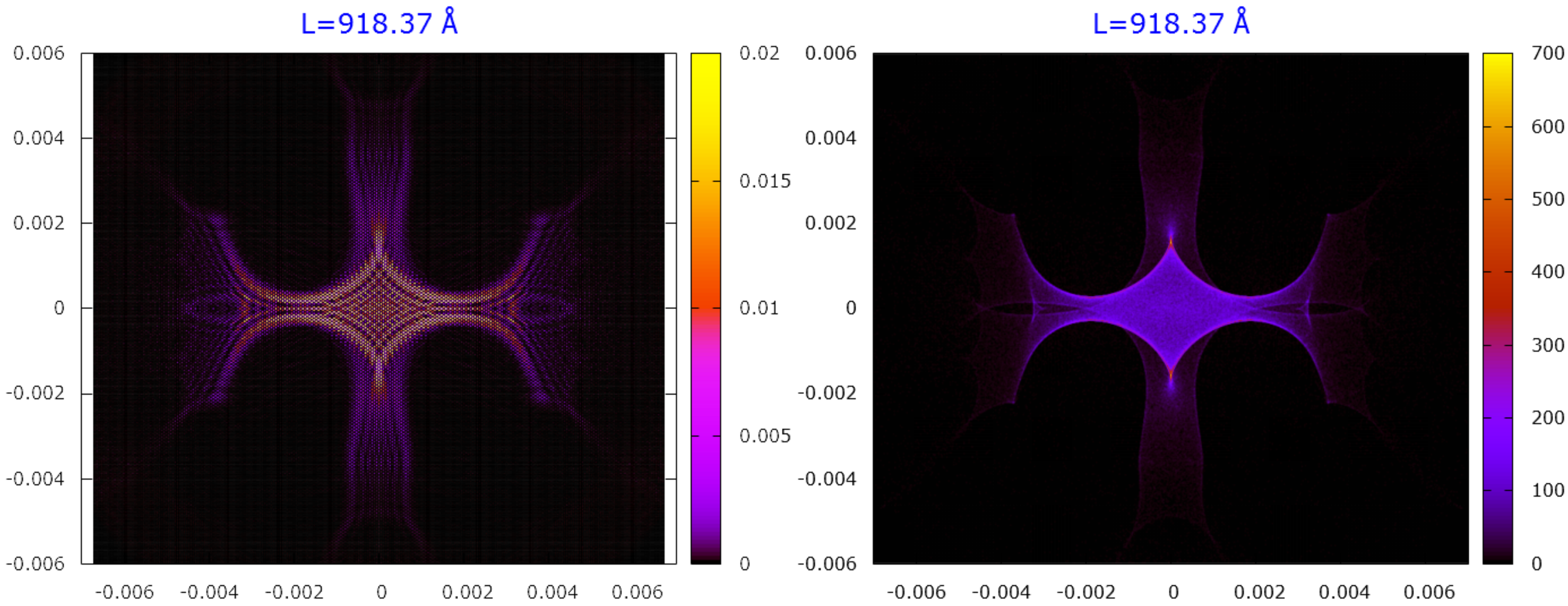


electrons 5MeV Si <100> 50-1600Å

Quantum angular distributions of electrons in ultrathin Si <100> crystal

electrons 5MeV Si <100> 50-1600Å

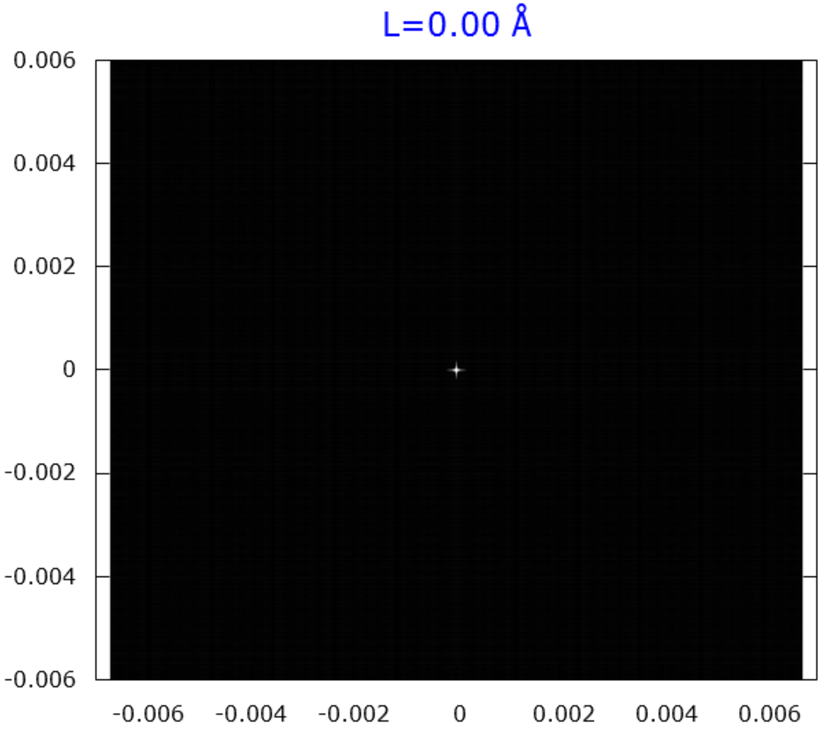
Quantum and Classical angular distributions of 2 MeV protons in ultrathin Si <110> crystal, Ψ=0

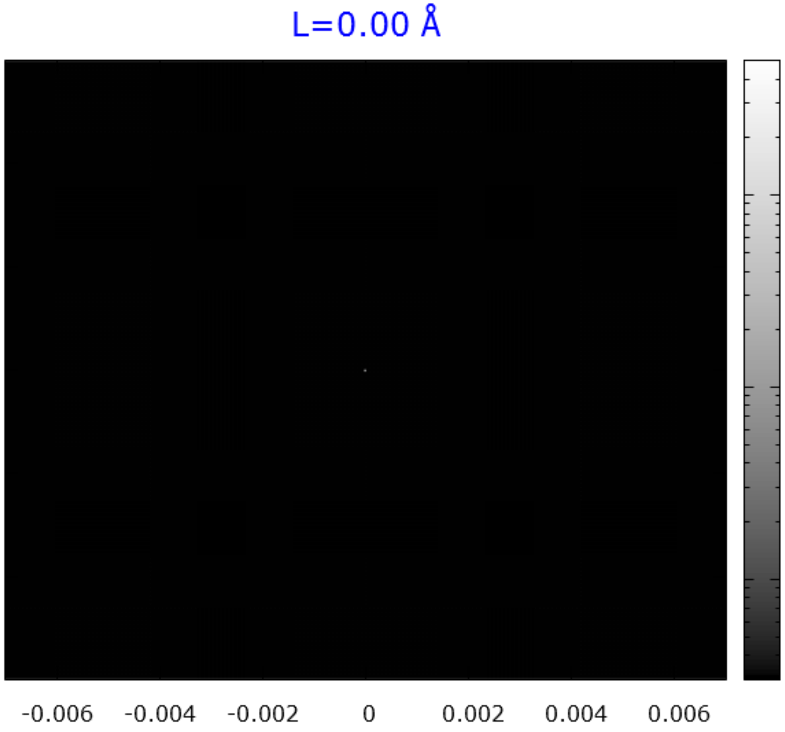


quantum classical

Experiment: M. Mothapothula et al. NIM B283 (2012) 29 (Fig. 2)

Quantum and Classical angular distributions of 2 MeV protons in ultrathin Si <110> crystal

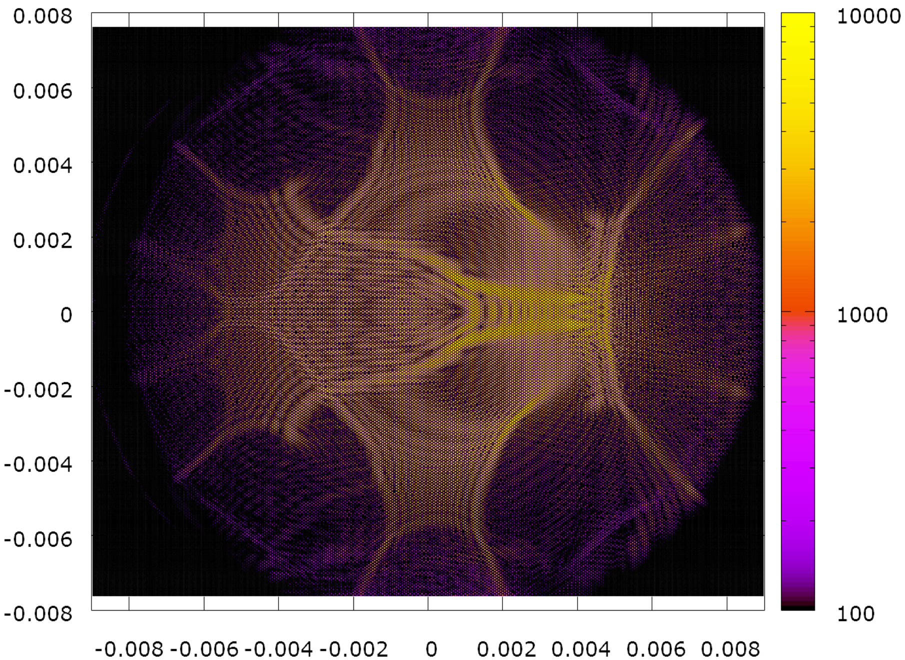


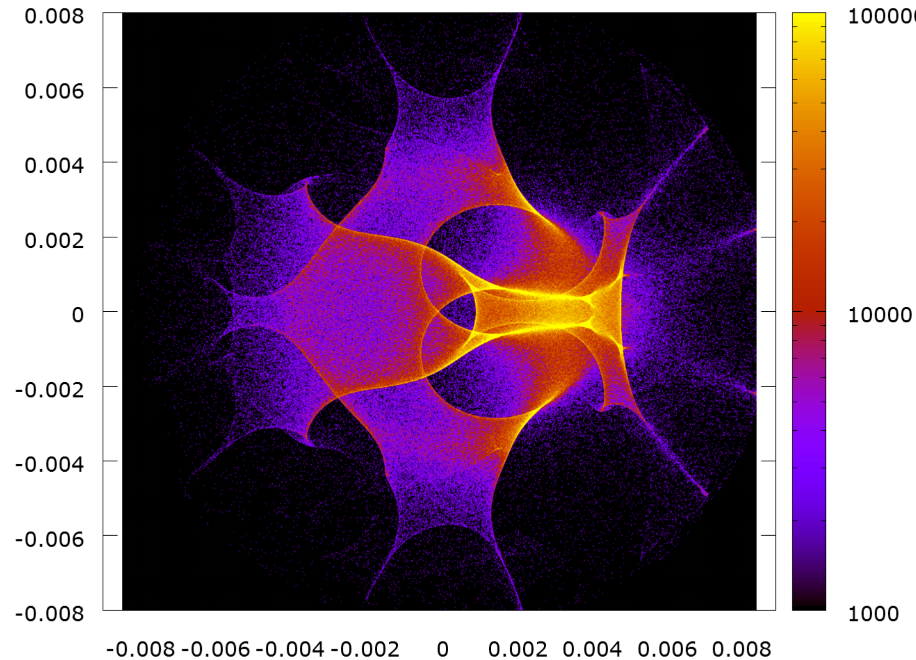


quantum

classical

Quantum and Classical angular distributions of 2 MeV protons in ultrathin Si <110> crystal $\Psi=\Psi_c/2$



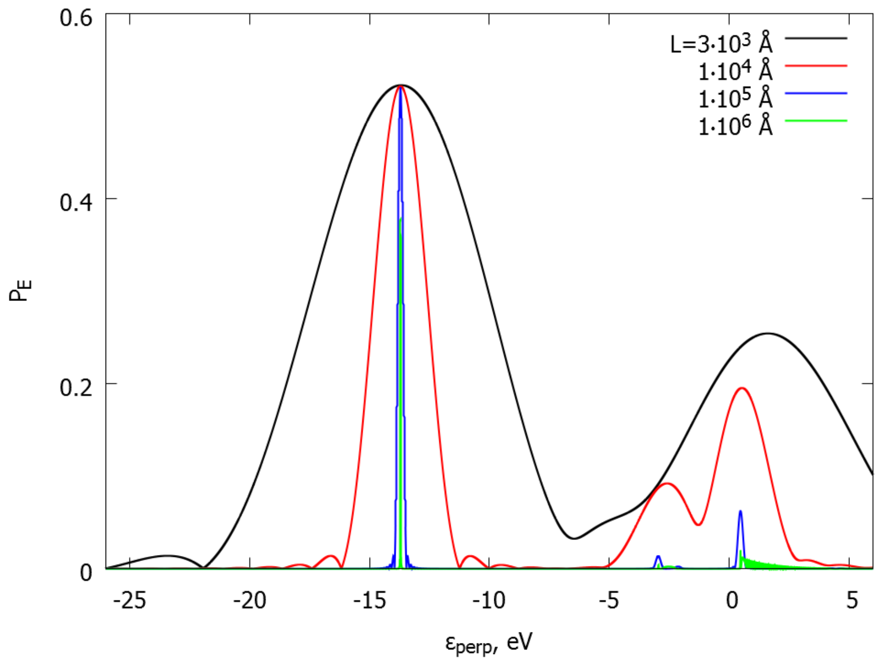


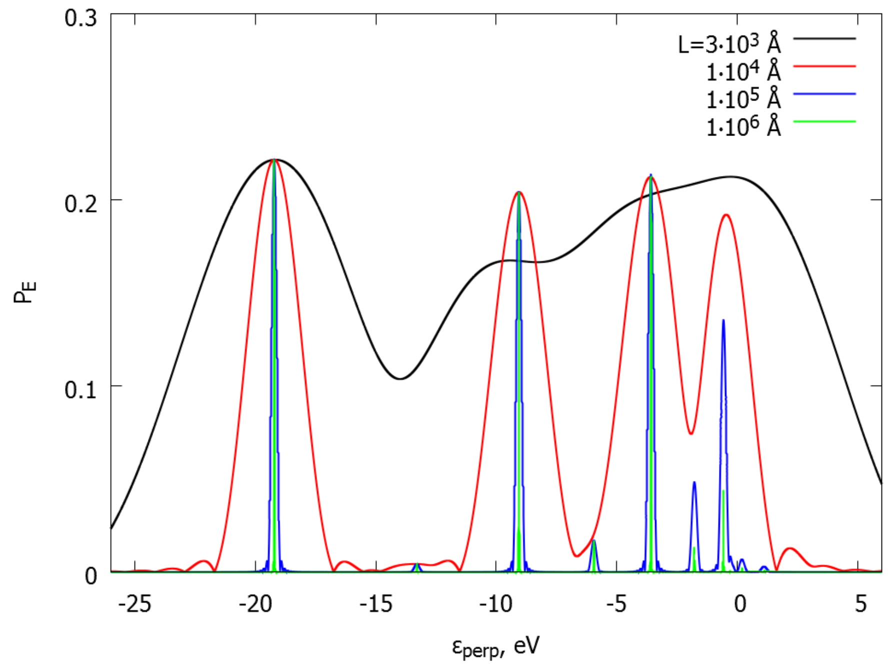
quantum

classical

Experiment: M. Mothapothula et al. NIM B283 (2012) 29 (Fig. 7)

Bound states levels for 4 MeV and 50 MeV electrons at different thicknesses in (110) Si crystal





$E_{kin} = 4MeV$

$E_{kin} = 50MeV$

bound states levels $$P_E = \frac{1}{T}\int_0^T dt e^{\frac{i}{\hbar}\varepsilon t} \int d^2\rho\, \psi^*(\rho, t=0)\psi(\rho, t)$$

Conclusions

- Quantum and classical theories of scattering

- Transitional region from ultrathin to thick crystals (from channeling absence to channeling presence)

- Quantun and classical effects at scattering (coherence, interference, rainbow, …)

- Possibility of experimental observation of quantum effects

- Radiation in transitional region of thickness

- How do quantum levels and zones appear at regular motion and dynamical chaos?
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-
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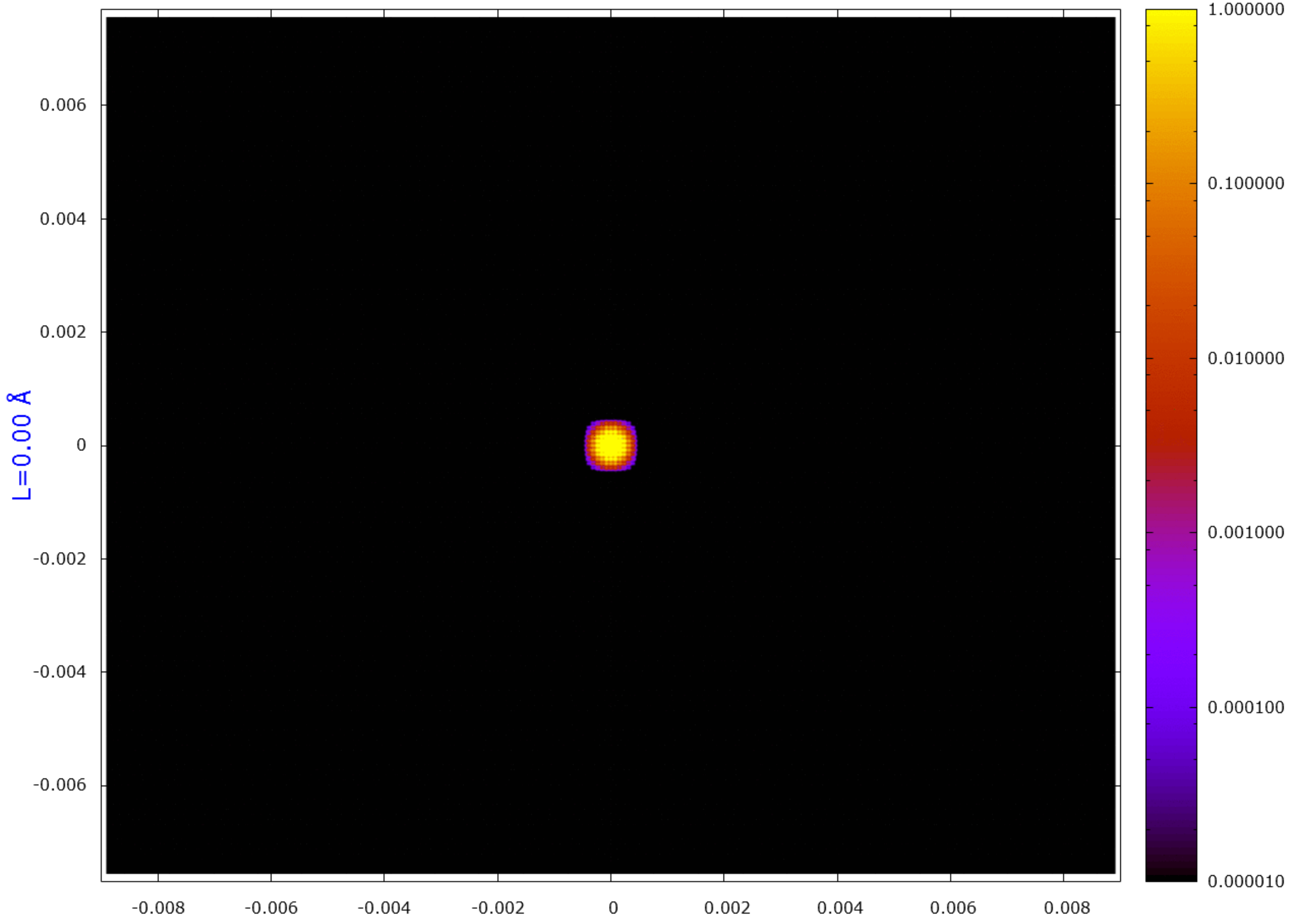
Thank you for attention!



Quantum: p^+ $\varepsilon = 2MeV$ $Si\langle 110 \rangle$ $0 < L < 2000\,\mathring{A}$