Kinetics of relativistic electrons passing through quasi-periodic fields

Eugene Bulyak, Nikolay Shul’ga

NSC KIPT, Kharkov Ukraine

Channeling 2016 Sirmione del Garda Sept. 27
Challenge of hard x–rays and gamma rays

High intensity, $\gtrsim 10^{12} \ldots 10^{15}$ photons/sec

<table>
<thead>
<tr>
<th></th>
<th>$E_{\text{phot}}$ MeV</th>
<th>$E_{\text{el}}$ GeV</th>
<th>based on</th>
<th>phot/e$^-$/pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>polar $e^+$ LC</td>
<td>5...10</td>
<td>120...150</td>
<td>linac+undul</td>
<td>50...300</td>
</tr>
<tr>
<td>polar $e^+$ LC</td>
<td>15...20</td>
<td>1.0...1.3</td>
<td>CR+laser</td>
<td>$\sim 10^{-2}$</td>
</tr>
<tr>
<td>NRF anal</td>
<td>1...3</td>
<td>0.3...0.7</td>
<td>CR/ERL+las</td>
<td>$10^{-3} \ldots 10^{-2}$</td>
</tr>
<tr>
<td>medic x–rays</td>
<td>0.03...0.1</td>
<td>0.03...0.1</td>
<td>CR/ERL/lin+las</td>
<td>$10^{-6} \ldots 10^{-5}$</td>
</tr>
<tr>
<td>XFEL</td>
<td>0.005...0.02</td>
<td>1...3</td>
<td>linac+undul</td>
<td>1000</td>
</tr>
</tbody>
</table>

CR – Compton ring; ERL – energy recovered linac

XFEL requires fine tuning

Kinetics of electron bunches is an issue: recoils should not be neglected
Problem setup: ultra-relativistic electrons, quantum recoils

\[ f_0(\gamma) \quad f(z, \gamma) \]

Motivation:
- ILC positron source: effect of the undulator on the beam parameters
- ILC alternative Compton positron source – performance
- FEL energy limitations
- Lack of theory corresponding to \( x \lesssim 1 \), drawbacks of the diffusive approximation

Problem setup:
- Periodic force with given envelope – undulator, laser pulse
- Radiation: statistically independent photons, given spectra
- Recoils decrease electrons’ energy
- Goal: evolution of initially given electron spectrum along the field
- Special attention – front end of the field
Kinetic equation for electron spectrum \( f(z, \gamma) \)

Landau 1944; Akhiezer, Shul’ga 1996; Khokonov 2004

System of units: \( m_e = c = \hbar = 1 \), initial distribution \( f_0(\gamma) \equiv f(z = 0, \gamma) \)

\[
\frac{\partial}{\partial z} f(z, \gamma) = \int [f(z, \gamma + \omega)W(z, \gamma + \omega, \omega) - f(z, \gamma)W(z, \gamma, \omega)] \, d\omega
\]

where \( W(z, \gamma, \omega) \) is the probability density

Approximation: \( \gamma \gg 1, \omega_{\text{max}} \ll \gamma \), \( W(z, \gamma, \omega) = \psi(z)w(\gamma, \omega) \approx \psi(z)w(\omega) \)

Kinetic equation casts into

\[
f' = f \star w - f
\]

with \( f \star w \equiv \int f(\gamma + \omega)w(\omega)d\omega \) cross correlation; \( x = \int_0^z \psi(z') \, dz' \) number of emitted photons.
Fourier transform of electron spectrum evolution then the inverse

\[
\hat{f} = \hat{f}_0 e^{x(\hat{\omega} - 1)} = \sum_{n=0}^{\infty} \frac{e^{-x}x^n}{n!} (\hat{f}_0 \hat{\omega}^n)
\]

\[
f(x, \gamma) = \sum_{n=0}^{\infty} \frac{e^{-x}x^n}{n!} F_n(\gamma),
\]

with \( F_0(\gamma) = f_0(\gamma) = f(x = 0, \gamma) \) the initial distribution (spectrum),

\[
F_n(\gamma) = \int_{-\infty}^{\infty} F_{n-1}(\gamma + \omega) w(\omega) \, d\omega
\]

mean energy \( \bar{\gamma} \) variance \( (\gamma - \bar{\gamma})^2 \) skewness \( (\gamma - \bar{\gamma})^3 \) \( \Rightarrow \) linear of \( x \)

\[
\bar{\gamma}(x) = \bar{\gamma}_0 - x \bar{\omega}; \quad \text{Var}[\gamma](x) = \text{Var}[\gamma_0] + x \bar{\omega}^2; \quad \text{Sk}[\gamma](x) = \text{Sk}[\gamma_0] - x \bar{\omega}^3
\]

Relation to distance \( z \) along the axis: \( z \rightarrow x(z) = \int_0^z \psi(z') \, dz' \).
Evolution in general

Electrons’ $n$th central moment linear of recoil’s $n$th raw moment and the average number of recoils

$$\overline{(\gamma - \bar{\gamma})^n} = (\gamma_0 - \bar{\gamma}_0)^n + (-1)^n x \bar{\omega}^n ; \ n > 1$$

- Contribution of the initial distribution, $f_0$, decays as $e^{-x}$
- Tail on the left (negative skewness)
- Asymmetry $\sim 1/\sqrt{x}$
Max photon energy (=recoil) scaled as $\omega_{\text{max}} \propto \gamma^2 \times \lambda_{\text{Comp}}/\lambda_u$

Smallness of recoils

$$\frac{\omega_{\text{max}}}{\gamma} \approx G\gamma \frac{\lambda_{\text{Comp}}}{\lambda_u} \ll 1$$

Keeping the terms of second–order smallness and set $\gamma = \bar{\gamma}$ we get

$$\bar{\gamma}_{\text{cor}} = \bar{\gamma}_0 - x\bar{\omega}_0 \left(1 - \frac{2x\bar{\omega}_0}{\bar{\gamma}_0}\right)$$

$$\text{Var}[\gamma] = \text{Var}[\gamma_0] + x\bar{\omega}_0^2 \left(1 - \frac{4x\bar{\omega}_0}{\bar{\gamma}_0}\right)$$
Dipole radiation. Specific states
Initial $\delta(\gamma - 2000) \omega_* = 40$

$m$th state properties
- normalized $\int f_m d\gamma = 1$
- symmetric about mean = median = mode = $\gamma_0 - m\bar{\omega}$
- compact support, $\text{supp}[f_m] = m \text{supp}[\omega]$
Dipole radiation. Aggregate spectra analytic vs. simulation. $f_0$ excluded
Dipole harmonic
Compton source, 1 GeV + 1 eV

for $x \gg 1$ max density

$\gamma_{\text{mode}} \approx \overline{\gamma} + \frac{\omega^3}{2\omega^2}$
Tapering = variation $K(z)$ to keep photon’s frequency resonant

$$\frac{[\gamma(z) + \kappa]^2}{1 + K^2(z)} = \text{const} = \frac{\gamma_0^2}{1 + K_0^2}, \quad \kappa \equiv \frac{\omega^3}{2\omega^2}$$

(helical undulator, $z$ – in periods)

Iterative procedure

$$1 + K^2(z) = \frac{1 + K_0^2}{\gamma_0^2} \left[ \gamma_0 + \kappa - \epsilon \int_0^z K^2(z') \, dz' \right]^{2}, \quad \epsilon \approx \frac{4\pi\alpha_{fs}}{3}$$

converges fast

Initial blocks must be tapered with respect to evolution of electrons’ spectrum

Electron energy spread must not exceed

\[ \frac{\Delta \gamma}{\gamma} \approx \sqrt{\text{Var}[\gamma]} = \frac{1}{2N_{\text{und}}} \]

Coherent length (number of ‘coherent’ periods) does not exceed

\[ \frac{L_{\text{coh}}}{\lambda_u} \approx 2 \left( \frac{\lambda_u}{K \gamma \lambda_C} \right)^{2/3} \propto \gamma^{-2/3} \]

the gain length

\[ \frac{L_g}{\lambda_u} \propto \gamma \]

Max ‘coherent’ electron energy \( \gamma_{mc} \) ⇒ \( L_g = L_{\text{coh}} \) Max energy of photons \( \propto \gamma_{mc}^2 \)
Summary and Conclusion

**Summary**

- Evolution of spectrum of electrons that undergo small quantum losses was studied.
- Electrons’ spectrum may be presented as the Poisson–weighted sum of the specific $n$-spectrum in domain of number of recoils.
- *Centered moments* of the spectrum are linear on number of recoils and on *raw moments* of recoil spectrum.
- Quantum nature of losses induces negative skewness of spectrum.
- Results of the theory are in agreement with both empirical (numerical) data and existing approximate theories.

**Conclusions**

- Dilution of the el. spectrum will limit performance (brightness) of x– and gamma–rays.
- Negative skewness decreases ‘quantum life time’ of the beam circulating in Compton rings.
- Derived expressions are suitable for computer codes.
Simulations – ILC undulator: \( K = 0.92 \) vs \( K = 0.46 \) at 150 GeV \( f_0 = \delta(\gamma - \gamma_0) \), 15 000 particles

Relative deviation \( = \frac{\gamma - \gamma_0}{\gamma_0} \)

**K=0.92 (300 gammas)**

- Mean = -0.019; st.dev = 1.29E-3;
- Min = -0.024; Max = -0.014
- Min = mean - 0.005; Max = mean + 0.005

**K=0.46 (70 gammas)**

- Mean = -0.0046; st.dev = 6.48E-4;
- Min = -0.0077; Max = -0.0023
- Min = mean - 0.0031; Max = mean + 0.0023
longitudinal phase space

- electrons escape the separatrix downward, in the ‘tail direction’
  - separatrix cuts out the tail
- rate of losses $\propto$ bunch density $\times$ laser density
- red band width $\propto$ tail length $\times$ synchrotron period

We proposed to mitigate quantum losses via *the asymmetric cooling*
Model setup

Laser radiation field exists at $z \geq 0$

Spread vs. # scatterings