Kinetics of relativistic electrons passing through quasi-periodic fields

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Challenge of hard x–rays and gamma rays
High intensity, $\gtrsim 10^{12} \ldots 10^{15}$ photons/sec

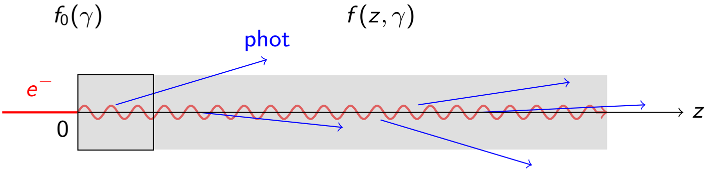
	$E_{\rm phot}$ MeV	$E_{\rm el}$ GeV	based on	phot/e^-/pass
polar e^+ LC	$5 \ldots 10$	$120 \ldots 150$	linac+undul	$50 \ldots 300$
polar e^+ LC	$15 \ldots 20$	$1.0 \ldots 1.3$	CR+laser	$\sim 10^{-2}$
NRF anal	$1 \ldots 3$	$0.3 \ldots 0.7$	CR/ERL+las	$10^{-3} \ldots 10^{-2}$
medic x–rays	$0.03 \ldots 0.1$	$0.03 \ldots 0.1$	CR/ERL/lin+las	$10^{-6} \ldots 10^{-5}$
XFEL	$0.005 \ldots 0.02$	$1 \ldots 3$	linac+undul	1000

CR – Compton ring; ERL – energy recovered linac

XFEL requires fine tuning

Kinetics of electron bunches is an issue: recoils should not be neglected

Problem setup: ultra–relativistic electrons, quantum recoils



problem setup

- periodic force with given envelop – undulator, laser pulse
- radiation: statistically independent photons, given spectra
- recoils decrease electrons' energy
- goal: evolution of initially given electron spectrum along the field special attention – front end of the field

motivation

- ILC positron source: effect of the undulator on the beam parameters
- ILC alternative Compton positron source – performance
- FEL energy limitations
- lack of theory corresponding to $x \lesssim 1$, drawbacks of the diffusive approximation

Kinetic equation for electron spectrum $f(z,\gamma)$
Landau 1944; Akhiezer, Shul'ga 1996; Khokonov 2004

System of units: $m_e = c = \hbar = 1$, initial distribution $f_0(\gamma) \equiv f(z=0,\gamma)$

$$\boxed{\frac{\partial}{\partial z} f(z,\gamma) = \int \left[f(z,\gamma+\omega) W(z,\gamma+\omega,\omega) - f(z,\gamma) W(z,\gamma,\omega) \right] \mathrm{d}\omega}$$

where $W(z,\gamma,\omega)$ is the probability density
Approximation: $\gamma \gg 1,\ \omega_{\max} \ll \gamma,\ \ W(z,\gamma,\omega) = \psi(z) w(\gamma,\omega) \approx \psi(z) w(\omega)$
Kinetic equation casts into

$$\boxed{f'_x = f \star w - f}$$

with $f \star w \equiv \int f(\gamma+\omega) w(\omega) d\omega$ cross correlation; $x = \int_0^z \psi(z')\,\mathrm{d}z'$ number of emitted photons.

Fourier transform of electron spectrum evolution then the inverse

$$\hat{f} = \hat{f}_0 \, \mathrm{e}^{x(\check{w}-1)} = \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-x} x^n}{n!} (\hat{f}_0 \check{w}^n) \qquad f(x,\gamma) = \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-x} x^n}{n!} F_n(\gamma) ,$$

with $F_0(\gamma) = f_0(\gamma) = f(x=0,\gamma)$ the initial distribution (spectrum),

$$F_n(\gamma) = \int_{-\infty}^{\infty} F_{n-1}(\gamma+\omega) w(\omega) \, \mathrm{d}\omega$$

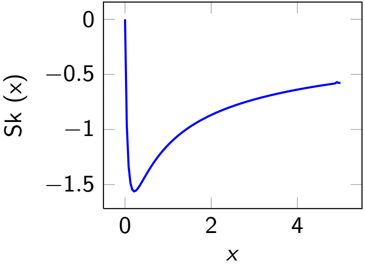
mean energy $\overline{\gamma}$ variance $\overline{(\gamma-\overline{\gamma})^2}$ skewness $\overline{(\gamma-\overline{\gamma})^3}$ $\Rightarrow$ linear of x

$\overline{\gamma}(x) = \overline{\gamma_0} - x\,\overline{\omega}$; $\mathrm{Var}[\gamma](x) = \mathrm{Var}[\gamma_0] + x\,\overline{\omega^2}$; $\mathrm{Sk}[\gamma](x) = \mathrm{Sk}[\gamma_0] - x\,\overline{\omega^3}$

Relation to distance z along the axis: $z \rightarrow x(z) = \int_0^z \psi(z')\,\mathrm{d}z'$.

Evolution in general

Pearson's skewness $\frac{\overline{(\gamma-\overline{\gamma})^3}}{\overline{(\gamma-\overline{\gamma})^2}^{3/2}}$



- Electrons' nth central moment linear of recoil's nth raw moment and the average number of recoils

$$\overline{(\gamma-\overline{\gamma})^n} = \overline{(\gamma_0-\overline{\gamma_0})^n} + (-1)^n x\,\overline{\omega^n}\,; \;\; n > 1$$

- Contribution of the initial distribution, f_0, decays as e^{-x}
- Tail on the left (negative skewness)
- Asymmetry $\sim 1/\sqrt{x}$

Max photon energy (=recoil) scaled as $\omega_{\rm max} \propto \gamma^2 \times \lambda_{\rm Comp}/\lambda_{\rm u}$
Smallness of recoils

$$\frac{\omega_{\rm max}}{\gamma} \approx G\gamma \frac{\lambda_{\rm Comp}}{\lambda_{\rm u}} \ll 1$$

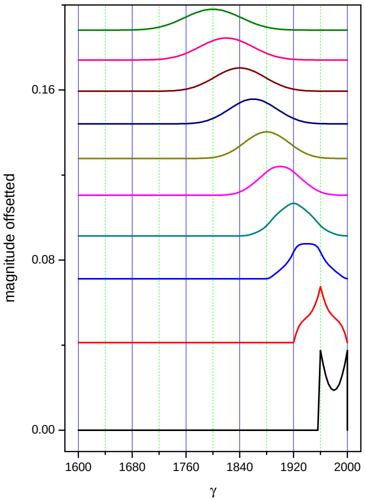
Keeping the terms of second–order smallness and set $\gamma = \overline{\gamma}$ we get

$$\overline{\gamma}_{\rm cor} = \overline{\gamma_0} - x\overline{\omega_0}\left(1 - \frac{2x\overline{\omega_0}}{\overline{\gamma_0}}\right)$$

$$\mathrm{Var}[\gamma] = \mathrm{Var}[\gamma_0] + x\overline{\omega_0^2}\left(1 - \frac{4x\overline{\omega_0}}{\overline{\gamma_0}}\right)$$

Dipole radiation. Specific states

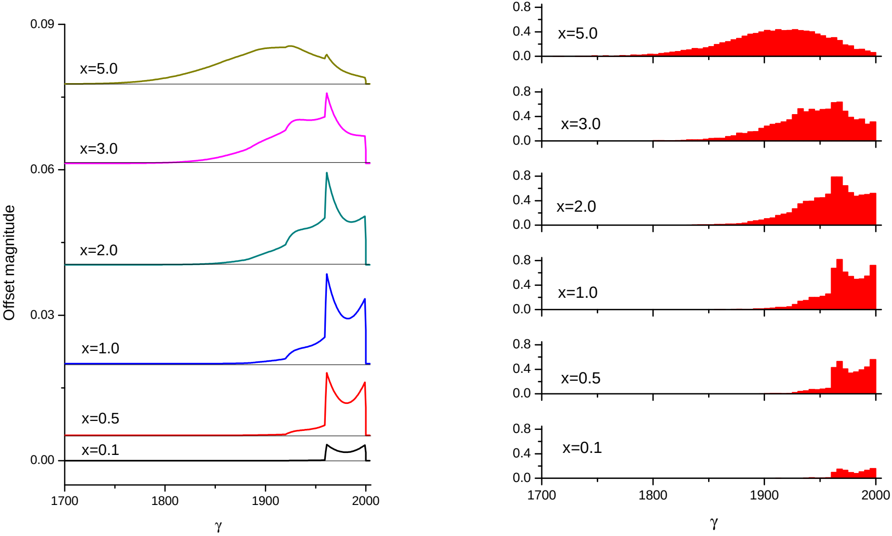
Initial $\delta(\gamma - 2000)$ $\omega_* = 40$



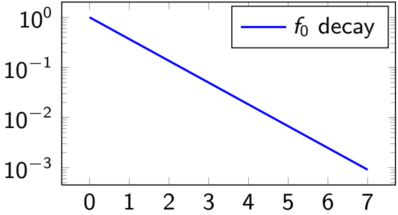
*m*th state properties

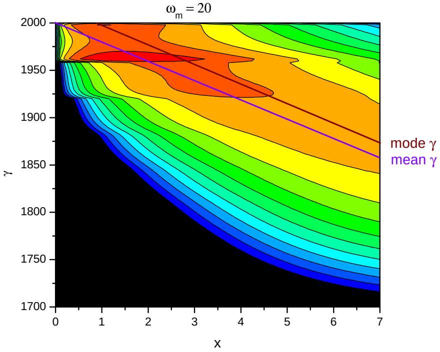
- normalized $\int f_m \, \mathrm{d}\gamma = 1$
- symmetric about
  $\text{mean} = \text{median} = \text{mode} = \gamma_0 - m\,\overline{\omega}$
- compact support, $\mathrm{supp}[f_m] = m\,\mathrm{supp}[\omega]$

Dipole radiation. Aggregate spectra
analytic vs. simulation. f_0 excluded



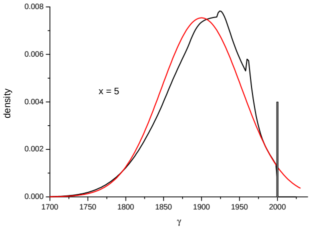
Dipole harmonic
Compton source, 1 GeV + 1 eV

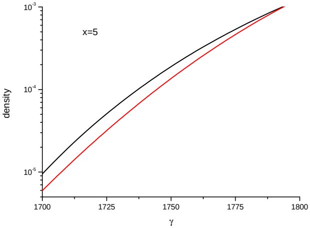




for $x \gg 1$ max density

$$\gamma_{\text{mode}} \approx \overline{\gamma} + \frac{\overline{\omega^3}}{2\overline{\omega^2}}$$





Application for Undulator Tapering

Tapering = variation $K(z)$ to keep photon's frequency resonant

$$\frac{[\overline{\gamma}(z)+\kappa]^2}{1+K^2(z)} = \mathrm{const} = \frac{\overline{\gamma_0}^2}{1+K_0^2}\ , \quad \kappa \equiv \frac{\overline{\omega^3}}{2\overline{\omega^2}}$$

(helical undulator, z – in periods)
Iterative procedure

$$1+K^2(z) = \frac{1+K_0^2}{\overline{\gamma_0}^2}\left[\overline{\gamma_0}+\kappa-\epsilon\int_0^z K^2(z')\,\mathrm{d}z'\right]^2\ , \quad \epsilon \approx \frac{4\pi\alpha_{\mathrm{fs}}}{3}$$

converges fast
Initial blocks must be tapered with respect to evolution of electrons' spectrum
c.f. A.Mak *et.al.* arXiv:1608.08108v1 [physics.acc-ph] 29 Aug 2016

Limits of Coherency
Pellegrini C., Marinelli A. and Reiche S. Rev. Mod. Phys.(2016)

Electron energy spread must not exceed

$$\frac{\Delta\gamma}{\gamma} \approx \frac{\sqrt{\mathrm{Var}[\gamma]}}{\gamma} = \frac{1}{2N_{\mathrm{und}}}$$

Coherent length (number of ‘coherent’ periods) does not exceed

$$\frac{L_{\mathrm{coh}}}{\lambda_{\mathrm{u}}} \approx 2\left(\frac{\lambda_{\mathrm{u}}}{K\gamma\lambda_{\mathrm{C}}}\right)^{2/3} \propto \gamma^{-2/3}$$

the gain length

$$\frac{L_g}{\lambda_{\mathrm{u}}} \propto \gamma$$

Max ‘coherent’ electron energy $\gamma_{mc} \Rightarrow L_g = L_{\mathrm{coh}}$ Max energy of photons $\propto \gamma_{mc}^2$

Summary and Conclusion

summary

- Evolution of spectrum of electrons that undergo small quantum losses was studied
- Electrons' spectrum may be presented as the Poisson–weighted sum of the specific *n*-spectrum in domain of number of recoils
- *Centered moments* of the spectrum are linear on number of recoils and on *raw moments* of recoil spectrum
- Quantum nature of losses induces negative skewness of spectrum
- Results of the theory are in agreement with both empirical(numerical) data and existing approximate theories

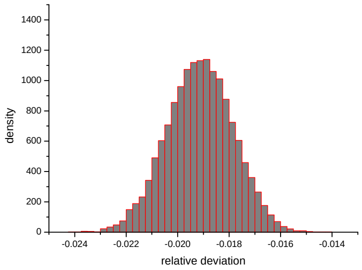
conclusions

- Dilution of the el. spectrum will limit performance (brightness) of x– and gamma–rays
- Negative skewness decreases 'quantum life time' of the beam circulating in Compton rings
- Derived expressions are suitable for computer codes

Simulations – ILC undulator: $K = 0.92$ vs $K = 0.46$ at 150 GeV $f_0 = \delta(\gamma - \gamma_0)$, 15 000 particles

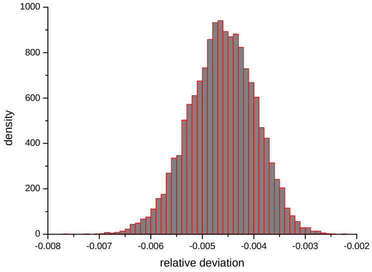
relative deviation $\equiv \frac{\gamma - \gamma_0}{\gamma_0}$

K=0.92 (300 gammas)



mean = - 0.019; st.dev = 1.29E-3;
min = -0.024; max = -0.014
min=mean-0.005; max=mean+0.005

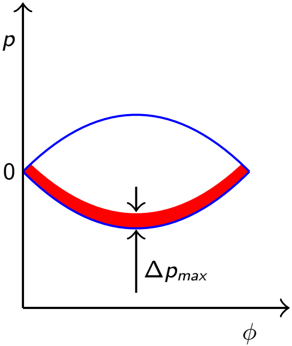
K=0.46 (70 gammas)



mean = - 0.0046; st.dev = 6.48E-4;
min = -0.0077; max = -0.0023
min=mean-0.0031; max=mean+0.0023

Where tails matter: Quantum lifetime in Compton rings

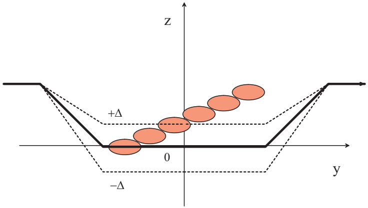
longitudinal phase space



- electrons escape the separatrix downward, in the 'tail direction'
  separatrix cuts out the tail
- rate of losses $\propto$ bunch density$\times$laser density
- red band width $\propto$ tail length$\times$synchrotron period

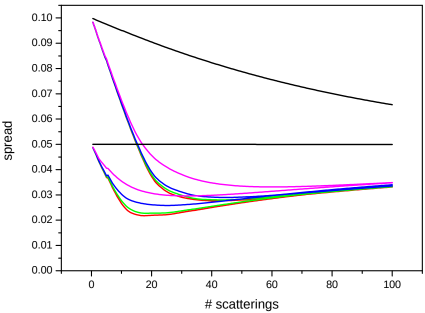
We proposed to mitigate quantum losses via *the asymmetric cooling*

Asymmetric (Fast) Cooling. E.Bulyak, J.Urakawa, F.Zimmermann 2011–2013



Model setup

Laser radiation field exists at $z \geq 0$



Spread vs. # scatterings