Generation of Plasmons by quantum charged fast oriented particle

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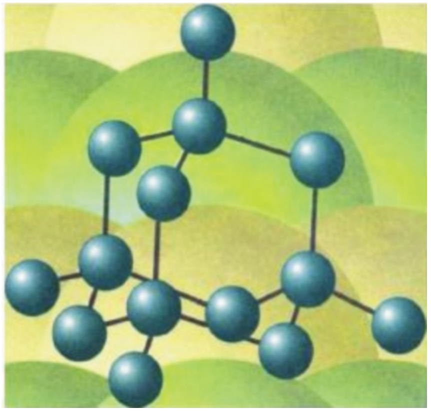
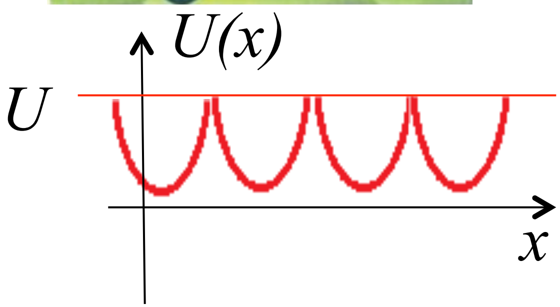
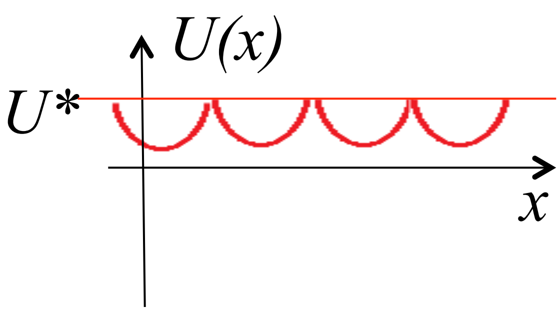






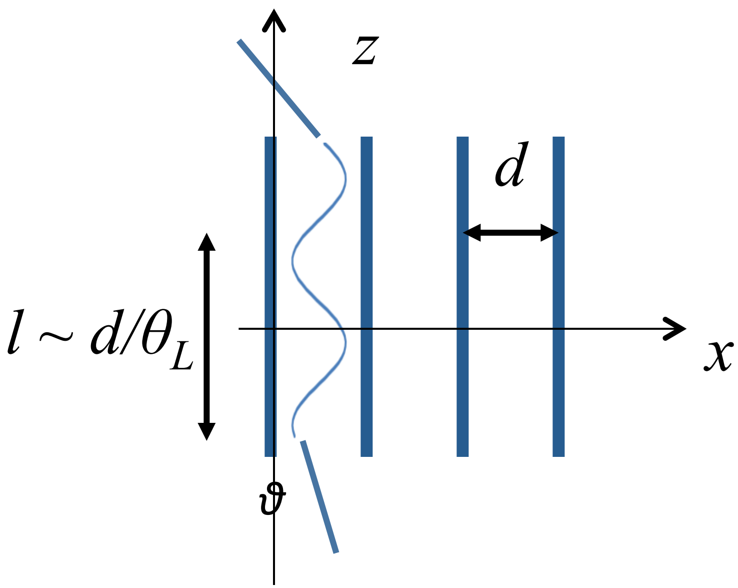
Standard approach:

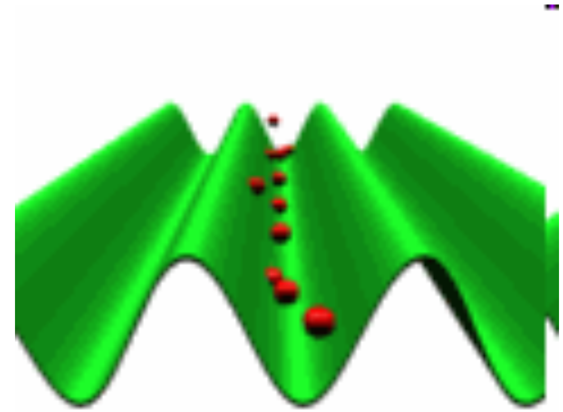
$U(x) = \int dydz(Ze/r)exp(-r/R_{atom})$

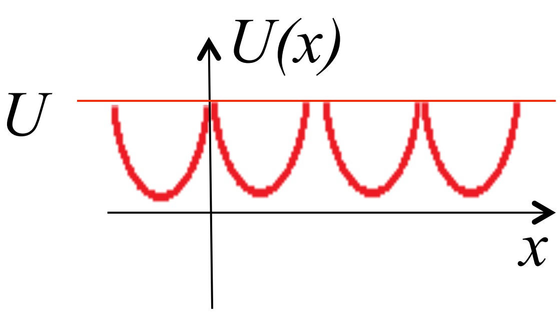
In covalence crystals (C, Si) 4 electrons are far away from nucleus and evenly distributed

$U^*(x) = \int dydz(Z-4)e/r)exp(-r/R_{ion}) < U(x)$

Planar channeling - thick crystal







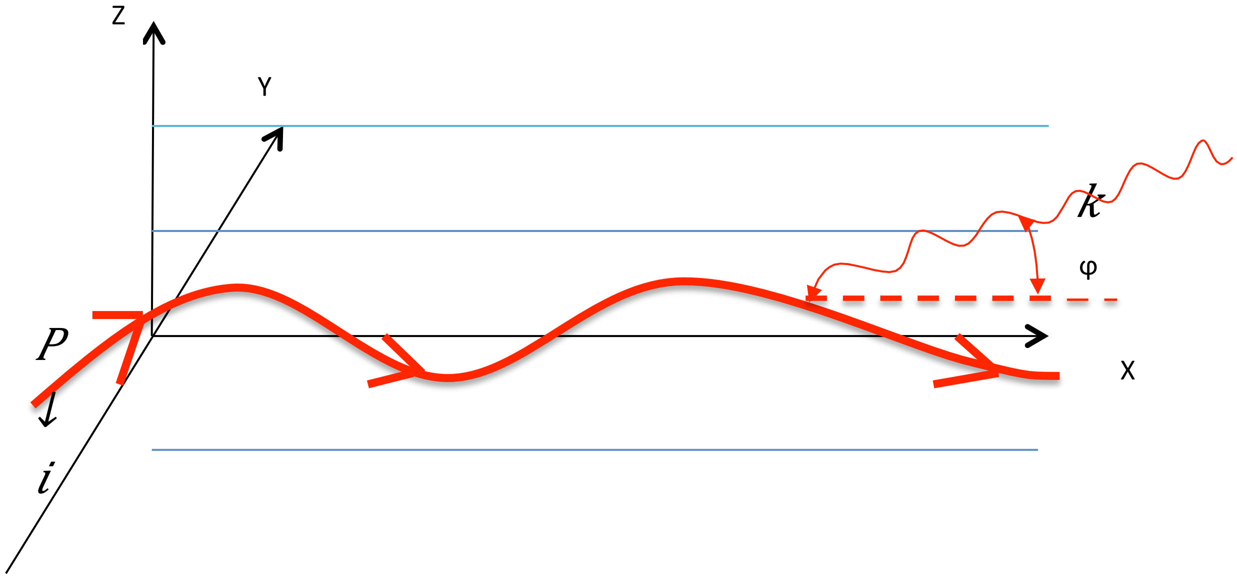
$\vartheta < \vartheta_L = (2U/E)^{1/2}$ Lindhard angle

$E >> mc^2$; $U \sim 20 - 40$ eV

d – interplane distance;

l – period of trajectory

$$P_i = P_i(x) + P_{i\perp}(\theta) = P_0\left(1-\theta_0^2/2\right) i + P_0\,\theta_0\, j$$



$P_{i\perp}^2/2m \lessapprox \langle U \rangle$, $\quad P_0^2\,\theta_0^2/2m \lessapprox \langle U \rangle$, $\quad \theta_0 \le 2m\langle U \rangle / P_o^2 = \theta_L^2$

"Accompanying" reference (coordinate) system "

$$P_x' = \frac{P_x - \frac{V}{c^2}}{\sqrt{1 - \frac{V^2}{C^2}}} = 0 \qquad \Rightarrow V = \frac{P_x}{E} c^2$$

$$P_y' = P_y$$
$$P_z' = P_z$$
$$\Rightarrow P_\perp' = P_\perp$$

$$E' = E - V P_x / \sqrt{1 - V^2 / C^2} \qquad \Rightarrow m_0 c^2 + P_\perp^2 / 2 m_0$$

In “accompanying” system

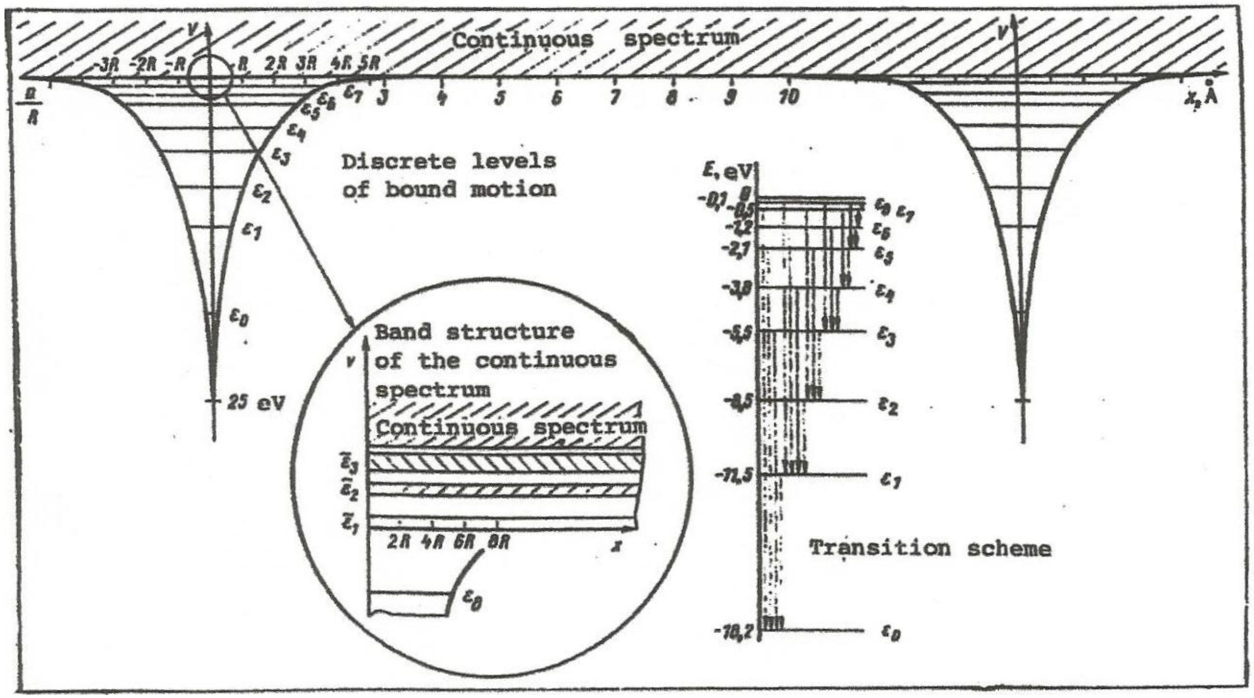
$$\omega=\omega_0\sqrt{1-V^2/C^2}\,/\,(1+Vk/\omega_0)=\omega_0\sqrt{1-V^2/C^2}\,/\,(1+V/C\cos\psi)$$

$$\psi=\pi-\varepsilon$$

$$\omega=\omega_0\sqrt{1-V^2/C^2}\,/\,(1-V/C+V/C\,\varepsilon^2/2)$$

$$\Rightarrow\omega=\omega_0\,1/\gamma^{-1}+\varepsilon^2/2\,\gamma$$

•Structure of energy bands and radiative transitions of 56-MeV electrons channeled along the (110) plane in Si



$$\overline{\Delta E} \sim \frac{U_{acc}}{N} \sim \frac{Uo\,\gamma}{\sqrt{\gamma}} \sim U_0\sqrt{\gamma}$$

$$U_0(Si) \approx 25eV \qquad R \approx 0{,}2\ \dot{A}$$

$$\overline{\omega_{12}}(lab) \sim \gamma^{3/2}$$

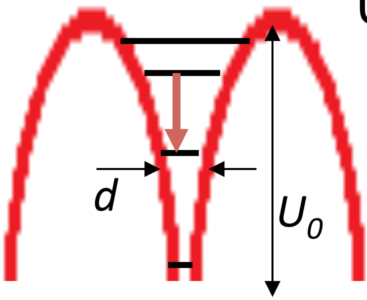
$$\omega_{max}(lab) \sim U_0\gamma^2$$

PLASMON DISPERSION

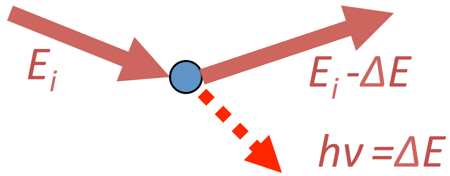
$$\mathrm{Im}\left[-\frac{1}{\varepsilon(\vec{q},\omega)}\right]=\frac{\pi\omega_p^2}{2\omega^2(\vec{q})}\delta\left(\omega-\omega(\vec{q})\right);$$

$$\omega^2(\vec{q})=\omega_p^2+\left(\frac{\vec{q}^{\,2}}{2m}\right)^2.$$

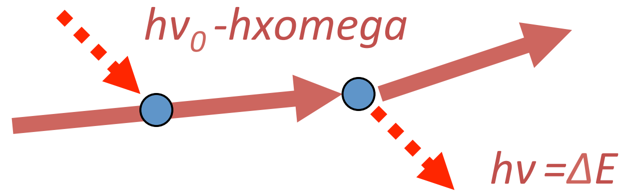
Channeling plasmon-photon radiation conditions



Usual spontaneous radiation



Plasmon “wings” in radiation



1. Potential in lab. system: $U_0 \sim 20eV$; $d \sim 0{,}2\text{-}0{,}3$ A (Si);
   in accomponying system ($v_x = 0$): $U = U_0\,(E/mc^2)$

2. Number of levels: $N \sim P_{z\,max} d / h \sim (EU_0)^{1/2} d / hc$
3. Distance between levels in acc. system: $\Delta E \sim U/N \sim (EU_0)^{1/2}(h/mcd)$

4. Plasmon energy in acc. system:

$$hxomega = 2hxomega_0/(mc^2/E + E\psi^2/mc^2) < 2h\nu_0 E/mc^2$$

5. In resonance conditions radiation “wings” can be very effective:

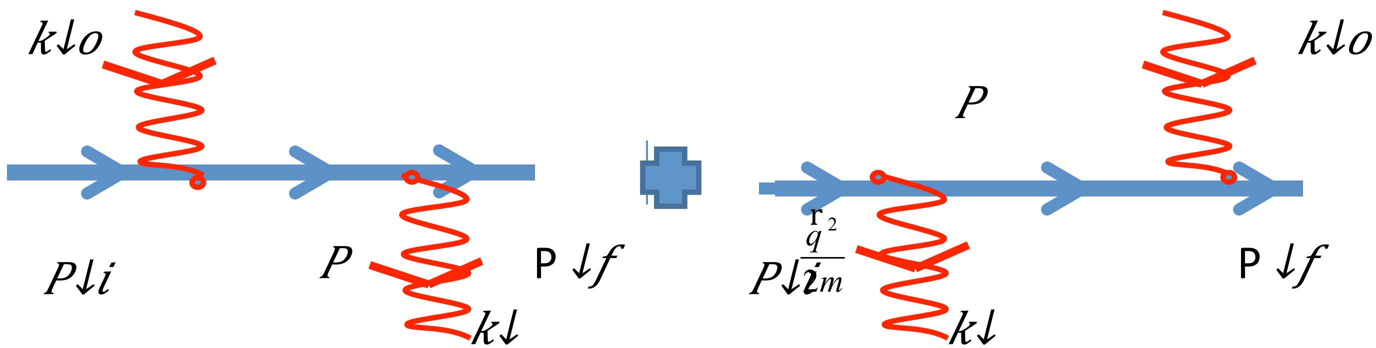
$$\Delta E \sim (EU_0)^{1/2}(h/mcd) = h\nu < 2h\nu_0 E/mc^2 = 2hE/mc\lambda_0$$

6. Resonance can be reached by correctly orienting laser beam, if:

$$E/U_0 > (\lambda_0/2d)^2 \sim 10^{7\text{-}8}$$

Plasmon and photon excitations by "bound" electrons

Richard P. FEYMAN diagrams



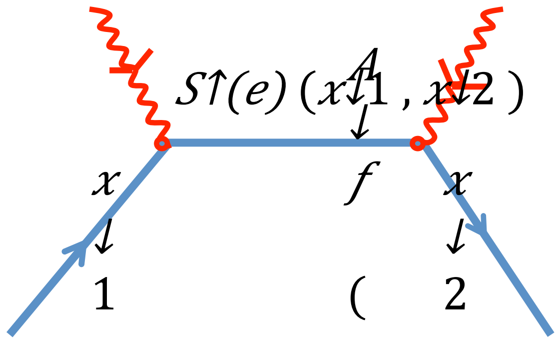
$$P_i + k_0 = P_f + k$$

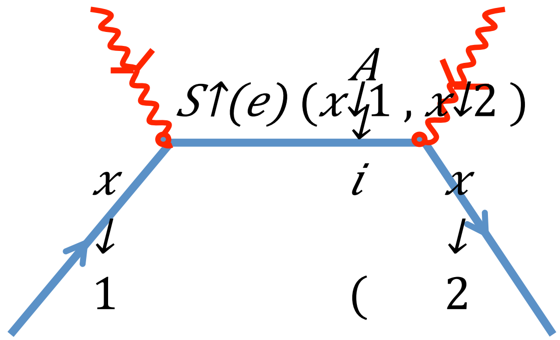
$$P_i \cos\theta_0 + \frac{\hbar\omega_0}{c}\cos\psi = P_f \cos\theta_f + \frac{\hbar\omega}{c}\cos\psi_f$$

$$E_i + \hbar\omega_0 = E_f + \hbar\omega$$

$$\hbar\omega = E_i - E_f + \hbar\omega_0$$

Generation of longitudinal excitation (plasmon) and a photon by a “bound” electron





$$A_i(x) = A_i(r) e^{-i\omega_i t} = e_i/\sqrt{2\omega_i}\, e^{i(k_i r - \omega_i t)}$$

$$A_f(x) = A_f(r) e^{i\omega_f t} = e_f/\sqrt{2\omega_f}\, e^{-i(k_f r - \omega_f t)}$$

$$\psi_i(x) = \psi_i(r) e^{-iE_i t}$$

$$\psi_f(x) = \psi_f(r) e^{-iE_f t}$$

Generation of scalar excitations in the crystal by fast charged particles

$$dw_{if} = \iint \overline{j_{if}^{*(0)}(x)\, j_{if}^{(0)}(x')} < \Phi(x)\Phi(x')\, d^4x\, d^4x'\, d\nu$$

$$w_{if} = \sum_{\omega}\sum_{\vec{q}\in BZ}\sum_{\vec{q}'\in BZ}\sum_{\vec{G}\vec{G}'}\iint \overline{j_{if}^{*(0)}(x)\, j_{if}^{(0)}(x')}\exp\left(i\left(\vec{q}+\vec{G}\right)\vec{r}\right)\exp\left(i\left(\vec{q}'-\vec{G}'\right)\vec{r}'\right)\times$$

$$\times\exp\left[i(\omega+i\delta)(t-t')\right]d^4x\, d^4x'\frac{4\pi}{\left(\vec{q}+\vec{G}\right)^2}\frac{4\pi e^2}{\left(\vec{q}'-\vec{G}'\right)^2}S_{ee}\left(\vec{q}+\vec{G},\vec{q}'-\vec{G}',\omega\right).$$

Loss of energy channeled particles upon excitation of the crystal

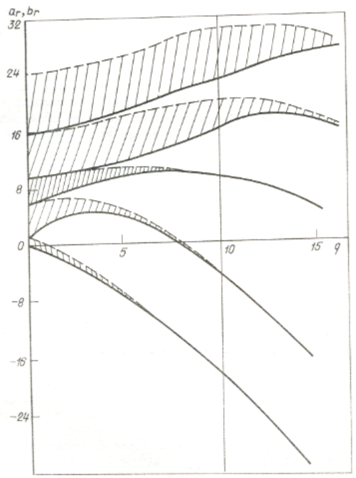
$$-\frac{dE}{dx} = \sum_{\kappa\kappa'}\int_0^{\infty}\frac{d\omega\omega}{2\pi}\int\frac{d^3\vec{q}}{(2\pi)^3}\int dq'_x\,\frac{m^2}{EP_z\upsilon_z}\left|C_\kappa(P_x)\right|^2\times$$

$$\times\frac{4\pi e^2}{q^2}\frac{4\pi}{q_x'^2+q_y^2+q_z^2}\cdot\frac{\mathrm{Im}\,\varepsilon^{-1}\left(q_x,q'_x,q_y,q_z,\omega\right)}{1-\exp(-\omega\beta)}\int_{-\infty}^{+\infty}dx\psi_{\kappa'}(x)\exp(iq_x x)\psi_\kappa(x)\times$$

$$\times\int_{-\infty}^{+\infty}dx'\psi_\kappa(x')\exp(-iq'_x x')\psi_{\kappa'}(x')\,\delta(\left(m^2+P_z^2+P_y^2-\kappa_1^2\right)^{1/2}-\left(m^2+(P_z-q_z)^2+\left(P_y-q_y\right)^3-\kappa_2^2\right)^{1/2}-\omega).$$

Orientation dependence of the band structure of relativistic positrons in single crystal

The band spectrum transverse motion of a fast oriented particle in the approximation of a sinusoidal crystal potential (band spectrum allocated to 25MeV energy particles). Lower zone border and the upper border of zones vs pulse (in dimensionless units) for the crystallographic plane (110) in silicon Si.

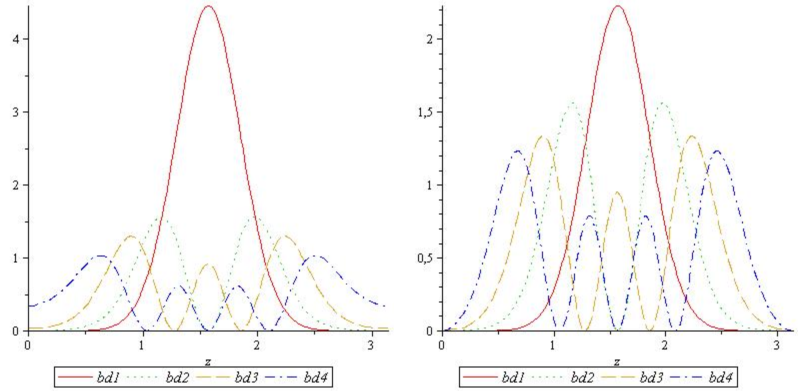


$$V(x) = \bar{V} + 2\sum_{n=1,2} V_{4n,0,0}\cos(4Gx).$$

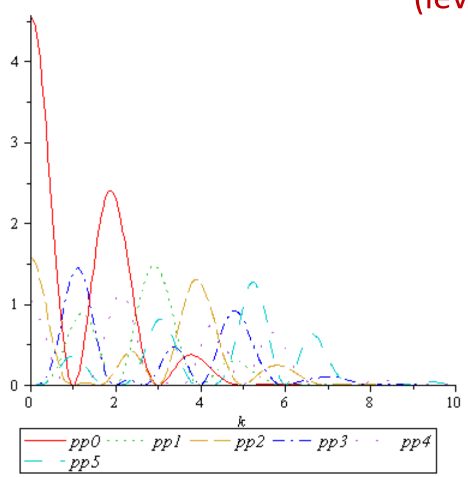
$$\frac{\partial^2 U}{\partial S^2} + (\vartheta - 2q\cos 2S)U(S) = 0,$$

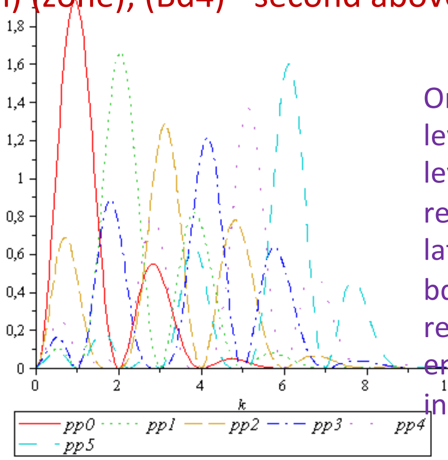
$$\vartheta = E_{\perp}^2 / 4G^2\hbar^2c^2 - \frac{8EV_0}{2\hbar^2c^2G^2};$$

$$q = \frac{EV_G}{2\hbar^2c^2G^2}; \quad S = 2Gx; \quad G \equiv G_{мин} = 2\pi / d;$$



Squares module of even (a) and odd (b) wave functions of positrons with an energy of 28 MeV in the planar channel (110) in a single crystal Si. (BD1) - first deep sub-barrier level; (Bd2) - the second level in the middle of the channel; (BD3) - the first above- barrier zone (level) (zone); (Bd4) - second above- barrier zone.





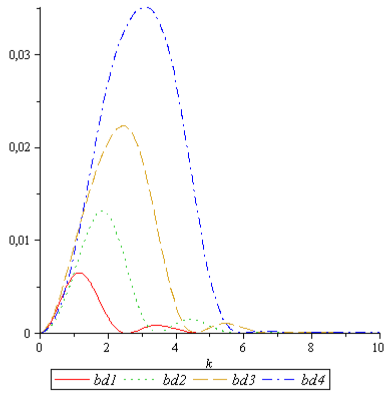
Orientation dependence of the probability of the population of the levels of the transverse motion of the positron l for even and odd levels depending on the angle of incidence of the positron in a crystal relative to the plane (110) in the Si. Angle θ is measured in reciprocal lattice vectors. Levels of oriented particles are numbered with index bd. On the x-axis the wave vector is specified as a fraction of the reciprocal lattice vector, with five of the reciprocal lattice vectors at energy 28 MeV correspond approximately to an Lindhardt angle of incidence of the positron.

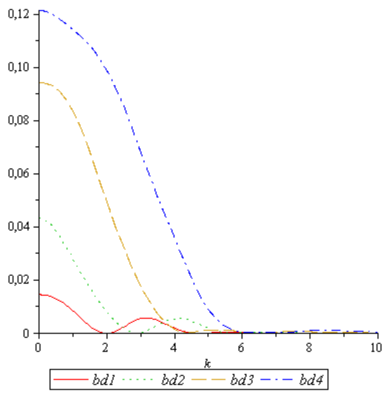
Orientation dependence

Squared modulus of the matrix element of the transition positron to the even and odd levels depending on the angle of incidence of the positron in a crystal.

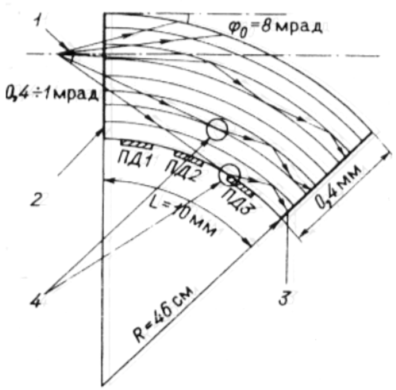
Angle θ is measured in reciprocal lattice vectors.

Levels of oriented particles are numbered with index bd. On the x-axis of the wave vector is specified as a fraction of the reciprocal lattice vector, with five of the reciprocal lattice vectors at energy 28 MeV correspond approximately to an Lindhard angle of incidence of the positron.



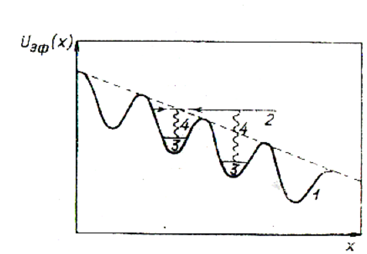


Schematic of the experiment to capture into channeling state in a volume of a curved crystal : 1 proton beam; 2-input end of the crystal; 3 output end of the crystal; 4 trapping region into channeling in the crystal; L - length along the beam



Graph of the effective potential profile in a curved crystal near the point of trajectories circulation of fast charged particles

$$\left[2\pi/(pa)\right]\sqrt{R/R_0} > 1\ \%$$



Volume capture of fast charged particles in a bent crystal

$$f_{\theta_0}(\theta) \approx \sqrt{\frac{2i^3 p}{\pi\theta_0}}\, 4L\theta_0 \left\{ \exp\left[\frac{ipL}{2}\left(\theta^2-\theta_0^2\right)-1\right]\right\} / \, pL\left(\theta^2-\theta_0^2\right)$$

$$-\frac{dE_\perp}{dl}(r) = \frac{2\pi Ze^2}{\upsilon}\int \frac{d^3q}{q^2} d_\omega(qv) \sum_G \operatorname{Im}\varepsilon^{-1}(q, q-G, \omega)\,\delta(\omega - qv)\exp(iGr)$$

$$\eta = \mathrm{V}\sigma/\sigma = \int_0^{\theta_0-\mathrm{V}\theta_0} \frac{|f|^2\, d\theta \cdot 2\pi}{\sigma} \approx \int_{pL\theta_0\mathrm{V}\theta_0/2}^{\infty} \sin^2 x / x^2 dx / \pi \approx \int_0^{\infty} dx \sin^2 x / x^2 \pi \approx 0.046$$

ENERGY LOSSES OF THE FAST CHARGED PARTICLE WITH THE ACCOUNT OF THE POTENTIAL CONFIGURATION

$$\frac{dE}{dl} = \frac{2}{3\pi}\frac{2Ry}{a_0}\frac{1}{\alpha}ug\left(u,\alpha\right),$$

$$g\left(u,\alpha\right) = \ln\left(3\pi n^2 / \alpha\right)/u^3; \qquad u = V / V_F; \quad \alpha = l^2 / \mathrm{h}V_F.$$

$$n \approx 2\cdot 10^{23}\,\text{см}^{-3} \qquad V_F \approx \sqrt{2\varepsilon_F / m_{\text{эл}}} \approx 1{,}6\cdot 10^6 \qquad u \approx V_0 / V_F \approx 10^2$$

$$\alpha = l^2 / \mathrm{h}V_F = \left(l^2 / \mathrm{h}c\right)\left(c / V_F\right) \approx \left(1/137\right)10^2 \approx 1.$$

$$\left(dE / dl\right):\ 10^{-3}\,\text{эВ} / A^0.$$

Motion of the fast particle in the bent crystal

$$m\ddot{\vec r} = -\nabla_{\vec r} U_{cr}\left(\vec r\right) + \vec F_K\left(\vec r\right),$$

$$\vec F_{\perp K}\left(\vec r\right) = -\frac{dE_\perp}{d\vec l}\left(\vec r\right) = \frac{2\pi Z e^2}{v}\int \frac{d^3\vec q}{q^2}\int d\omega\, \hbar\, \vec q_\perp \vec v_\perp^{(0)}\left(t\right)\times$$

$$\times\sum_{\vec G} \mathrm{Im}\,\varepsilon^{-1}\left(\vec q, \vec q + \vec G, \omega\right)\delta\left(\omega - \vec q\vec v^{(0)}\left(t\right)\right)\exp\left(i\vec G\vec r^{(0)}\left(t\right)\right),$$

$$m\ddot r = -\partial U_{cr}\left(\vec r\right)/\partial r + F_{\perp êèëü}\left(\vec r\right) + M^2/mr^3.$$

$$m\ddot r = -Kr - F_r + F,$$

Motion of the fast particle in the bent crystal

$$\ddot{\tilde{r}} = (U_0 / ma)\sin(\tilde{r} / a) - \frac{F_r}{m} + M^2 / m^2 \tilde{r}^3, \qquad \tilde{r} = R + r'$$

$$\ddot{r} a = (2\pi U_0 / ma)\sin(2\pi r + 2\pi R / a) - \frac{F_r}{m} + M^2 / m^2 (ra + R)^3,$$

$$r = r' / a \qquad \ddot{r} a \frac{ma^2 / 2\pi U_0}{ma^2 / 2\pi U_0} = (2\pi U_0 / ma)\sin(2\pi r + 2\pi R / a) - \frac{F_r}{m} + M^2 / m^2 (ra + R)^3,$$

$$\ddot{r} = \sin(2\pi r + 2\pi R / a) - b + H / (ra / R + 1)^3$$

$$T = \sqrt{ma^2 / 2\pi U_0} \qquad b = F_r \frac{a}{2\pi U_0}$$

PARAMETERS

$$H = \frac{1}{2\pi}\frac{a}{R} mv^2 / U_0$$

$$mv^2 / U_0 \ ? \ \ 1$$

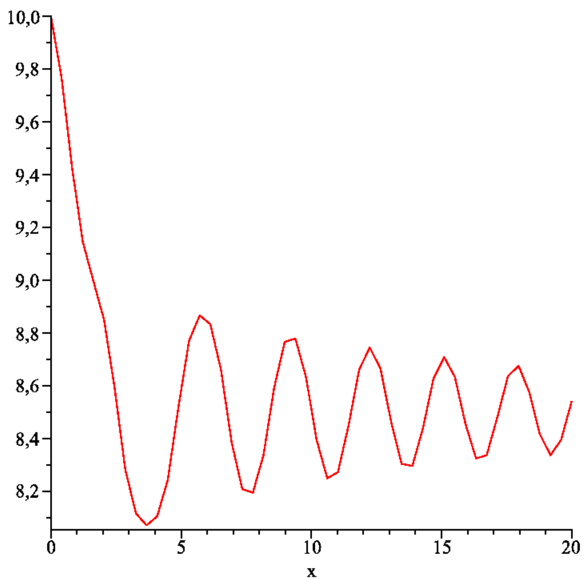
$$a / R = \ 1$$

$$mv^2 / U_0 \approx mc^2 / U = 9 \cdot 10^8 eV / 25eV = 4 \cdot 10^7 \ ? \ \ 1$$

$$b = F_r \frac{a}{2\pi U_0} = \frac{10^7 \cdot 1.6 \cdot 10^{-19} \cdot 3 \cdot 10^{-10}}{6.28 \cdot 24 \cdot 1.6 \cdot 10^{-19}} = \frac{10^7 \cdot 3 \cdot 10^{-10}}{6.28 \cdot 24} = \frac{1}{6.28 \cdot 8} \cdot 10^{-3} = 2 \cdot 10^{-5}$$

$$H = 3 \cdot 10^{-10} \cdot 4 \cdot 10^7 = 1.2 \cdot 10^{-2}$$

Volume capture

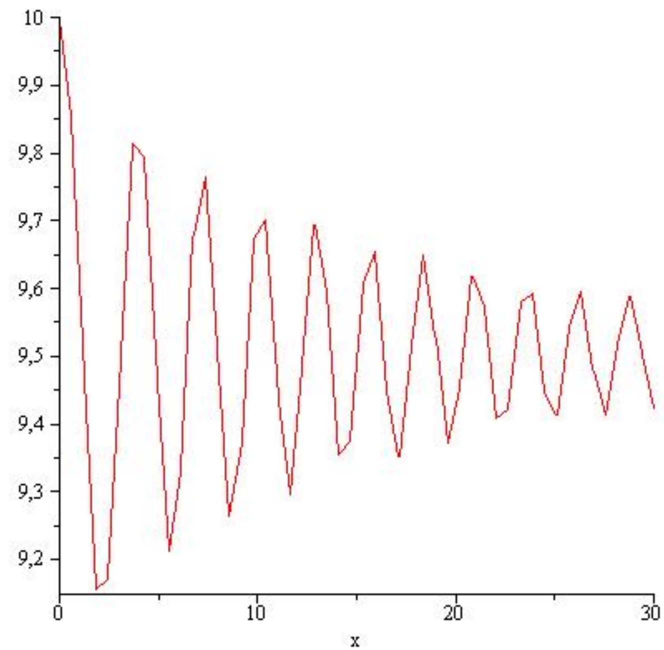


Conditions

$y(0) = 10$
$y'(0) = -.5$

Parameters

$H = .1e\text{-}1$
$R = 3000000000$
$b = .1$



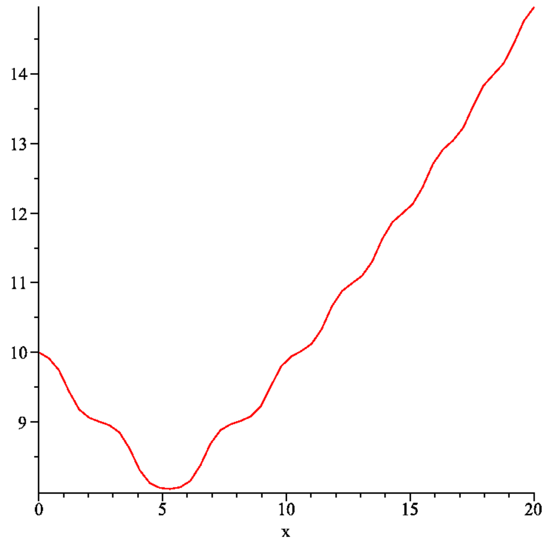
Conditions

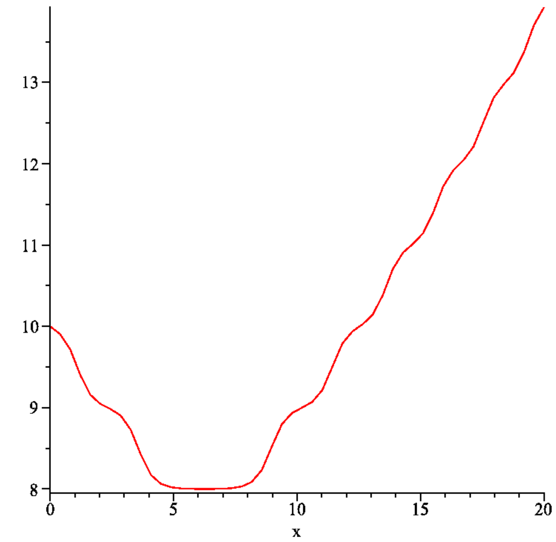
$y(0) = 10$
$y'(0) = -.18$

Parameters

$H = .2e\text{-}1$
$R = 3000000000$
$b = .1$

Volume reflection





$y(0) = 10$
$y'(0) = -.18$

$H = .1e\text{-}1$
$R = 3000000000$
$b = .2e\text{-}5$

$y(0) = 10$
$y'(0) = -.2$

$H = .1e\text{-}1$
$R = 3000000000$
$b = .2e\text{-}7$

RESUME

1.The theory of the generation of the single-particle and collective excitations in a crystal by a fast charged quantum oriented relative to the crystallographic axes particle is constructed.

2.The dependence of the intensity of the generation of excitations in the crystal depending on the level of the transversal both sub barrier and over barrier movement is obtained.

3. It is shown that the loss of transverse energy by the fast quantum charged particle moving in the potential of the curved crystal is leading to the effect of the volume capture of such particles in the crystal.

4. Mathematical modeling of the various modes both of the volume capture and volume reflection of fast charged particles in a bent crystal is carried out.