BCM-2.0 – the New Version of Computer Code
“Basic Channelling with Mathematica©”

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Some codes are based on the concept of the continuous potential
Other group codes use the scheme of binary collisions
Motivation

The first version of the computer code allowed calculating

- planar channeling potential function
- Fourier components of the potential function
- classical trajectory of planar channeled charged particles in crystals
- wave function of the planar channeled particles
- transverse quantum states initial populations
Newly developed different packages of this code were successfully applied to following problems:

- Flux dynamics and angular distributions of relativistic electrons and positrons passing through the thin and half-wave crystals, including mirroring
- Depth oscillations of electronuclear reactions caused by relativistic planar channelled electrons: quantum versus classical calculations
- Cherenkov radiation from relativistic electrons in a crystal
- Calculation of Cherenkov radiation angular distributions from channeled relativistic electrons (positrons) and heavy ions
- Optical radiation from channeled relativistic heavy ions in vicinity of the Cherenkov angle
- Asymmetry of the angular distribution of radiation of channeled relativistic electrons in optically transparent crystals
- Angular distribution features of Channeling radiation in the optical range
- PXRC (parametric X-Radiation at channeling) and its quantum features
- Radiation energy loss of channeled relativistic electrons in a crystal
- Channeling radiation from electrons in a half-wave crystal
- Positron source via electron-positron pair production by channeling radiation
- Orbital angular momentum of channeling radiation from relativistic electrons
- **Equation of motion**

\[ \gamma m\ddot{x} = F_x = -\frac{\partial U(x)}{\partial x}, \quad \gamma m\ddot{z} = 0 \]

- **Initial conditions**

\[ x(0) \equiv x_0 \]

\[ u_x(0) = c\sqrt{1 - \frac{1}{\gamma^2}} \sin(\theta) \]
Flux Dynamics and Angular Distributions of Relativistic Electrons and Positrons Passing Through the Thin and Half-Wave Crystals, Including Mirroring
Flux Dynamics and Angular Distributions of Relativistic Electrons and Positrons Passing Through the Thin and Half-Wave Crystals, Including Mirroring

Electrons, $L=0.58 \, \mu m$, $\theta=0.0^\circ$, $\Delta \theta=0.09 \, \text{mrad}$

$z, \, \mu m$
Flux Dynamics and Angular Distributions of Relativistic Electrons Passing Through the Thin and Half-Wave Crystals, Including Mirroring

Beam intensity distribution 255 MeV electrons in (111) Si 0.91 µm crystal

<table>
<thead>
<tr>
<th>Experiment (SAGA LS)</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam intensity</td>
<td>Beam intensity</td>
</tr>
<tr>
<td></td>
<td>distribution</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Energy, MeV</th>
<th>Plane</th>
<th>Crystal thickness, µm</th>
<th>Deflection angle, mrad</th>
<th>Deflection efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAGA-LS</td>
<td>255 MeV e²</td>
<td>Unbent Si(111)</td>
<td>0.91</td>
<td>0.45 mrad</td>
</tr>
<tr>
<td></td>
<td>255 MeV e²</td>
<td>Unbent Si(220)</td>
<td>0.74</td>
<td>0.41 mrad</td>
</tr>
<tr>
<td>MAMI</td>
<td>855 MeV e²</td>
<td>Bent Si(111)</td>
<td>30.5</td>
<td>0.91 mrad</td>
</tr>
<tr>
<td>SLAC</td>
<td>3.35 GeV e²</td>
<td>Bent Si(111)</td>
<td>60 µm</td>
<td>0.40 mrad</td>
</tr>
<tr>
<td></td>
<td>6.3 GeV e²</td>
<td>Bent Si(111)</td>
<td>60 µm</td>
<td>0.40 mrad</td>
</tr>
</tbody>
</table>

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Channeling Radiation From Electrons in a Half-wave Crystal

CR spectra and typical trajectories

Electrons, L=0.74 μm, θ=0°

Electrons, L=0.74 μm, θ=θc/2

Channeling Radiation From Electrons in a Half-wave Crystal

CR spectra and typical trajectories


Y. Takabayashi, V.G. Bagrov, Y.P. Pivovarov, O.V. Bogdanov, T.A. Tukhfatullin
Cherenkov Radiation from Relativistic Ions in a Crystal

Intensity of Cherenkov radiation

\[
\frac{dI}{d\omega d\Omega} = \omega \cdot L \cdot \left( \frac{Ze \cdot \sin \vartheta}{c} \right)^2 \cdot f_{TF} (\theta, \omega)
\]

\[
f_{TF} (\theta, \omega) = \frac{1}{\Delta \vartheta_{TF}} \cdot \left( \frac{\sin x}{x} \right)^2; \quad x = \frac{\pi}{\Delta \vartheta_{TF}} \left( \cos \vartheta - \frac{1}{\beta n} \right)
\]

The width of Tamm-Frank distribution

\[
\Delta \vartheta_{TF} = \frac{\lambda}{nL};
\]

Cherenkov angle

\[
\cos \vartheta_C = \frac{1}{\beta n}
\]

Ionization energy loss

\[
-\frac{dE}{dx} = 2\pi z^2 \rho N_A r^2 m_ec^2 Z \frac{1}{A \beta^2} \left( \ln \left( \frac{2m_ec^2 \beta^2 \gamma^2}{I} \right) - \beta^2 - \frac{\delta}{2} \right)
\]
Cherenkov Radiation from Relativistic Ions in a Crystal

Au ions with initial energies $E = 905$ MeV/u in a LiF radiator, wave length range 380–740 nm

Cherenkov Radiation from Relativistic Ions in a Crystal

$E_0 = 10-3000 \text{ GeV/u, ion}^{197}\text{Au, radiator LiF, } L = 1 \text{ cm}$

$E_0 = 10 \text{ GeV/u}$

$E_0 = 20 \text{ GeV/u}$

$E_0 = 30 \text{ GeV/u}$

$E_0 = 170-3000 \text{ GeV/u}$

$\theta, \text{ mrad}$

$E_0 = 10 \text{ GeV/u}$

$E_0 = 20 \text{ GeV/u}$

$E_0 = 30 \text{ GeV/u}$

$E_0 = 170 - 3000 \text{ GeV/u}$

$\theta, \text{ mrad}$

Cherenkov Radiation from Relativistic Ions in a Crystal
Parametric X-radiation at Channeling and its Quantum Features

**PXRC at planar channeling**

The PXRC appears when an electron passes through a crystal in the channeling regime. The channeling means that the electron is in a bound state with the crystal plane.

- $\beta$: is the Bragg angle
- $\mathbf{g}$: is the reciprocal lattice vector
- $\mathbf{v}$: is the electron velocity

**PXRC experiment (SAGA LS)**

- Random orientation
- (220) planar channeling

$\delta = 0.06$

**PXRC simulation**

- $d^3 N / dz d\theta_x d\theta_y \times 10^{-10}$, ph/sr/Å

**Interband** transitions ($i \neq f$) ⇒
Diffracted Channeling Radiation

**Intraband** transitions ($i = f$) ⇐
Parametric X-radiation at channeling

Nitta suggested
- the form factors of channeled states to be equal approximately to 1
- $|F_{ii}|^2 \approx 1$

Angular distribution of PXRC does not differ from that of PXR

Experiment:
- SAGA-LS (JETP Letters, 2012)

A difference exists

Motivation to re-calculate PXRC angular distribution

**Theory:**

**difference of PXR and PXRC**

The PXRC angular distribution = sum over populated quantum states (bands)

\[
\frac{d^3 N_{\text{PXRC}}}{d\theta_x d\theta_y dz} = dN_{\text{PXR}} \sum_n P_n(\theta_\circ) |F_{nn}|^2
\]

Initial population of the n-th energy level (band)

\[
P_n(\theta_\circ) = \frac{1}{d} \left| \int_{-d/2}^{d/2} \exp(ik_y \theta_\circ y) \phi_n(y) dy \right|^2
\]

**Number n**  
**Form-factors** $|F_{nn}|^2$  
**Population** $P_n(\theta_\circ)$

**Relativistic factor** $\gamma$

**Silicon (220)**

- Number of channeled states
- $I^\text{max}_{\text{PXRC}} = \frac{d}{d\theta_x d\theta_y dz}$
- at these points appear the new **odd** quantum states

- (220) Si  
  $\theta_\circ=16.1^\circ$  
  $\theta_0=0$

**Diamond (220)**

- Number of channeled states
- $I^\text{max}_{\text{PXRC}} = \frac{d}{d\theta_x d\theta_y dz}$
- at these points appear the new **odd** quantum states

- (220) C  
  $\theta_\circ=16.1^\circ$  
  $\theta_0=0$

- band influence in (220) C - $0$

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The electronuclear reaction yield is proportional to the function (Yu.M. Filimonov, Yu.L. Pivovarov and S.A. Vorobiev, Nuclear Physics, 47 3 (1988) 894-895):

$$\omega_k(z,\sigma) = \int F(\rho, z, \sigma)\delta(\rho - \bar{\rho}_k(z))d\rho$$

Flux density of electrons $E=255$ MeV (quantum calculation)

Flux dynamics of electrons $E=255$ MeV (classical calculation)

Depth oscillations of electronuclear reactions caused by relativistic planar channelled electrons: quantum versus classical calculations

Electronuclear reaction yield (quantum calculation)

\[ Y(z) = \int_{-d/2}^{d/2} F(x, z)|\Psi(x, z)|^2 dx \]

Electronuclear reaction yield (classical calculation)

\[ Y(z) = \int_{-d/2}^{d/2} F(x, z)P(x, z)dx \]
Radia?on energy loss of channeled relativistic electrons in a crystal

Angle-of-incidence dependence of the total yield of channeled radiation in a thin crystal:
- Planar and axial channeling for the (100), (110) (111) planes and <100>, <110> axes
- Electrons and positrons
- 155-855 MeV energy range (extendable)
- Si, C, W crystals
- Initial electron/positron beam angular divergence is taken into account

Angle-of-incidence dependence of the total yield of the CR can be used for the thin crystal alignment in more complicated channeling experiments and, even more, for diagnostics of angular spread of moderately relativistic electron/positron beams

\[
\Delta E = \frac{2e^2}{3c^3} \int_0^T \frac{a^2(t) - \left[\frac{v(t) a(t)}{c^2}\right]^2}{(1 - v^2(t) / c^2)^3} dt
\]

Angle-of-incidence dependence of the total yield of channeling radiation of electrons at (100) planar channeling in Si

The energy spectra of positrons, produced by the radiation from 200 MeV electrons in W: (a, b) axial CR; (c, d) BS. Solid lines correspond to converter thickness 0.35 cm; dashed – 0.71 cm.

- Energy spectra and the total yield of the positrons calculated for the hybrid scheme of positron source using the channeling radiation from 200 – 1600 MeV electrons and thin amorphous converter

- Comparison energy spectra and the total yield of the positrons for cases of channeling radiation and the bremsstrahlung is carried out

- Studies on hybrid positron source using channeling radiation and a thin W amorphous convertor are extended to the case of more thick radiators

- Positron beam emittance and the influence of dechanneling processes in thick radiator crystal are in progress

Conclusions

- New version of computer code BCM–2.0 is developed.
- Newly developed different packages of this code were successfully applied to simulate scattering, radiations, electron-positron pairs creation and other effects connected with channelling of relativistic particles in aligned crystal.
THANK FOR YOUR ATTENTION!