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BCM-2.0 – the New Version of Computer Code "Basic Channelling with Mathematica©"

S.V. Abdrashitov, O.V. Bogdanov, K.B. Korotchenko, Yu.L. Pivovarov, E.I. Rozhkova, T.A. Tukhfatullin, Yu. L. Eikhorn

National Research Tomsk Polytechnic University, Tomsk 634050, Russia

The 7th International Conference
CHANNELING 2016

Motivation

- **Several software packages based on various model approaches have been developed recently for numerical simulations of the channelling process** (G.B. Sushko, V.G. Bezchastnov, I.A. Solovyov, A.V. Korol, W. Greiner, A.V. Solovyov, J. Comput. Phys. 252 (2013) 404, E. Bagli, V. Guidi, Nucl. Instr. Meth. B 309 (2013) 124, A. I. Sytov, V.V. Tikhomirov, Nucl. Instr. Meth. B 355 (2015) 383)
- **Some codes are based on the concept of the continuous potential**
- **Other group codes use the scheme of binary collisions**
- **"Basic Channelling with Mathematica©" BCM–1.0 is the computer code developed by the authors for solving numerous problems related to channelling** (O.V. Bogdanov, E.I. Fiks, K.B. Korotchenko, Yu.L. Pivovarov T.A. Tukhfatullin, J. Phys.: Conf. Ser. 236 (2010) Article number 012029)

Motivation

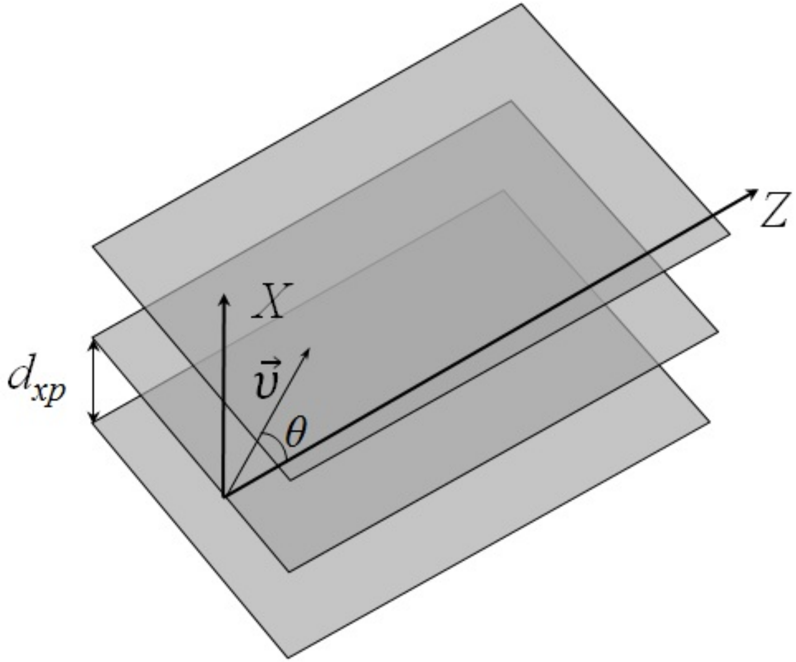
The first version of the computer code allowed calculating

- **planar channeling potential function**
- **Fourier components of the potential function**
- **classical trajectory of planar channeled charged particles in crystals**
- **wave function of the planar channeled particles**
- **transverse quantum states initial populations**

Motivation

Newly developed different packages of this code were successfully applied to following problems:

- **Flux dynamics and angular distributions of relativistic electrons and positrons passing through the thin and half-wave crystals, including mirroring**
- **Depth oscillations of electronuclear reactions caused by relativistic planar channelled electrons: quantum versus classical calculations**
- **Cherenkov radiation from relativistic electrons in a crystal**
- **Calculation of Cherenkov radiation angular distributions from channeled relativistic electrons (positrons) and heavy ions**
- **Optical radiation from channeled relativistic heavy ions in vicinity of the Cherenkov angle**
- **Asymmetry of the angular distribution of radiation of channeled relativistic electrons in optically transparent crystals**
- **Angular distribution features of Channeling radiation in the optical range**
- **PXRC (parametric X-Radiation at channeling) and its quantum features**
- **Radiation energy loss of channeled relativistic electrons in a crystal**
- **Channeling radiation from electrons in a half-wave crystal**
- **Positron source via electron-positron pair production by channeling radiation**
- **Orbital angular momentum of channeling radiation from relativistic electrons**



- **Equation of motion**

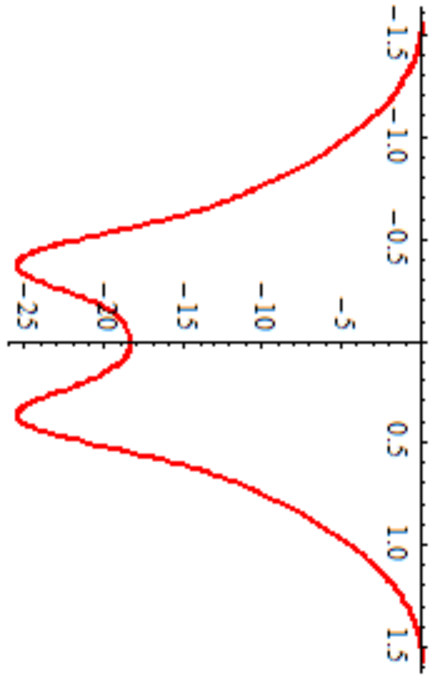
$$\gamma m\ddot{x} = F_x = -\frac{\partial U(x)}{\partial x}, \quad \gamma m\ddot{z} = 0$$

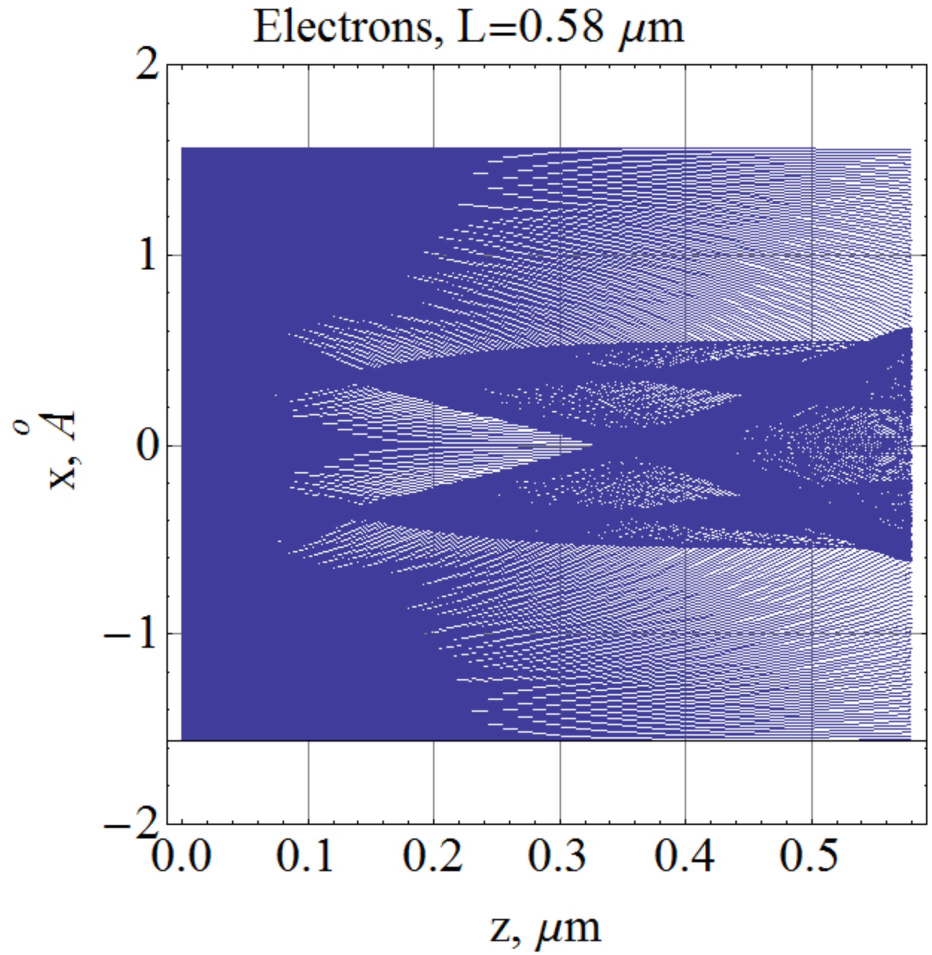
- **Initial conditions**

$$x(0) \equiv x_0$$

$$\upsilon_x(0) = c\sqrt{1-\frac{1}{\gamma^2}}\sin(\theta)$$

Flux Dynamics and Angular Distributions of Relativistic Electrons and Positrons Passing Through the Thin and Half-Wave Crystals, Including Mirroring





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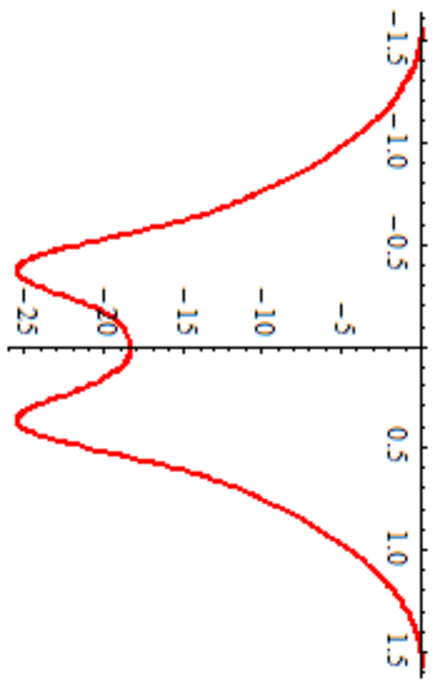
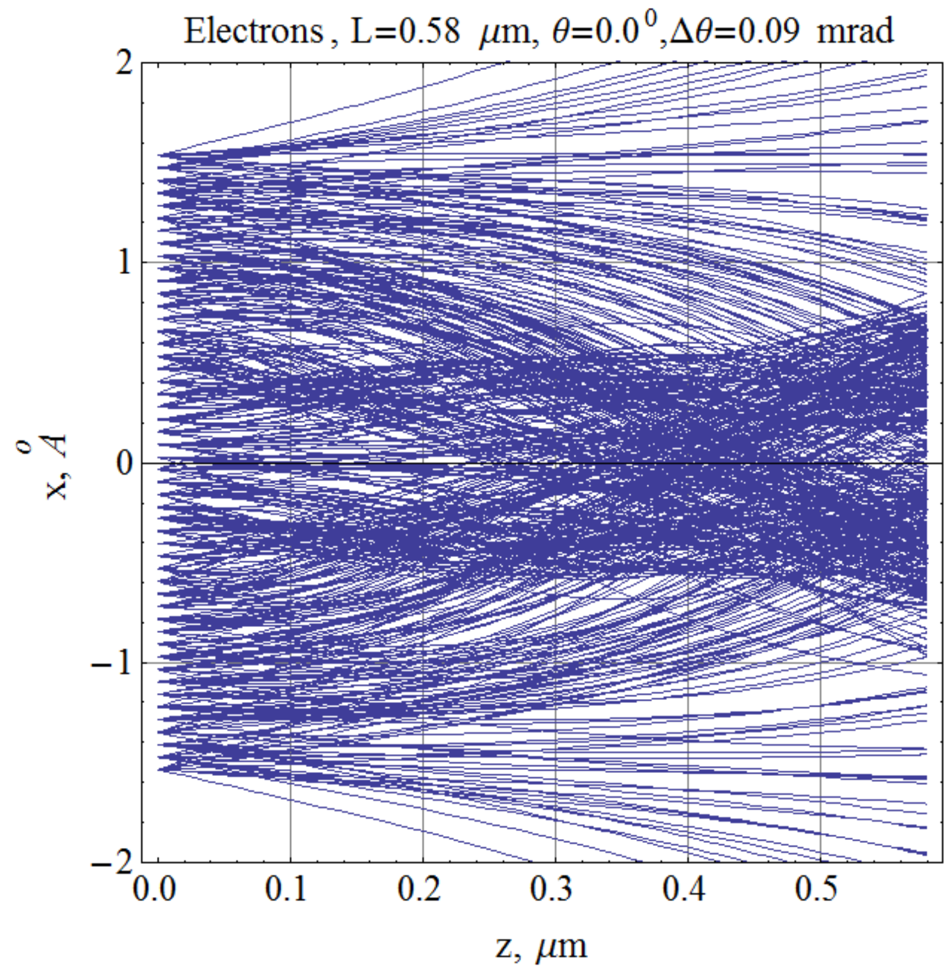
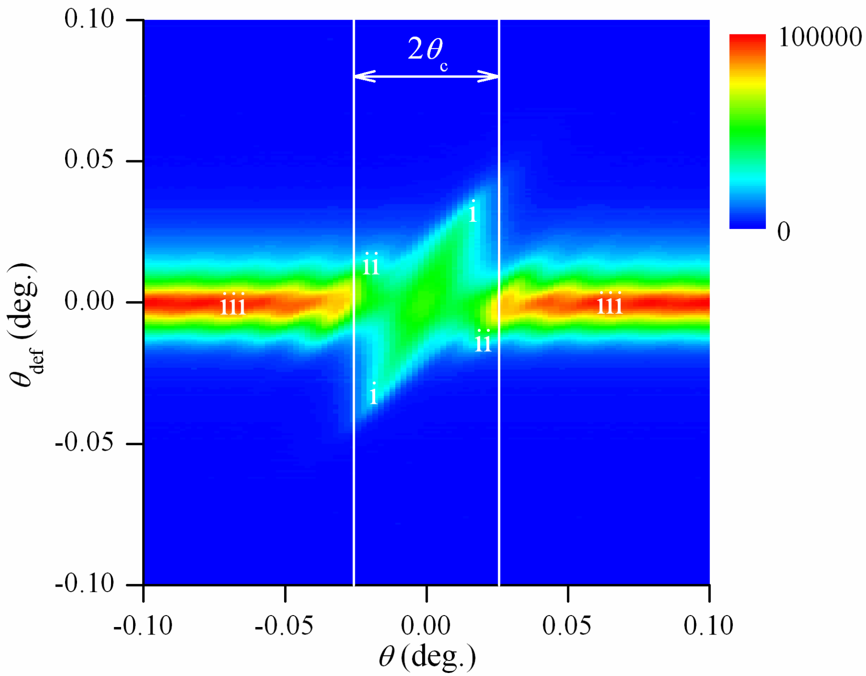


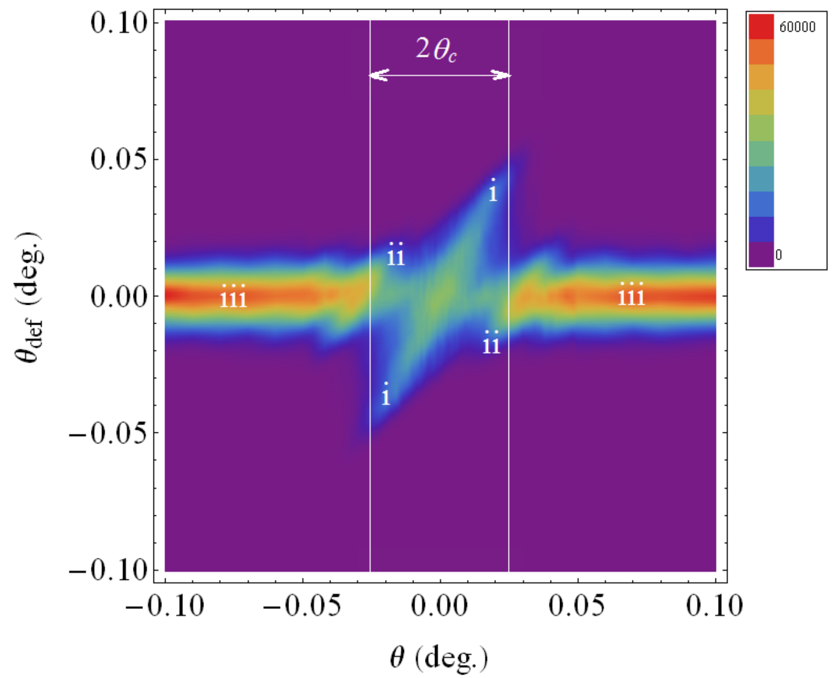


Beam intensity distribution 255 MeV electrons in (111) Si 0.91 μm crystal

Experiment (SAGA LS)



Simulation

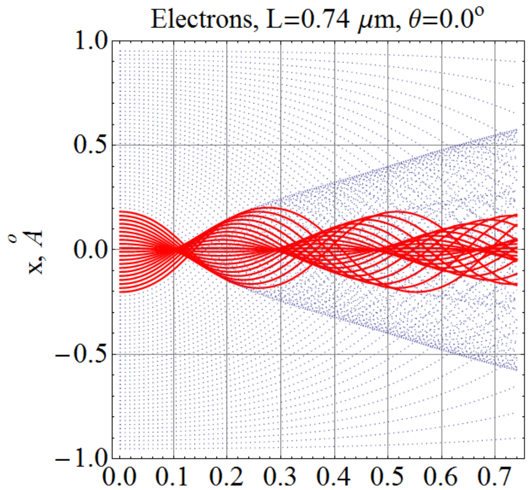


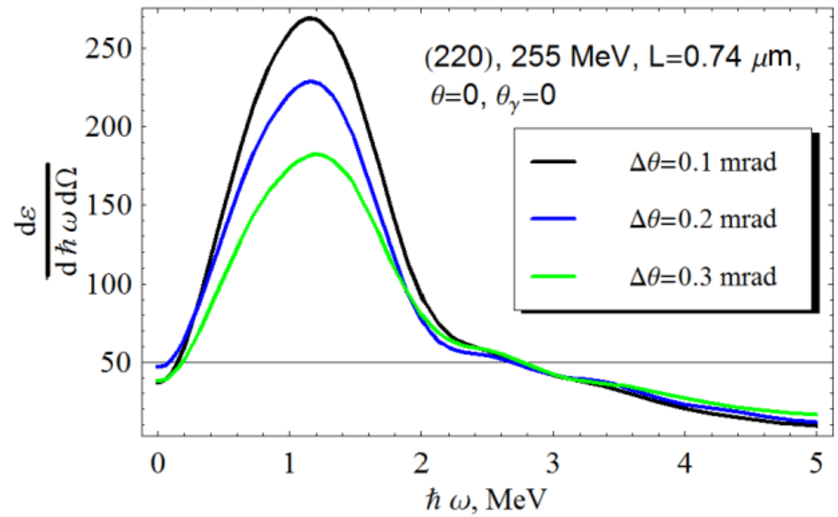
	Energy	Plane	Crystal thickness	Deflection angle	D e fl e c ti o n efficiency
SAGA-LS	255 MeV e?	Unbent Si(111)	0.91 ?m	0.45 mrad	29?
	255 MeV e?	Unbent Si(220)	0.74 ?m	0.41 mrad	22?
MAMI	855 MeV e?	Bent Si(111)	30.5 ?m	0.91 mrad	20?
SLAC	3.35 GeV e?	Bent Si(111)	60 ?m	0.40 mrad	22?
	6.3 GeV e?	Bent Si(111)	60 ?m	0.40 mrad	22?

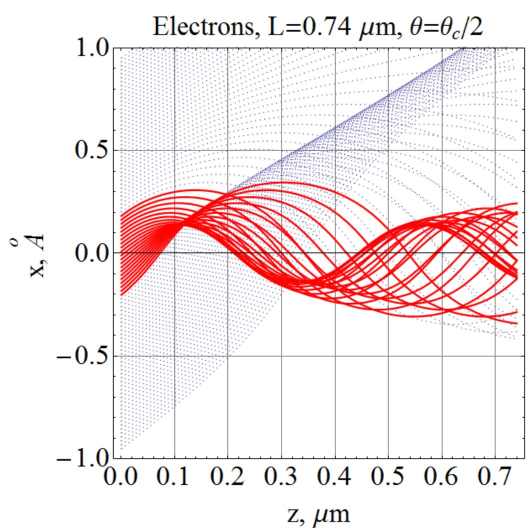
Y.Takabayashia, Yu.L.Pivovarov, T.A.Tukhfatullin Observation of sub-GeV electrons mirrored by ultrathin crystalline Si// Physics Letters B751(2015) 453–457

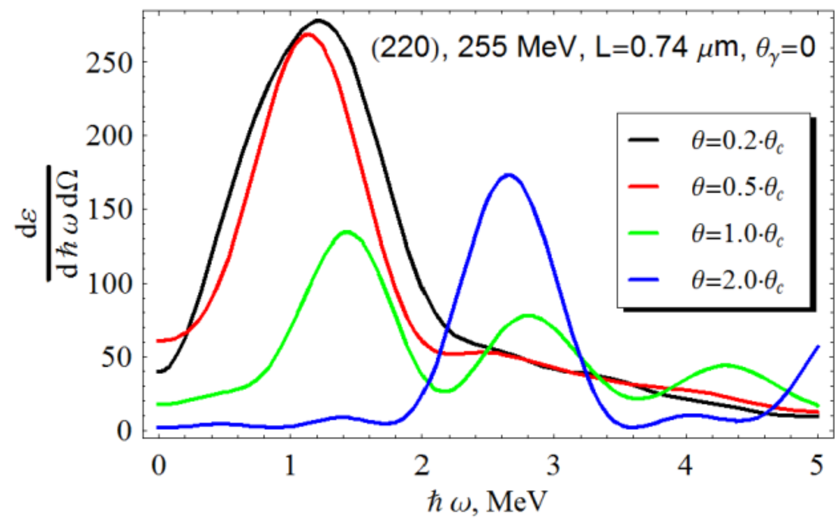
Channeling Radiation From Electrons in a Half-wave Crystal

CR spectra and typical trajectories









Y. Takabayashi, V.G. Bagrov, Y.P. Pivovarov, O.V. Bogdanov, T.A. Tukhfatullin Angular distributions of relativistic electrons under channeling in half-wavelength crystal and corresponding radiation// Nucl. Instr. and Meth. in Phys. Res. B 355 (2015) 188-192.

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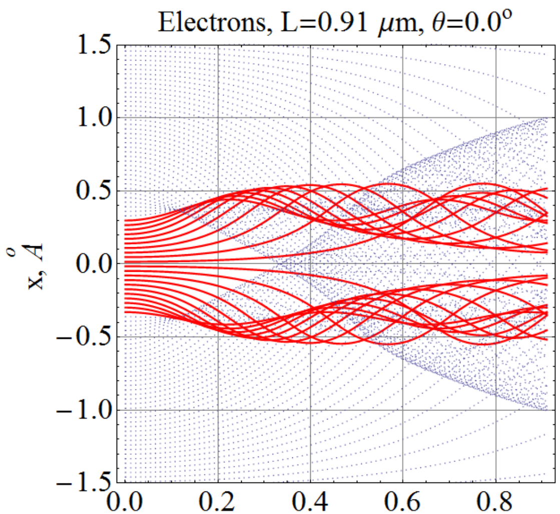
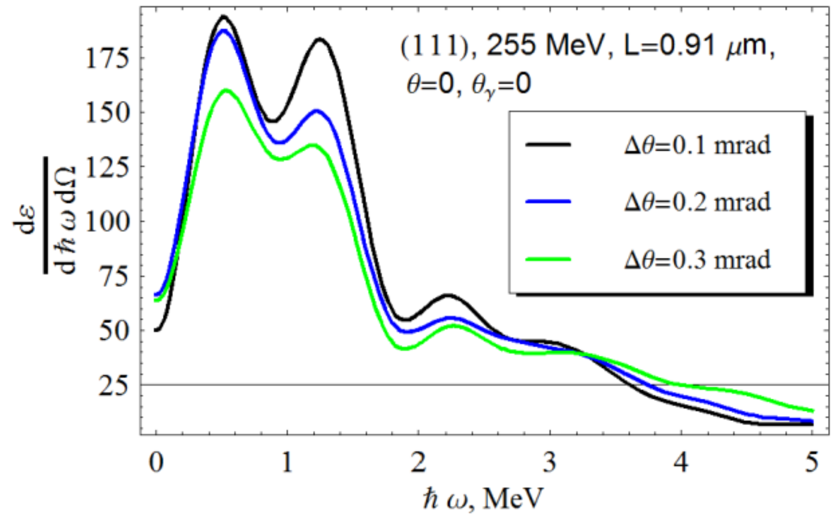
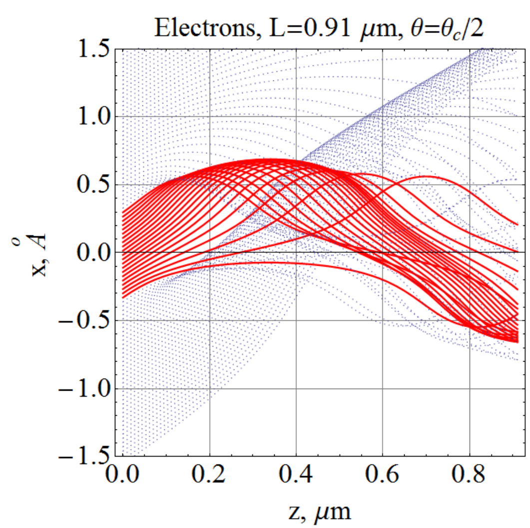
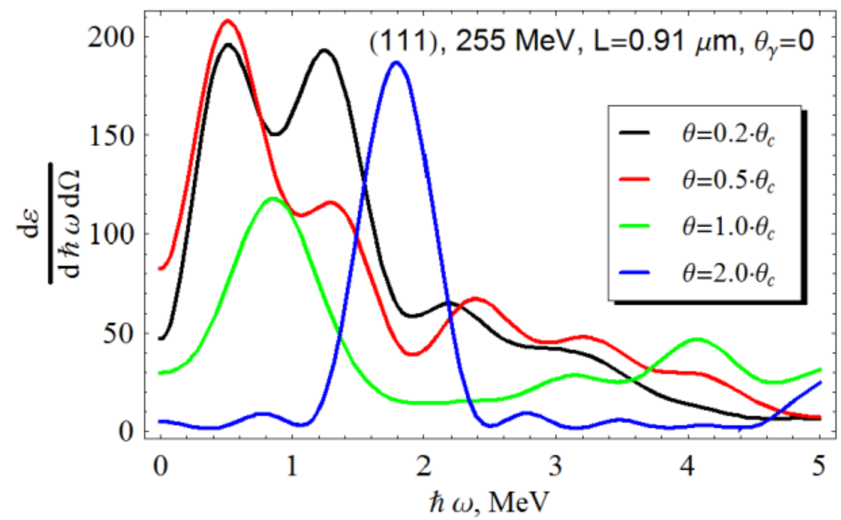
CR spectra and typical trajectories









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Y. Takabayashi, V.G. Bagrov, Y.P. Pivovarov, O.V. Bogdanov, T.A. Tukhfatullin Angular distributions of relativistic electrons under channeling in half-wavelength crystal and corresponding radiation// Nucl. Instr. and Meth. in Phys. Res. B 355 (2015) 188-192.

Cherenkov Radiation from Relativistic Ions in a Crystal

Intensity of Cherenkov radiation

$$\frac{dI}{d\omega d\Omega} = \omega \cdot L \cdot \left(\frac{Ze \cdot \sin \vartheta}{c} \right)^2 \cdot f_{TF}(\theta, \omega)$$

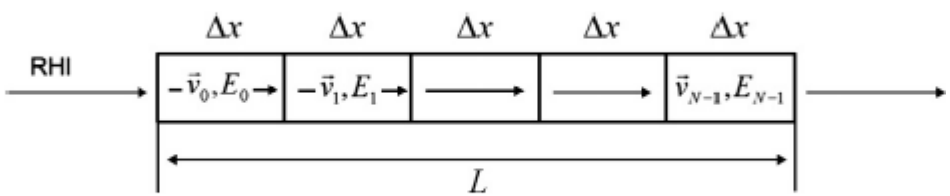
$$f_{TF}(\theta,\omega) = \frac{1}{\Delta\vartheta_{TF}} \cdot \left(\frac{\sin x}{x} \right)^2; \; x = \frac{\pi}{\Delta\vartheta_{TF}} \left(\cos\vartheta - \frac{1}{\beta n} \right)$$

The width of Tamm-Frank distribution $\Delta\vartheta_{TF} = \frac{\lambda}{nL};$

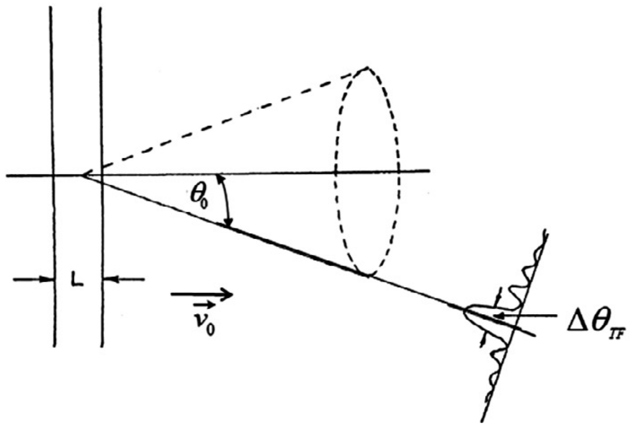
Cherenkov angle $\cos\vartheta_C = 1/\beta n$

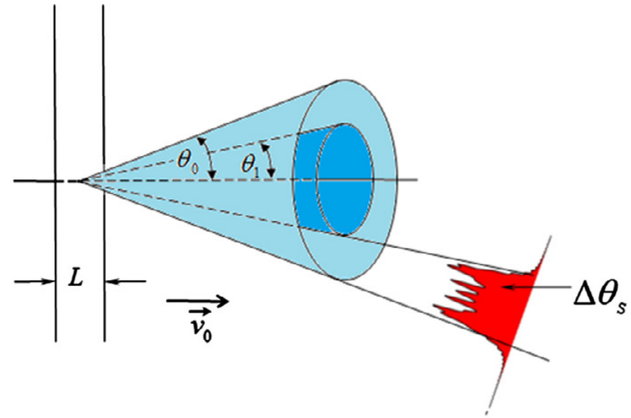
Ionization energy loss

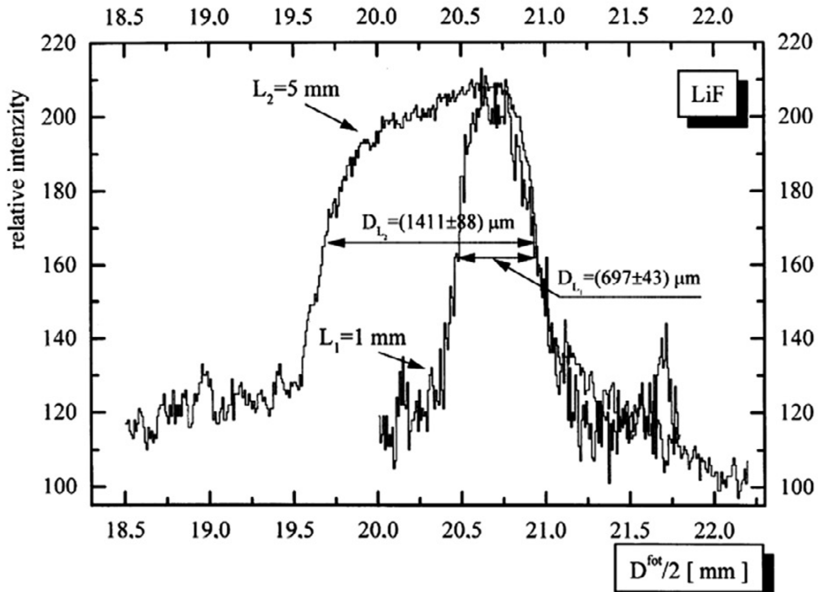
$$-\frac{dE}{dx} = 2\pi z^2 \rho N_A r_e^2 m_e c^2 \frac{Z}{A} \frac{1}{\beta^2} \left(\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I} \right) - \beta^2 - \frac{\delta}{2} \right).$$

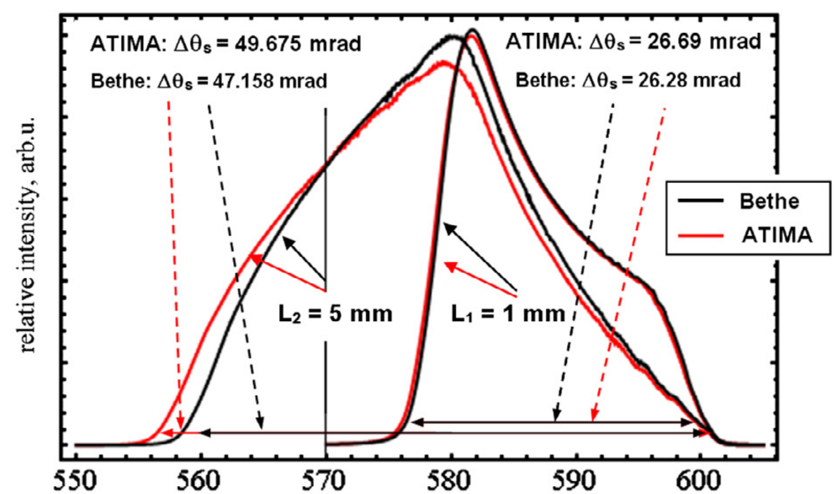


Cherenkov Radiation from Relativistic Ions in a Crystal









Au ions with initial energies E = 905 MeV/u in a LiF radiator, wave length range 380–740 nm

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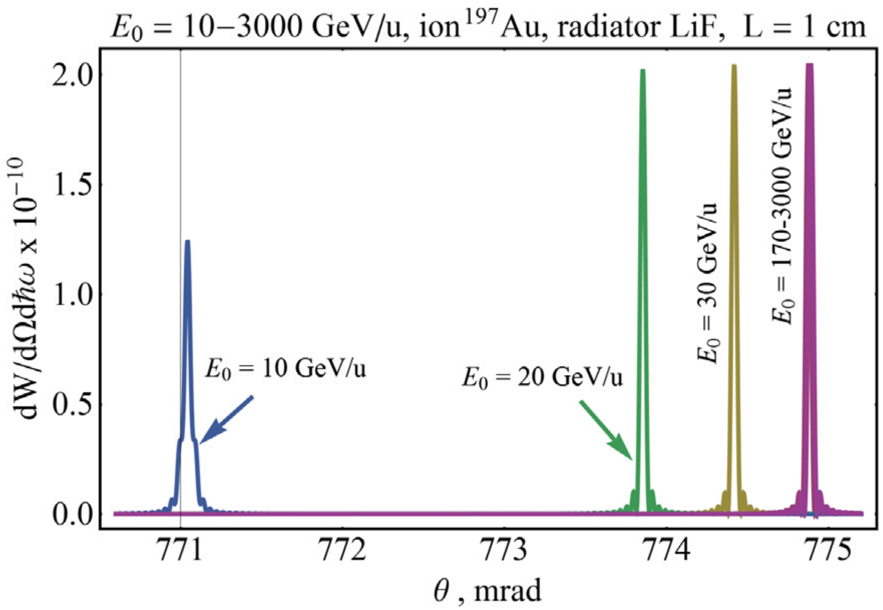
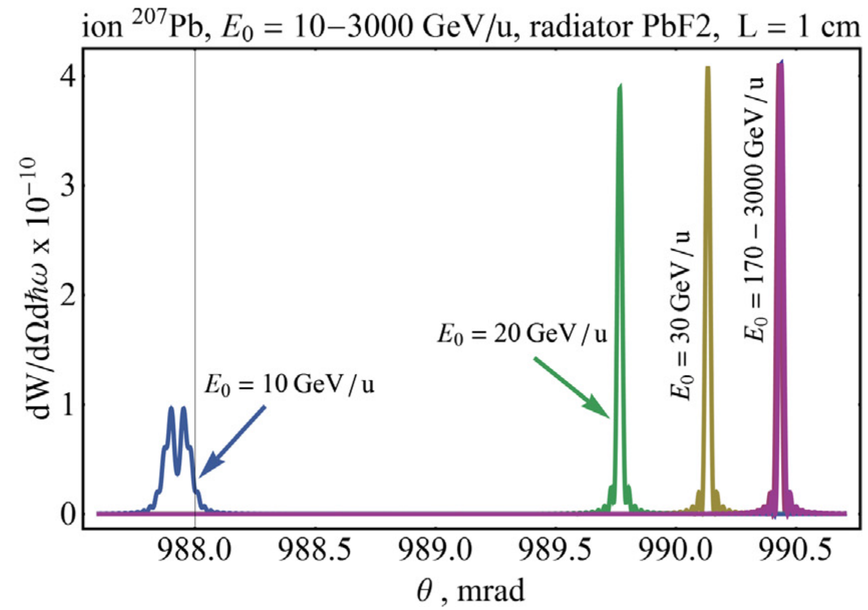
E.I. Fiks, Yu.L. Pivovarov, O.V. Bogdanov, H. Geissel, C. Scheidenberger, Influence of slowing down in the radiator on the Cherenkov radiation angular distributions from relativistic heavy ions at FAIR, SPS and LHC energies// Nuclear Instruments and Methods in Physics Research B 309 (2013) 146–150

Cherenkov Radiation from Relativistic Ions in a Crystal





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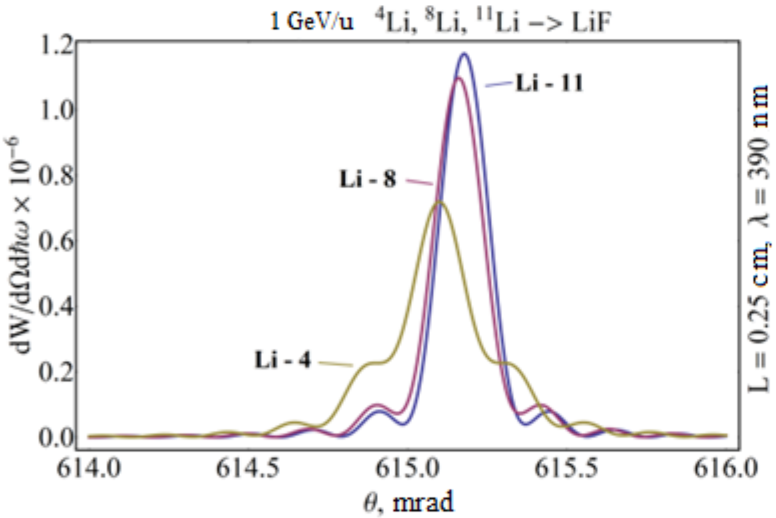
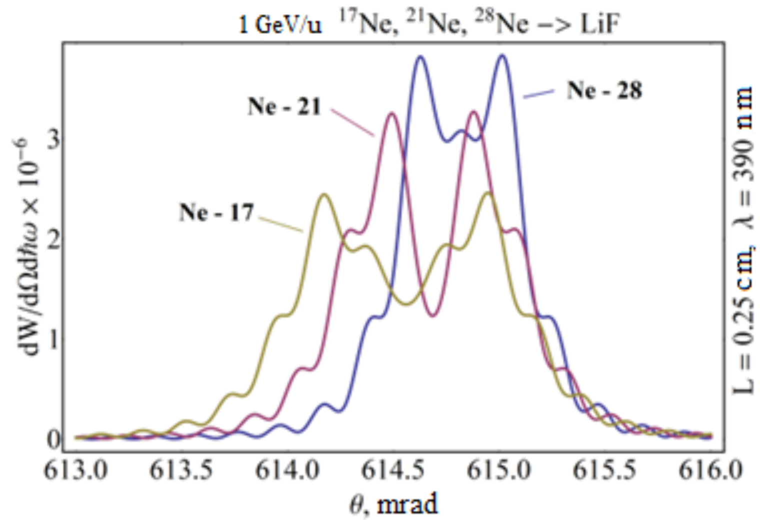
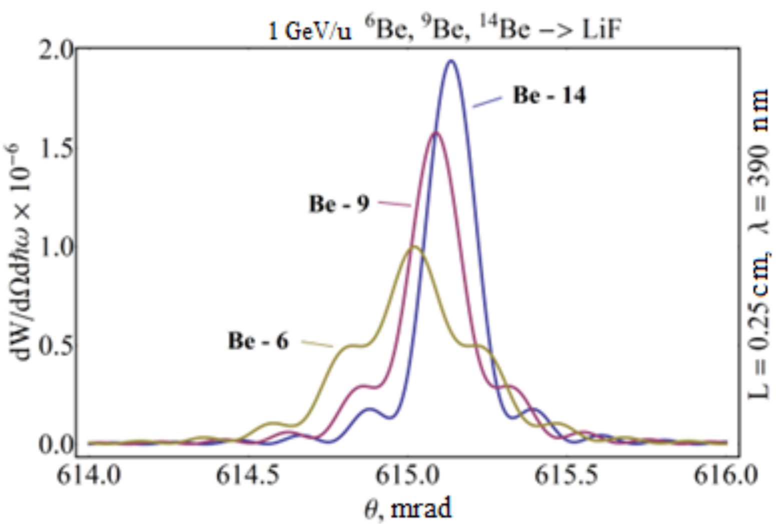
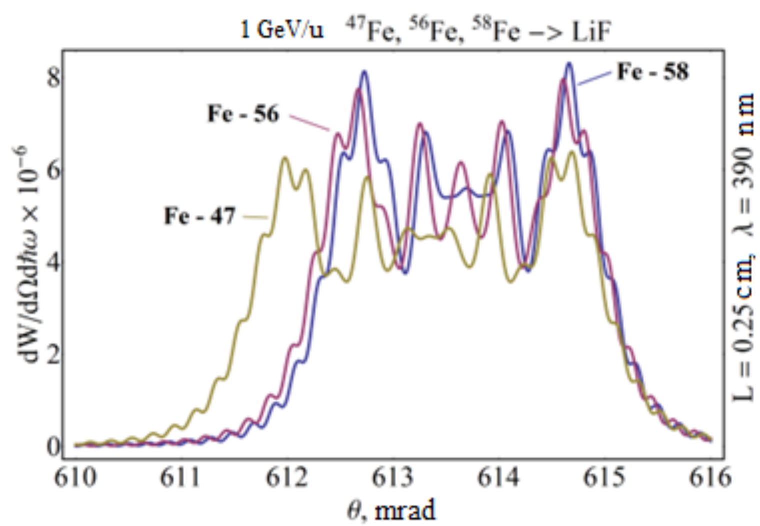




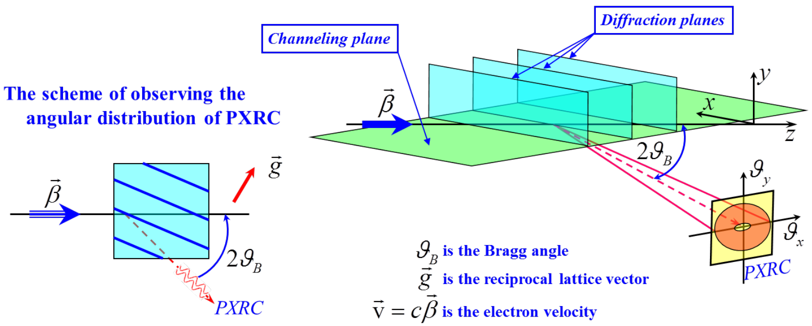




Parametric X-radiation at Channeling and its Quantum Features

PXRC at planar channeling

The PXRC appears when an electron passes through a crystal in the channeling regime. The channeling means that the electron is in a bound state with the crystal plane



Interband transitions ($i \neq f$) ⇒ Diffracted Channeling Radiation

⇐ **Intraband** transitions ($i = f$) Parametric X-radiation at channeling

Nitta suggested ➡ the form factors of channeled states to be equal approximately to 1 $|F_{ii}|^2 \approx 1$

⬇

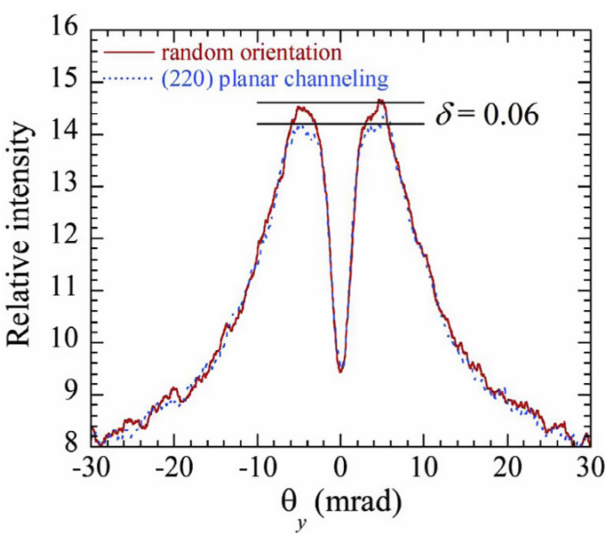
Angular distribution of PXRC **does not differ** from that of PXR

Experiment: SAGA-LS (JETP Letters, 2012) ➡ **a difference exists** !

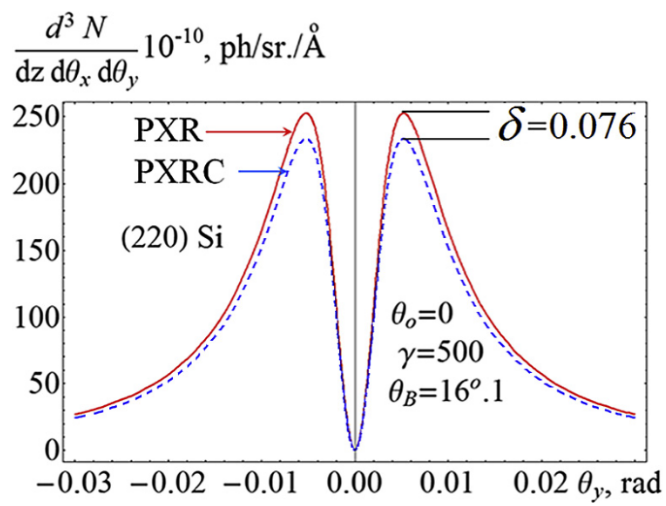
⬇

Motivation to **re-calculate** PXRC angular distribution

PXRC experiment (SAGA LS)



PXRC simulation



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K.B. Korotchenko, Yu.L. Pivovarov, Y. Takabayashi, Experimental and theoretical study of PXRC (Parametric X-Radiation at Channeling) from 255 MeV electrons in Si// Nuclear Instruments and Methods in Physics Research B 309 (2013) 25–29

PXRC (Parametric X-radiation at Channeling) and its Quantum Features

Theory: difference of PXR and PXRC

The PXRC angular distribution = sum over populated quantum states (bands)

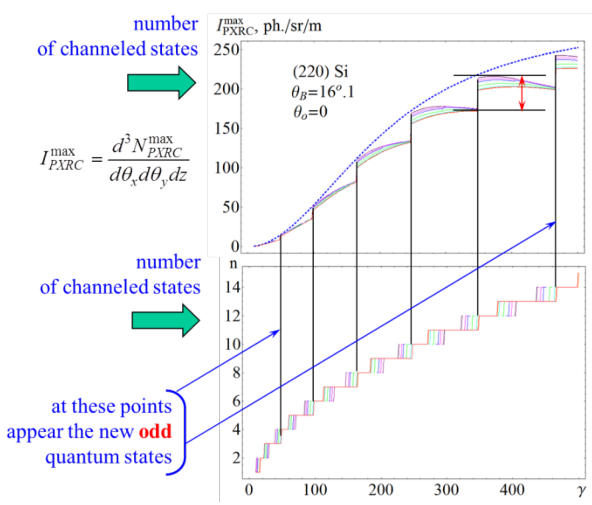
$$\frac{d^3 N_{PXRC}}{d\theta_x d\theta_y dz} = dN_{PXR} \sum_n P_n(\theta_\circ) \, | F_{nn} |^2$$

Initial population of the n-th energy level (band)

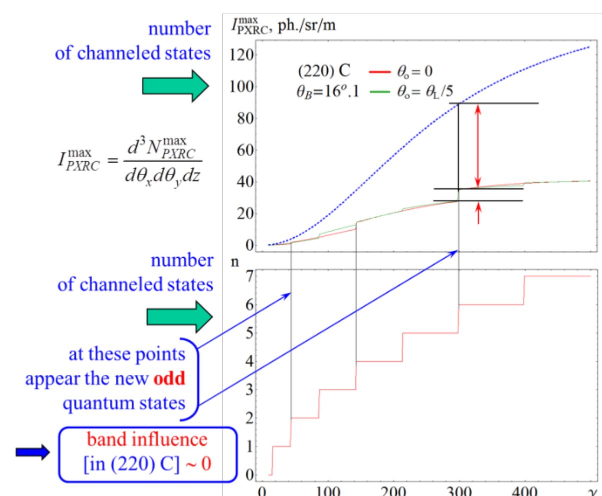
$$P_n(\theta_\circ) = \frac{1}{d} \left| \int_{-d/2}^{d/2} \exp(i k_y \theta_\circ y) \phi_n(y) dy \right|^2$$

Relativistic factor γ → **Number n**, **Form-factors $|F_{nn}|^2$**, **Population $P_n(\theta_\circ)$** of quantum states (bands)

Silicon (220)



Diamond (220)



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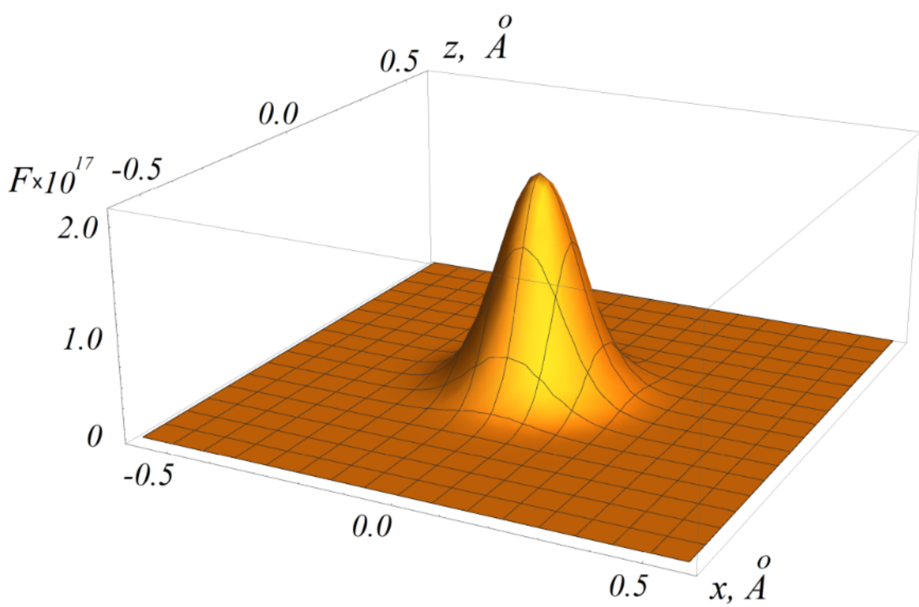
K.B. Korotchenko, Yu.L. Pivovarov, Y. Takabayashi, Experimental and theoretical study of PXRC (Parametric X-Radiation at Channeling) from 255 MeV electrons in Si// Nuclear Instruments and Methods in Physics Research B 309 (2013) 25–29

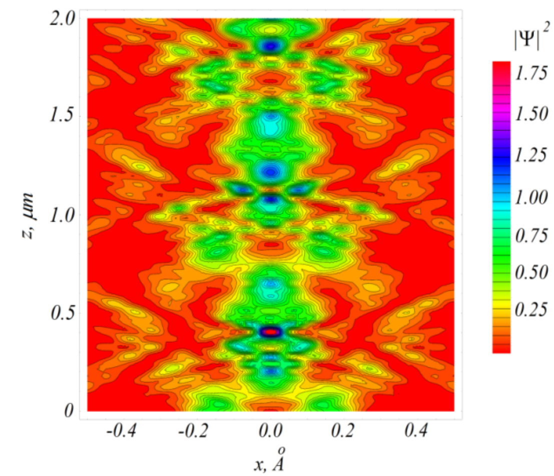
The electronuclear reaction yield is proportional to the function (Yu.M. Filimonov, Yu.L. Pivovarov and S.A. Vorobiev, Nuclear Physics, 47 3 (1988) 894-895):

$$\omega_k(z,\sigma) = \int F(\vec{\rho}, z, \sigma)\delta(\vec{\rho} - \vec{\rho}_k(z))d\vec{\rho}$$

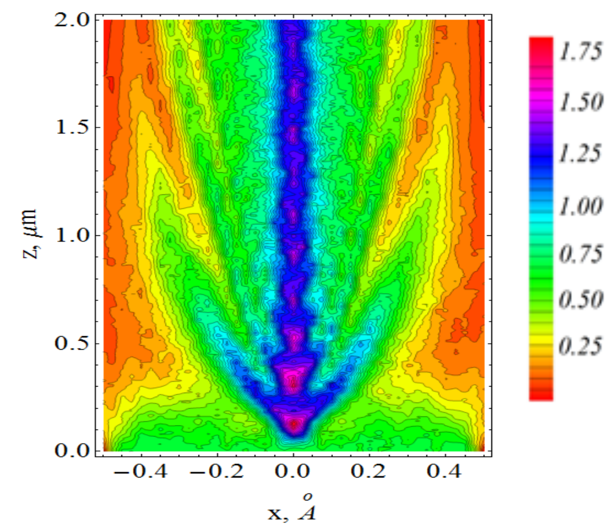


L. C. Feldman, J. W. Mayer, Fundamentals of surface and thin film analysis, North-Holland, 1986

Flux density of electrons E=255 MeV (quantum calculation)



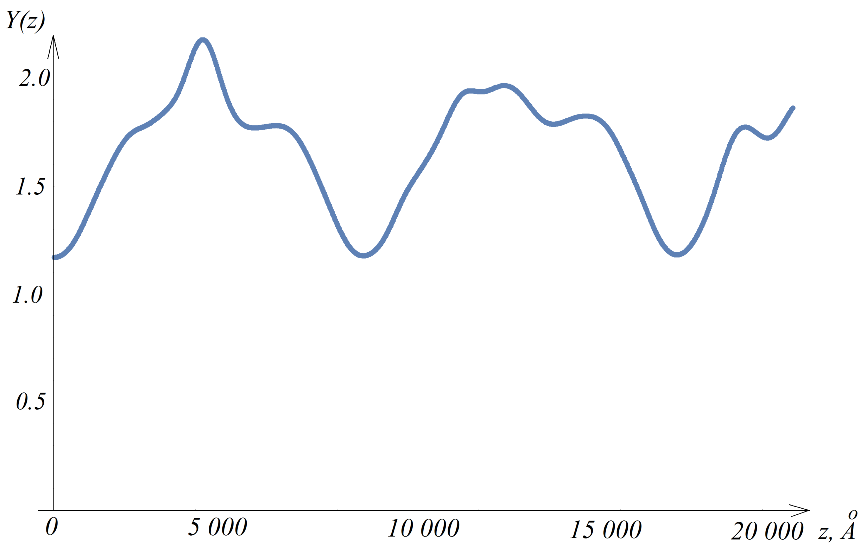
Flux dynamics of electrons E=255 MeV (classical calculation)



Depth oscillations of electronuclear reactions caused by relativistic planar channelled electrons: quantum versus classical calculations

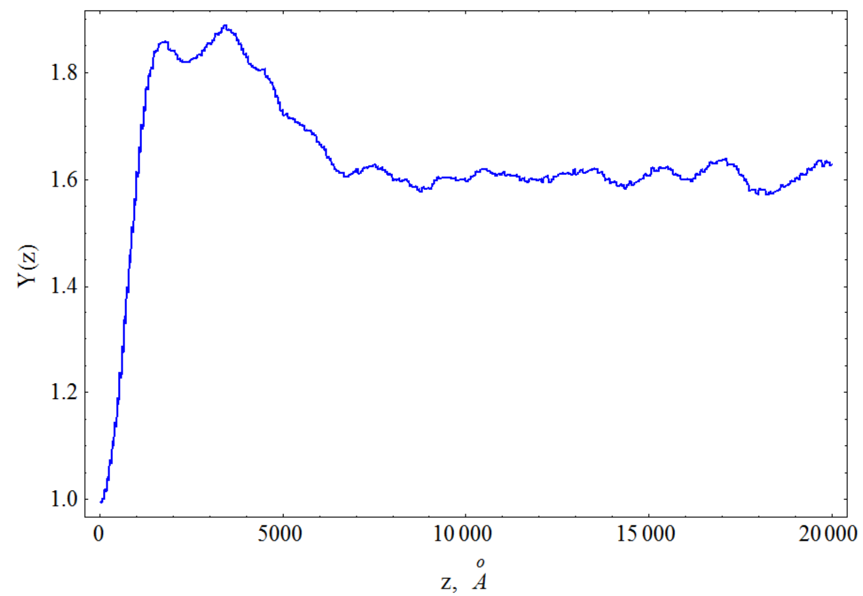
Electronuclear reaction yield (quantum calculation)

$$Y(z) = \int_{-d/2}^{d/2} F(x,z) \left|\Psi(x,z)\right|^2 dx$$



Electronuclear reaction yield (classical calculation)

$$Y(z) = \int_{-d/2}^{d/2} F(x,z) P(x,z) dx$$



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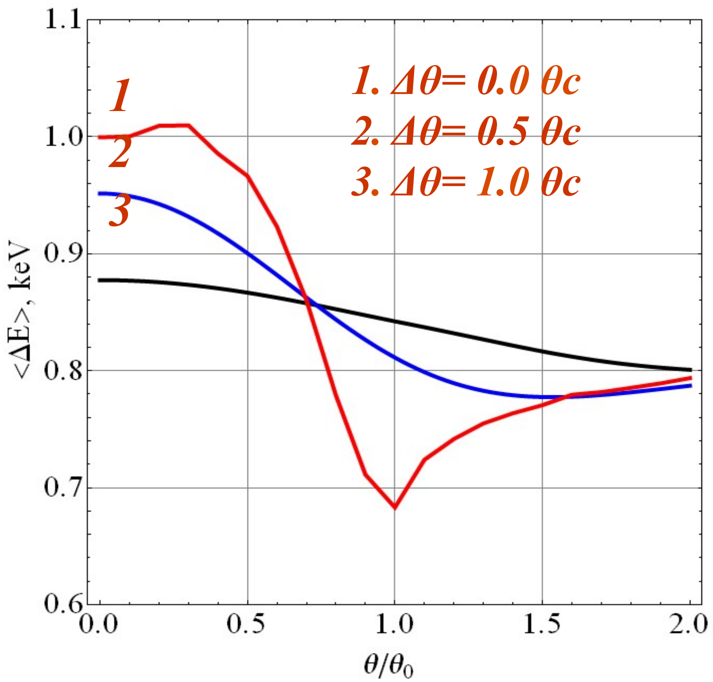
Radiation energy loss of channeled relativistic electrons in a crystal

$$\Delta E = \frac{2e^2}{3c^3}\int_0^T \frac{a^2(t) - \left[\mathbf{v}(t)\mathbf{a}(t)/c^2\right]^2}{(1-v^2(t)/c^2)^3}\,dt$$



Angle-of-incidence dependence of the total yield of channeling radiation of electrons at (100) planar channeling in Si

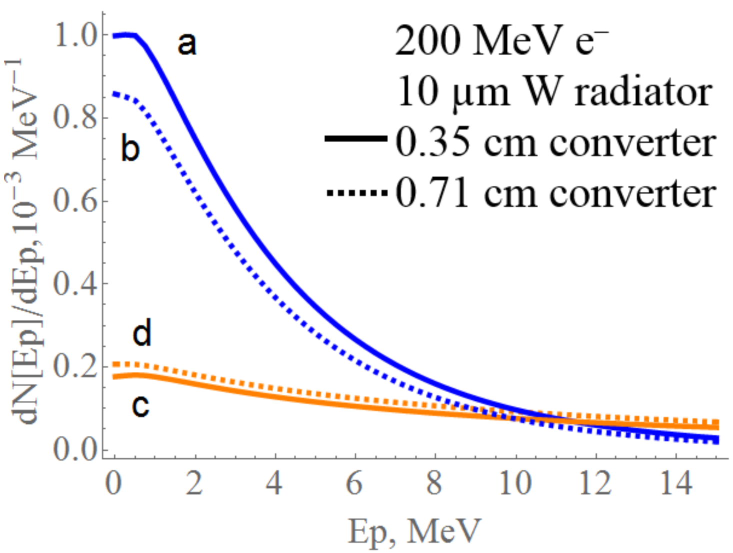
- **Angle-of-incidence dependence of the total yield of channeling radiation in a thin crystal:**
  - **Planar and axial channeling for the (100), (110) (111) planes and <100>, <110> axes**
  - **Electrons and positrons**
  - **155-855 MeV energy range (extendable)**
  - **Si, C, W crystals**
  - **Initial electron/positron beam angular divergence is taken into account**

- **Angle-of-incidence dependence of the total yield of the CR can be used for the thin crystal alignment in more complicated channeling experiments and, even more, for diagnostics of angular spread of moderately relativistic electron/positron beams**

S.V. Abdrashitov, O.V. Bogdanov, S.B. Dabagov, Yu.L. Pivovarov, T.A. Tukhfatullin, Total yield of channeling radiation from relativistic electrons in thin Si and W crystals // Nuclear Instruments and Methods in Physics Research B 309 (2013) 59-62

Positron source via e$^-$ - e$^+$ pair production by channeling radiation

$$\frac{dN_P}{dE_P} = n \cdot L_C \int \frac{1}{E_\gamma} \frac{dW}{dE_\gamma dL} \cdot L \cdot \frac{d\sigma(Z, E_P, E_\gamma)}{dE_P} dE_\gamma$$



The energy spectra of positrons, produced by the radiation from 200 MeV electrons in W: (a, b) axial CR; (c, d) BS. Solid lines correspond to converter thickness 0.35 cm; dashed – 0.71 cm.

- Energy spectra and the total yield of the positrons calculated for the hybrid scheme of positron source using the channeling radiation from 200 – 1600 MeV electrons and thin amorphous converter
- Comparison energy spectra and the total yield of the positrons for cases of channeling radiation and the bremsstrahlung is carried out
- Studies on hybrid positron source using channeling radiation and a thin W amorphous convertor are extended to the case of more thick radiators
- Positron beam emittance and the influence of dechanneling processes in thick radiator crystal are in progress

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S.V. Abdrashitov, O.V. Bogdanov, Y.L. Pivovarov, S.B. Dabagov, T.A. Tukhfatullin Hybrid scheme of positron source at SPARC_LAB LNF facility// Nucl. Instr. and Meth. in Phys. Res. B 355 (2015) 65–68.

Conclusions

- New version of computer code BCM–2.0 is developed.
- Newly developed different packages of this code were successfully applied to simulate scattering, radiations, electron-positron pairs creation and other effects connected with channelling of relativistic particles in aligned crystal.

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THANK FOR YOUR ATTENTION!