A METHOD FOR CALCULATION OF EMITTED RADIATION IN STRAIGHT AND BENT CRYSTALS BASED ON THE BAIER KATKOV QUASICLASSICAL METHOD

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Channeling 2016
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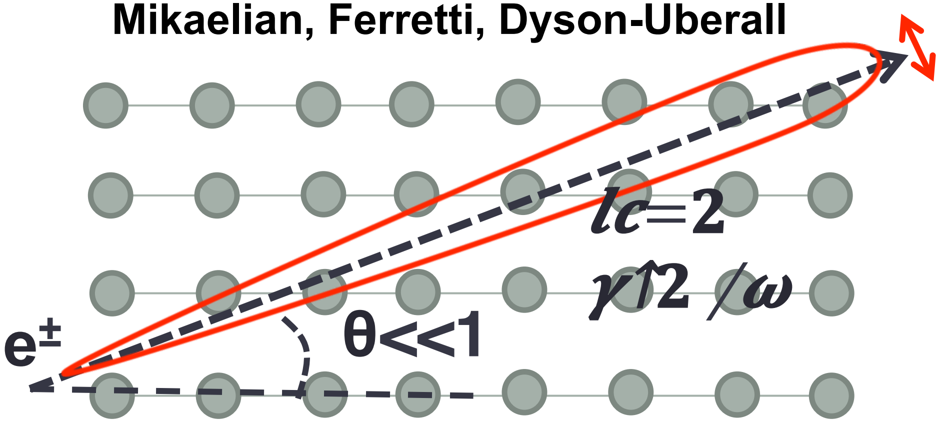
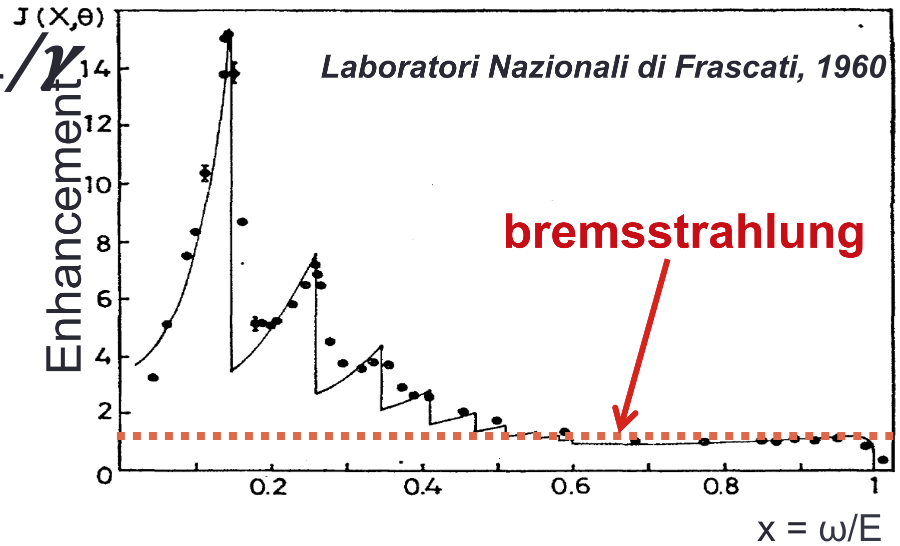
Outlook

- Introduction of radiation processes in crystals;
- Development of an algorithm to compute the radiation generation in oriented crystals based on the Baier and Katkov method;
- Comparison with experiments at intermediate energies (1 GeV $e^{\pm}$);
- Comparison with experiments at ultra-high energies (100 GeV $e^{\pm}$);
- Conclusions.

Enhancement of bremsstrahlung radiation in aligned crystals

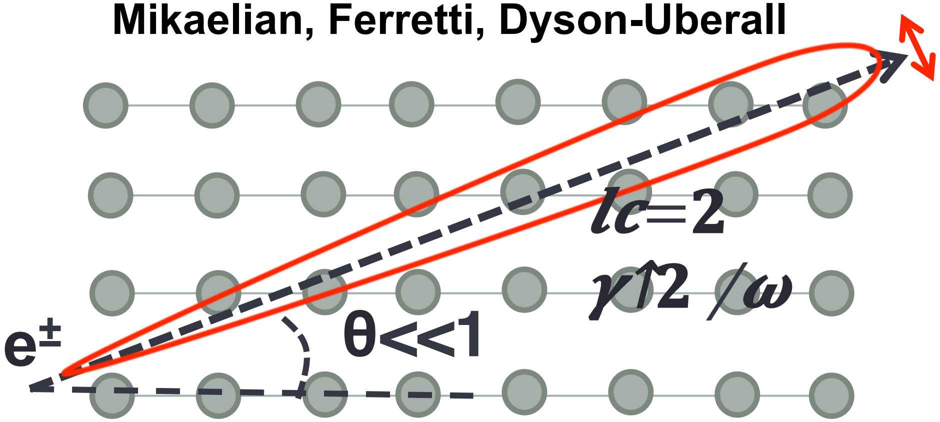
Coherent Bremsstrahlung (1950s) Ter-Mikaelian, Ferretti, Dyson-Uberall

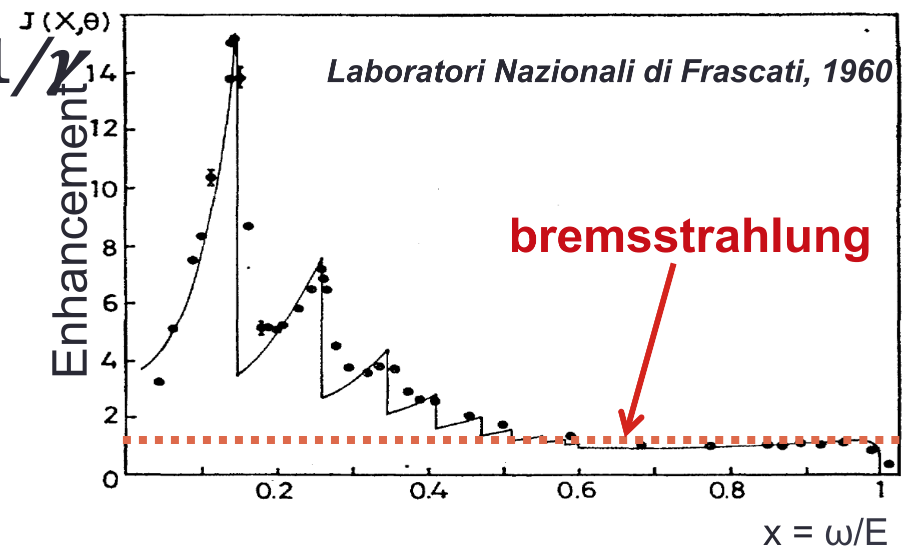


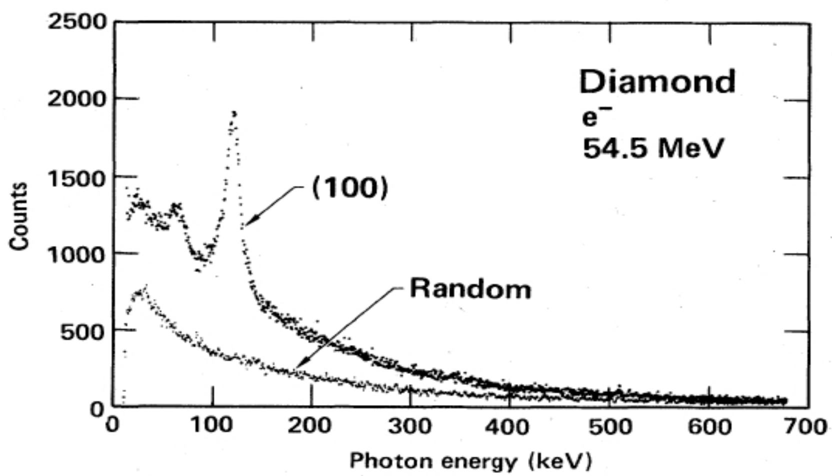


Enhancement of radiation in aligned crystals

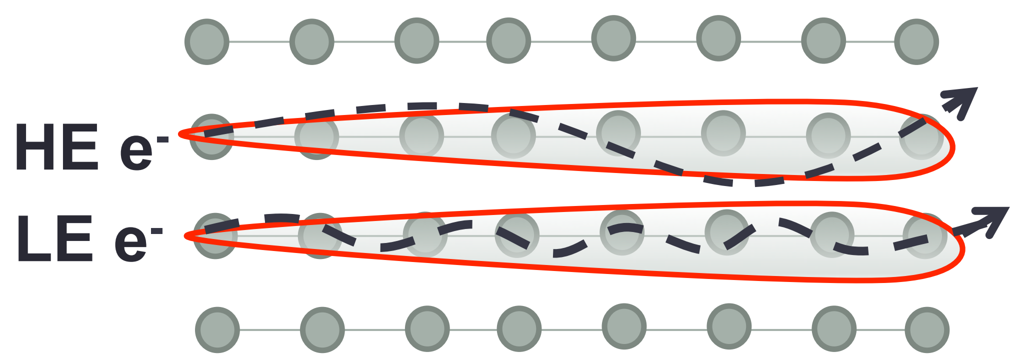
Coherent Bremsstrahlung (1950s) Ter-Mikaelian, Ferretti, Dyson-Uberall







M. Kumakhov, Physics Letters A 57, 17 (1976).

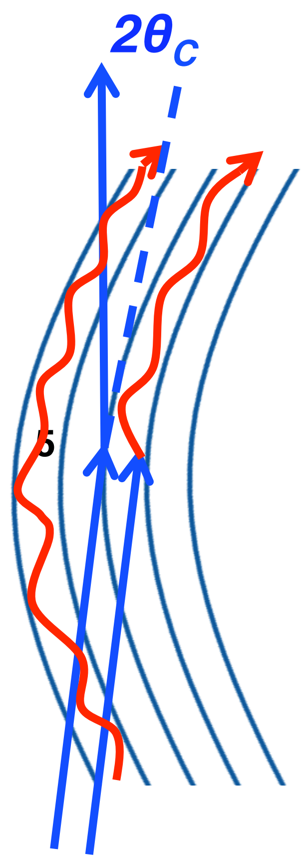


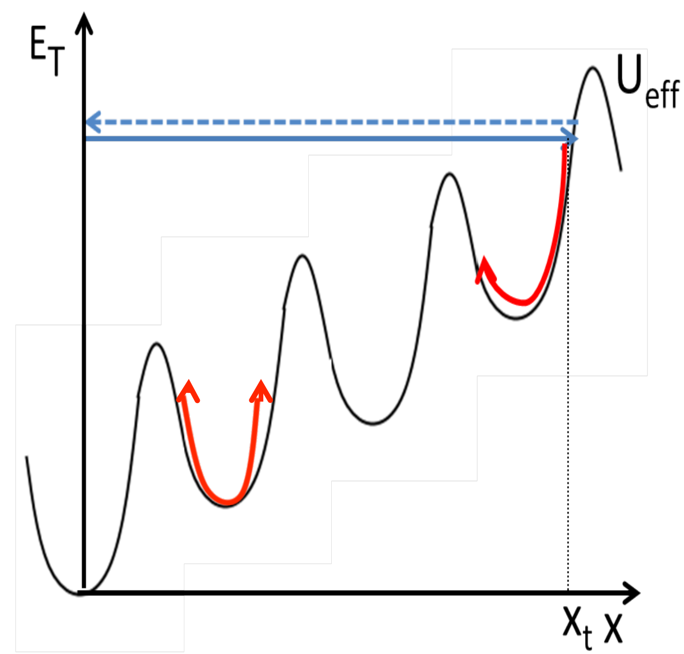
Channeling radiation (Kumakhov)

Channeling and related effects in a bent crystal

- ✓ **Tsyganov (1976):** channeling in bent crystals;
- ✓ **Taratin and Vorobiov (1987):** Volume Reflection of over-barrier particles.

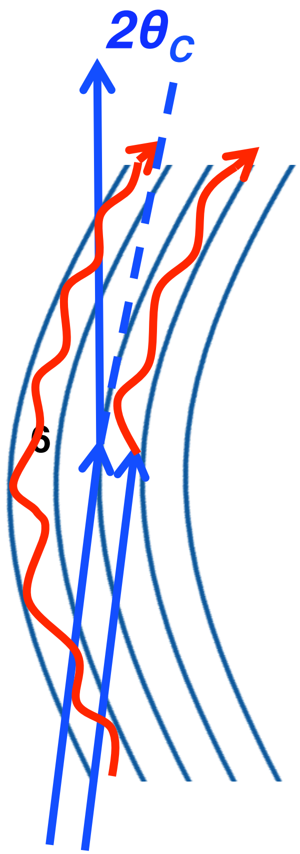
$$U_{eff}(x) = U(x) + \frac{pv}{R}x$$



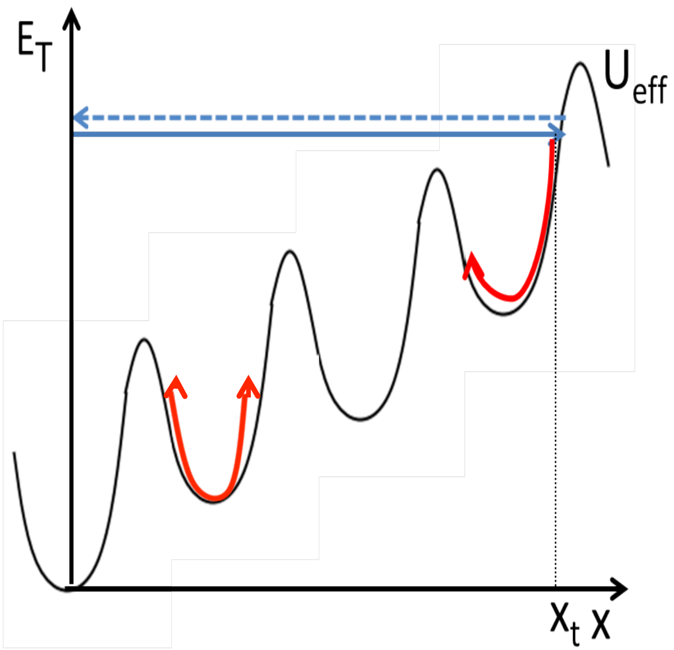


Channeling and related effects in a bent crystal

- ✓ **Tsyganov (1976):** channeling in bent crystals;
- ✓ **Taratin and Vorobiov (1987):** Volume Reflection of over-barrier particles.
- Under VR **the angle between the particle trajectory and crystalline planes changes during the motion**
- **Need of a general method for radiation computation**



$$U_{eff}(x) = U(x) + \frac{pv}{R}x$$



Baier-Katkov quasiclassical operator method (1967-1968)

General method for calculation of radiation generated by $e^{\pm}$ in an external field

The electromagnetic radiated energy is evaluated with the BK formula:

$$dE/d^3k = \omega dN/d^3k \quad \alpha/4\pi^2 \iint dt_1\, dt_2 \, [(E^2 + E'^2)(v_1 v_2 - 1) + \omega^2/\gamma^2]/2E'^2 \, e^{-ik'(x_1 - x_2)} \qquad (1)$$

where the integration is made over the classical trajectory.

Why classical trajectory?

2 types of quantum effects :

- the quantization of particle motion $\sim\hbar\omega_0/E$
  In crystals: **negligible for electron/positron energy >10-100 MeV**
- the **quantum recoil** of the particle when it radiates a photon with energy $\hbar\omega\sim E$
  NOT negligible for electron/positron energy >50 GeV

An algorithm for radiation in crystals

Integration of the BK formula

SMALL ANGLE APPROXIMATION: Since the angle between particle trajectories and crystal planes or axes is small and at ultrarelativistic energies the radiation angle 1/γ is much smaller than unity the particle velocity **v** and photon momentum **k** can be represented in the form :

$$\mathbf{v}(t) \simeq \mathbf{v}_\perp(t) + \mathbf{e}_z \left[1 - 1/2\gamma^2 - v_\perp^2(t)/2\right],$$
$$\mathbf{k} = \mathbf{n}\omega \simeq \mathbf{e}_\perp \omega\theta + \mathbf{e}_z \omega \left(1 - \theta^2/2\right),$$

where the angle $\theta \ll 1$ represents the radiation angle. The formula (1) can be rewritten as:

$$\frac{dE}{d^3k} \sim \frac{\alpha}{8\pi^2} \frac{\varepsilon^2 + \varepsilon'^2}{\varepsilon'^2} \omega^2 C, \qquad (2)$$

where $$C = |\, \boldsymbol{I}_\perp \,|^2 + \gamma^{-2} \frac{\omega^2}{\varepsilon^2 + \varepsilon'^2} \,|\, J \,|^2 \qquad (3)$$

V. Guidi, L. Bandiera, V. Tikhomirov, Phys. Rev. A 86 (2012) 042903]

L. Bandiera, et al., Nucl. Instrum. Methods Phys. Res., Sect. B 355, 44 (2015).

An algorithm for radiation in crystals

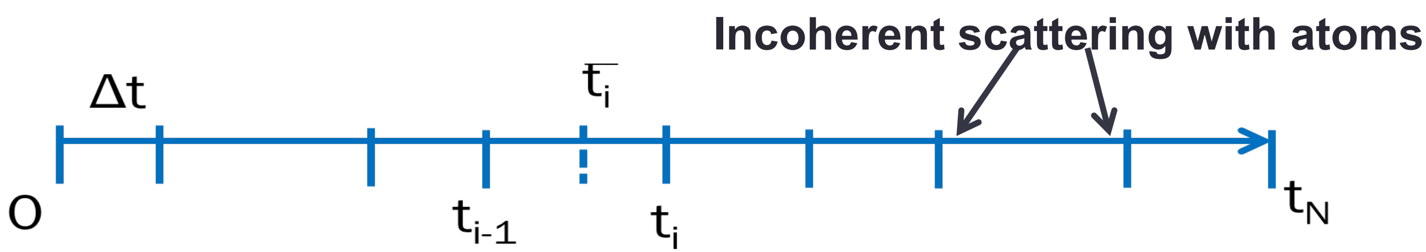
Integration of the BK formula

SMALL ANGLE APPROXIMATION: the integrals of eq. (1) can be represented as follows:

$$\left\{ \begin{array}{c} J \\ \boldsymbol{I}_\perp \end{array} \right\} = \int_{-\infty}^{\infty} \left\{ \begin{array}{c} 1 \\ (\mathbf{v}_\perp(t) - \boldsymbol{\theta}) \end{array} \right\} \exp\left[i\phi(t)\right] dt, \qquad (4)$$

being $\phi(t) = k' x(t) = \omega'[t - \mathbf{n}\ \mathbf{r}(t)] \simeq \frac{\omega'}{2} \int^{t} \left[1/\gamma^2 + (\mathbf{v}_\perp(t) - \boldsymbol{\theta})^2\right],$ and $\omega' = \omega\varepsilon/\varepsilon'.$

ACCOUNT OF INCOHERENT SCATTERING:

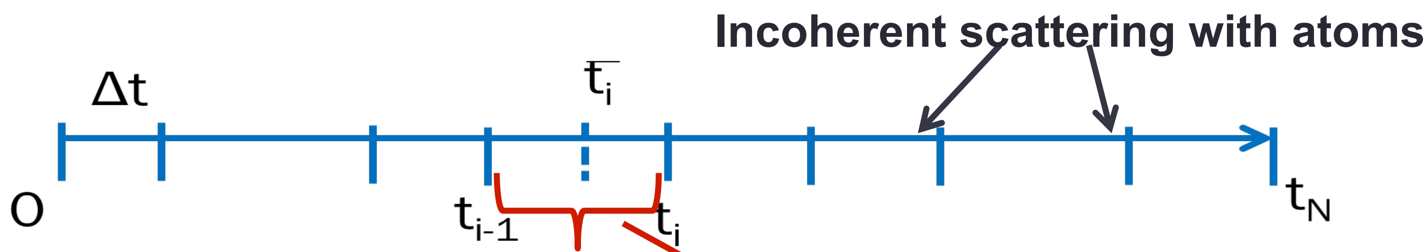


The **particle trajectory is then divided in N small steps**, within which the particle trajectory is calculated through the integration of equation of motion in the continuous potential. **At the end of each step the scattering by nuclei and electrons is sampled** and the transverse velocity for the i-step becomes

$$\mathbf{v}_{\perp,\mathbf{i}} \rightarrow \mathbf{v}_{\perp,\mathbf{i}} + \theta_{\mathbf{s},\mathbf{i}}$$

An algorithm for radiation in crystals

Integration of the BK formula



In order to improve the convergence of its integration over *t* and ***θ*** (photon emission angle), the integrals of eq. 4 are computed as follows after an integration by parts:

$$J \approx i \sum_{i=1}^{N} \left\{ \exp\left[i\phi(t_i)\right] \left[\frac{1}{\dot{\phi}_{t_i+0}} - \frac{1}{\dot{\phi}_{t_i-0}}\right] - \exp\left[i\phi(\bar{t}_i)\right] \left[\left.\frac{2\ddot{\phi}}{\dot{\phi}^3}\right|_{\bar{t}_i} \sin\left(\left[\phi(t_i-0) - \phi(t_{i-1}+0)\right]/2\right) \right] \right\}$$

If incoherent scattering is switched off, it is go to zero.

$$\dot{\phi}(t < t_i) = \frac{\omega'}{2}\left[1/\gamma^2 + (\mathbf{v}_\perp(t) - \boldsymbol{\theta})^2\right], \qquad \ddot{\phi}(t) = \omega'\left(\mathbf{v}_\perp(t) - \boldsymbol{\theta}\right)\dot{\mathbf{v}}_\perp(t),$$

$$\dot{\phi}(t_i + 0) = \frac{\omega'}{2}\left[1/\gamma^2 + (\mathbf{v}_\perp(t) + \theta_{\mathbf{s,i}} - \boldsymbol{\theta})^2\right], \qquad \dot{\mathbf{v}}_\perp = -\frac{1}{\varepsilon}\frac{\partial U(\mathbf{r})}{\partial \mathbf{r}_\perp},\ U(\mathbf{r}) \text{ being the continuous potential.}$$

The contributions of the trajectory ends are not taken into account, thus neglecting the soft contribution of transition radiation.

The integration over θ leads to the radiation spectral intensity, ωdN/dω.

COMPARISON WITH EXPERIMENTS

GeV energy range:

- No quantum correction;
- Thin crystals -> single photon emission.

RADCHARM++

The algorithm for direct integration of the BK formula has been included in the RADCHARM++ routine [1] , which is an expansion of the DYNECHARM++ code [2] (*see E. Bagli talk today at 11:20*)

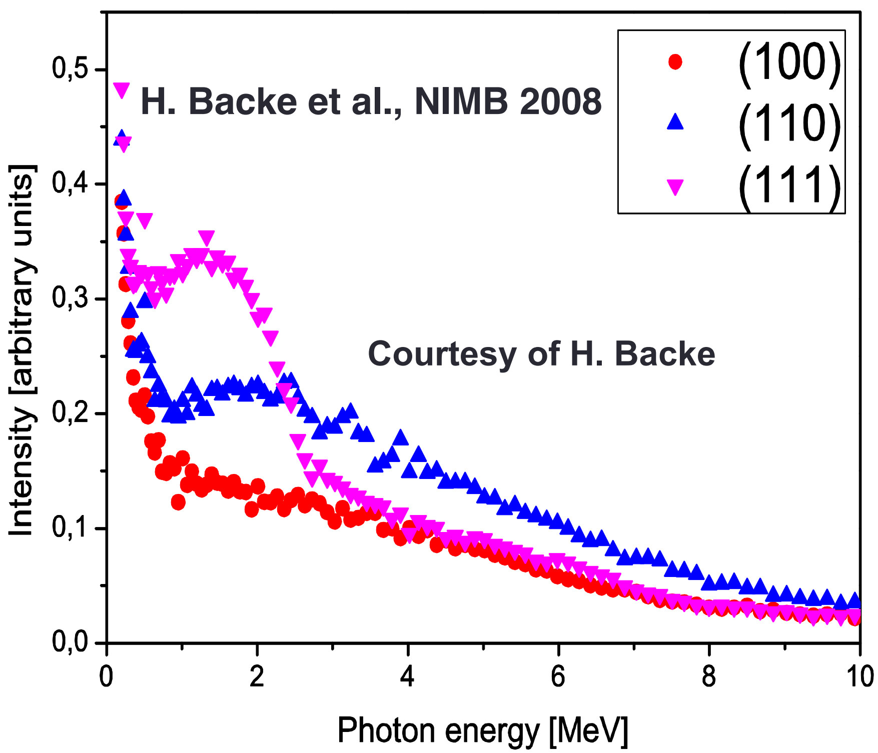
- The electrical characteristic of the crystal are evaluated by using the atomic form factors from x-ray diffraction data;
- Numerical integration of the classical equation of motion of particle trajectories under the continuum potential approximation;
- At the end of each step the multiple and single scattering by nuclei and electrons is sampled.

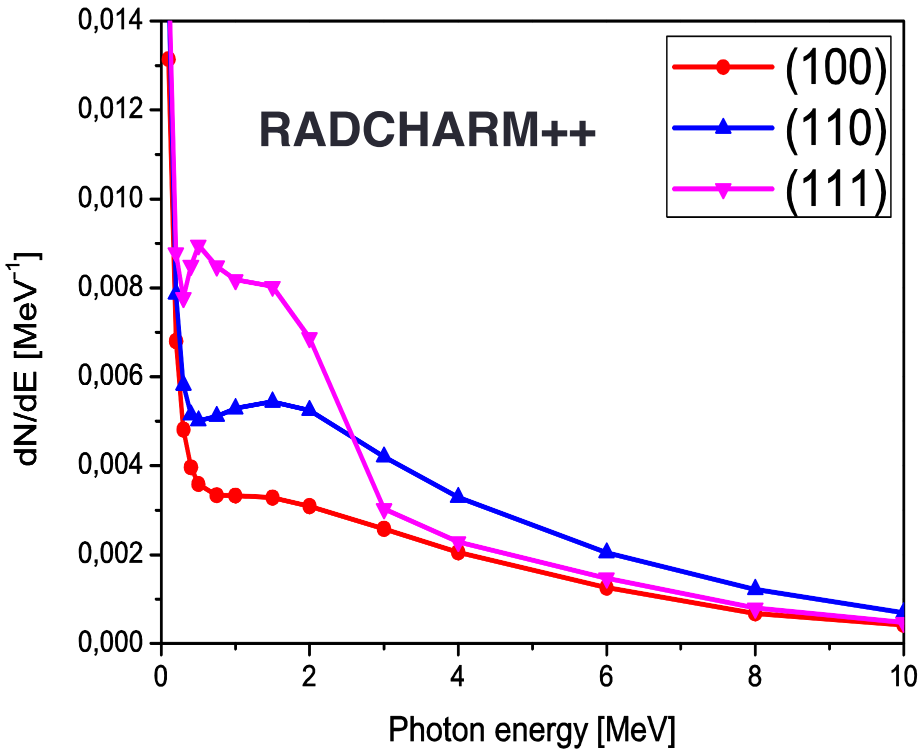
DYNECHARM++ has already been implemented in Geant4 [3]. The RADCHARM++ can also be implemented to include the bremsstrhalung radiation enhancement in crystals.

[1] L. Bandiera, et al., Nucl. Instrum. Methods Phys. Res., Sect. B 355, 44 (2015).
[2] E. Bagli, V. Guidi, Nucl. Instr. and Meth. in Phys. Res. Section B 309 (2013) 124
[3] E. Bagli, M. Asai, D. Brandt, et al. Eur. Phys. J. C (2014) 74: 2996.

Simulation of e.m. radiation emitted by ultrarelativistic electrons in the field of any crystal plane

Comparison with past experiments performed at the Mainzer Mikrotron with 855 MeV electrons interacting with a 175 μm straight Si crystal

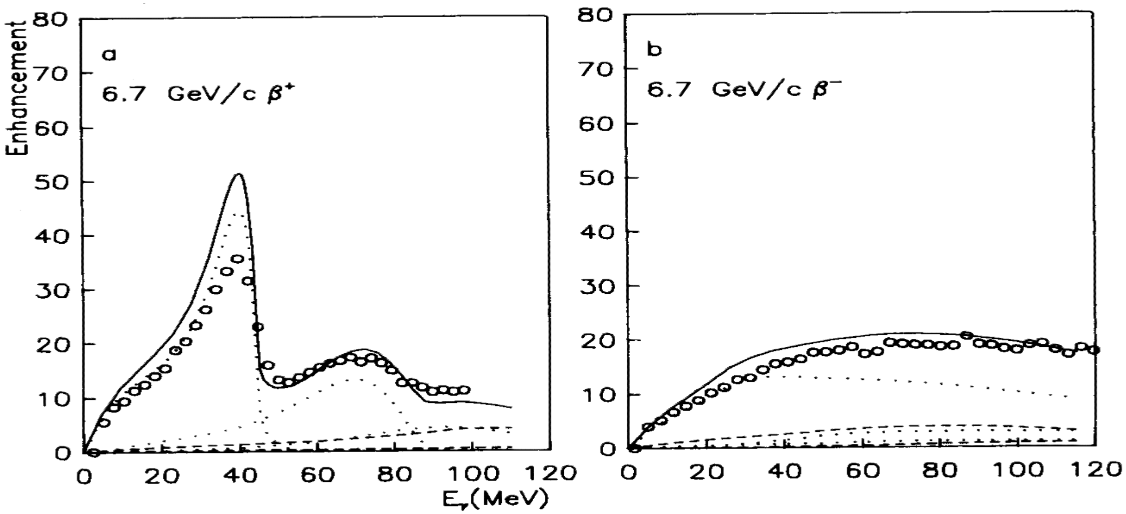




L. Bandiera, et al., Nucl. Instrum. Methods Phys. Res., Sect. B 355, 44 (2015).

Simulation of e.m. radiation emitted by both positive and negative particles

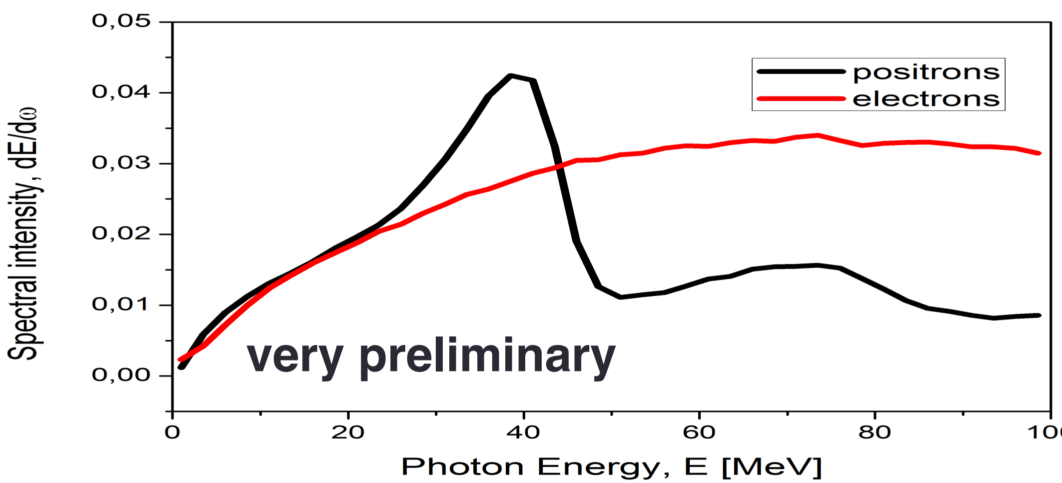
Comparison with past experiments at CERN: 6.7 GeV positrons/electrons channeling in a 0.1 mm thick Si (110)



Enhancement with respect to random

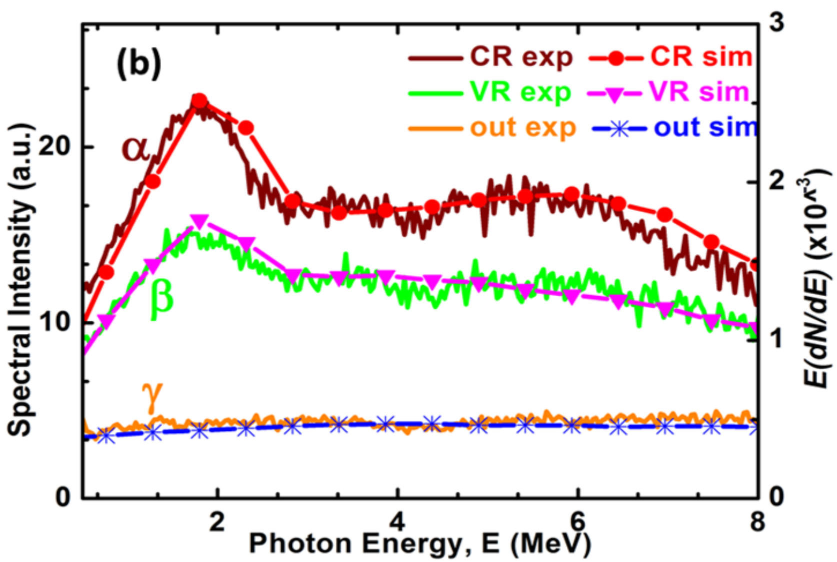
J. Bak et al., Nucl.Phys. B254(1985)491.

Spectral Intensity, $dE/d\omega = \omega dN/d\omega$ (ideal case)

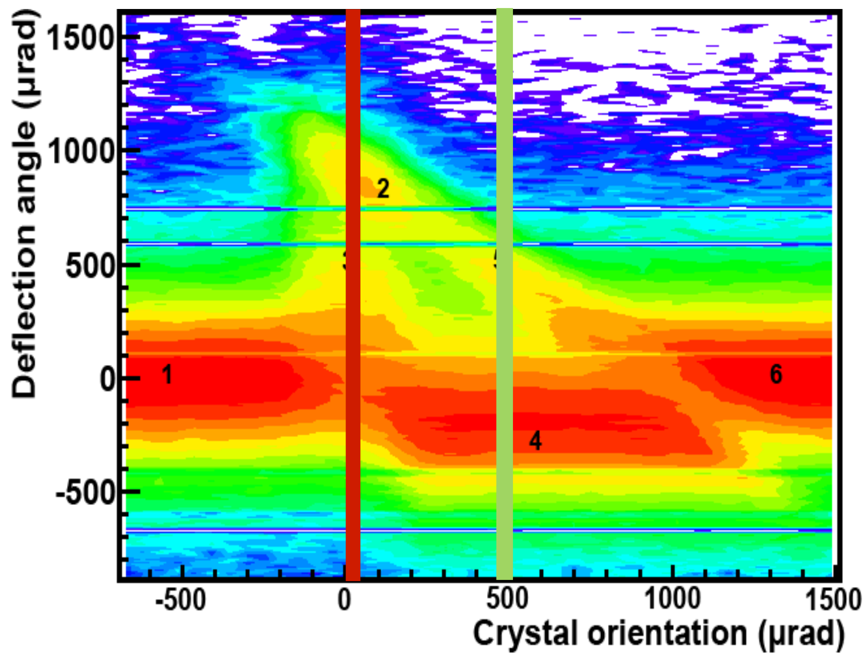


Simulation of e.m. radiation emitted by ultrarelativistic electrons in a bent crystal

Comparison with experiment performed at the Mainzer Mikrotron with 855 MeV electrons interacting with a 30.5 μm bent Si crystal along the (111) planes



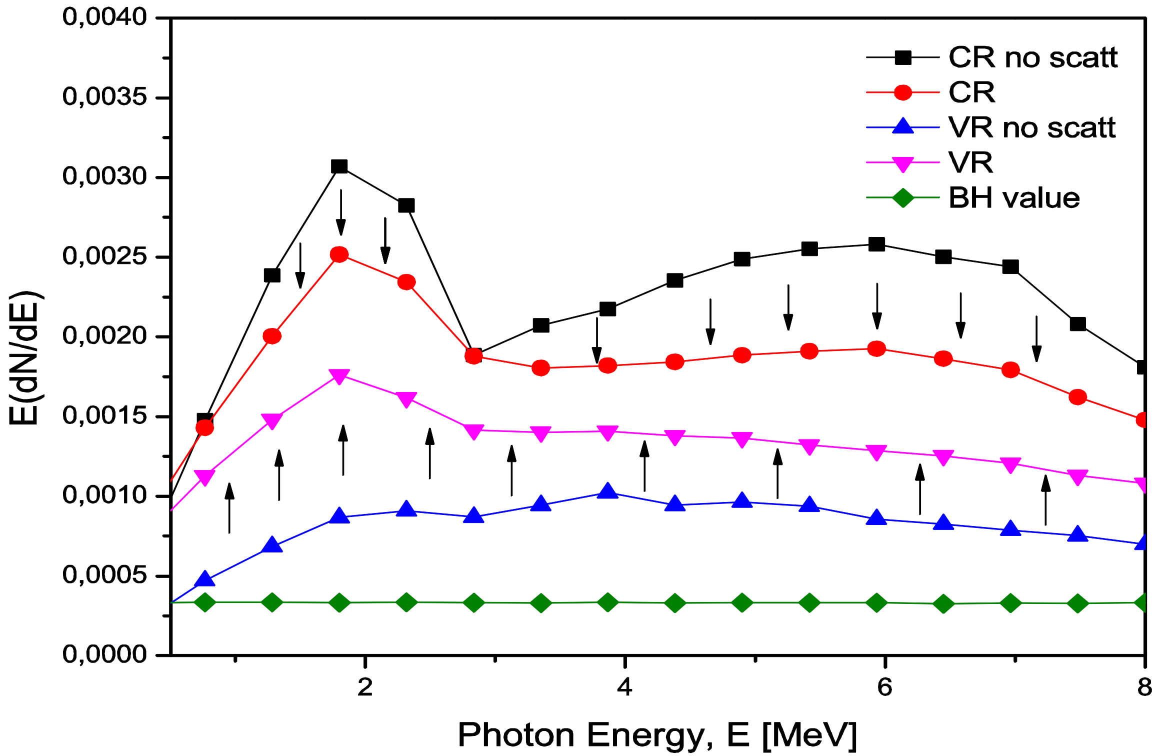
RADCHARM++



Channeling at <zero> position
Volume reflection at +490 μrad
OUT: not aligned at 8 mrad

L. Bandiera et al., Phys. Rev. Lett. 115 (2015) 025504.

Simulation of the contribution to radiation of incoherent scattering



Simulation of the contribution of the scattering with nuclei and electrons to radiation spectral intensity [E(dN/dE)].

COMPARISON WITH EXPERIMENTS

100 GeV energy range:

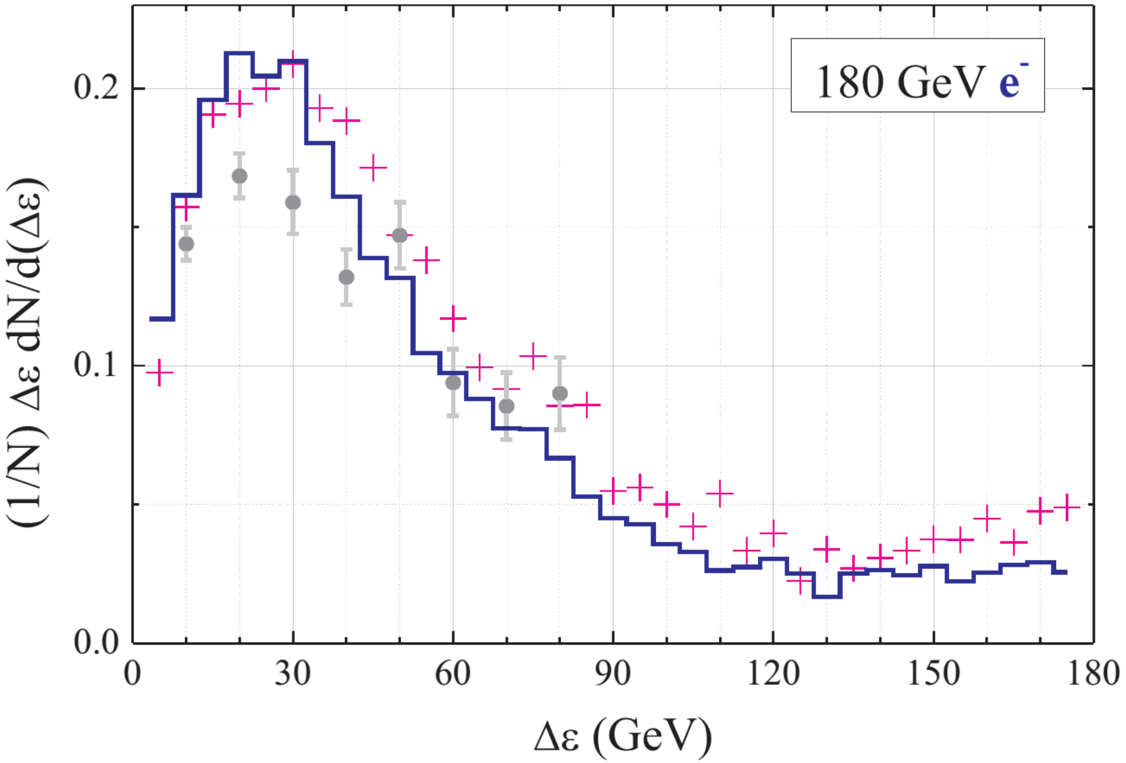
- Quantum correction;
- Multiple photon emission.

Multi photon emission

- In principle, the BK formula should be integrated along the whole particle trajectory.
- At very-high energy, the total probability of radiation may exceed unity -> **multiple photon emission!**
- **Separation of particle trajectory in intermediate lenghts** > coherence length and << typical distance between two sequential photon emission points. **Total probability of radiation on such trajectory part does not exceed 0.1**.
- The trajectory-part ends are neglected as the interference between them.

V. Guidi, L. Bandiera, V. Tikhomirov, Phys. Rev. A 86 (2012) 042903.

Simulation of PLANAR volume reflection



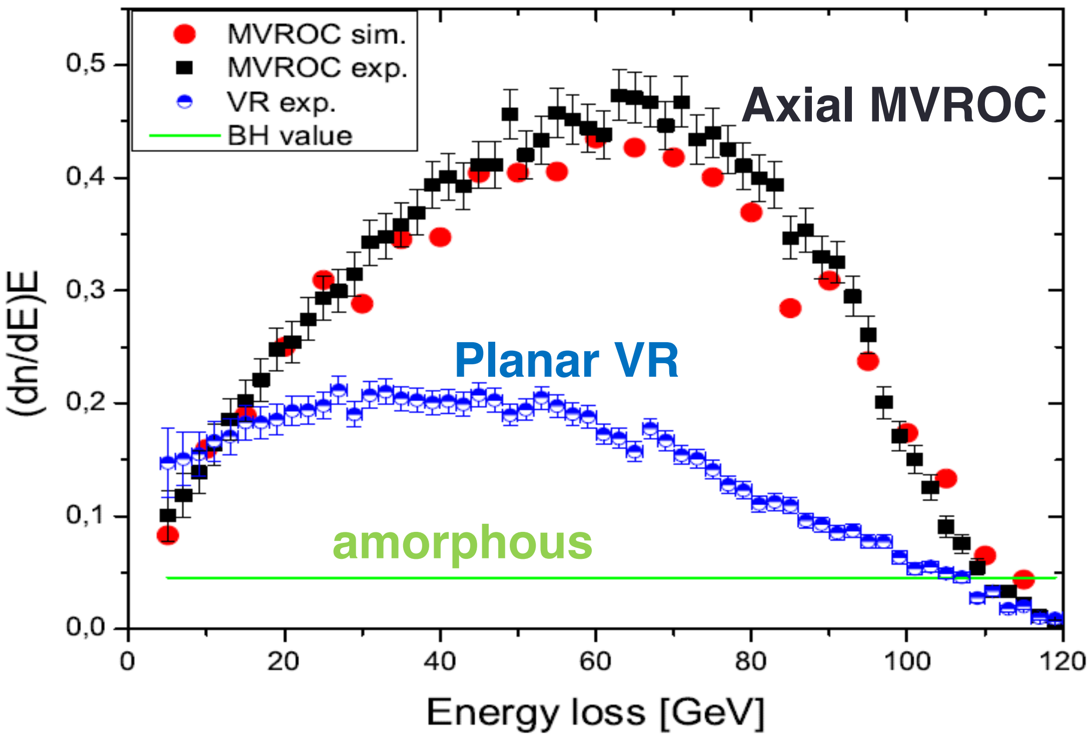
Si Crystal parameters:
Length = 0.84 mm;
Bending radius = 12 m;
Plane (111)
Beam divergence:
σ_x = 25 μrad and σ_y = 46 μrad
Incidence angle:
Θ_{X0} = 40 μrad

Photon energy ≥ 1 GeV has been selected.

Energy loss spectral intensities: (dn/dE)*E of 180 GeV/c volume reflected electrons

Experiment: W. Scandale, et al.,Phys.Rev.A79, 012903 (2009).
Simulation: V. Guidi, L. Bandiera, V. Tikhomirov, Phys. Rev. A 86 (2012) 042903

Simulation of AXIAL multi-volume reflection



Si Crystal parameters:
Length = 2mm;
Bending radius = 2.7 m;
Plane (110)
Axis <111>
Beam divergence:
σ_x = 50 μrad and σ_y = 65 μrad
Incidence angle:
Θ_{X0} = 365 μrad ≈ half bending
Θ_{Y0} misaligned for planar VR
= 205 μrad for axial MVROC

Photon energy ≥ 1 GeV has been selected.

Energy loss spectral intensities: (dn/dE)*E of 120 GeV/c single and multi-reflected electrons

L. Bandiera et al., Phys. Rev. Lett. 111(2013) 255502

Summarizing

- An algorithm to compute of radiation emitted by relativistic $e^{\pm}$ in crystals based on the Baier-Katkov method has been presented;

- Such algorithm has already been implemented in existing Monte Carlo codes for simulation of particle trajectories in crystals;

- Comparison with experiments show a very good agreement in a wide energy range (from 1 GeV to 100 GeV);

- Such a method can be inserted in the most general toolkits for the simulation of the passage of particles through matter, such as Geant 4, as an implementation of the radiation processes in oriented crystalline structures.