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**Stimulated Resonance
Radiation
of Channeling Particles**



Dabagov S.B., Kalashnikov N.P.





Stimulated Resonance Radiation of Channeling Particles

^{1,2,3} Dabagov S.B., ² Kalashnikov N.P.

¹ Laboratori Nazionale di Frascati, Istituto Nazionale di Fisica Nucleare, Italy.

² National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Moscow, Russia.

³RAS, P.N. Lebedev Physical Institute, Moscow, Russia



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- Introduction
- The interaction potential of charged particles with crystalline lattice atoms
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INTRODUCTION



The motion of channeling particles in the accompanying coordinate system can be considered as an one-dimensional oscillator (in the case of planar channeling), or as a two-dimensional atom (in the case of axial channeling [1]). The transversal motion of the channeling particle is characterized by discrete spectrum. The occupation probability of transversal motion levels depends on the entrance angle of charged particle relative to the crystallographic axis [2-4]. Passing through the crystal charged channeling particle undergoes periodic action of the lattice atoms [5-7] with basic frequency $\omega = \gamma v/d$, where d is the lattice constant, v and γ are the velocity and the Lorentz-factor of the channeling particles [6]. If the frequency of an external periodic interference coincides with the frequency of the transition of the moving channeling particles from one quantized state of transversal motion in another, the resonant excitation of the channeling particles is possible, i.e. similar to the excitation of atomic electrons by the periodic field of monochromatic electromagnetic waves [8-9]. In the report the resonance conditions are discussed and the induced resonance radiation spectrum of channeled particles is analyzed.

INTRODUCTION

The channeling particles from a quantum point of view are characterized by discrete spectrum of transversal motion [2-5].

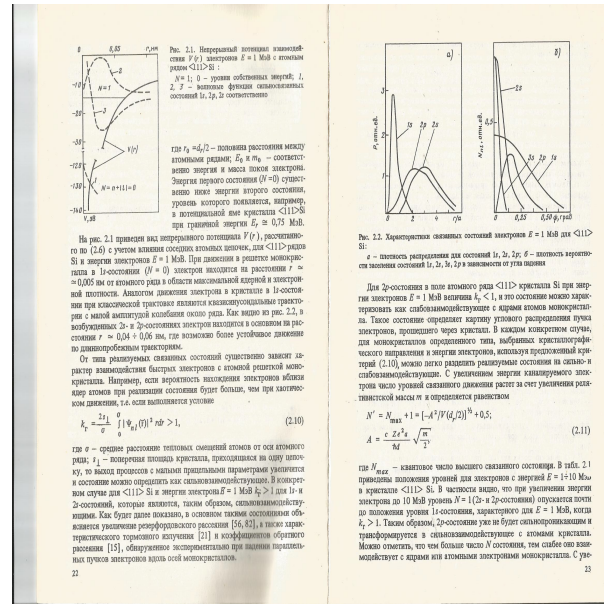


Fig. 1.1

The continuous average potential $U(r)$ of the interaction of 1 MeV energy electrons with the atomic row $\langle 111 \rangle$ of single crystal Si: $N = 1; 0$ are levels its transversal energies; 1, 2, and 3 are wave functions of strongly bound states of 1s, 2s, 2p, respectively [3-5].

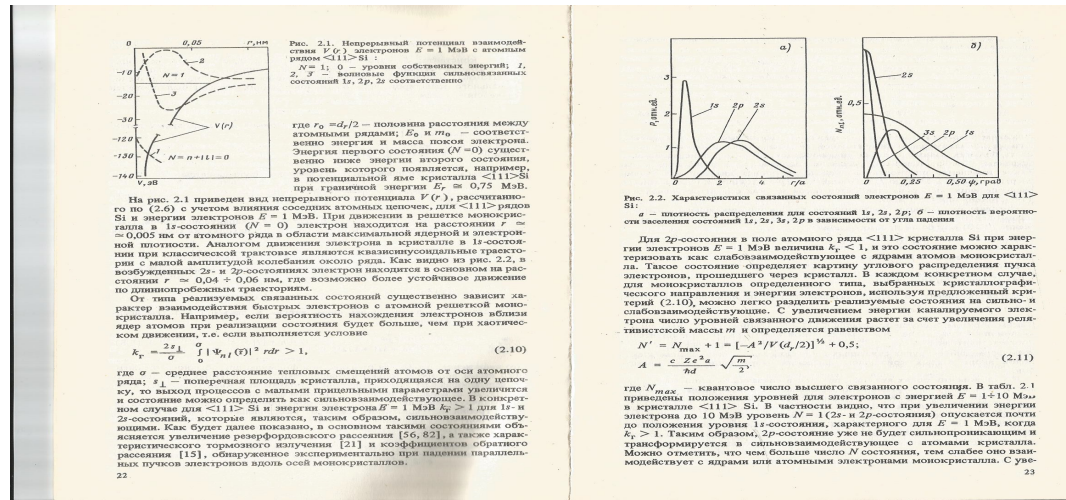


Fig. 1.2

The characteristics of the bound states of electrons with 1 MeV energy which are interacting with an atomic row $\langle 111 \rangle$ of the single crystal Si :
 a) – probability density of the population of states 1s, 2p, 2s ;
 b) - probability density of the population of states 1s, 2s, 3s, 2p depending on the entrance angle respectively the axis $\langle 111 \rangle$.



The interaction potential of charged particles with crystalline lattice atoms



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The interaction potential of charged particles with crystalline lattice atoms

- The periodic lattice potential is written in the form:

$$V(\mathbf{R}) = \sum_{\mathbf{g}} V_{\mathbf{g}} \exp(-i \mathbf{g} \cdot \mathbf{R}), \quad (2.1)$$

where \mathbf{g} is the reciprocal lattice vector; \mathbf{R} is the radius-vector of the channeling particle.

We use an expression $\mathbf{R} = \mathbf{v} t + \mathbf{r}$, where \mathbf{r} is the radius-vector of the channeling particle (in the accompanying coordinate system moving with longitudinal velocity of the

channeling particle \mathbf{v}). By using the expressions for a transverse and longitudinal components

of the reciprocal lattice vector as \mathbf{g}_{\perp} and \mathbf{g}_{\parallel} ($\mathbf{g}_{\parallel} = l/a$, $l=0,1,2,\dots$), we get



The interaction potential of charged particles with crystalline lattice atoms



If we assume that the interaction of charged particle with an isolated lattice atom describes by the screened Coulomb potential

$$U(r) = (Z_1 Z_2 e^2 / r) \exp(-r/R_0), \quad (2.3)$$

where Z_1 is the charge of the lattice atom nucleus, Z_2 is charge of the incident particles, $R_0^{-1} = me^2 (Z_1^2/3 + Z_2^2/3) / \hbar^2$ is the reciprocal screening radius, then the average potential of isolated row will look [2]



The interaction potential of charged particles with crystalline lattice atoms



The eigenvalues of energy for the potential (2.4) is expressed by the expression

$$E_{\perp} = -\left(\frac{Z_1 Z_2 e^2}{R} \right)^2 \frac{R^2}{2} \frac{E}{2a^2} (n + |m| + 1/2)^2. \quad (2.10)$$

The number of the electron bound motion levels is determined by the relation

$$N' = N_{\max} + 1 = \left[\frac{Z_1^2 e^4 R^2}{2E} \frac{U(d/2)}{d^2} \right]^{1/2} + 1/2, \quad (2.11)$$

where N_{\max} is the upper bound state quantum number.

It would be interesting to consider the case when there are only two energy levels of the transversal motion. For simplicity, we assume that the nonrelativistic electron ($\gamma \approx 1$) moves in the regime of the axial channeling. The channeling electrons which interact with the atomic row of the single crystal Si are characterized by the quantum states 1s, 2s.



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The V.V. Okorokov effect. Estimation of the resonance velocity of the channeling particles



The external electromagnetic field of the crystalline lattice (2.2) (with $l \neq 0$) is the cause induced transitions between quantized levels of transverse motion of channeling particles.

The first harmonic of the potential (2.2) corresponds to the frequency

$$\omega = v/d \quad \omega_{\downarrow n} = 2\pi/d \cdot vn / \sqrt{1 - v^2/c^2} \\ = 2\pi v/d \gamma_n, \quad (3.1)$$

If an external periodic action frequency coincides with the transition frequency of the channeling particles from one quantized state of transversal motion in another, then the resonant excitation of channeling particles is possible. This effect is analogous with the excitation of the atomic electrons by the periodic field of monochromatic electromagnetic waves [2,7,8].

For example, if the channeling particle is moving in a potential type (2.4) the transition between adjacent levels of the transversal motion will be characterized by the difference of energies:

$$\Delta\varepsilon = \hbar \omega (2n+1) \pi \omega (Z_1 Z_2 e^2) \omega$$



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The scattering of a photon with a frequency ω_1 , a wave vector k_1 and a polarization e_1 by a channeling particle is considered. A channeling particle is described by the initial wave function $\psi_i(x) = \psi_i(r) e^{-i\varepsilon_i t}$. As a result of the interaction the photon with a frequency ω_2 , a wave vector k_2 , a polarization e_2 is created and the electron turns into the state with the wave function $\psi_f(x) = \psi_f(r) \cdot e^{-i\varepsilon_f t}$.

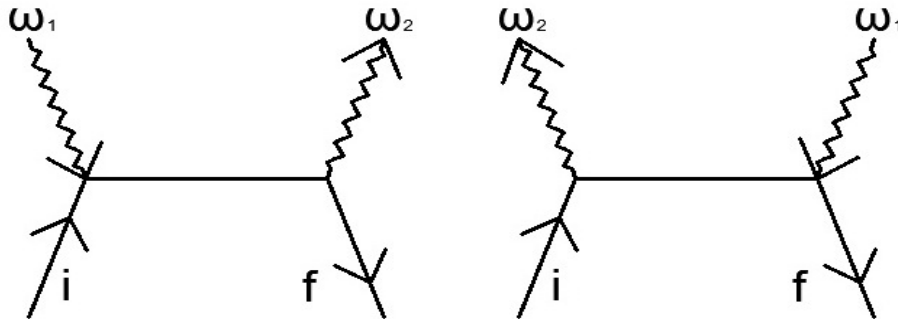


Fig. 4.1. Feynman diagrams describing the matrix element of the second order.

The photon states is characterized by the potentials

$$\begin{aligned}
 A_{\downarrow 1}(x) &= e_{\downarrow 1} / \sqrt{2\omega_{\downarrow 1}} e^{i(k_{\downarrow 1} r - \omega_{\downarrow 1} t)}, \\
 A_{\downarrow 2}(x) &= e_{\downarrow 2} / \sqrt{2\omega_{\downarrow 2}} e^{i(k_{\downarrow 2} r - \omega_{\downarrow 2} t)}.
 \end{aligned}
 \tag{4.1}$$

The expression for the matrix element of the transition can be written as follows:

$$\langle f | \mathcal{S}^{\uparrow}(2) | i \rangle = 2\pi i U \delta(\epsilon_{\downarrow i} + \omega_{\downarrow 1} - \epsilon_{\downarrow f} - \omega_{\downarrow 2}),$$

where $U = -2\pi\alpha / \sqrt{\omega_{\downarrow 1} \omega_{\downarrow 2}} \sum_{s \uparrow} \{ 1/\epsilon_{\downarrow s} - \epsilon_{\downarrow i} -$

$$\omega_{\downarrow 1} / \epsilon_{\downarrow i} \hat{L}_{\downarrow 2} e^{i(k_{\downarrow 2} r - \omega_{\downarrow 2} t)} - \omega_{\downarrow 2} / \epsilon_{\downarrow i} \hat{L}_{\downarrow 1} e^{i(k_{\downarrow 1} r - \omega_{\downarrow 1} t)}$$

Considering that the photon wavelength is large compared with the channel size, the photon differential cross-section scattering can be associated with the transition matrix element U by the relation:

$$d\sigma = 2\pi |U|^2 \delta(\epsilon_{\downarrow i} + \omega_{\downarrow 1} - \epsilon_{\downarrow f} - \omega_{\downarrow 2}) d\Omega_3 k_{\downarrow 2} / (2\pi)^{\uparrow 3}, \quad (4.4)$$

where

$$U = -2\pi\alpha\sqrt{\omega_{\downarrow 1}\omega_{\downarrow 2}} \sum_{s\uparrow} \{ \langle f | \mathbf{re}_{\downarrow 2} | s \rangle \langle s | \mathbf{re}_{\downarrow 1} | i \rangle + 1/\epsilon_{\downarrow i} - \epsilon_{\downarrow s} + \omega_{\downarrow 1} \langle f | \mathbf{re}_{\downarrow 1} | s \rangle \langle s | \mathbf{re}_{\downarrow 2} | i \rangle \} \quad (4.5)$$

$$\begin{aligned}
 u = & -2\pi\alpha\sqrt{\omega_1 \omega_2} \sum_s \langle f | \mathbf{r} \mathbf{e}_2 | s \rangle \langle s | \mathbf{r} \mathbf{e}_1 | i \rangle \\
 & / (\varepsilon_s - \varepsilon_i - \omega_1 - i/2 \Gamma_s), \quad (4.8)
 \end{aligned}$$

where the summation extends to the all states with energy ε_s .

Therefore, the differential scattering cross-section can be written as:

$$\begin{aligned}
 d\sigma = & \omega_1 \omega_2 \frac{d\Omega_2}{4\pi} \left| \sum_s \langle f | \mathbf{Q} \mathbf{e}_2 | s \rangle \langle s | \mathbf{Q} \mathbf{e}_1 | i \rangle \right|^2 \\
 & / (\varepsilon_s - \varepsilon_i - \omega_1)^2 + \Gamma_s^2 / 4 \quad (4.9)
 \end{aligned}$$

The total cross-section can be obtained by integrating the expression (4.9) over the angles, by averaging over the polarizations of the incident photon and by summing over the polarization



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Conclusion



The results of this study confirmed that orientation structure of atoms in the crystal lattice has a significant influence on the processes of excitation of passing through the single crystal of charged particles. In particular, there is a process induced resonant evolution of the occupation probability of the transversal motion levels of the channeling particles (electrons), which is accompanied by stimulated radiation of the channeling particles. The resonance condition depends on the specific parameters of the approximation of continuous atomic chain potential. Perhaps this mechanism may explain the emission spectrum peaks at small frequency ($\omega \ll E$) by nonrelativistic electrons ($\beta \ll 1$) [Korobochko Yu.S., Kosmach V.F. and Minaev V.I. Zh.Eksp.Teor.Fiz. V.48,1965, p.1248]



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Thanks for your attention!

