



Tomsk Polytechnic University, 30 Lenin Ave., 634050 Tomsk, Russia

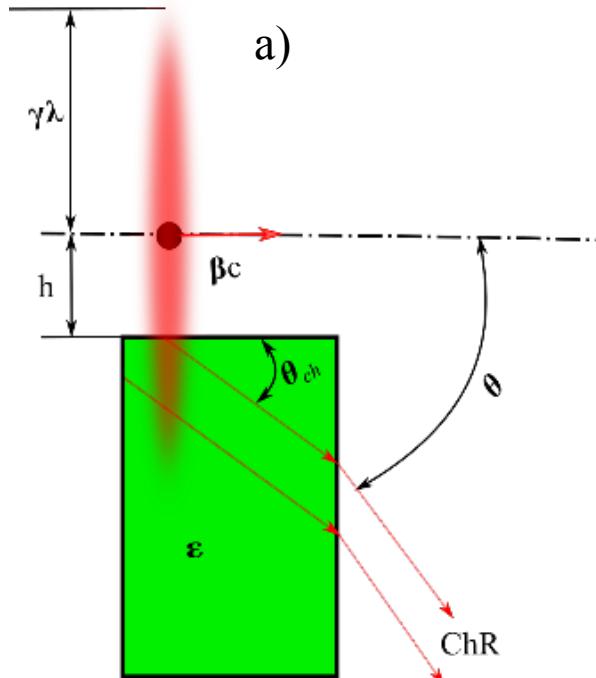
A.Potylitsyn, S. Gogolev



Angular distribution of coherent Cherenkov radiation from a
tilted bunch passing through a slit in target



The Cherenkov mechanism may be realized for charge passing in vacuum near a dielectric target. For the high Lorentz-factor γ and if the condition $h \leq \gamma\lambda$ is fulfilled (h is the impact parameter, λ is the ChR wavelength) ChR can be produced.



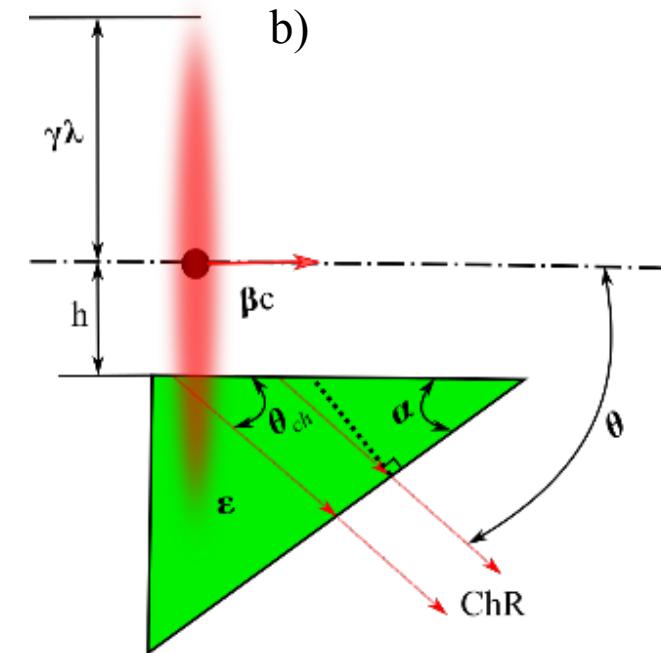
$$\cos\theta_{ch} = \frac{1}{\beta\sqrt{\epsilon}}$$

$$\theta_{ch} = \arccos\left(\frac{1}{\beta\sqrt{\epsilon}}\right)$$

$$\text{Fresnel's law } \sin\theta = \sqrt{\epsilon} \sin\theta_{ch}$$

$$\theta = \arcsin\left(\sqrt{\epsilon} \sin(\theta_{ch})\right)$$

If $\epsilon < 2 - \gamma^{-2}$ then the geometry a) can be realized



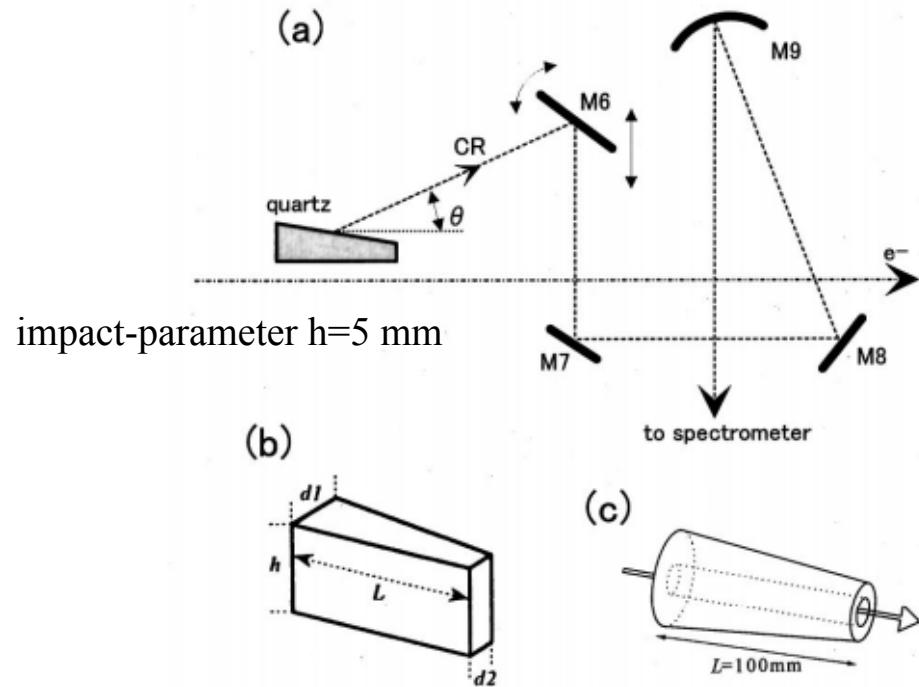
$$\theta = 90^\circ - \alpha - \arcsin\left(\sqrt{\epsilon} \cos(\alpha + \theta_{ch})\right)$$





Experiments

T. Takahashi Physical Review E., v.62 (2000) 8606



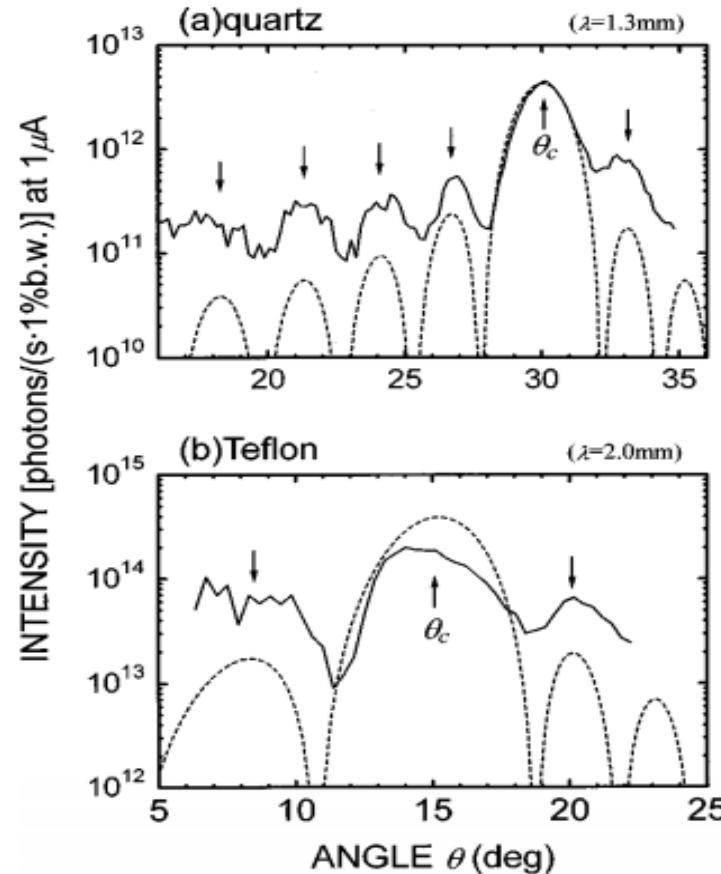
The schematic view in the vacuum chamber. (a) The sectional diagram of the optical components, (b) the block of quartz, and (c) the cone of Teflon with the cylindrical hole of 7 mm. (M6, M7, M8) plane mirrors; (M9) a spherical mirror; and (e^-) electron beam. The values of dimensions in (b) are listed in Table.

The experimental conditions of the electron beam.

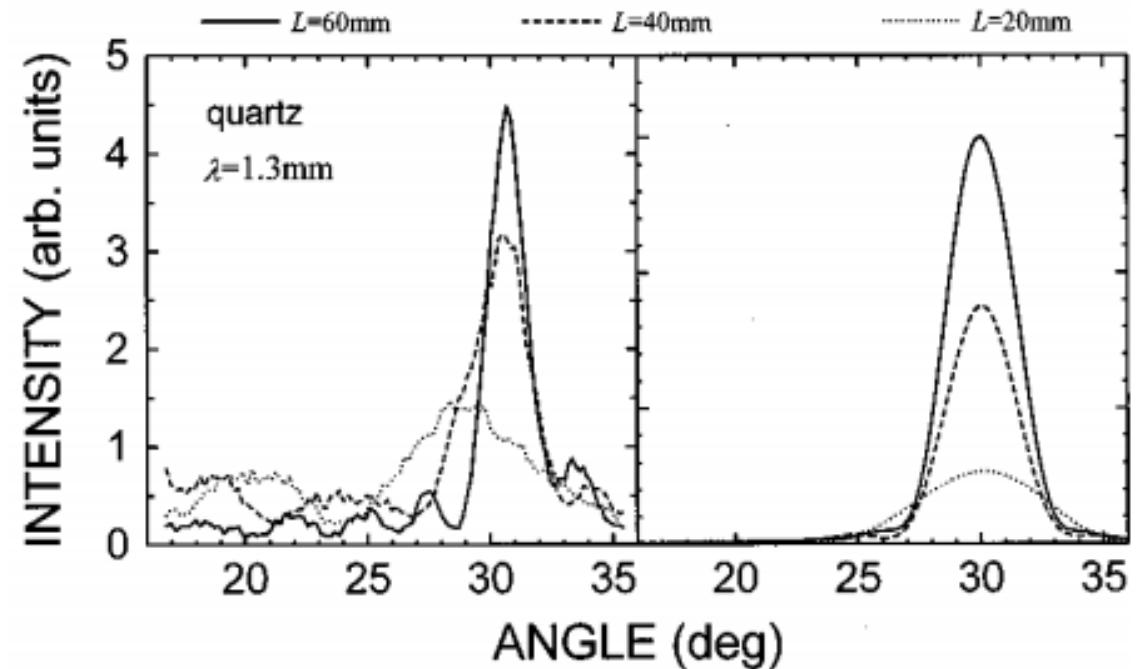
| | |
|--|-------------------|
| Electron energy (MeV) | 150 |
| Energy spread (%) | 0.5 |
| Accelerating rf (GHz) | 2.856 |
| Duration of a burst (μ s) | 2 |
| Repetition rate (pulses/s) | 150 |
| Average beam current (μ A) | 1 |
| Number of electrons per bunch | 7.2×10^6 |
| Transverse size $2\rho_0$ (mm) | 7.0 |
| Angular divergence 2Ψ (mrad) | 4.6 |
| Longitudinal bunch length $2\sigma_0$ (mm) | 0.21 |

Dimensions of the quartz.

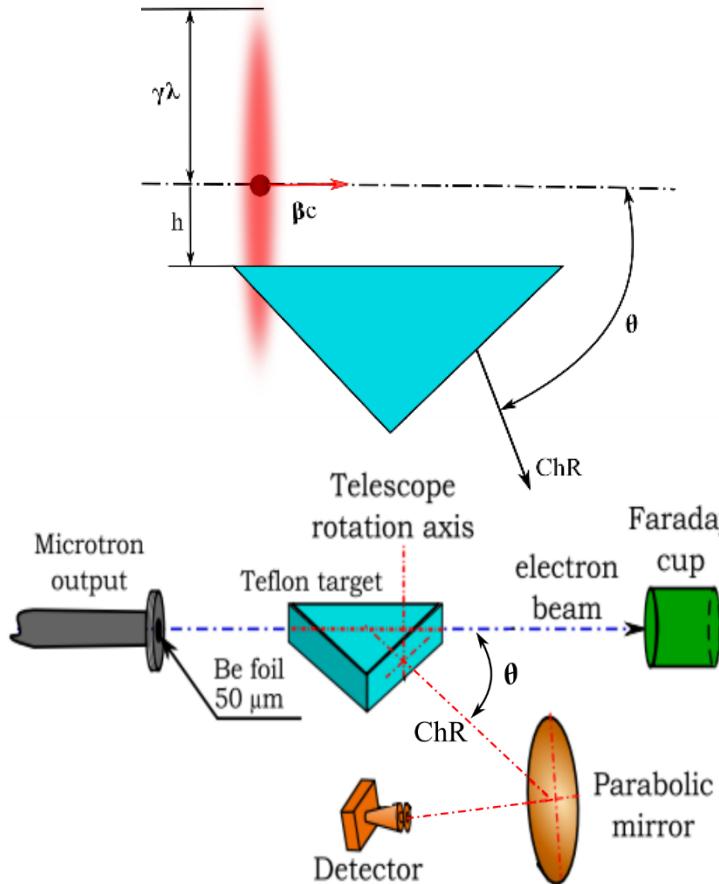
| L (mm) | d_1 (mm) | d_2 (mm) | h (mm) |
|----------|------------|------------|----------|
| 60 | 15 | 9 | 40 |
| 40 | 13.2 | 9.2 | 40 |
| 20 | 9.4 | 7.4 | 40 |



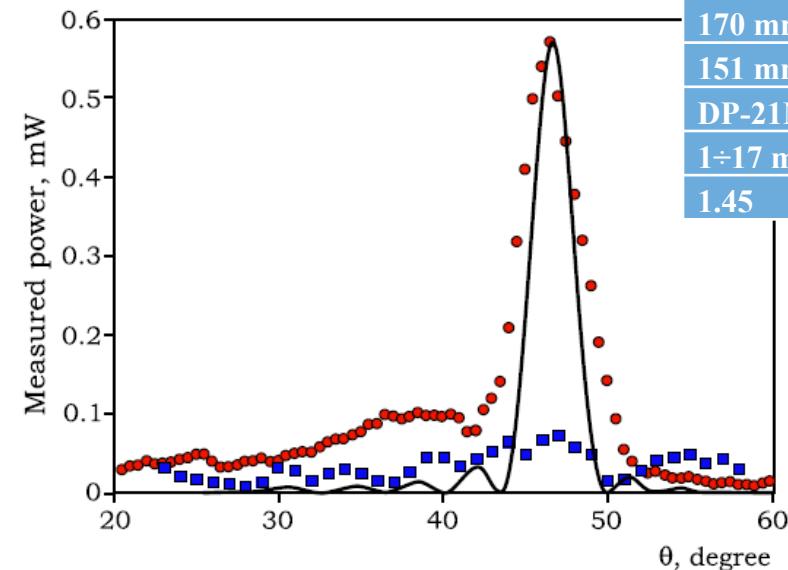
The angular distribution of radiation from the quartz of 60 mm long at $\lambda=1.3$ mm and the Teflon of 100 mm long at $\lambda=2.0$ mm. The data are plotted on a logarithmic scale in order to visualize satellite peaks in the angular distribution



The dependence of the angular distribution on the length of quartz. The solid, broken, and dotted curves represent the data for quartz of 60, 40, and 20 mm long, respectively. The curves at the right-hand side show the theoretical calculation.



Experimental scheme and some definitions.

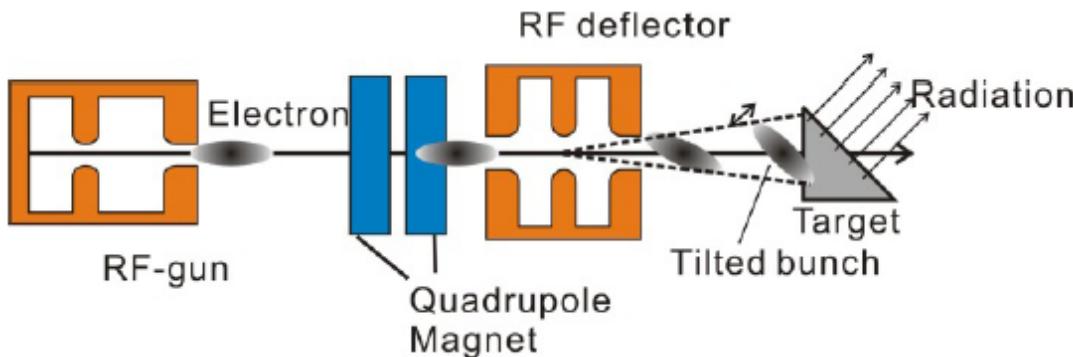


| 6.1 Mev ($\gamma \approx 12$) | electron energy |
|---------------------------------|--------------------------------------|
| 30 mA | average beam current |
| 10^8 | maximal bunch population |
| 10526 | Bunches in a train |
| 1.1 mm | longitudinal size (rms) of electrons |
| $4 \times 4 \text{ mm}^2$ | transverse sizes of electron beam |
| 4 μs | train duration |
| 25 mm | impact-parameter |
| 170 mm | Diameter of paraboloidal mirror |
| 151 mm | focal distance |
| DP-21M | detector |
| 1÷17 mm | viewed a range of wavelengths |
| 1.45 | Teflon refractive index |

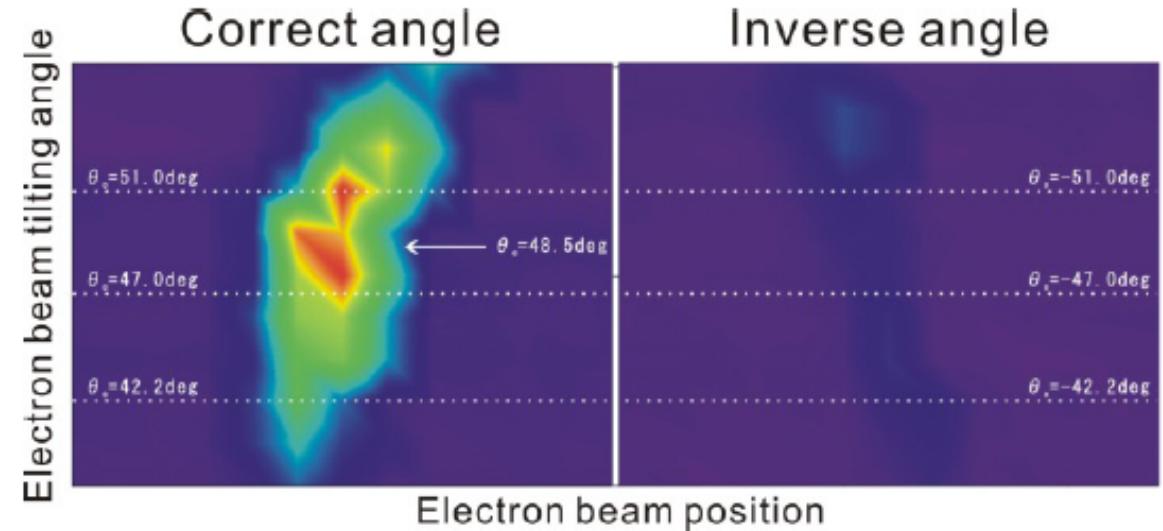
The angular dependence of CChR:
 the red circles – horizontal polarization component,
 the blue squares – vertical polarization component,
 the solid line – theoretical simulation.



| 5 Mev ($\gamma \approx 10$) | electron energy |
|-------------------------------|-----------------------------|
| quasi-optical | detector |
| 0.1÷2 THz | viewed a range of frequency |
| 1.52 | TOPAS refractive index |
| 48.5 deg | Cherenkov angle |
| width 1 mm | Prizm target size |
| thickness 1 mm | |



Experimental setup for coherent Cherenkov radiation by using tilted electron bunch.



THz intensity with 1 THz as a function of the electron bunch position and electron bunch tilting angle

Incoherent Cherenkov radiation



$$\frac{d^2W_{ChR}}{d\omega d\Omega} = cr'^2 \left| \overset{\text{III}}{E}_{vac}(r, \omega) \right|^2 \quad (1)$$

$$\overset{\text{III}}{E}_{vac}(r, \omega) = \frac{1}{|\epsilon|^2} \left(\left| \sqrt{\epsilon} F_E \right|^2 \left| H_P^R(r, \omega) \right|^2 + \left| F_H \right|^2 \left| H_\perp^R(r, \omega) \right|^2 \right) \quad (2)$$

$$H_P^R(r, \omega) = \sqrt{H_z^R(r, \omega)^2 + \left(H_x^R(r, \omega) \sin(\phi) + H_y^R(r, \omega) \cos(\phi) \right)^2} \quad (3)$$

$$H_\perp^R(r, \omega) = H_x^R(r, \omega) \cos(\phi) - H_y^R(r, \omega) \sin(\phi) \quad (4)$$

$$F_H = \frac{2\epsilon \cos \theta}{\epsilon \cos \theta + \sqrt{\epsilon - \sin^2 \theta}} \quad (5)$$

$$F_E = \frac{2 \cos \theta}{\cos \theta + \sqrt{\epsilon - \sin^2 \theta}} \quad (6)$$





$$H_R(k_x, y, z, \omega) = \frac{(\varepsilon - 1)\omega}{2c} \frac{e^{i\frac{\omega}{c}r'\sqrt{\varepsilon}}}{r'} k \times \int_0^L \left(\int_{a/2}^{a/2+H} E_e(k_x, y, z, \omega) e^{-i(k_y y + k_z z)} dy + \int_{-a/2-H}^{-a/2} E^{*e}(k_x, y, z, \omega) e^{-i(k_y y + k_z z)} dy \right) dz \quad (7)$$

$$E^{*e}(k_x, y, z, \omega) = -\frac{ie}{2\pi\beta c \sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \left\{ \sqrt{\varepsilon}\beta\gamma n_x, -i\sqrt{1+\varepsilon(\beta\gamma n_x)^2}, \gamma^{-1} \right\} e^{i\frac{\omega}{\beta c}z} e^{y\frac{\omega}{\beta c\gamma}\sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \quad (8)$$

$$E_e(k_x, y, z, \omega) = -\frac{ie}{2\pi\beta c \sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \left\{ \sqrt{\varepsilon}\beta\gamma n_x, i\sqrt{1+\varepsilon(\beta\gamma n_x)^2}, \gamma^{-1} \right\} e^{i\frac{\omega}{\beta c}z} e^{-y\frac{\omega}{\beta c\gamma}\sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \quad (9)$$

$$\frac{r}{k} = n\sqrt{\varepsilon} \frac{\omega}{c}, \quad r = \frac{1}{\sqrt{\varepsilon}} \left\{ \sin(\theta)\sin(\phi), \sin(\theta)\cos(\phi), \sqrt{\varepsilon - \sin^2(\theta)} \right\} \quad (10)$$



$$\frac{d^2W_{ChR}}{d\omega d\Omega} = \frac{e^2 \beta^2 \cos^2(\theta)}{4\pi^2 c (1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2)} \left| \frac{\left(\varepsilon - 1 \right) \left(e^{\frac{i2\pi L(1-\beta\sqrt{\varepsilon-\sin^2(\theta)})}{\lambda\beta}} - 1 \right)^2}{\varepsilon (1 - \beta \sqrt{\varepsilon - \sin^2(\theta)})} \right|^2$$

Spectral and angular distribution of the radiation, calculated using the method of polarization currents [D. V. Karlovets, A. P. Potylitsyn, Phys. Lett. A 373, 1988 (2009).]

$$\left[\left| \frac{\varepsilon}{\varepsilon \cos(\theta) + \sqrt{\varepsilon - \sin^2(\theta)}} \right|^2 \left| \left(\Phi_2 i \gamma \cos(\phi) \sqrt{\varepsilon - \sin^2(\theta)} \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2} + \Phi_1 (\sin(\theta) \cos^2(\phi) + \sin(\theta) \sin^2(\phi)) \gamma^2 (1 - \beta^2 - \beta \sqrt{\varepsilon - \sin^2(\theta)}) \right) \right|^2 + \left| \frac{\sqrt{\varepsilon}}{\varepsilon \cos(\theta) + \sqrt{\varepsilon - \sin^2(\theta)}} \right|^2 \right. \\ \left. \left| \sin^2(\theta) \sin^2(\phi) \gamma^2 \left(\Phi_1 \gamma \sin(\theta) \cos(\phi) \beta + i \Phi_2 \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2} \right)^2 + \left(\sin(\phi) \left(\Phi_1 \sin(\theta) \cos(\phi) + i \Phi_2 \gamma \sqrt{\varepsilon - \sin^2(\theta)} \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2} \right) - \right. \right. \right. \\ \left. \left. \left. - \Phi_1 \cos(\phi) \sin(\theta) \sin(\phi) \gamma^2 (1 - \beta^2 - \beta \sqrt{\varepsilon - \sin^2(\theta)}) \right)^2 \right| \quad (11) \right]$$

$$\Phi_{1,2} = \frac{\exp \left(-\frac{a\pi(-i\gamma \sin(\theta) \cos(\phi) \beta + \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2})}{\gamma \lambda \beta} \right) \left(1 - \exp \left(-\frac{2\pi H(-i\gamma \sin(\theta) \cos(\phi) \beta + \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2})}{\gamma \lambda \beta} \right) \right)_+}{(-i\gamma \sqrt{U_{ny}} \beta + \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2})} \\ \pm \frac{\exp \left(-\frac{a\pi(i\gamma \sin(\theta) \cos(\phi) \beta + \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2})}{\gamma \lambda \beta} \right) \left(1 - \exp \left(-\frac{2\pi H(i\gamma \sin(\theta) \cos(\phi) \beta + \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2})}{\gamma \lambda \beta} \right) \right)_+}{(i\gamma \sin(\theta) \cos(\phi) \beta + \sqrt{1 + \gamma^2 \sin^2(\theta) \sin^2(\phi) \beta^2})} \quad (12)$$



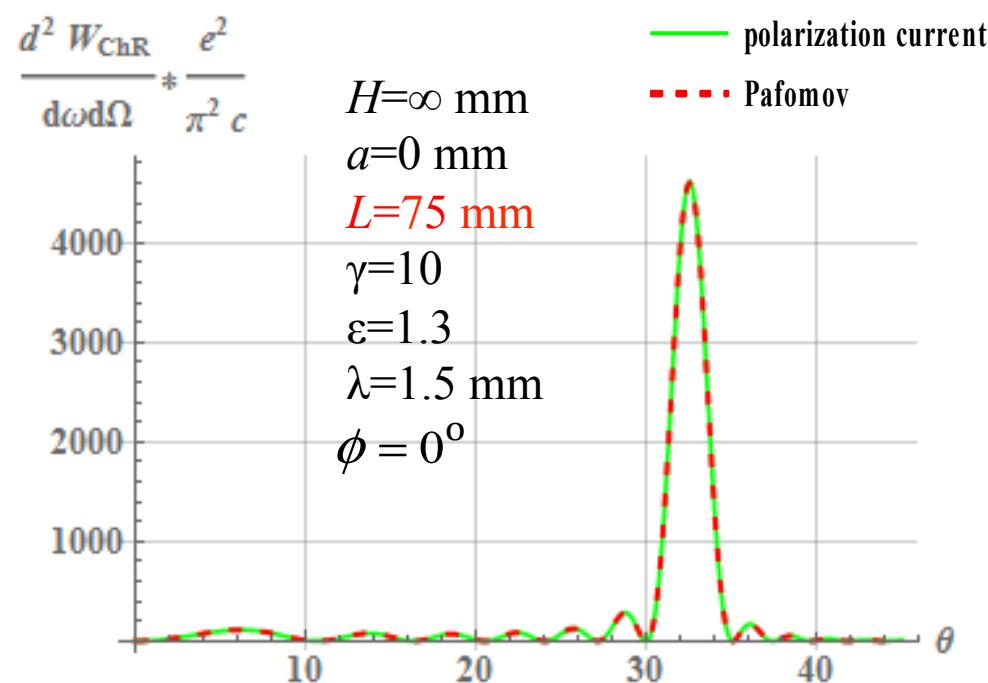
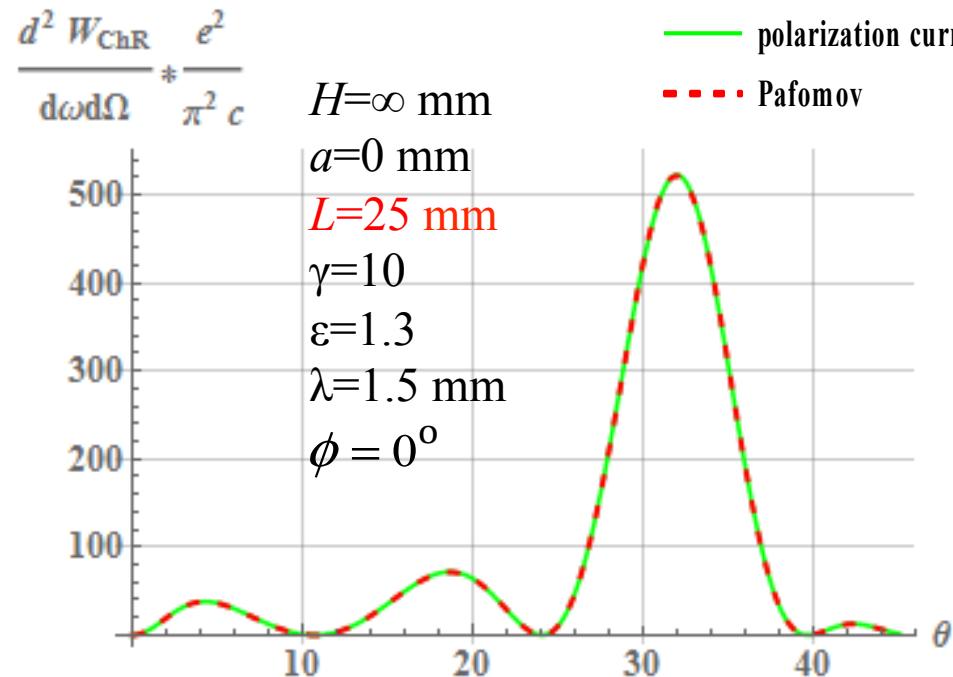
Spectral and angular distribution of the radiation, calculated using method of images

$$\begin{aligned}
& \frac{d^2 W_{ChR}}{d\omega d\Omega} = \frac{e^2 \beta^2 \sin^2(\theta) \cos^2(\theta) |(\varepsilon - 1)|^2}{\pi^2 c \left| (1 - \beta^2 \cos^2(\theta)) (1 - \beta^2 (\varepsilon - \sin^2(\theta))) \right|^2} \\
& \left(e^{-\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} - \varepsilon \cos(\theta) \right) \left(1 - \beta \sqrt{\varepsilon - \sin^2(\theta)} \right) \left(1 - \beta^2 + \beta \sqrt{\varepsilon - \sin^2(\theta)} \right) \right. \\
& \left. e^{\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} + \varepsilon \cos(\theta) \right) \left(1 + \beta \sqrt{\varepsilon - \sin^2(\theta)} \right) \left(1 - \beta^2 - \beta \sqrt{\varepsilon - \sin^2(\theta)} \right) \right. \\
& \left. - \left(e^{\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} + \varepsilon \cos(\theta) \right) - e^{-\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} - \varepsilon \cos(\theta) \right) \right) \right. \\
& \left. + \left(e^{\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} + \varepsilon \cos(\theta) \right) - e^{-\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} - \varepsilon \cos(\theta) \right) \right) \right. \\
& \left. - \frac{2e^{\frac{i2\pi L}{\lambda \beta}} \sqrt{\varepsilon - \sin^2(\theta)} (1 + \beta \cos(\theta)) (1 - \beta \cos(\theta) - \beta^2 \varepsilon)}{\left(e^{\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} + \varepsilon \cos(\theta) \right) - e^{-\frac{i2\pi L \sqrt{\varepsilon - \sin^2(\theta)}}{\lambda}} \left(\sqrt{\varepsilon - \sin^2(\theta)} - \varepsilon \cos(\theta) \right) \right)^2} \right) \quad (13)
\end{aligned}$$





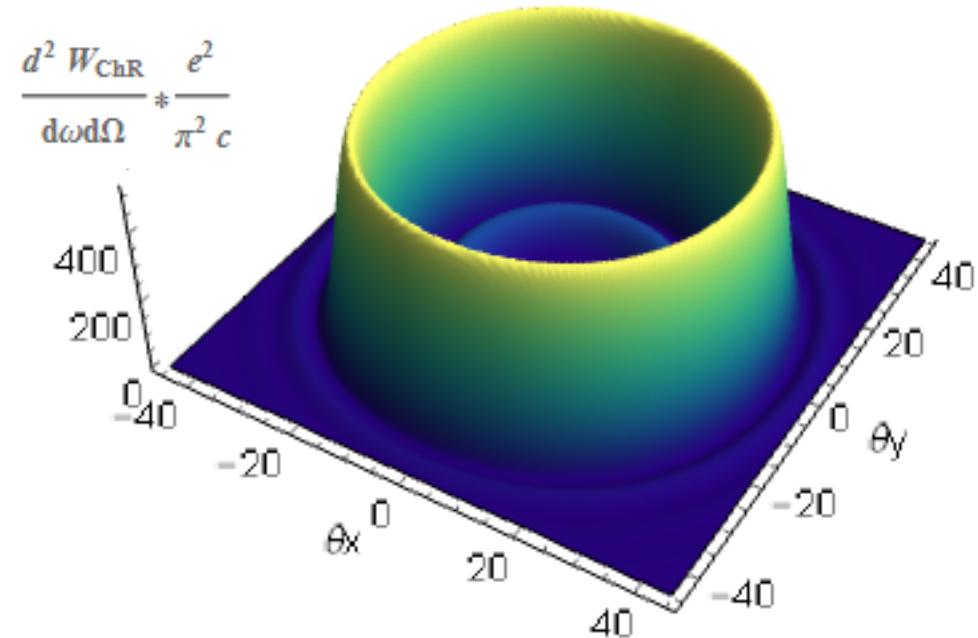
Comparison of both models



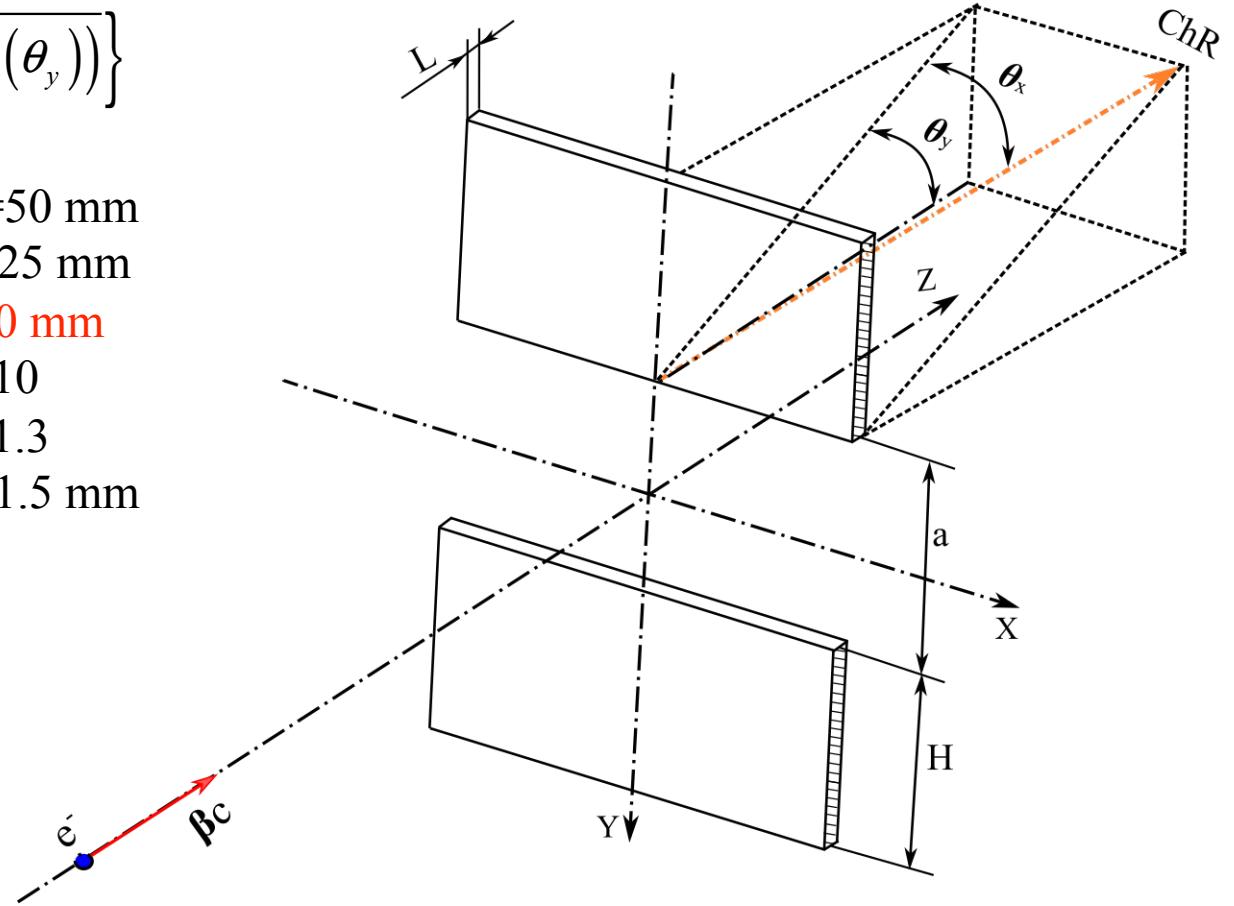
Angular distribution of incoherent ChR

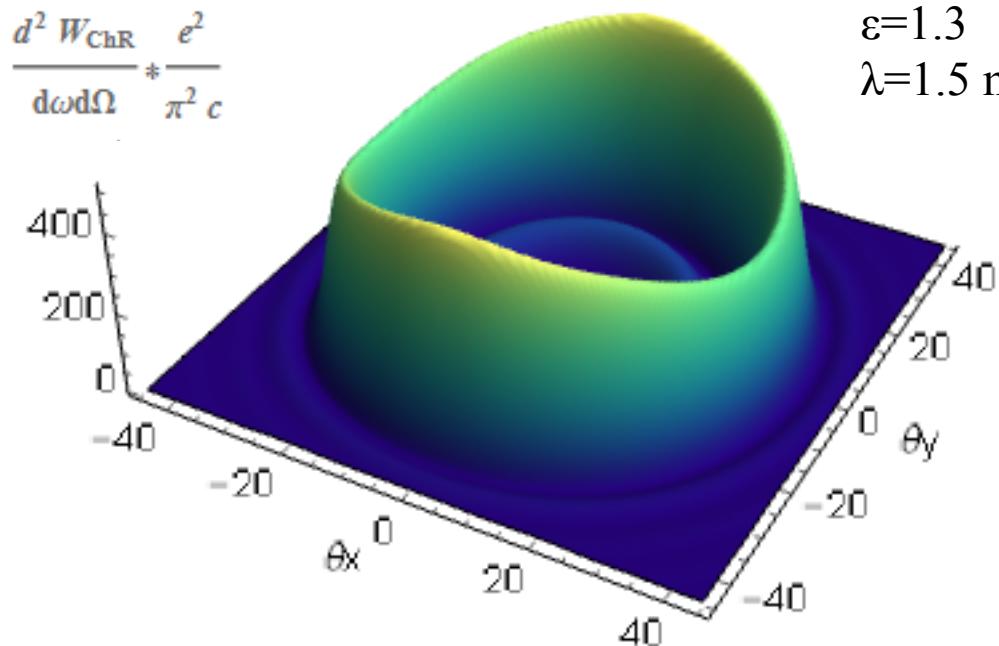


$$\mathbf{r} = \frac{2\pi}{\lambda} \left\{ \sin(\theta_x), \cos(\theta_x) \sin(\theta_y), \sqrt{\varepsilon - (1 - \cos^2(\theta_x) \cos^2(\theta_y))} \right\}$$

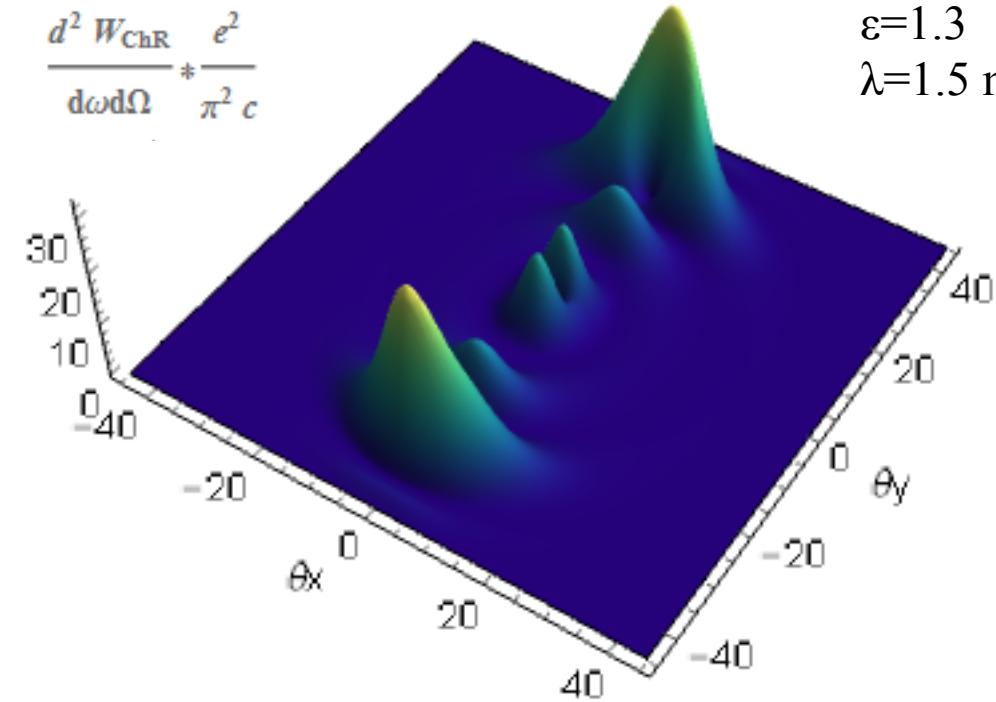


$H=50$ mm
 $L=25$ mm
 $a=0$ mm
 $\gamma=10$
 $\varepsilon=1.3$
 $\lambda=1.5$ mm





$H=50$ mm
 $L=25$ mm
 $a=0.1$ mm
 $\gamma=10$
 $\varepsilon=1.3$
 $\lambda=1.5$ mm



$H=50$ mm
 $L=25$ mm
 $a=5$ mm
 $\gamma=10$
 $\varepsilon=1.3$
 $\lambda=1.5$ mm

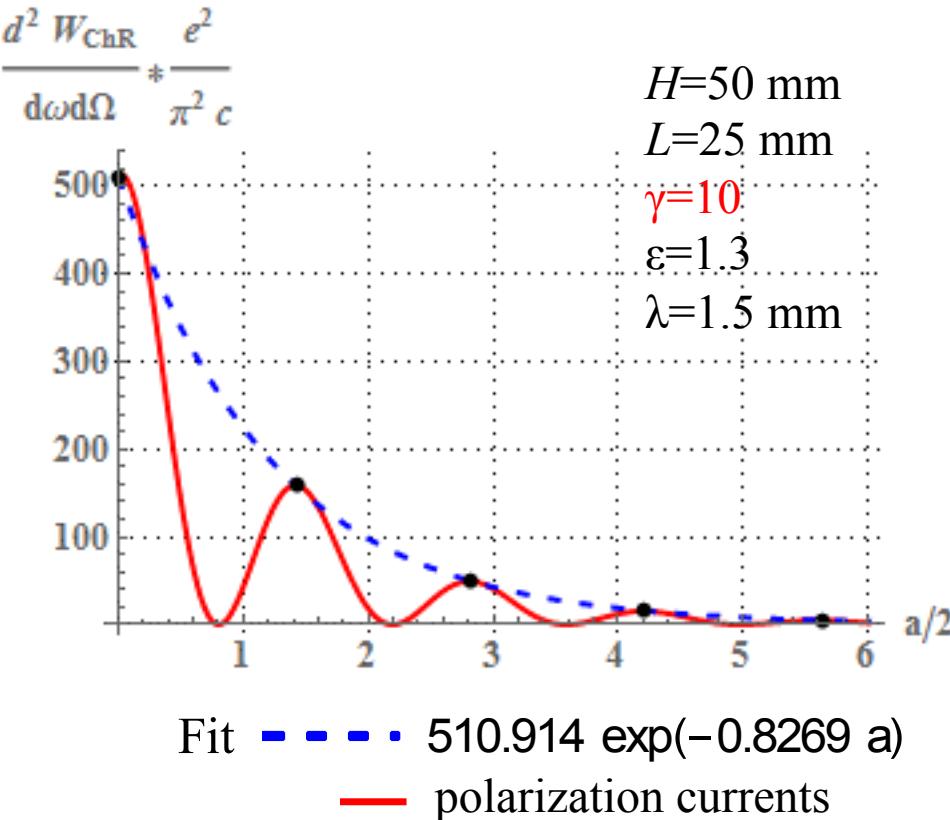




see, Norihiro Sei, Takeshi Sakai, et. al., Physics Letters A, 379 (2015) 2399–2404

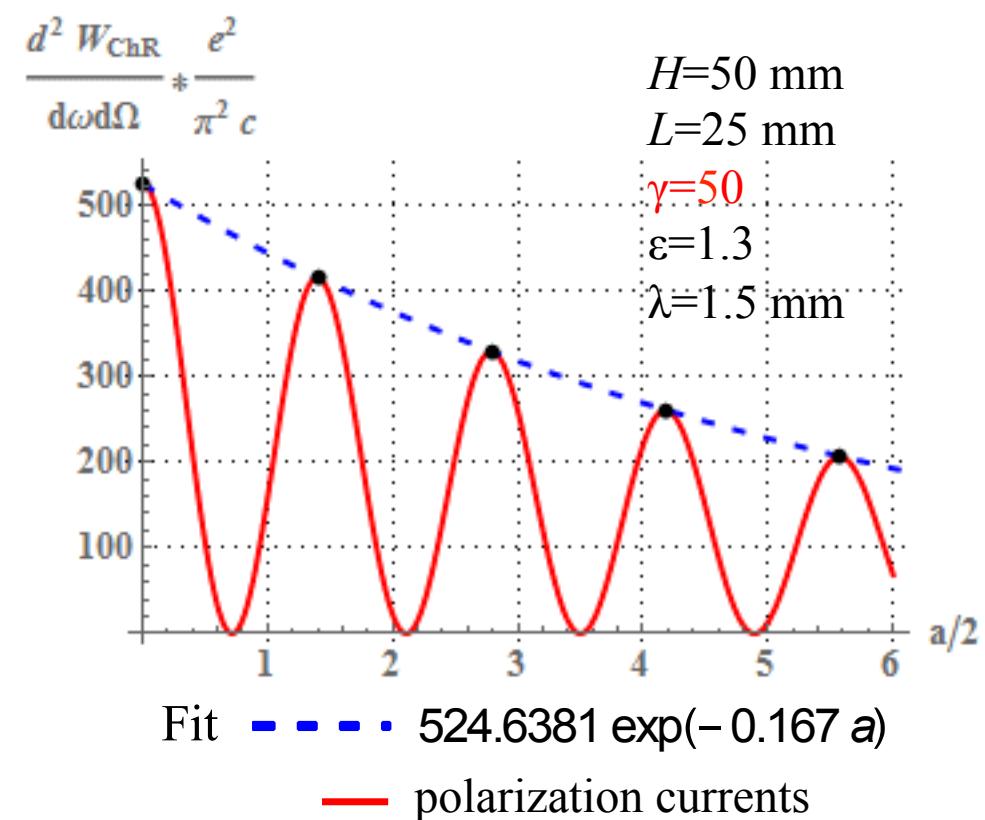
can be approximated as

$$\exp\left(\frac{-4\pi a}{\gamma\lambda\beta}\right) = \exp\left(-\frac{4\pi a}{10 \cdot 1.5 \cdot 0.9949}\right) = \boxed{\exp(-0.8419 a)}$$



$$\exp\left(\frac{-4\pi a}{\gamma\lambda\beta}\right)$$

$$\exp\left(\frac{-4\pi a}{\gamma\lambda\beta}\right) = \exp\left(-\frac{4\pi a}{50 \cdot 1.5 \cdot 0.9998}\right) = \boxed{\exp(-0.1675 a)}$$





Coherent Cherenkov radiation

The spectral-angular density of Coherent Cherenkov radiation (CChR) from a bunch with population N :

$$\frac{d^2W_{CChR}}{d\omega d\Omega} = N \left(1 + (N-1)F(k) \right) \frac{d^2W_{ChR}}{d\omega d\Omega}, \text{ Form factor } F(k) = \left| \int_{-\infty}^{\infty} \rho(r) \exp \left(-i \left[\frac{\omega}{c} \sin(\theta) \sin(\phi)x + \frac{\omega}{c} \sin(\theta) \cos(\phi)y + \frac{\omega}{c\beta} z \right] \right) dr \right|^2,$$

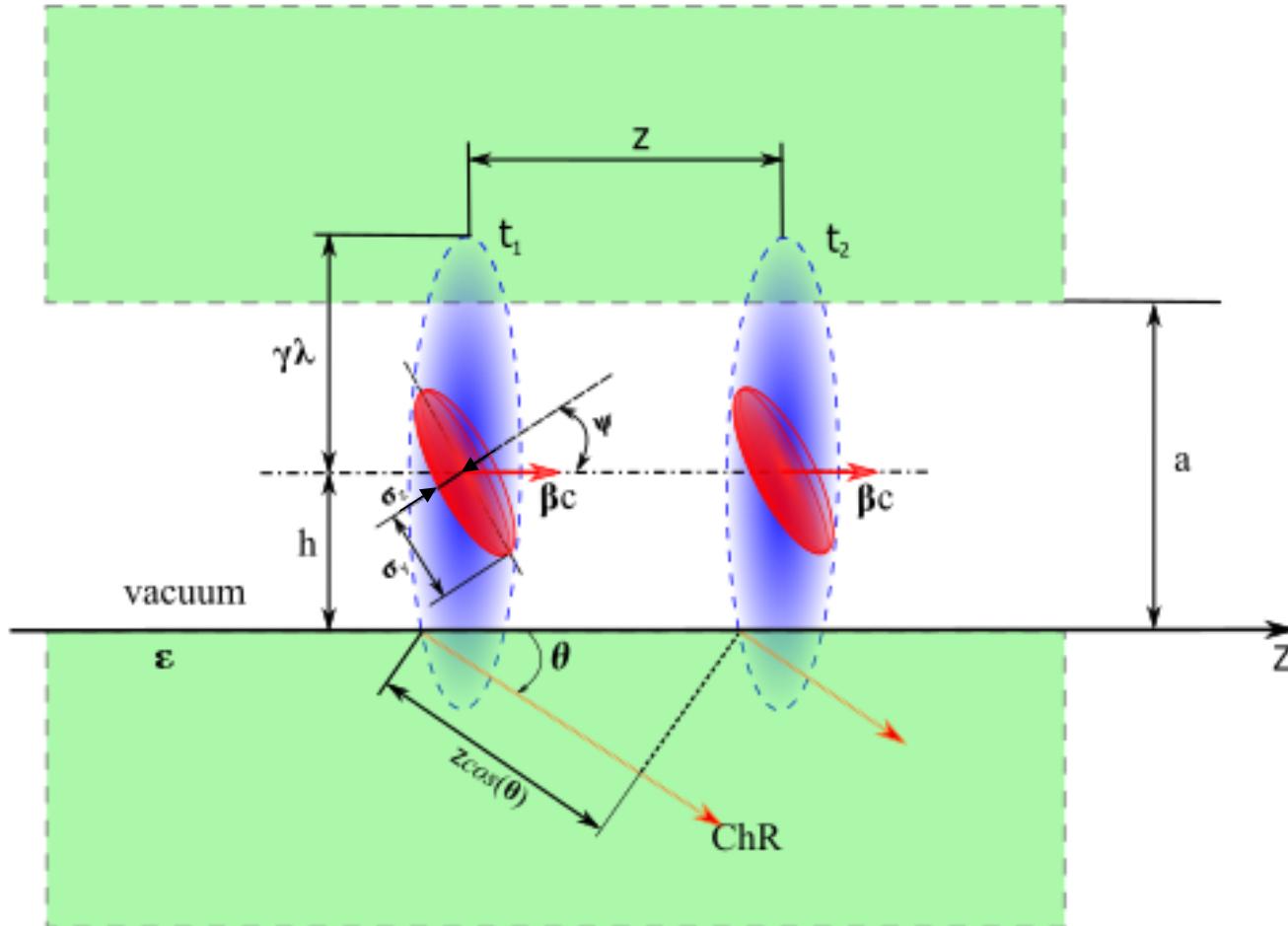
$\frac{d^2W_{ChR}}{d\omega d\Omega}$ is the spectral-angular density for a single charge.

$\rho(\vec{r})$ is a charge distribution of electrons in the bunch is a Gaussian,

$$\rho(r) = \rho(x, y, z) = \frac{1}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp \left(-\frac{1}{2} \left[\left(\frac{x}{\sigma_x} \right)^2 + \left(\frac{y}{\sigma_y} \right)^2 + \left(\frac{z}{\sigma_z} \right)^2 \right] \right). \quad (*)$$



Form factor a tilted electron bunch with distribution of electrons in the bunch is a Gaussian:



$$V\phi = \frac{2\pi}{\lambda} \left(x \sin(\theta) \sin(\phi) + y \sin(\theta) \cos(\phi) + \frac{z}{\beta} \right)$$





Form factor for tilted electron bunch

$$F(\mathbf{k})^{\mathbf{r}} = \left| \int_{-\infty}^{\infty} \rho(r)^{\mathbf{r}} \exp\left(-i\left[\frac{\omega}{c} \sin(\theta) \sin(\phi)x + \frac{\omega}{c} \sin(\theta) \cos(\phi)y + \frac{\omega}{c\beta} z\right]\right) dr \right|^2,$$

$$\rho(r)^{\mathbf{r}} = \frac{1}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y \cos(\psi) - z \sin(\psi)}{\sigma_y}\right)^2 + \left(\frac{y \sin(\psi) + z \cos(\psi)}{\sigma_z}\right)^2 \right] \right). \quad (**)$$

$$F(\mathbf{k})^{\mathbf{r}} = \exp\left(-k_x^2 \sigma_x^2 + \frac{1}{2} \left[- (k_y^2 + k_z^2) (\sigma_y^2 + \sigma_z^2) - (k_y^2 - k_z^2) (\sigma_y^2 - \sigma_z^2) \cos(2\psi) + 2k_y k_z (\sigma_y^2 - \sigma_z^2) \sin(2\psi) \right] \right), \quad (14)$$

$$\{k_x, k_y, k_z\} = \frac{2\pi}{\lambda} \left\{ \sin(\theta) \sin(\phi), \sin(\theta) \cos(\phi), \frac{1}{\beta} \right\} \quad (15)$$

$$\{k_x, k_y, k_z\} = \frac{2\pi}{\lambda} \left\{ \sin(\theta_x), \cos(\theta_x) \sin(\theta_y), \frac{1}{\beta} \right\} \quad (16)$$





Angular distribution of coherent Cherenkov radiation from a tilted bunch passing through a target

$$H=50 \text{ mm}$$

$$L=25 \text{ mm}$$

$$a=0 \text{ mm}$$

$$\gamma=10$$

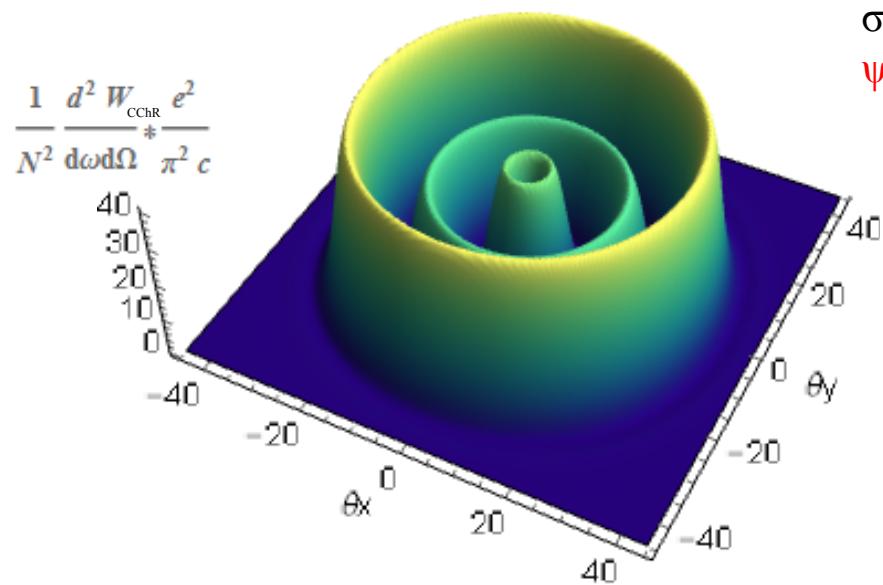
$$\epsilon=1.3$$

$$\lambda=1.5 \text{ mm}$$

$$\sigma_x=\sigma_y=707 \mu\text{m}$$

$$\sigma_z=100 \mu\text{m}$$

$$\psi=0 \text{ deg}$$



$$H=50 \text{ mm}$$

$$L=25 \text{ mm}$$

$$a=0 \text{ mm}$$

$$\gamma=10$$

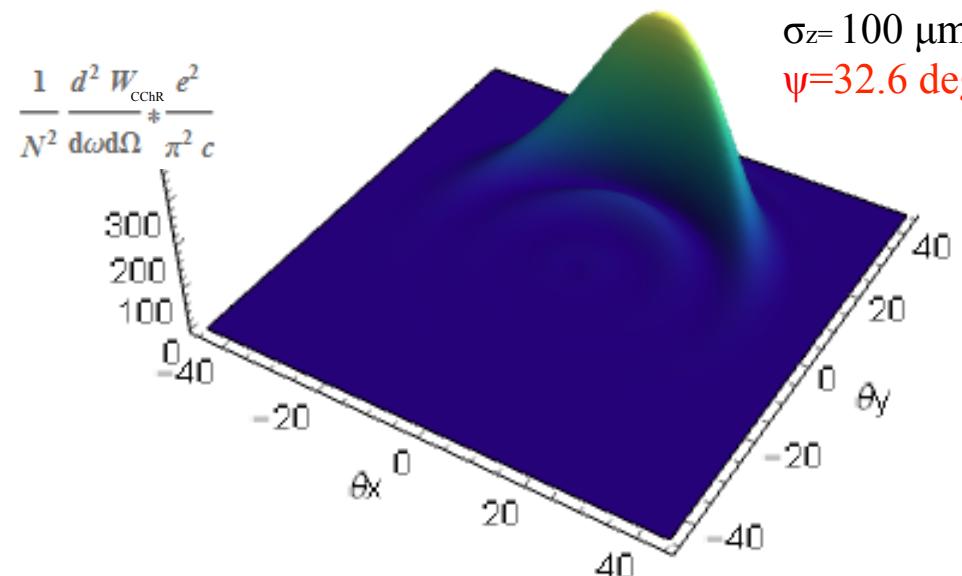
$$\epsilon=1.3$$

$$\lambda=1.5 \text{ mm}$$

$$\sigma_x=\sigma_y=707 \mu\text{m}$$

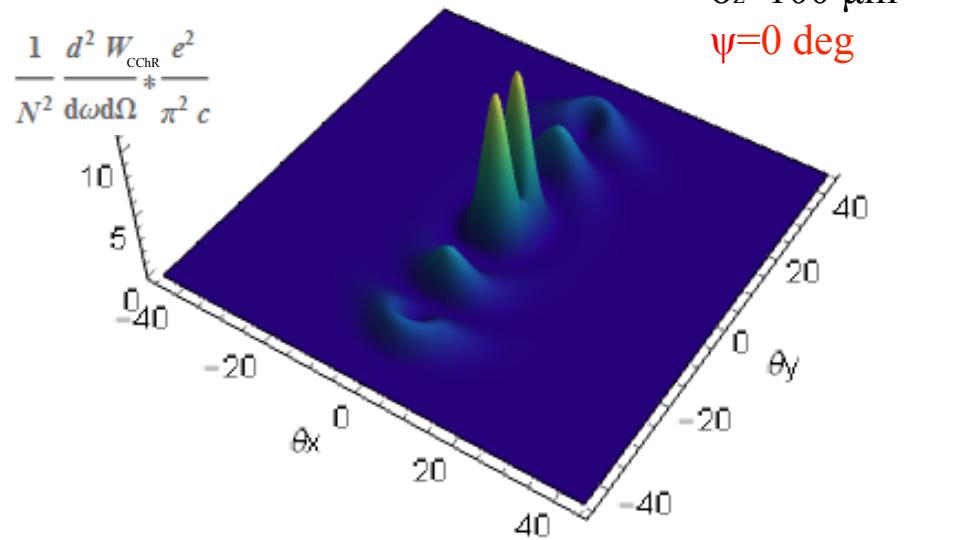
$$\sigma_z=100 \mu\text{m}$$

$$\psi=32.6 \text{ deg}$$

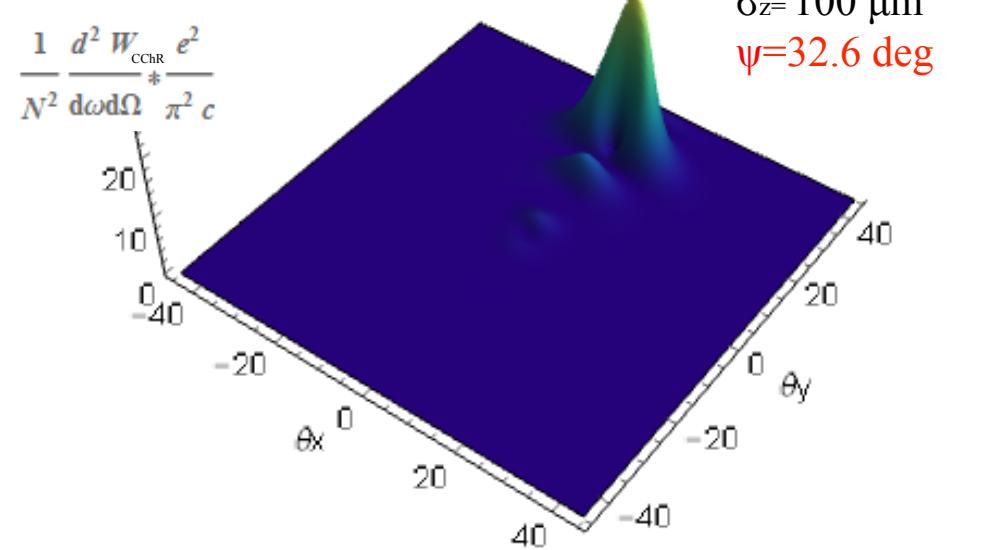




Angular distribution of coherent Cherenkov radiation from a tilted bunch passing through a slit in target



$H=50$ mm
 $L=25$ mm
 $a=5$ mm
 $\gamma=10$
 $\epsilon=1.3$
 $\lambda=1.5$ mm
 $\sigma_x=\sigma_y=707$ μm
 $\sigma_z=100$ μm
 $\psi=0$ deg

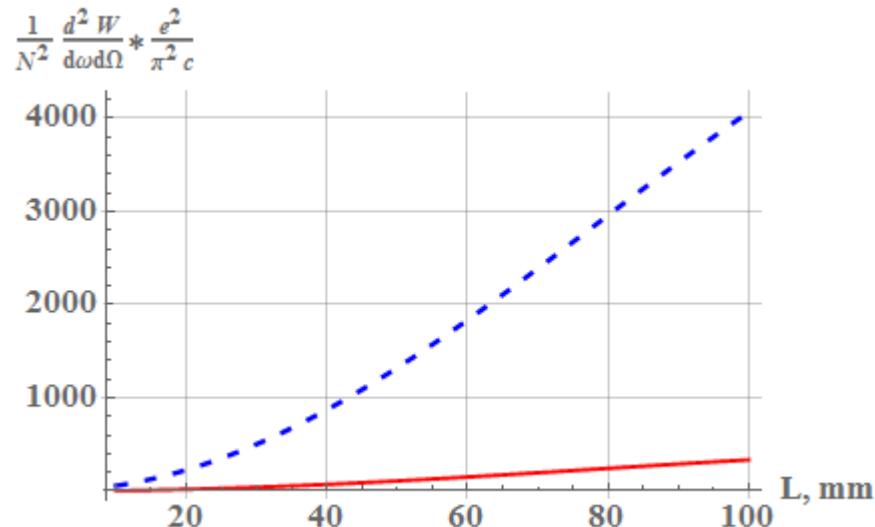


$H=50$ mm
 $L=25$ mm
 $a=5$ mm
 $\gamma=10$
 $\epsilon=1.3$
 $\lambda=1.5$ mm
 $\sigma_x=\sigma_y=707$ μm
 $\sigma_z=100$ μm
 $\psi=32.6$ deg



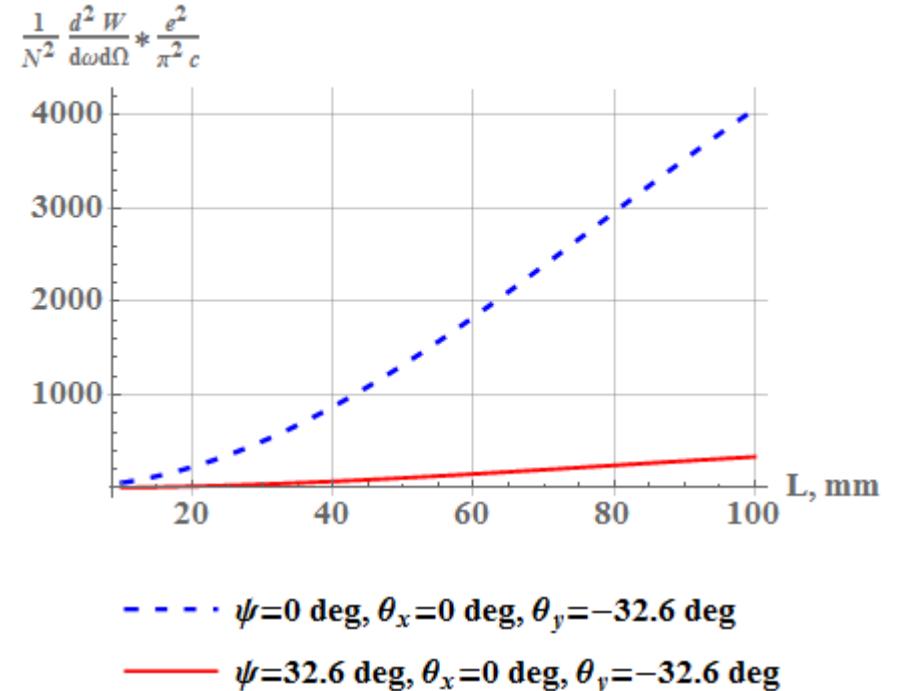


$$-iV\varphi = -i \frac{2\pi}{\lambda} \left[\sin(\theta_x) x_b + \frac{z_b}{\beta} \right] - \frac{2\pi}{\beta\gamma\lambda} \sqrt{1 + (\beta\gamma\sin(\theta_x))^2} y_b$$



$\cdots \psi=0 \text{ deg}, \theta_x=0 \text{ deg}, \theta_y=32.6 \text{ deg}$
 $\text{--- } \psi=32.6 \text{ deg}, \theta_x=0 \text{ deg}, \theta_y=32.6 \text{ deg}$

$H=50 \text{ mm}$
 $a=0 \text{ mm}$
 $\gamma=100$
 $\epsilon=1.3$
 $\lambda=1.5 \text{ mm}$
 $\sigma_x=\sigma_y=707 \mu\text{m}$
 $\sigma_z=100 \mu\text{m}$



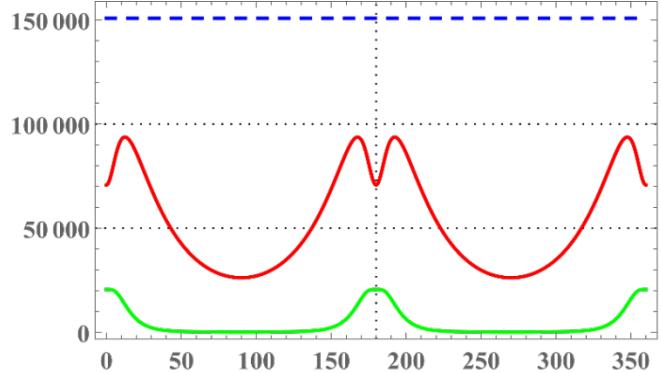
$\cdots \psi=0 \text{ deg}, \theta_x=0 \text{ deg}, \theta_y=-32.6 \text{ deg}$
 $\text{--- } \psi=32.6 \text{ deg}, \theta_x=0 \text{ deg}, \theta_y=-32.6 \text{ deg}$



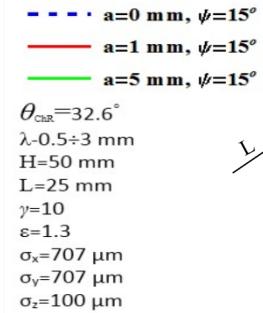
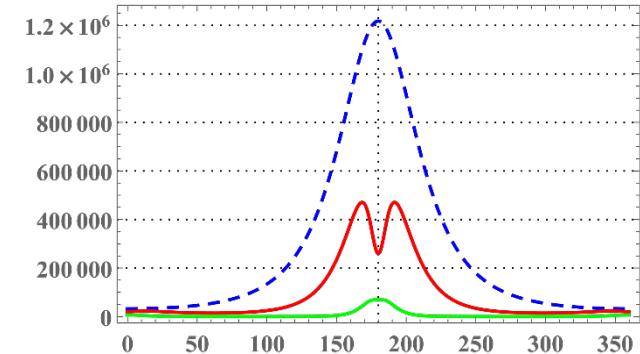


Coherent Cherenkov radiation from a tilted bunch

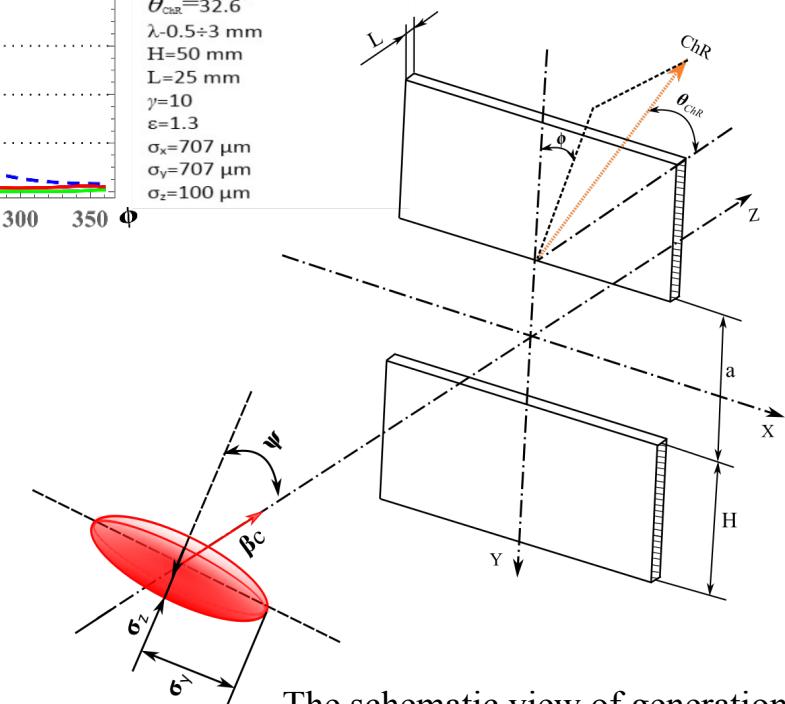
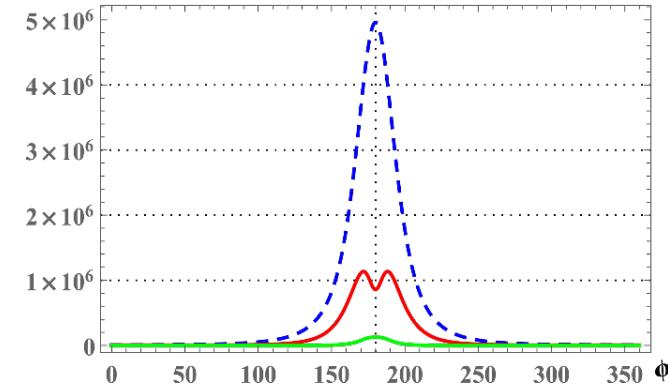
$$\frac{1}{N^2} \frac{dW}{d\Omega} * \frac{e^2}{\pi^2 c}$$



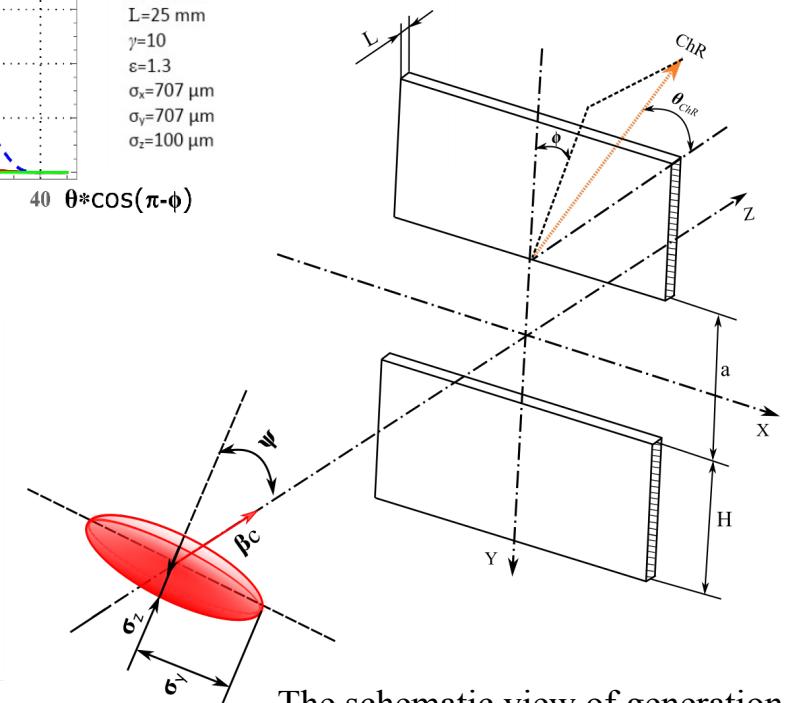
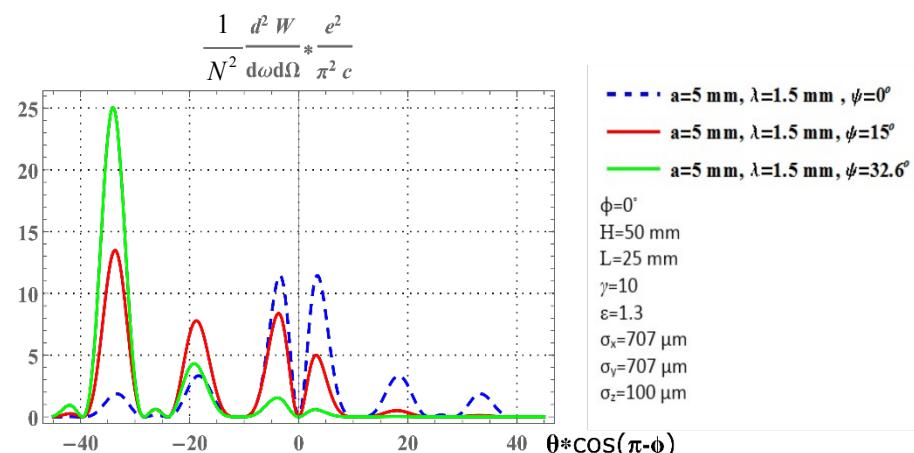
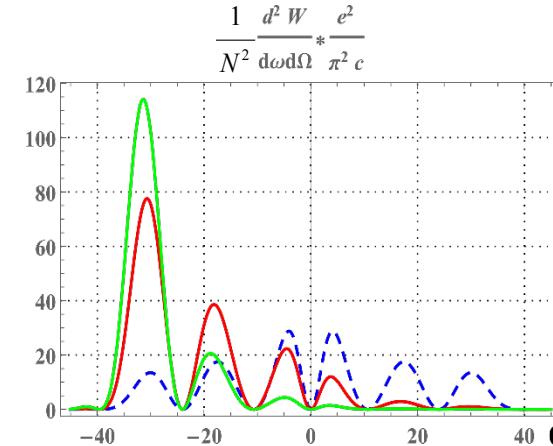
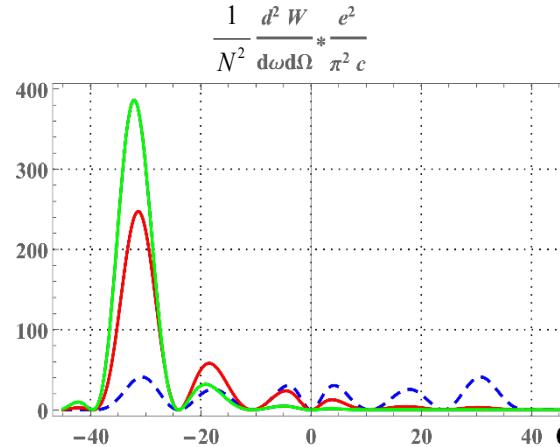
$$\frac{1}{N^2} \frac{dW}{d\Omega} * \frac{e^2}{\pi^2 c}$$



$$\frac{1}{N^2} \frac{dW}{d\Omega} * \frac{e^2}{\pi^2 c}$$

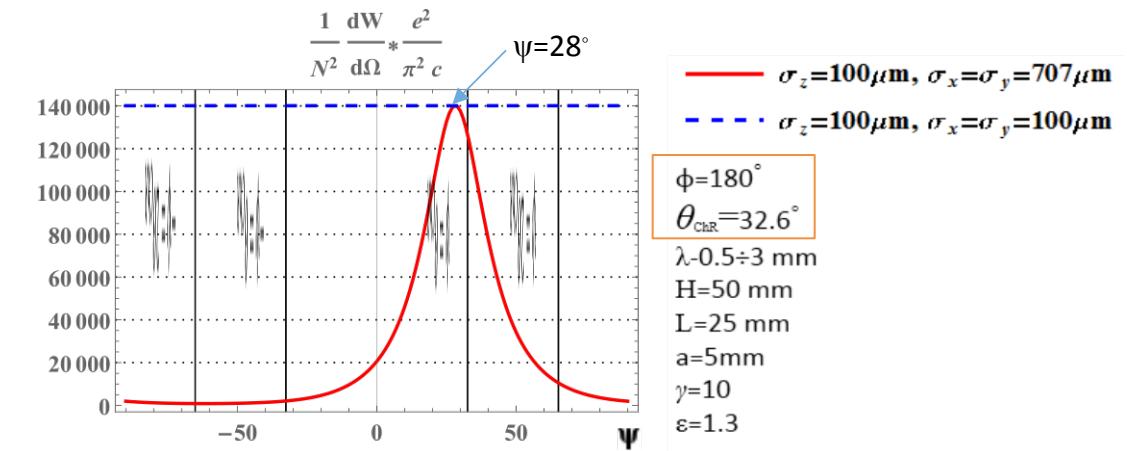
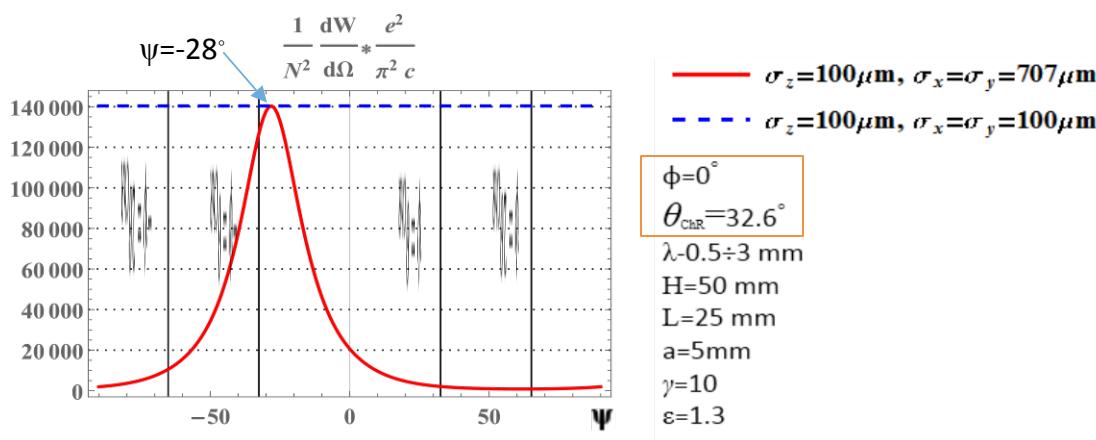
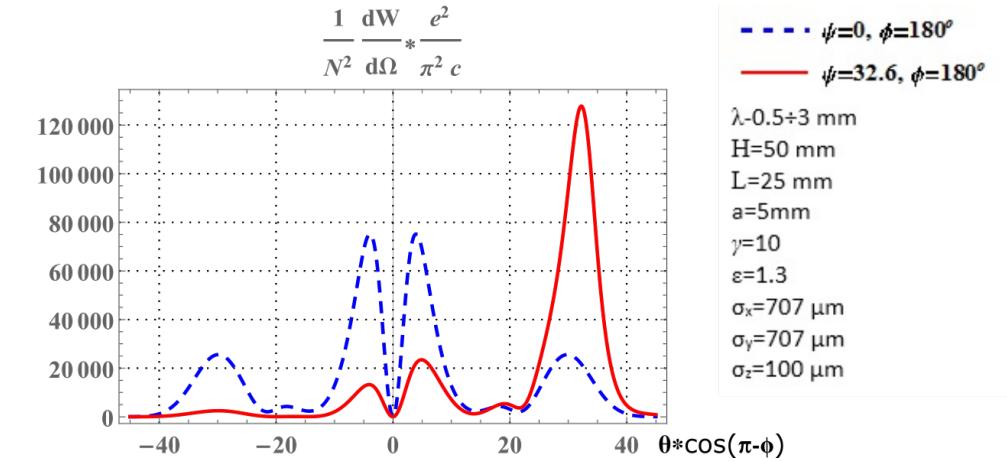
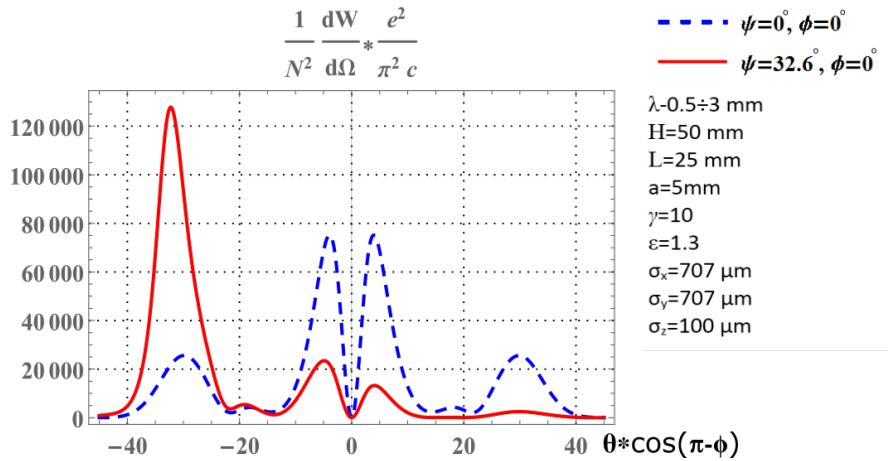


Coherent Cherenkov radiation from a tilted bunch





Coherent Cherenkov radiation from a tilted bunch





Conclusion

1. The developed model allows to simulate the spectral-angular distribution of ChR generated by short electron bunches for which $\sigma_t = \sqrt{\sigma_x^2 + \sigma_y^2}$? σ_z for any target geometry.
2. ChR produced by an ultrashort electron bunch with the axes tilted relative to the bunch velocity possesses the strong azimuthal asymmetry.
3. Simulation results show that the maximal ChR yield is placed in the plane coinciding with the bunch axes and confirm the experimental data.
4. The observed effect can be used to produce the intense THz radiation beam concentrated in the narrow angular range in contrast with the conventional ChR where an intensity is distributed along the cone surface with the opening angle θ_{ChR}





THANK FOR YOUR ATTENTION!

