Smith-Purcell Radiation From a Cylindrical Grating

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The 7th International Conference "Charged & Neutral Particles Channeling Phenomena“, Sirmione-Desenzano del Garda, September 25-30

Outline

- Problem setup
- Induced charge and radiation from a charge rotating around a conducting cylinder
- Electromagnetic fields and the radiation intensity from a cylindrical grating
- Numerical examples
- Conclusions

Problem setup

- Investigations of the Smith-Purcell effect mainly consider the radiation sources moving along straight trajectories

- In this case the radiation mechanism is purely Smith-Purcell one (possibly with an additional Cherenkov radiation if the source moves in a medium)

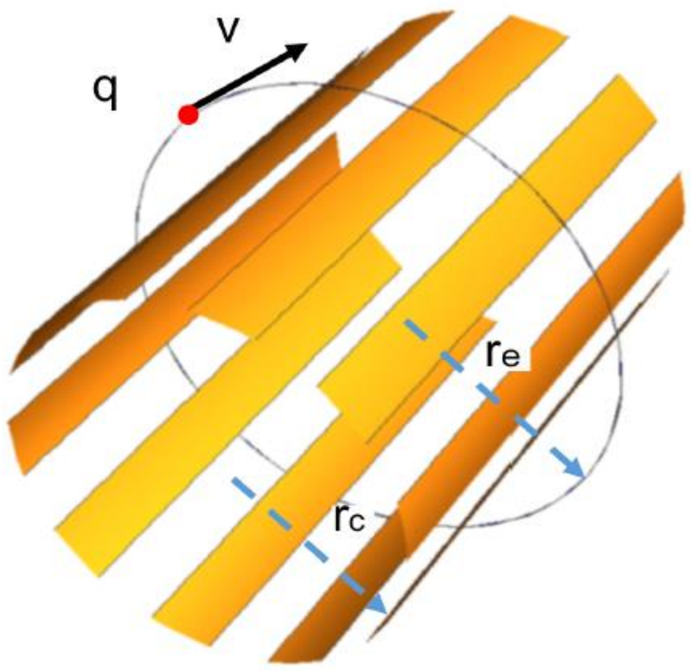
- We consider a problem with two types of radiation mechanisms acting together:

  Synchrotron and Smith-Purcell

- Though the primary source of the radiation for both the synchrotron and Smith-Purcell emissions is the electromagnetic field of the charged particle, the Smith-Purcell radiation is formed by the medium as a result of its dynamic polarization by the field of the moving charge

Problem setup

- Point charge rotating around a metallic diffraction grating on a cylindrical surface

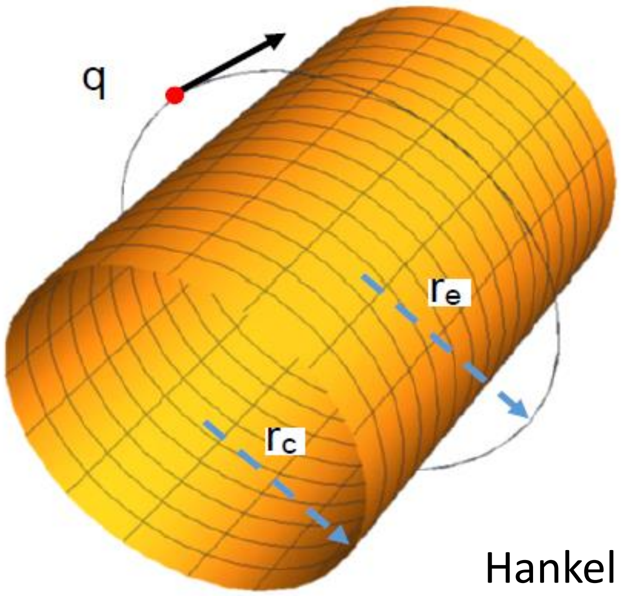


- Grating consists metallic strips with width *a* and with the separation *b*
- System is immersed into a homogeneous medium with the dielectric permittivity *ε*
- Effect of the grating on the radiation intensity is approximated by the surface currents induced on the strips by the field of the rotating charge

- This approach has been employed previously for planar gratings (see A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko, *Diffraction Radiation from Relativistic Particles*)

Induced charge and current on a cylinder

- In order to find the induced currents first consider a charge rotating around a conducting cylinder



- Problem is exactly solvable and closed analytic expressions are obtained for the electric and magnetic fields

- Having the fields one can find the surface charge and current densities

- *Charge density*: $\sigma(\varphi, z, t) = \sum_{n=-\infty}^{+\infty} e^{in(\varphi - \omega_0 t)} \int_{-\infty}^{+\infty} dk_z\, e^{ik_z z} \sigma_n(k_z)$

Hankel function of the first kind

$$\sigma_n(k_z) = -\frac{q r_e}{8\pi^2 r_c^2} \sum_{\alpha=\pm 1} \frac{H_{n+\alpha}(\lambda r_e)}{H_{n+\alpha}(\lambda r_c)} \qquad \lambda = \sqrt{n^2 \omega_0^2 \varepsilon / c^2 - k_z^2}$$

- *Current density*: $j_{sn2}(k_z) = v' \sigma_n(k_z), \quad v' = \omega_0 r_c = v r_c / r_e$

- Properties: $r_c \int_0^{2\pi} d\varphi \int_{-\infty}^{+\infty} dz\, \sigma(\varphi, z, t) = -q \qquad \sigma(\varphi, z, t)|_{r_e \to r_c} = -q\delta(\varphi - \omega_0 t)\, \delta(z) / r_c$

Radiation intensity for a solid cylinder

- Angular density of the radiation intensity on a given harmonic for a charge rotating around a conducting cylinder

$$\frac{dI_n^{(c)}}{d\Omega} = \frac{q^2 n^2 \omega_0^2}{8\pi\sqrt{\varepsilon}c}\beta^2 \left[|B_{n+1} - B_{n-1}|^2 + |B_{n+1} + B_{n-1}|^2 \cos^2\theta \right] \quad \beta = v\sqrt{\varepsilon}/c,\ d\Omega = \sin\theta d\theta d\varphi$$

$$B_{n+\alpha} = J_{n+\alpha}(\lambda r_e) - \frac{H_{n+\alpha}(\lambda r_e)}{H_{n+\alpha}(\lambda r_c)} J_{n+\alpha}(\lambda r_c) \qquad \lambda = \frac{n\omega_0}{c}\sqrt{\varepsilon}\sin\theta$$

- In the limit $r_e \to r_c$ the radiation intensity vanishes as $(r_e/r_c - 1)^2$
- In this limit the field of the charge is compensated by the field of its image on the cylinder surface
- For small angles θ: $\frac{dI_n^{(c)}}{d\Omega} \approx \left[1 - (r_c/r_e)^{2(n-1)}\right]^2 \frac{dI_n^{(0)}}{d\Omega},$ $\qquad \frac{dI_n^{(0)}}{d\Omega} \approx \frac{q^2\omega_0^2}{\pi c\sqrt{\varepsilon}} \frac{(n\beta/2)^{2n}}{\Gamma^2(n)} \sin^{2(n-1)}\theta$
- At the angle = 0 the radiation intensity at zero angle vanishes for all harmonics including *n* = 1

Radiation intensity in a homogeneous medium

Induced charge and current on grating

- Current density induced on the strips of the cylindrical grating

$$j_l^{(\mathrm{s})} = v_l' \sigma^{(\mathrm{s})}(\varphi, z, t)\delta(r - r_c), \quad v_l' = v'\delta_{2l}$$

- Surface charge density

$$\sigma^{(\mathrm{s})}(\varphi, z, t) = \begin{cases} \sigma(\varphi, z, t), & m\varphi_1 \leq \varphi \leq m\varphi_1 + \varphi_0 \\ 0, & \text{otherwise} \end{cases}, \quad m = 0, 1, 2, \ldots, N-1$$

$$\varphi_0 = a/r_c, \ \varphi_1 = (a+b)/r_c \qquad N = \frac{2\pi r_c}{a+b}$$

← number of the periods of the grating

- Electric and magnetic fields are decomposed as: $F_l(x) = F_l^{(0)}(x) + F_l^{(\mathrm{s})}(x)$

$F_l^{(0)}(x)$ ← Fields in the absence of grating

$F_l^{(\mathrm{s})}(x)$ ← Fields induced by grating

Electric and magnetic fields

- Fourier expansion of the fields: $F_l^{(\mathrm{s})}(x) = \sum_{n,m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dk_z\, e^{i(n+mN)\varphi - in\omega_0 t + ik_z z} F_{nml}^{(\mathrm{s})}(k_z, r)$

- Fourier components for the magnetic field:

$$H_{nml}^{(\mathrm{s})}(k_z, r) = \frac{qvk_z}{4ci^{l-1}} \sum_{\alpha=\pm1} \alpha^{l-1} B_{n,m}^{(\alpha)} H_{n+mN+\alpha}(\lambda r), \quad l = 1, 2$$

$$H_{nm3}^{(\mathrm{s})}(k_z, r) = \frac{iqv\lambda}{4c} \sum_{\alpha=\pm1} \alpha B_{n,m}^{(\alpha)} H_{n+mN}(\lambda r). \qquad r > r_c$$

- Fourier components for the electric field:

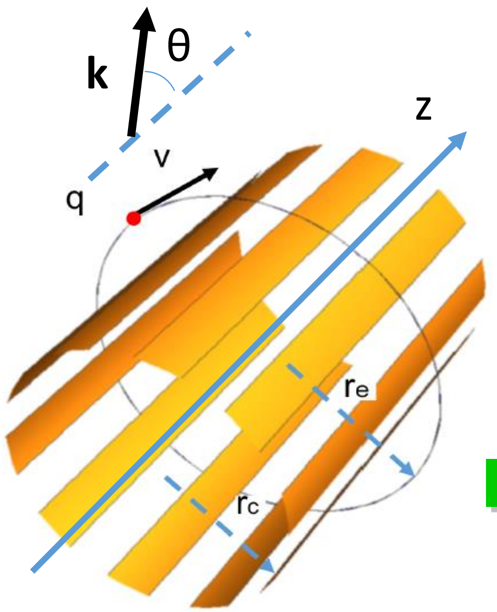
$$E_{nml}^{(\mathrm{s})}(k_z, r) = \frac{i^{-l} qv}{8\varepsilon n\omega_0} \sum_{\alpha=\pm1} \alpha^l \left[\left(n^2\omega_0^2\varepsilon/c^2 + k_z^2\right) B_{n,m}^{(\alpha)} - \lambda^2 B_{n,m}^{(-\alpha)} \right] H_{n+mN+\alpha}(\lambda r), \, l = 1, 2$$

$$E_{nm3}^{(\mathrm{s})}(k_z, r) = \frac{qvk_z\lambda}{4\varepsilon n\omega_0} \sum_{\alpha=\pm1} B_{n,m}^{(\alpha)} H_{n+mN}(\lambda r),$$

- Coefficients: $B_{n,m}^{(\alpha)} = -e^{-\frac{\pi i a}{a+b}m} S_m \frac{H_{n+\alpha}(\lambda r_e)}{H_{n+\alpha}(\lambda r_c)} J_{n+mN+\alpha}(\lambda r_c)$ $\qquad S_m = \frac{1}{\pi m} \sin\left(\frac{\pi m a}{a+b}\right)$

Radiation intensity

- Angular density of the radiation intensity on a given harmonic



$$\frac{dI_n^{(g)}}{d\Omega} = \frac{q^2\beta^2 n^2 \omega_0^2}{8\pi c\sqrt{\varepsilon}} \sum_{m=-\infty}^{+\infty} \left[\left| R_{n,m}^{(+1)} - R_{n,m}^{(-1)} \right|^2 + \cos^2\theta \left| R_{n,m}^{(+1)} + R_{n,m}^{(-1)} \right|^2 \right]$$

$$R_{n,m}^{(\alpha)} = \delta_{0m} J_{n+\alpha}(n\beta\sin\theta) - S_m \frac{H_{n+\alpha}(n\beta\sin\theta)}{H_{n+\alpha}(n\beta'\sin\theta)} J_{n+mN+\alpha}(n\beta'\sin\theta),$$

$$\beta = v\sqrt{\varepsilon}/c, \; d\Omega = \sin\theta d\theta d\varphi \qquad \beta' = r_c\beta/r_e$$

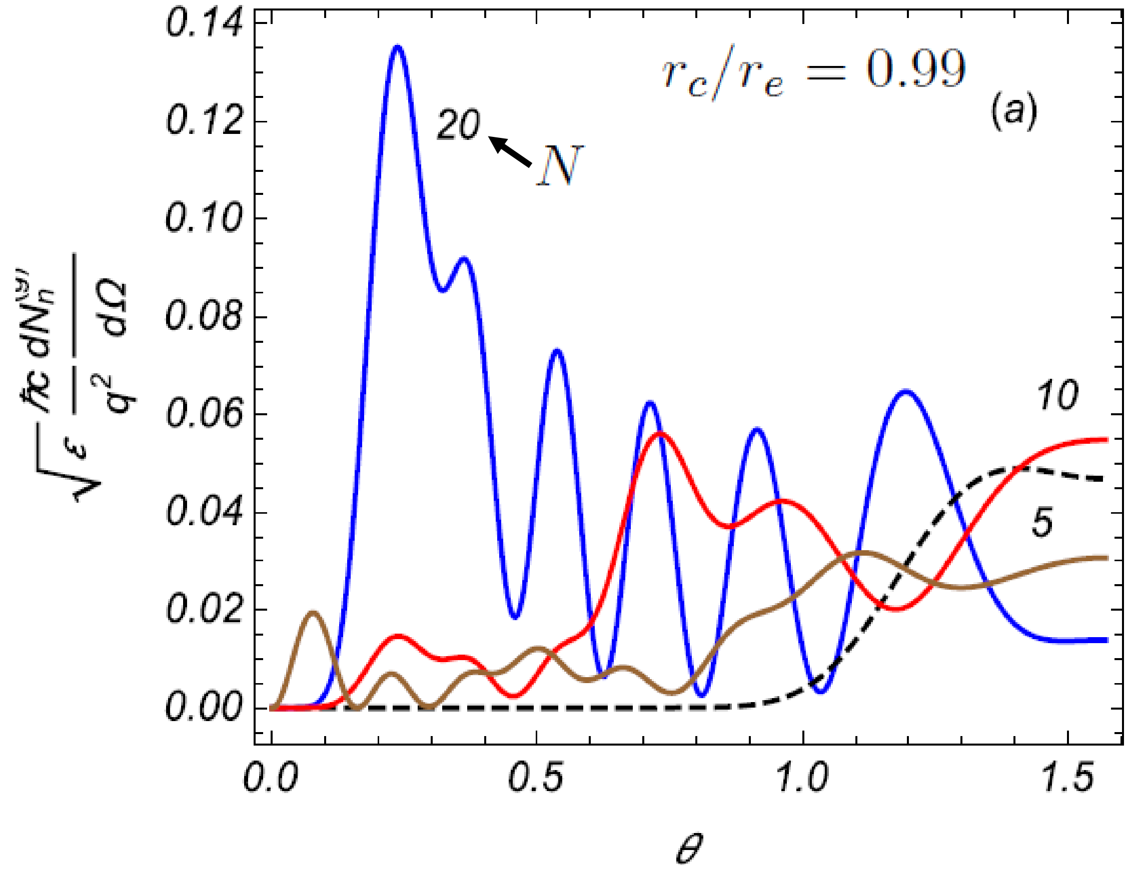
- For a = 0 the term m = 0 contributes only and the intensity is reduced to the one for the radiation in a homogeneous medium
- Another special case b = 0 corresponds to the radiation around a conducting cylinder
- In the geometry of diffraction grating the behavior of the radiation intensity on large values of the harmonic n can be essentially different from that for a charge rotating in the vacuum or around a solid cylinder
- For large values of n and N the main contribution to the radiation intensity comes from the term with the lowest value for $|n + mN + \alpha|$

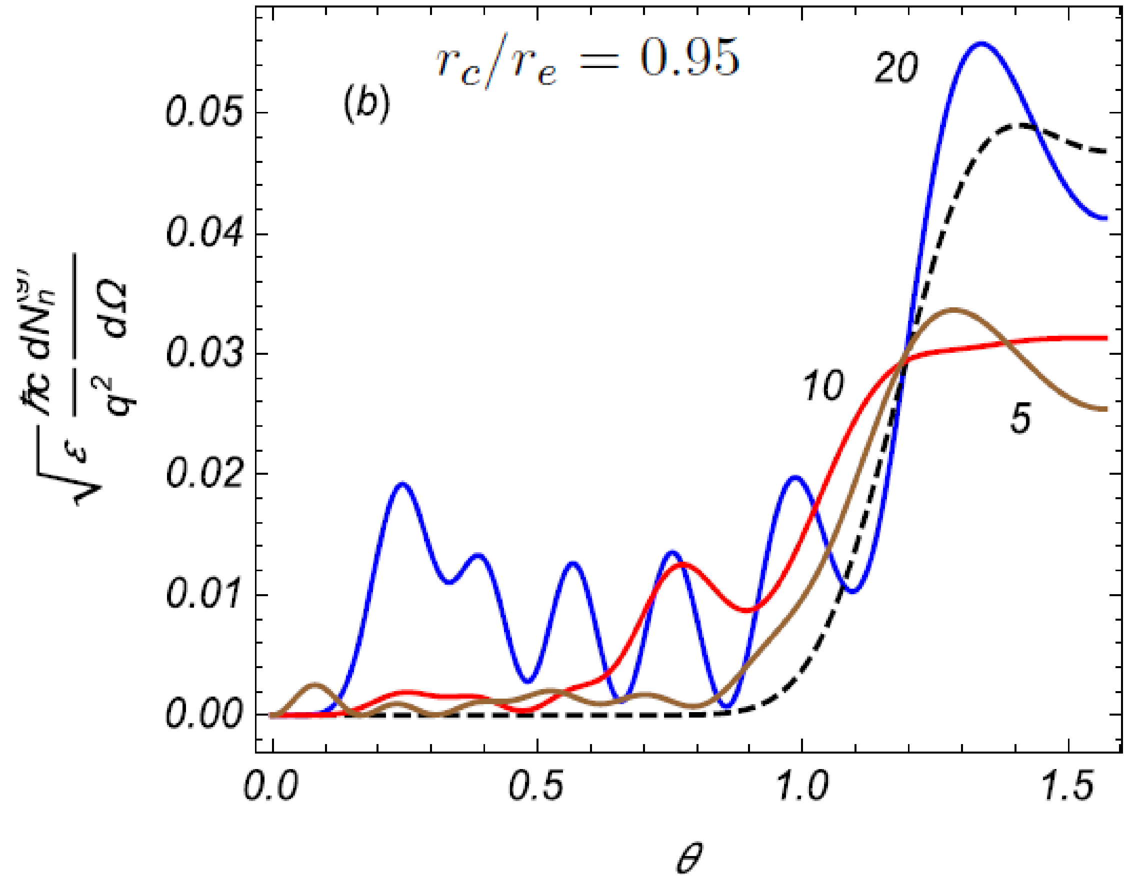
Numerical examples

Numerical results are presented for the angular density of the number of the radiated quanta per period of rotation

$$\frac{dN_n^{(g)}}{d\Omega} = \frac{T}{\hbar n \omega_0} \frac{dI_n^{(g)}}{d\Omega}$$

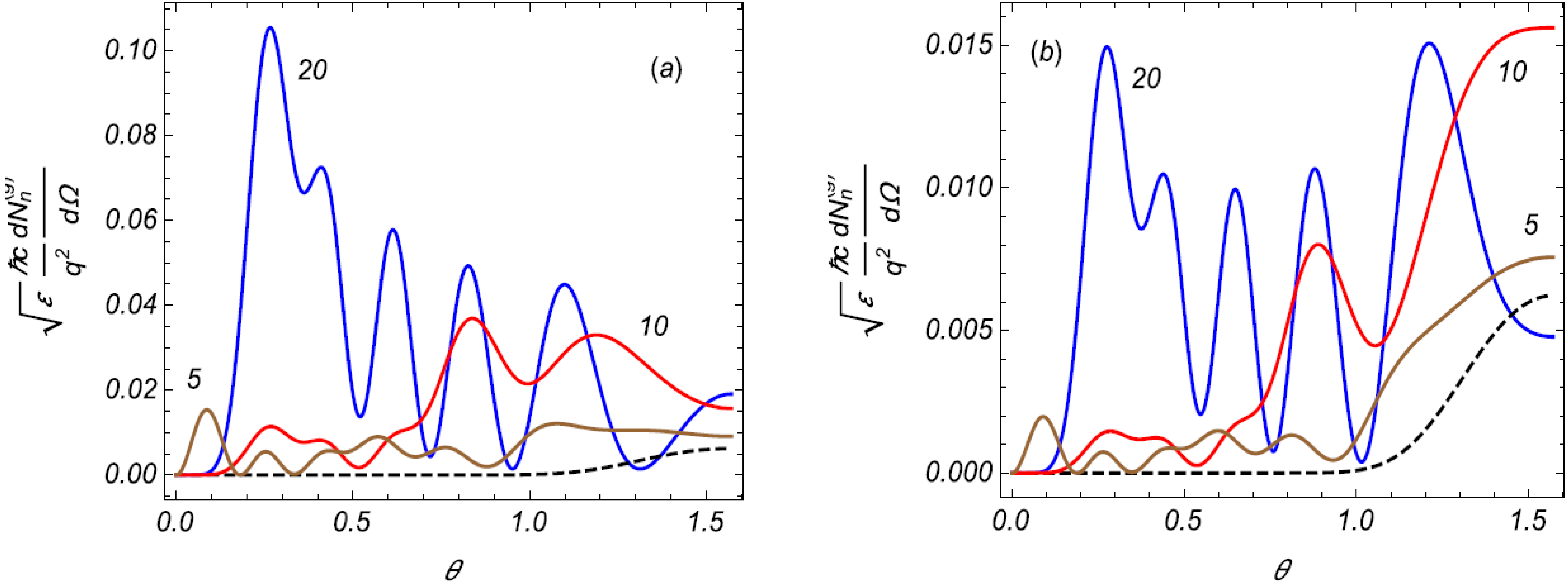
Energy = 2 MeV, harmonic number *n* = 25 $b/a = 1$ $\varepsilon = 1$





Numerical examples

Energy = 2 MeV

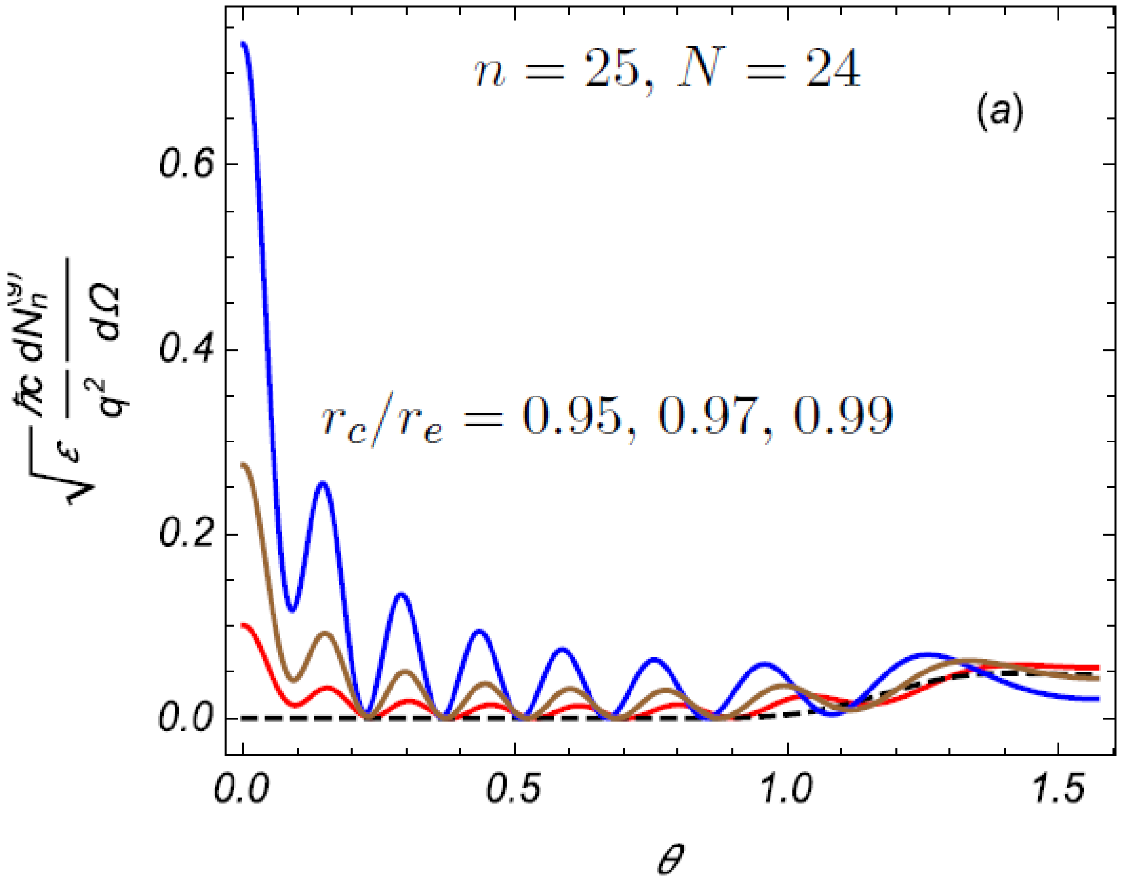


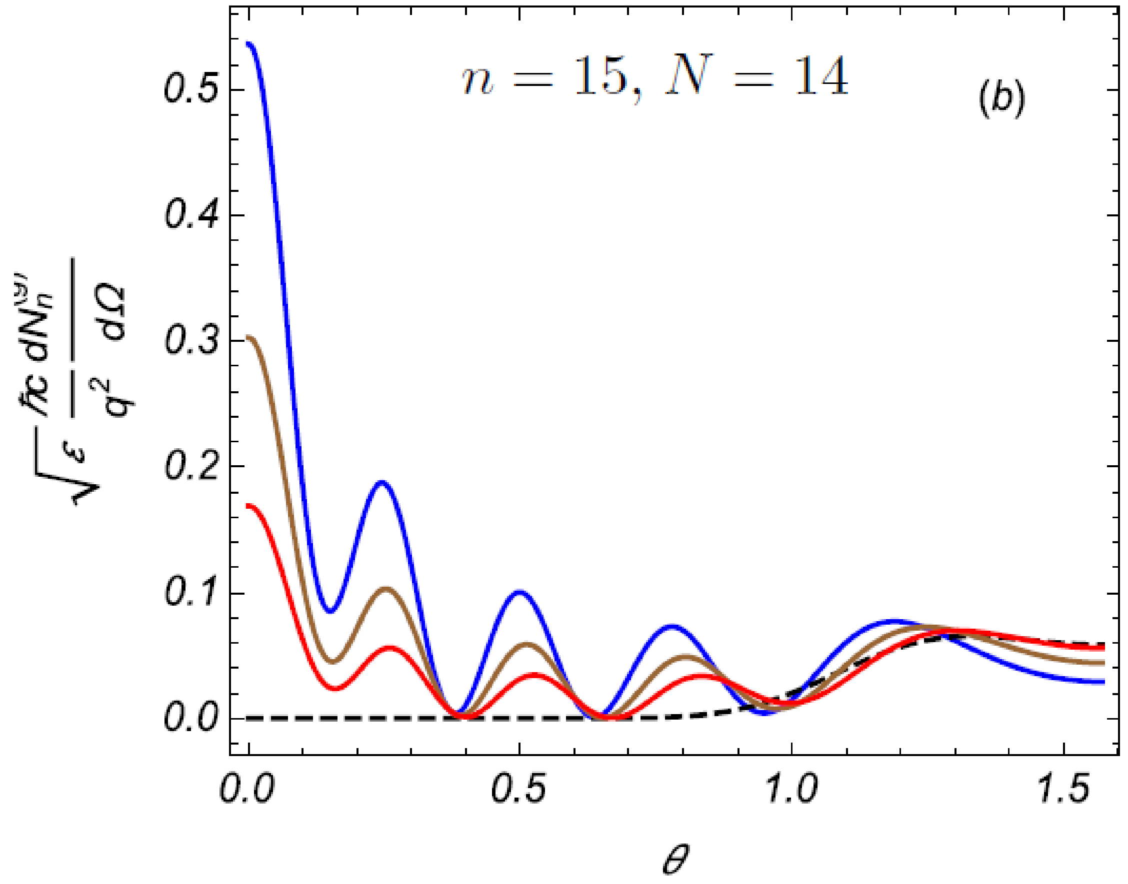
With decreasing energy the relative contribution of the synchrotron radiation decreases and the Smith-Purcell part is dominant

Numerical examples

For a given harmonic *n*, the most strong radiation at small angles θ is obtained for the number of periods *N* = *n*±1

Energy = 2 MeV

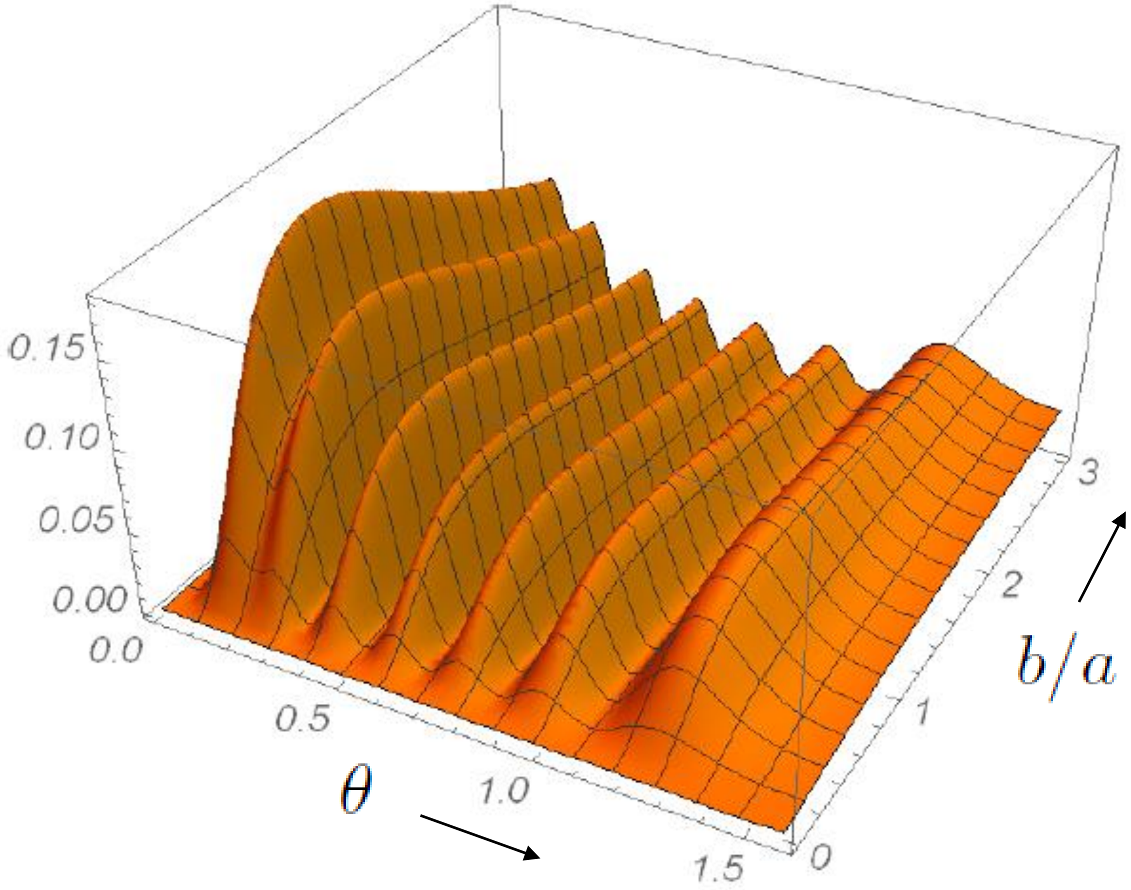




Numerical examples

Energy = 2 MeV

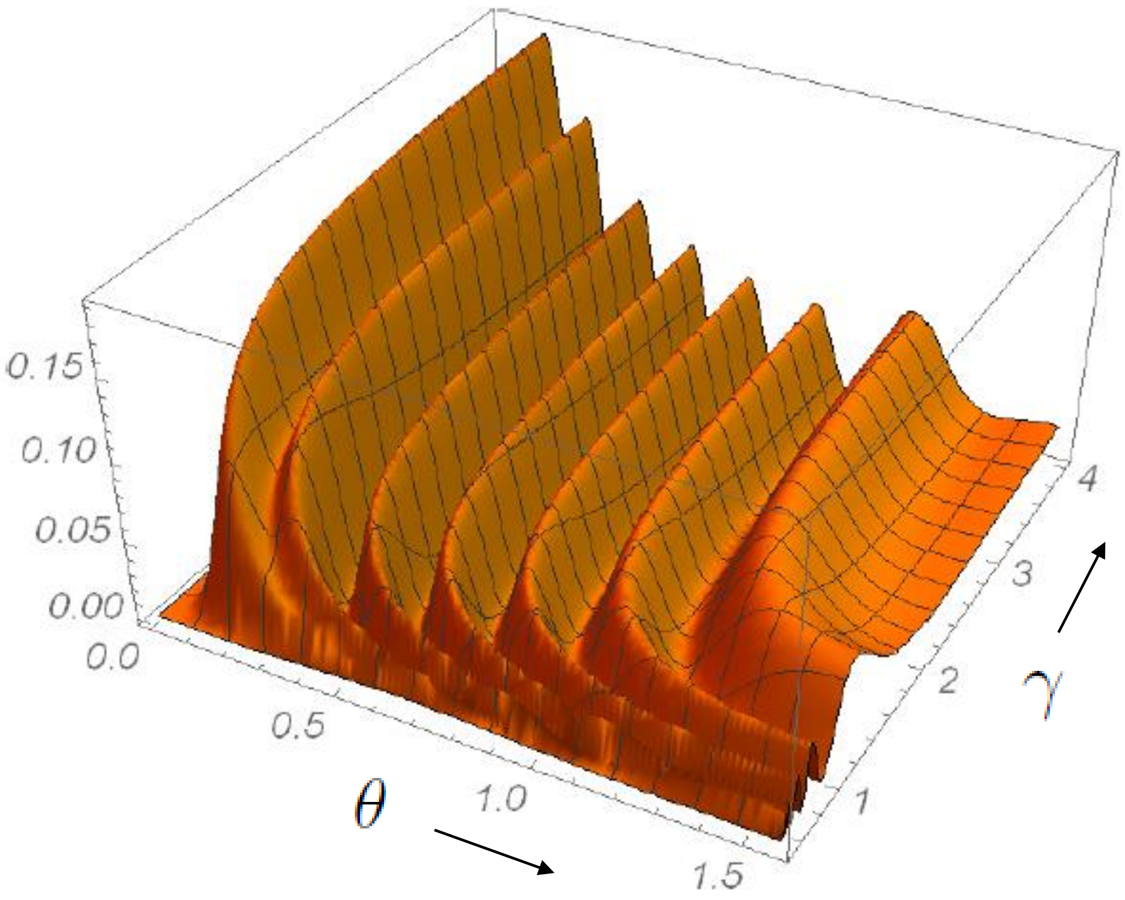
$N = 28,\ r_c/r_e = 0.99,\ \varepsilon = 1$



- In the limit *b/a* << 1 we recover the result for a charge rotating around a conducting cylinder
- In the opposite limit *b/a* >> 1 the result for the rotation in the vacuum is obtained
- locations of the angular peaks are not sensitive to the ratio *b/a*
- Radiation intensity takes its maximum value for *b/a* close to 1

Numerical examples

$N = 28,\ r_c/r_e = 0.99,\ \varepsilon = 1\ \ b/a = 1$



For $\gamma \gtrsim 3$ and for the angles not to close to θ=π/2 (rotation plane), the heights and the locations of the angular peaks in the radiation intensity are not sensitive to the value of the electron energy

Conclusions

- For a charge rotating around a cylindrical grating, the radiation intensity on a given harmonic contains an additional summation corresponding to the periodic structure along the azimuthal direction

- In two limiting cases $a \to 0$ and $b \to 0$ the corresponding expression coincides with the exact results for the radiation in a homogeneous medium and for the radiation from a charge rotating around a solid cylinder

- Unlike to these limiting cases, the radiation intensity for the geometry of diffraction grating on the harmonics $n > 1$ does not vanish for small angles

- Radiation intensity on large harmonics can be essentially different from that for a charge rotating in the vacuum or around a solid cylinder

- With decreasing energy, the relative contribution of the synchrotron radiation decreases and the Smith-Purcell part is dominant

- For large values of the radiation harmonic and of the number of periods in the diffraction grating, the locations of the angular peaks in the radiation intensity are not sensitive to the values of the ratio b/a

- For the angles not to close to the rotation plane and for high-energy particles, the heights and the locations of the angular peaks are not sensitive to the value of the charge energy