Towards the detection of strong field, Volkov resonances in electron-laser interactions

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Channelling 2016, Sirmione
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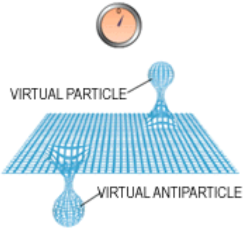
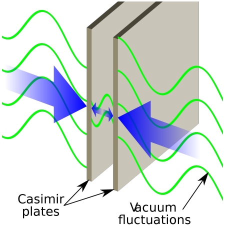
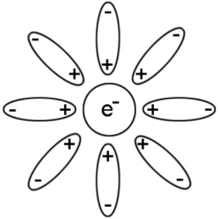
Vacuum properties in a strong electromagnetic field



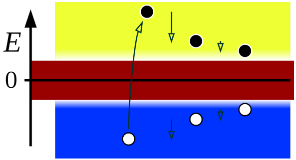
Heisenberg uncertainty



Casimir force



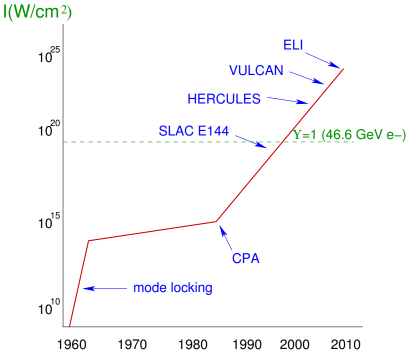
Bare/Dressed charge



Strong field - Vacuum Polarization

Where to find Strong fields?

- $E_{\text{crit}} = 1.32 \times 10^{18}$ V/m, $\Upsilon = E/E_{\text{crit}}$
- Magnetic field at surface of neutron star, $B \to B_{\text{crit}}$
- heavy ion collisions, $E \to 2Z\alpha E_{\text{crit}}$ (positron production spectrum not understood)
- Earths magnetic field in boosted frame of cosmic rays
- Electric field of a charge bunch at a future linear collider (in boosted frame)
- Inter-lattice fields in crystals
- ultra-intense LASERS



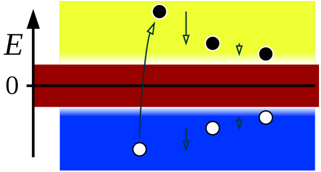
$$\Upsilon = \frac{2\gamma E_{\text{lab}}}{E_{\text{crit}}} \quad , \quad I = \frac{E_{\text{lab}}^2}{377\Omega} = \frac{E_{\text{crit}}^2}{4508\gamma^2}$$

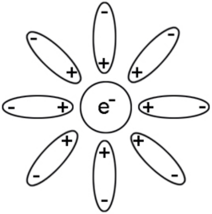
so $\Upsilon = 1$ for $I = 1.4 \times 10^{19}$ W/cm^2 and 46.6 GeV electrons

Synopsis

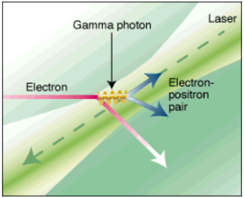
1. The quantum vacuum undergoes changes in the presence of a strong electromagnetic field
2. A QFT with a background electromagnetic field predicts extra peaks (resonances) in scattering cross-sections
3. A detailed study can show resonance peak locations and widths - depend on field strength, scattering kinematics
4. The lasers, electron beams and detectors to detect these predicted resonances are available now or coming online
5. Study progress - theoretical issues, specific experimental sites $\rightarrow$ experiment $\rightarrow$ new strong field quantum effects

Using a strong field to polarise the vacuum

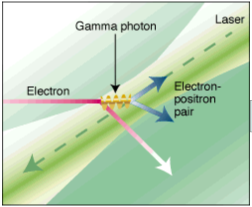




- the quantum vacuum starts to polarise as we apply an electromagnetic field
- The Schwinger limit ($E_{\text{cr}} = 1.32 \times 10^{18}$ V/m) - **but observable effects occur well below E_{cr}**
- What novel experimental effects can we expect as $E \to E_{\text{cr}}$
- How do we incorporate these vacuum changes into our theories?



Quantum Field Theory with a strong field



Lagrangian of interacting fields (background,Dirac,Maxwell)

↓

Euler-Lagrange equations

↓

Field equations of motion (Dirac+background)

↓

Wavefunction solutions

↓

Quanitsed particles

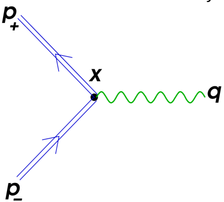
↓

Matrix Element

↓

Final state integration

Transition Probability



(W.H.) Furry Picture

- Separate gauge field into external A^{ext}_μ and quantum A_μ parts

$$\mathcal{L}^{\text{Int}}_{\text{QED}} = \bar\psi(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar\psi(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

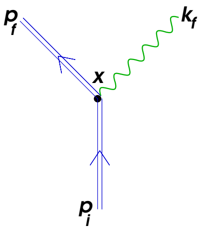
$$\mathcal{L}^{\text{FP}}_{\text{QED}} = \bar\psi^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \frac{1}{4}(F_{\mu\nu})^2 - e\bar\psi^{\text{FP}}\,A\!\!\!/\,\psi^{\text{FP}}$$

- Euler-Lagrange equation $\rightarrow$ new equations of motion requires exact (w.r.t. A^{ext}) solutions ψ^{FP}

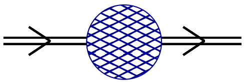
$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

- For certain classes of external fields (plane waves, Coloumb fields and combinations) exact solutions exist **[Volkov Z Physik 94 250 (1935), Bagrov and Gitman Exact solutions of Rel wave equations (1990)]**
- A QFT which is non-perturbative wrt external gauge field A^{ext} and perturbative wrt ψ^{FP}, A

Unstable Strong field particles & resonant transitions



- **Electrons decay in strong field Furry picture**
  - Background field dissipative medium
  - bound electron finite lifetime, Γ
  - probability of radiation, W
  - final state photons can become initial states of second order processes
- **Resonant transitions in propagator**
  - required by S-matrix analyicity
  - Optical theorem $W = \text{Im}(\Sigma)$
  - extra propagator poles leading to physically accessible resonances
  - related to energy level structure of vacuum



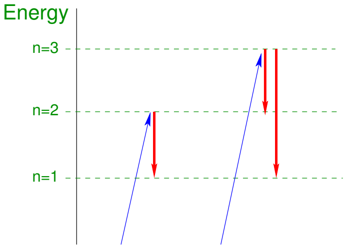
Need to consider a 2nd order FP process with a fermion propagator

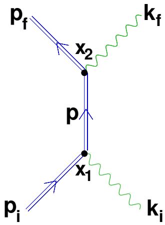
Strong field theory → vacuum quasi energy levels

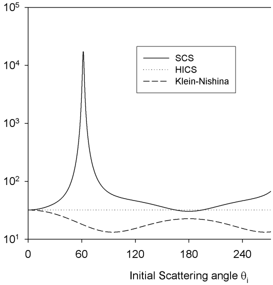
BOUND STATES
Resonant transition between
Zel'dovich energy levels in 2nd order processes

$$(q + nk)^2 = m_*^2$$

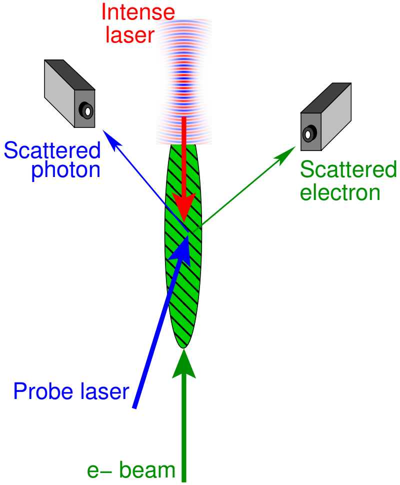
q = effective electron momentum
n = nth energy level
m_* = effective electron mass
k = LASER 4-momentum

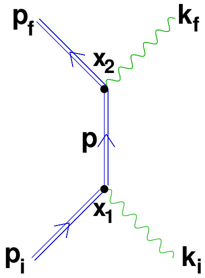






Experimental setup to detect new resonances





- Intense counter-propagating laser ($> 10^{19}$ W/cm^2)
- relativistic electron bunch
- probe laser variable energy, incident angle
- scans over scattering angles $\rightarrow$ resonances
- detectors of sufficient resolution

ELI-NP Strong field QED - beam parameters

$$\nu(=\eta=\xi=a_0)=\frac{e^2E^2}{\omega^2m^2}$$

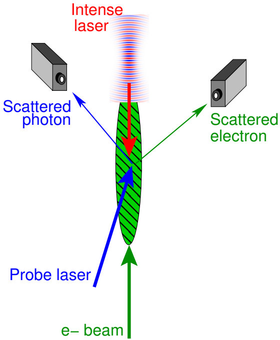
$$\chi=\frac{k\cdot p}{m^2}\approx\frac{2\gamma\omega}{m^2}$$

$$\Upsilon=\nu\frac{k\cdot p}{m^2}=\frac{\gamma E}{E_c}$$

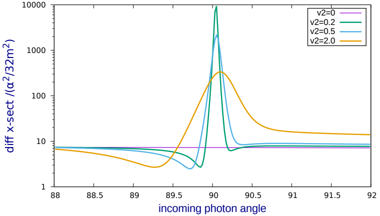
- The intensity parameter $\nu\approx 1$ non-linear regime
- The energy parameter quantum effects: recoil, self energy
- The strong field parameter field strength in electron rest frame

	electron	100 TW laser	1 PW laser
Energy	80-720 MeV	$\sim$ 1 eV	$\sim$ 1 eV
Energy spread (%)	0.1		
transverse beam size, ω_0 (μm)	7.5	5	5
bunch/pulse length	100-400 μm	22 fs	22 fs
Emittance ϵ_x (mm mrad)	0.2-0.6		
beam energy (J))		2.2	22
Charge (pC)	25-400		
Rep rate (Hz)	1	10	1
ν		$\sim$ 6	$\sim$ 19
χ		0.64-5.8$\times10^{-3}$	0.64-5.8$\times10^{-3}$
Υ		3.84-34.8$\times10^{-3}$	12.2-110.2$\times10^{-3}$

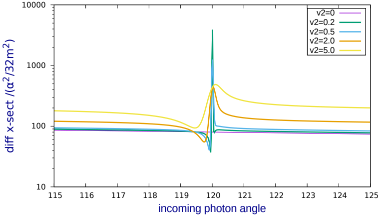
Resonances in probe laser angle scan



Probe laser 2 eV, intense laser 1 eV
electrons 10 MeV, detector at 160^o

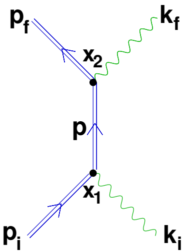


Probe laser 4 eV, intense laser 1 eV
electrons 40 MeV, detector at 175^o



- key ratio for resonance is $\frac{\omega_i}{\omega} \in \mathbb{Z}$
- resonances broadened by larger ν
- resonances suppressed by large electron energy

Dressed vertices & higher order traces



- double fermion lines are Volkov-type solutions
- Volkov E_p functions can be grouped around the vertex
- only need one new Feynman picture element - the dressed vertex

$$\gamma^{\mu}_{\mathsf{fpx}_2} = \int d^4x_2\, \bar{E}_{p_f}(x_2)\gamma^{\mu}E_{p_i}(x_2)\, e^{i(p_f+k_f-p)\cdot x_2}$$

- 2nd order trace has 4 dressed vertices. How many terms?

$$\sum |M_{\mathsf{fi}}|^2 \propto \mathrm{Tr}\left[(\not{p}_{\mathsf{f}}+m)\gamma^{\mu}_{\mathsf{fpx}_2}(\not{p}+m)\gamma^{\nu}_{\mathsf{ipx}_2}(\not{p}_{\mathsf{i}}+m)\bar{\gamma}_{\mathsf{fpx}_1\,\nu}(\not{p}+m)\bar{\gamma}_{\mathsf{fpx}_1\,\mu}\right]$$

- 4 channels x 4 γ x (2x2) E_{p} x (4x2) spin sum = 512 terms
- Higher order terms become intractable
- Feyncalc strategy no good for strong field particle generator
- Need schema for Furry pic trace simplification for any order

Fierz transformation for Volkov spinors

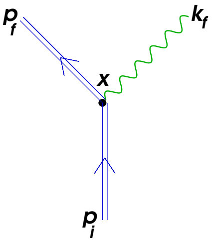
[*Hartin 2016, arXiv:1608.06527*]

define the Volkov spinor V_{px}

$$\Psi^{\mathrm{V}}_{\mathrm{prx}} = n_p \, [\not{\Pi}_{px} + m] \, \frac{\not{k}}{2k\cdot p} \, u_{\mathrm{pr}} \, e^{-i\Delta_{\mathrm{px}}} \equiv V_{\mathrm{pxr}} \, e^{-i\Delta_{\mathrm{px}}}$$

Fierz transformation applies to Volkov spinors

$$\sum_{\mathrm{rsr's'}} [\bar{V}_{\mathrm{frx}} \, \Gamma^{j}_{\mathrm{J}} \, V_{\mathrm{isx}}][\bar{V}_{\mathrm{is'x'}} \, \Gamma_{\mathrm{J}\ j} V_{\mathrm{fr'x'}}] = \sum_{\mathrm{rsr's'K}} F_{\mathrm{JK}} \left[\bar{V}_{\mathrm{frx}} \, \gamma^{\mu} \Gamma^{k}_{\mathrm{K}} \gamma_{\mu} \, V_{\mathrm{fr'x'}}\right] [\bar{V}_{\mathrm{is'x'}} \, \Gamma_{k\mathrm{K}} \, V_{\mathrm{isx}}]$$



example: amplitude for HICS

$$M_{\mathrm{fi}} = -ie \int d^4x \; \bar{\psi}^{\mathrm{V}}_{\mathrm{frx}} \, \not{A}_{\mathrm{fx}} \, \psi^{\mathrm{V}}_{\mathrm{isx}}$$

squared amplitude splits into two traces

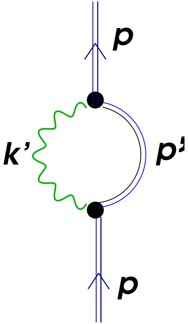
$$|M_{\mathrm{fi}}|^2 \propto \sum_{\mathrm{K}} F_{\mathrm{SK}} \; \mathrm{Tr}\left[\bar{V}_{\mathrm{fx}} \, \gamma^{\mu} \Gamma^{k}_{\mathrm{K}} \gamma_{\mu} \, V_{\mathrm{fx'}}\right] \mathrm{Tr}\,[\bar{V}_{\mathrm{ix'}} \, \Gamma_{k\mathrm{K}} \, V_{\mathrm{ix}}]$$

Loops in an external field

Electron self energy in a strong field
$\rightarrow$ resonance width

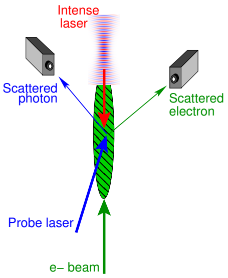
(Oleinik 1967, Ritus 1971
Baier, Katkov & Strakhenko)

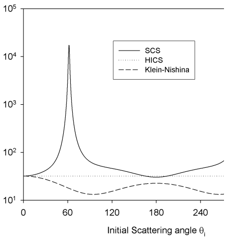
$$\Sigma^{\mathrm{FP}} = \int \frac{d^4k'\, d^4p'}{k'^2\,(p'^2+m^2)}\, \gamma^{\mathrm{FP}\mu}\,(\not{p}'+m)\,\gamma^{\mathrm{FP}}_{\mu}$$



- I need electron self energy in strong field. Easy, but...
- Are there divergences in the Furry picture (Rohrlich "there can be no NEW divergences")
- Are the Ward identities satisfied?
- Explicit calculation of leading order Furry picture loops with counter-terms
- Does the external field self-regulate the UV divergences?
- For $E > E_{\mathrm{cr}}$ unstable vacuum (Fradkin)
- Narozhnyi 1979 - external field enters into coupling constant - convergence of perturbation theory?

Summary: Strong field vacuum resonances in the lab!

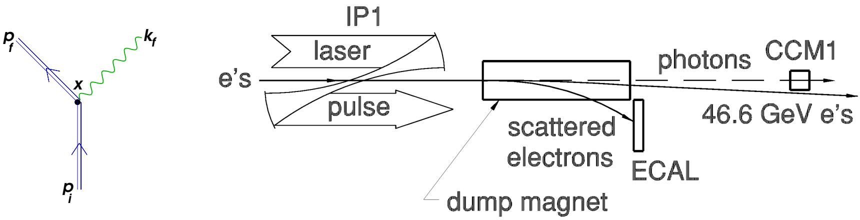




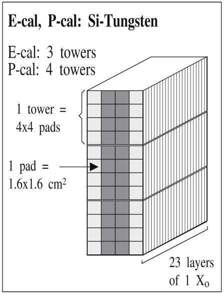
- Quantum vacuum shows new properties in a strong electromagnetic field
- QFT with background field predicts resonant cross-sections
- Experimental facilities can already perform tests of the theory
- Collaboration between strong field theorists & experimentalists is needed
  $\rightarrow$ Design experiment
- Goal is experimental proposal in 2 years followed by experiments
- **Discover new quantum effects and perform new tests of QFT in background fields**

BACKUP

SLAC E144 experiment 1995/6 I



- 46.6 GeV e- beam collides with focussed ND:YAG laser at $\theta = 17^o$
- measurements made of pulse width, spot size and laser energy to get Intensity $\approx 10^{18}$ W/cm^2
- electrons with $E \leq 30$ GeV detected by Si-Tungsten ECAL, $\frac{\sigma}{E} \approx \frac{0.25}{\sqrt{E(\text{GeV})}}$
- photons converted in Al and detected in gas Cherenkov, $\Delta N_\gamma = 10\%$

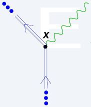


Historical timeline

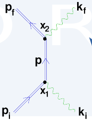
1930s — Dirac equation in external field, exact solution

1950s — QED with bound states, Furry Picture

1960s — 1^{st} order processes

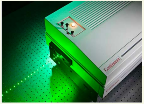


1990s — 2^{nd} order processes

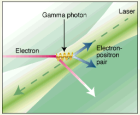


2010s

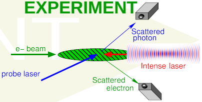
LASER



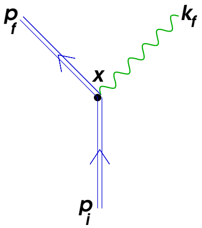
SLAC EXPT E144

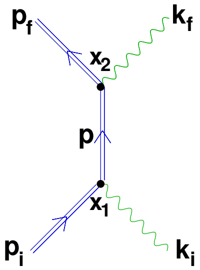


PROPOSED RESONANCE EXPERIMENT



Strong field laser-electron processes

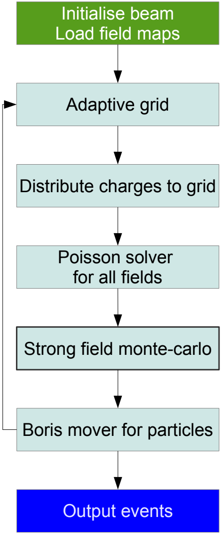




- **1st order intense field process:**
  - High Intensity Compton Scattering (HICS)
  - $p_i + nk \rightarrow p_f + k_f$
  - intense laser field included to all orders
  - final state photons can become initial states of second order processes
- **2nd order intense field processes:**
  - Stimulated Compton Scattering (SCS)
  - $p_i + k_i + nk \rightarrow p_f + k_f$
  - extra propagator poles leading to physically accessible resonances
  - related to energy level structure of vacuum

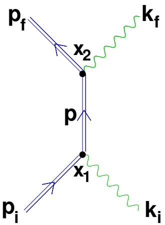
- Consider here the 2nd order process. 1st order is a background

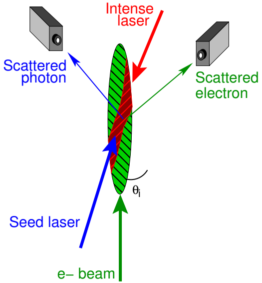
IPstrong - IFQFT monte carlo in a PIC el-mag solver



- Fortran 2003 with openMPI (Fortran 2008 has inbuilt GPU libraries)
- 3D electrostatic poisson solver (MPI)
- Strong field photon radiation
- output in multiple formats (stdhep, ascii)
- cross-check with GENSIS 1.3 FEL sim
- charge bunch/laser & laser/matter interaction

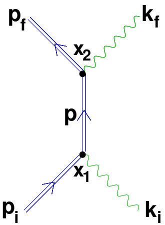
Furry Picture 2 vertex Compton scattering

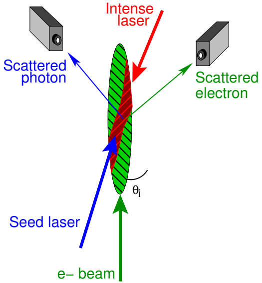




- Compton scattering in an intense electromagnetic field $\rightarrow$ Klein-Nishina with vanishing field
- resonances considered in special ref frame linearly polarised field and with electron self energy [Oleinik 1968]
- approx by electron in field of two electromagnetic fields [Fedorov 1975]
- considered with parallel photons, no resonances [Akhiezer & Merenkov 1985]

Furry Picture 2 vertex Compton scattering





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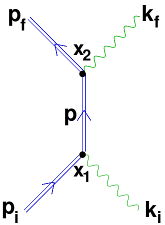
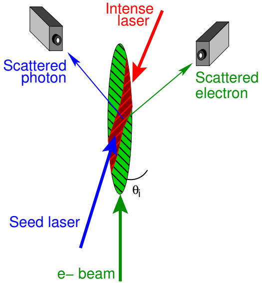
- circularly polarised field, no kinematic approximations, resonance correction by self energy inclusion in propagator [Hartin 1988, 2006]

Furry Picture 2 vertex Compton scattering





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- Experimental efficacy: resonances due to laser intensity, scattering angles, initial photon energy
- intense laser intensity, energy generally fixed. Gain extra tuning in initial state photon $\rightarrow$ resonance scans

theoretical aspects of the Furry Picture

- External field makes space-time inhomogeneous so propagator depends on separate space-time points rather than on the difference between them **[Berestetski Lifshitz Pitaevski, QED $\S 109$]**
- Normalised IN and OUT states can be formed and LSZ extended to include such states **[Meyer, J Math Phys 11 312 (1970)]**
- Vanishing field strength at $t = \pm\infty \rightarrow$ stable vacuum
- Vacuum can be polarised so must include tadpole diagrams **[Schweber Relativistic QFT $\S$15g]**
- Operator and path integral representations for generating functional **[Fradkin, QED in an unstable vacuum]**
- Physics in curved spacetimes $\equiv$ Physics in the Furry Picture
- Quantum Gravitation!

Furry Picture

$$\mathcal{L}_{\text{QED}}^{\text{Int}} = \bar{\psi}(i\not{\partial} - m)\psi - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(\not{A}^{\text{ext}} + \not{A})\,\psi$$

$$\mathcal{L}_{\text{QED}}^{\text{FP}} = \bar{\psi}^{\text{FP}}(i\not{\partial} - e\not{A}^{\text{ext}} - m)\psi^{\text{FP}} - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}}\not{A}\,\psi^{\text{FP}}$$

Equations of Motion

$$(i\not{\partial} - e\not{A}^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

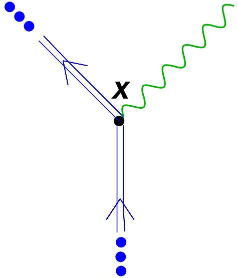
Wavefunction

$$\psi^{\text{FP}} = E_p\, e^{-ipx}\, u_p, \qquad E_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(e\not{A}^{\text{ext}}\not{k} + i2e(A^{e}\!\cdot p) - ie^2 A^{\text{ext}\,2}\right)\right]$$

Propagator

$$G^{\text{FP}} = \int \frac{d^4p}{(2\pi)^4}\, E_p(x)\, \frac{\not{p} + m}{p^2 - m^2}\, \bar{E}_p(x')\, e^{-ip\cdot x}\, u_p$$

Dressed verticies/vacuum



- double fermion lines are Volkov-type solutions
- Volkov E_p functions can be grouped around the vertex
- Only need one new Feynman picture element - the dressed vertex

$$\gamma_\mu^{\mathsf{FP}}(p_f, p_i, kx) = \int d^4x\, \bar{E}_{p_f}(x)\gamma_\mu E_{p_i}(x)\, e^{i(p_f - p_i + k_f)\cdot x}$$

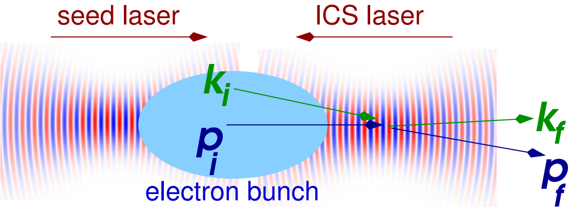
- momentum space vertex has contribution rk from external field

$$\gamma_\mu^{\mathsf{FP}}(p_f, p_i, k\cdot x) = \sum_{r=-\infty}^{\infty} \int_{-\pi}^{\pi} \frac{d\phi_v}{2\pi} \exp\left(ir\left[\phi_v - (k\cdot x)\right]\right)\gamma_\mu^e(p_f, p_i, \phi_v)$$

- Fourier transform of circularly polarised field leads to Bessel functions

$$\int_{-\pi}^{\pi} \frac{d\phi_v}{2\pi} \exp\left[ir\phi_v - z\sin\phi_v\right] = \mathrm{J}_r(z)$$

AXSIS Strong field interactions



- ICS laser field is strong ($a_0 \equiv \nu^2 \sim 1$) optical undulator
- electrons radiate according to strong field QED HICS process
- incoherent (ICS) mode and coherent (FEL) mode
- coupled seed laser field (to encourage FEL mode)
- seed laser induces 2nd order strong field QED Compton Scattering - resonant transitions

$$\omega_{fq} = \omega \, \frac{\gamma^2(1+\beta)^2}{\gamma^2(1+\beta)(1-\beta\cos\theta) + (a_0^2/2 + \omega\gamma(1+\beta))(1+\cos\theta)}$$