Measurements of multiple scattering of high energy protons in silicon crystals

by A.M. Taratin on behalf of UA9 collaboration

MS measurements were performed at the CERN SPS extracted beam
of 400 GeV/c protons with bent silicon crystals at their amorphous orientations

Reasons to study MS in crystals :

1. The ordered positions of atoms in crystals

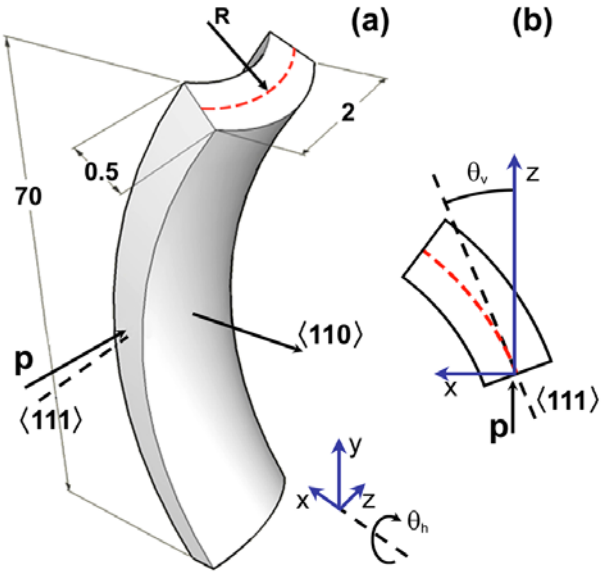
2. Importance of accurate representation of MS for protons in bent crystals for studies of crystal assisted collimation of the LHC beam

3. Existence of good tracking system for UA9 studies of crystal deflectors at CERN SPS external beam line

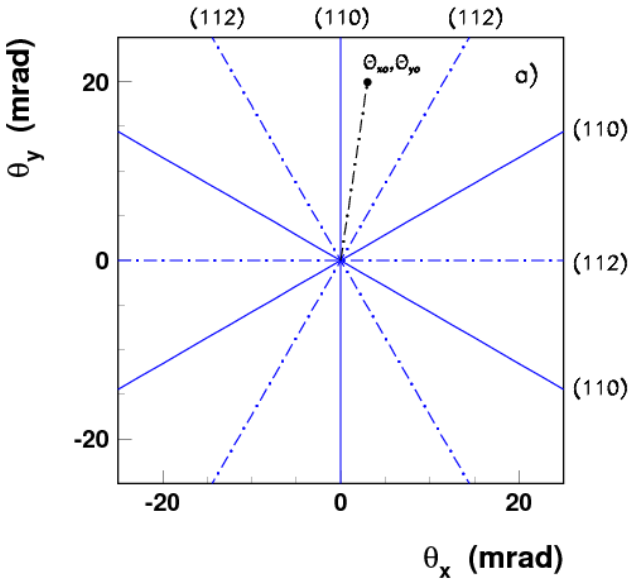
Silicon strip crystals for MS measurements

Three Si strip crystals (produced in Ferrara) with large faces parallel to (110) planes
entrance face is parallel to (111) planes

ST crystal



Angular space near <111> axis



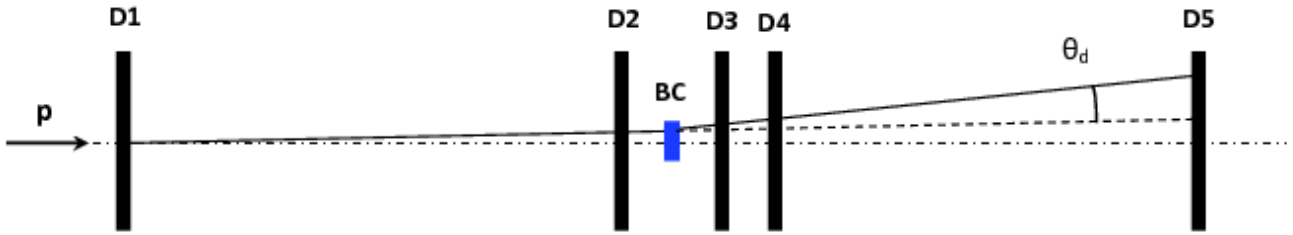
Crystal orientation should exclude situation when the beam is by chance near some crystal plane or axis

Crystal was bent to perform its orientation by observation of beam deflection
Small crystal bend does not change MS according to our simulations

Experimental layout for MS measurements of 400 GeV/c protons in crystals

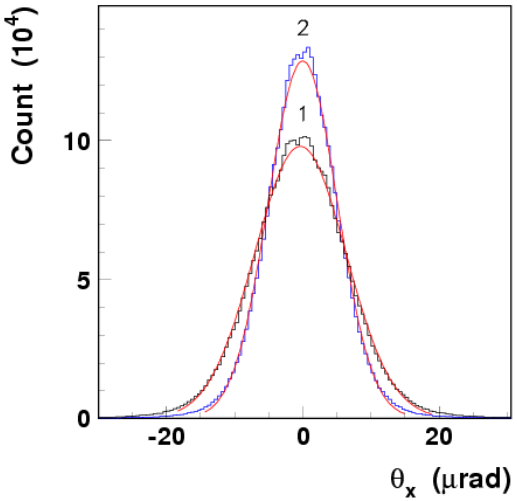
Five pairs of Si microstrip detectors D1-D5 with spatial resolution about 7 μm

Crystal was aligned by goniometer (2 μrad step) to be in AM position



Deflection angle = outgoing (D3-D5) – incoming (D1-D2)

Gaussian fits of the central 98% of the deflection angle distributions



RMS with crystal in AM position – σ_{am}

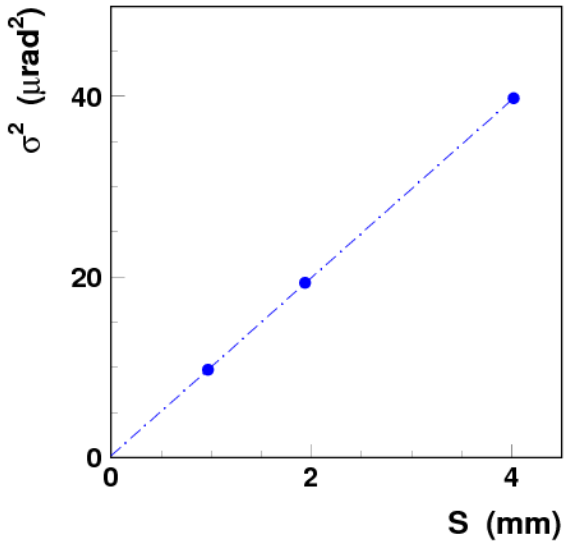
RMS without crystal – σ_{bg}

Distribution variance due to MS in crystal

$$\sigma^2 = \sigma_{am}^2 - \sigma_{bg}^2$$

Experimental results for MS of 400 GeV/c protons in Si crystals

MS measurements for Si crystals with S = 0.97, 1.94 and 4.02 mm

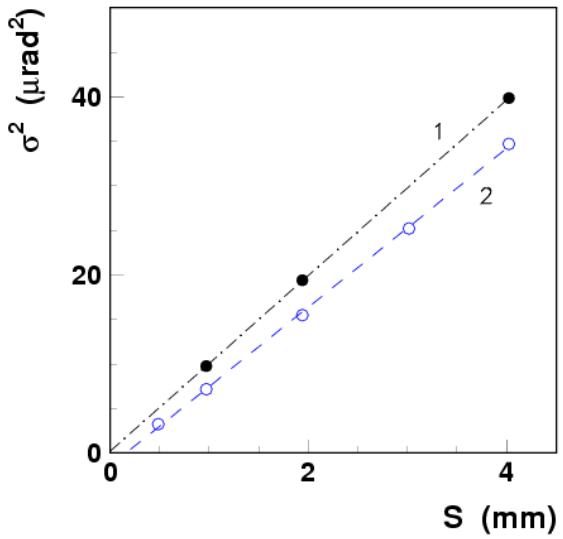


Linear fit gives the rate of variance increase - $d\sigma^2/dS$

Parameter	σ (μrad) Crystal 1	σ (μrad) Crystal 2	σ (μrad) Crystal 3	dσ²/dS (μrad²/mm)
Experiment	3.120±0.003	4.403±0.003	6.310±0.004	9.86±0.36

Comparison of experimental results with Moliere theory

Moliere distribution was calculated by numeric integration of its expression



Theoretical σ are smaller and variance increase rate is 10% smaller

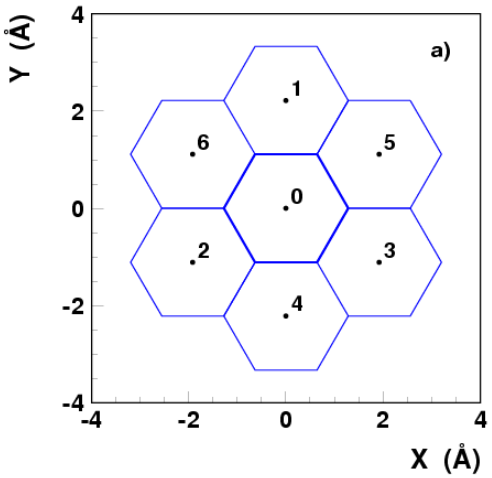
Parameter	σ (μrad) Crystal 1	σ (μrad) Crystal 2	σ (μrad) Crystal 3	dσ²/dS (μrad²/mm)
Experiment	3.120±0.003	4.403±0.003	6.310±0.004	9.86±0.36
Moliere theory	2.67±0.02	3.93±0.03	5.89±0.04	8.91±0.34

Simulation of MS in binary collision model (BCOL)

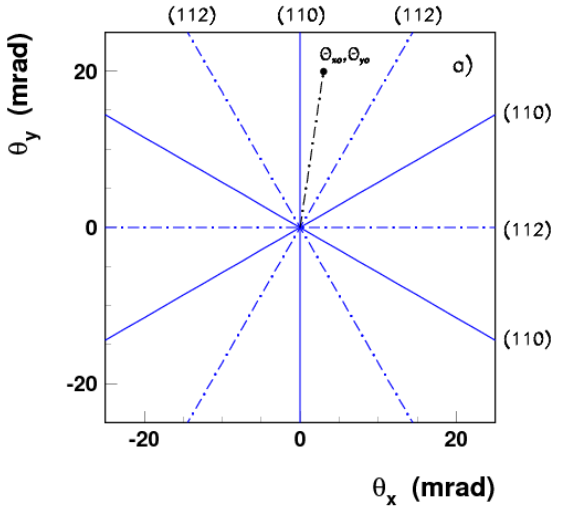
Entrance points are randomly and uniformly distributed on the symmetry cell

Beam direction (θ_{xo}, θ_{yo}) is far from <111> Si axis and main crystal planes

Symmetry cell along <111> axis



Angular space near <111> axis



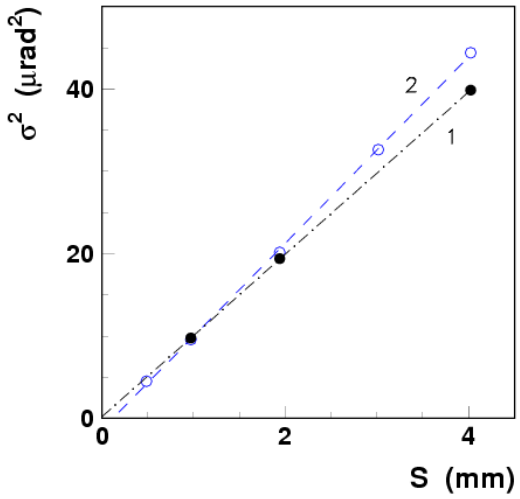
Scattering by the atom with Moliere potential in momentum approximation

$$\theta(r) = \frac{2Ze^2}{pva} \sum_{i=1}^{3} \alpha_i \beta_i K_1(\beta_i r / a)$$

Trajectory in binary collisions with the crystal atoms

Comparison of experimental results with BCOL simulations

Calculated angular distributions were fitted with Gaussians - σ



RMS from BCOL is 5.5% larger for the largest length and variance increase rate is 14% larger

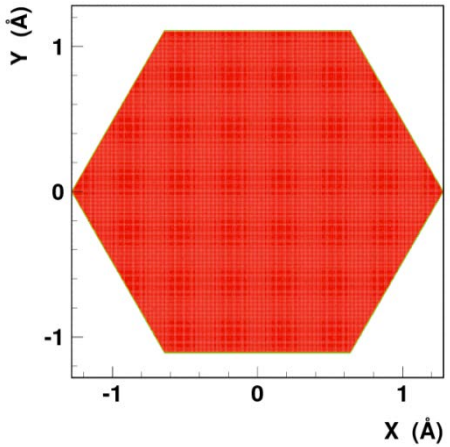
Parameter	σ (μrad) Crystal 1	σ (μrad) Crystal 2	σ (μrad) Crystal 3	dσ²/dS (μrad²/mm)
Experiment	3.120±0.003	4.403±0.003	6.310±0.004	9.86±0.36
Binary collisions	3.09±0.01	4.49±0.02	6.66±0.03	11.30±0.34

Direct simulation of multiple scattering in Si crystal

MS for a particle in the sequence of single scatterings with atoms

Impact parameters in scatterings are randomly and uniformly distributed on the crystal symmetry cell along <111> axis

<111> Si cell



It is similar to the usage of differential cross-section for screening potential

Number of scatterings along L

$N_L = L/d_a$

d_a – interatomic distance along S

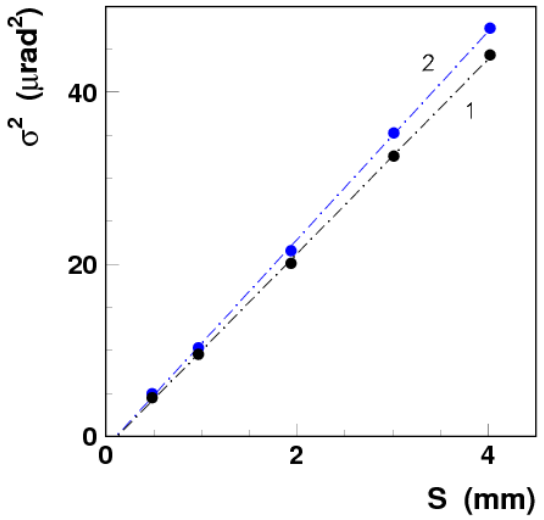
Resulted X-deflection in N scatterings is the sum

$$\theta_x(N) = \sum_{n=1}^{N_L} \theta(r_n) x_n / r_n \quad ,$$

$$r_n = (x_n^2 + y_n^2)^{1/2} \quad , \quad N_L = L/d_a \quad ,$$

Comparison of BCOL and direct simulations of MS in crystals

Calculated angular distributions were fitted with Gaussians - σ



RMS from direct simulations is 3.5% larger than from BCOL and variance increase rate is 7% larger

Parameter	σ (μrad) Crystal 1	σ (μrad) Crystal 2	σ (μrad) Crystal 3	dσ²/dS (μrad²/mm)
Binary collisions	3.09±0.01	4.49±0.02	6.66±0.03	11.30±0.34
Direct simulation	3.21±0.01	4.65±0.02	6.89±0.03	12.10±0.62

Conclusions

1. Good linear dependence of MS on crystal length has been observed

2. Experimental MS is a little greater than from Moliere theory;
variance increase rate with length is about 10% larger

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3. Distributions of positions across the symmetry cell for a single particle from BCOL are not uniform for short distances smaller than 1 μm

4. MS reduction in BCOL in comparison with direct simulations
may be explained by some collision correlations along short distances

5. RMS deflections due to MS of particles in crystals can be smaller by a few percent than for the same material in an amorphous state