

Betatron radiation as emittance diagnostics for plasma acceleration experiments

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Summary

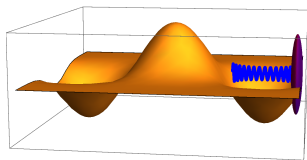
Betatron Radiation: Generalities

Transverse emittance measurement

Conclusions

Plasma acceleration

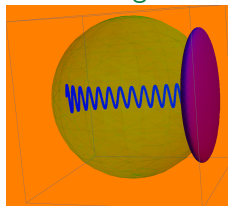
Linear/Quasi linear regime



Plasma Wakefields:

$$E_z(r, \zeta) = \frac{\sqrt{\pi}}{4} a_0^2 k_p c \tau_0 E_0 \exp \left[\frac{2r^2}{w_0^2} - \frac{k_p^2 c^2 \tau_0^2}{4} \right] \cos k_p \zeta$$
$$E_r(r, \zeta) = -\sqrt{\pi} a_0^2 \frac{c \tau_0 r}{w_0^2} E_0 \exp \left[\frac{2r^2}{w_0^2} - \frac{k_p^2 c^2 \tau_0^2}{4} \right] \sin k_p \zeta$$

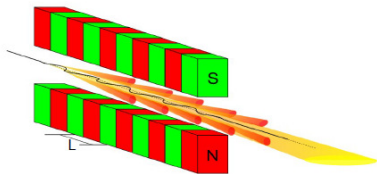
Bubble regime



Plasma Wakefields:

$$E_z(r, \zeta) = \frac{m}{3e} \omega_p^2 \zeta \cos k_p \zeta$$
$$E_r(r, \zeta) = \frac{m}{3e} \omega_p^2 r \sin k_p \zeta$$

Characteristics of the Betatron Radiation: wiggler analogy



The difference between a wiggler and an undulator resides in the anharmonic motion of the oscillating particles, due to high transverse velocities, which mirrors in a broadband spectrum, while the spectrum of an undulator is narrowband.

Wiggler parameters:

$$\lambda_W = \frac{L}{2\gamma_0^2} \left(1 + \frac{K_W^2}{2} + \gamma_0^2 \theta^2 \right)$$

$$K_W = 0.934 B[T] L[cm]$$

$$E_c = 3\gamma_0^2 K_W \hbar \omega_W$$

Plasma wiggler parameters:

$$\lambda_b = \frac{\lambda_\beta}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

$$K = \gamma k_\beta r_0$$

$$E_c = 3\gamma_0^2 K \hbar \omega_b$$

A method to measure the transverse emittance:

Definitions/1

$$\sqrt{\langle \Delta r^2 \rangle \langle \Delta \theta_d^2 \rangle - \langle \Delta r \Delta \theta_d \rangle^2}$$

$$P(r)$$

Def. geometric emittance

Transverse beam profile

$$\Theta(\theta_d)$$

Distribution of the angles
with respect to the
acceleration axis

$$\theta_d = \sqrt{\frac{\sqrt{1 + \frac{1}{2} \gamma_0^2 r_\beta^2 k_{\beta 0}^2}}{4 \gamma_0}} r_\beta k_p$$
$$\rightarrow r_\beta(\theta_d), r = r_\beta / \sqrt{2}$$

Correlation function

$$R(r) = rP(r)$$

Distribution of the
oscillation amplitudes

A method to measure the transverse emittance: Definitions/2

$$S_{\gamma, r_\beta}(E, \Omega)$$

Single particle spectrum

$$S(E) = \int dr_\beta d\gamma R(r_\beta) \Gamma(\gamma) d\Omega S_{\gamma, r_\beta}(E, \Omega)$$

Electron bunch spectrum

$$\int dr_\beta R(r_\beta) \sim \sum_i R_i$$

Trick

$$S(E_j) \equiv S_j = \sum_i R_i S_{ij}$$

Spectrum discretization

$$S_{ij} = \int \int d\Omega d\gamma \Gamma(\gamma) S_{\gamma, r_{\beta, i}}(E_j, \Omega)$$

Def. of the S matrix

$$S_j - \Sigma_j = \sum_{ij} R_i S_{ij} - \Sigma_j = 0$$

From the profile to the divergence

$$\omega_\beta = \omega_p / \sqrt{2\gamma}$$

Betatron frequency

$$\theta_d = \sqrt{\left\langle \left(\frac{dr}{dz} \right)^2 \right\rangle - \left\langle \left(\frac{dr}{dz} \right) \right\rangle^2} = \frac{\sqrt{2}}{2} r_\beta k_\beta$$

Single particle divergence

$$\gamma \sim \gamma_0$$

Linear correlation \rightarrow zero emittance

$$\gamma^* \sim \gamma_0 / \sqrt{1 + K_{\beta 0}^2 / 2}$$

Non linear correlation \rightarrow non-zero emittance

$$\Theta(\theta_d) \propto P(r(\theta_d))$$

Distribution of the correlated divergences

Experimental Results: Setup

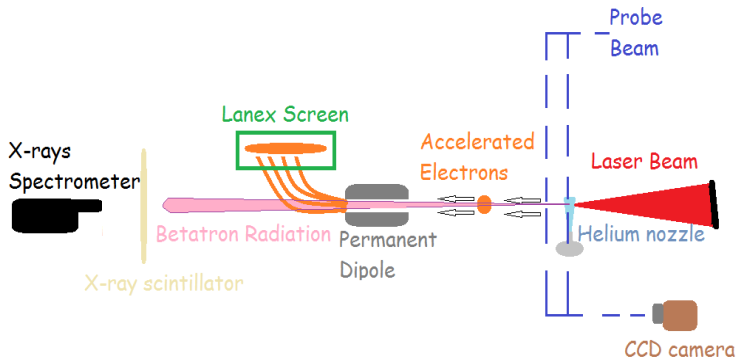


Figure: The main laser beam impinges on the supersonic helium jet, ionizing, channeling and generating plasma wakefields. The electrons, while accelerating, emit X-ray betatron radiation in the forward direction, detected by a CCD-X camera. The plasma channel is probed with interferometry. The electrons are damped out on the screen of the magnetic spectrometer.

Experimental Results: Plasma channel

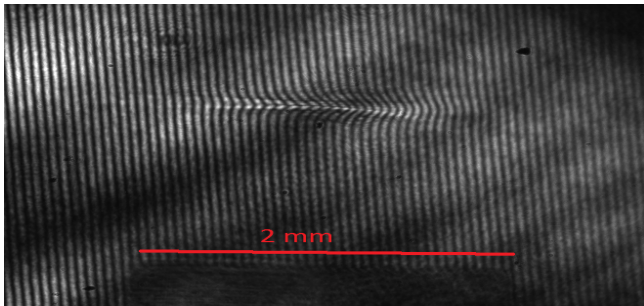


Figure: The Rayleigh length of the laser is evaluated $z_R \sim 100 \mu m$, while the detected high-intensity/high-density ($8 \times 10^{18} cm^{-3}$) region, where the acceleration occurs, is longer than one millimeter.

Index of refraction $\eta \sim 1 - \frac{\omega_p^2}{2\omega_0^2} \left(1 + \frac{\delta n_e}{n_0} - \frac{a^2}{4} \right)$

$a^2(x, y, z, t) \propto I(x, y, z, t)$ (**Laser Intensity**)

Experimental Results: Betatron Spectrum

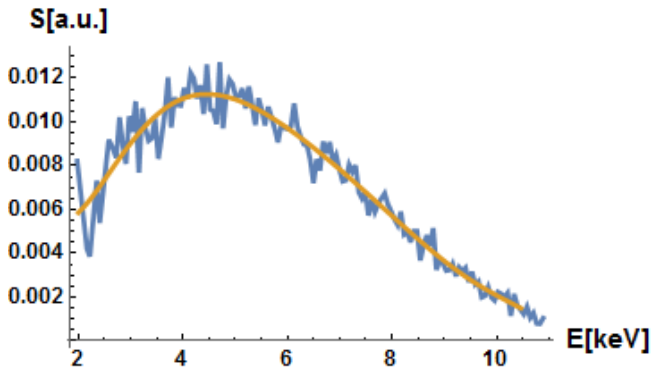


Figure: Betatron spectrum detected with a CCD-X camera working in single photon counting mode together with a polynomial fit.

Experimental Results: Electron Spectrum

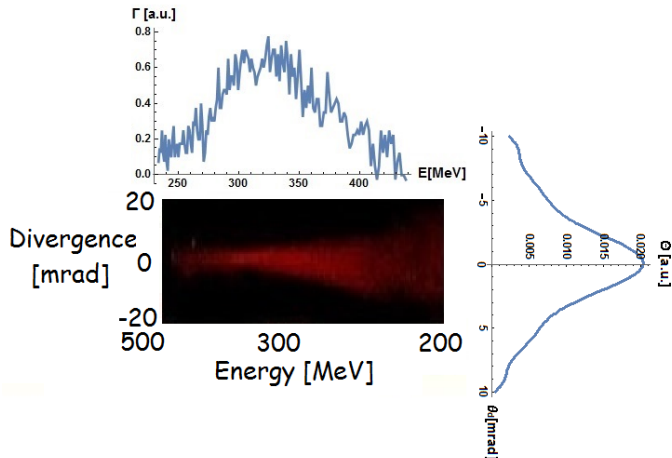


Figure: Electron energy spectrum relative to the electron bunch accelerated in the bubble regime. Central energy $\gamma_0 = 320$ MeV, energy spread $2\sigma_\gamma/\gamma_0 \sim 20$ %. The outlines of the energy spectrum (up) and of the beam divergence (right) are reported.

Experimental Results: Beam Profile Reconstruction

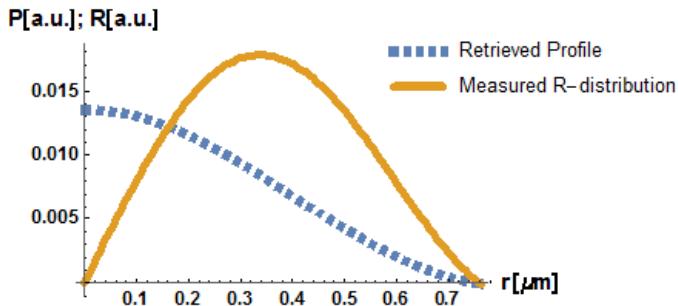


Figure: The beam profile $P(r)$ derived directly from the measured function $R(r_\beta)$. The rms betatron radius is found $\sigma_r \sim 0.42 \pm 0.04 \mu\text{m}$.

Experimental Results: Radiation Beam Distribution

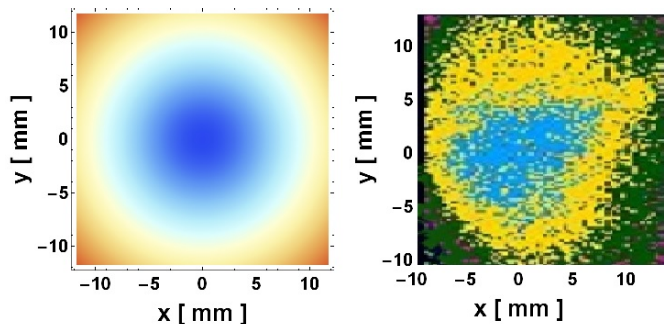


Figure: Betatron radiation spatial distribution: simulation (left) and measurement (right). Measured divergence of the radiation beam $\theta_{\beta}^{meas} = 8.2 \pm 0.3 \text{ mrad}$, simulation value $\theta_{\beta}^{sim} = 8.5 \text{ mrad}$.

Experimental Results: Distribution of the correlated divergences

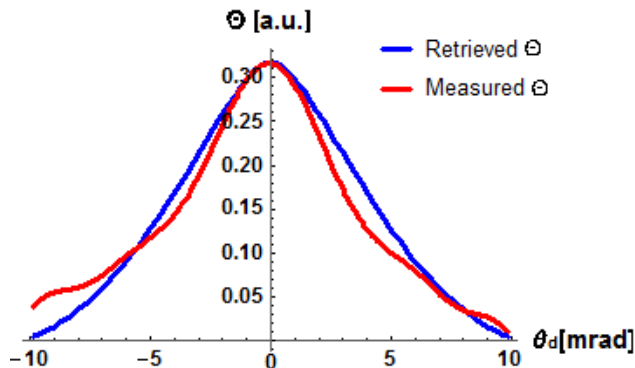


Figure: Comparison between the measured and the retrieved $\Theta(\theta_d)$ function. Corresponding values of beam divergence $\sigma_{\theta}^{meas} = 5.9 \pm 0.6 \text{ mrad}$ and $\sigma_{\theta}^{retr} = 5.2 \pm 0.6 \text{ mrad}$.

Experimental Results: Emittance Measurement

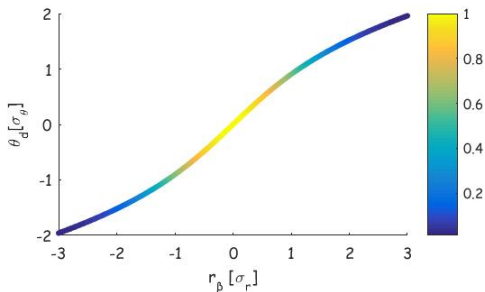


Figure: Reconstructed trace space density.

$$\epsilon_{r\beta} N = \gamma_0 \sqrt{(\sigma_\gamma/\gamma_0)^2 \sigma_r^2 \sigma_\theta^2 + \epsilon_{r\beta}^2} = (0.6 \pm 0.1) \text{ mm mrad}$$

Upper Limit: $\gamma_0 \sigma_r \sigma_\theta^{meas} = (1.5 \pm 0.3) \text{ mm mrad}$

Conclusions

- ★ We summarized typical features of betatron radiation
- ★ We presented a methodology able to reconstruct the transverse trace space of low emittance electron beams accelerated in the bubble regime. The single-shot measurement of both the betatron radiation spectrum and the electron energy spectrum can allow a complete measurement of the transverse emittance, including the correlation term.
- ★ We reported experimental results obtained at the SPARC-LAB test facility through the interaction of the ultra-short ultra intense $Ti : Sa$ laser FLAME with a He gas-jet target.

Thanks for your attention