

# ON THE MEASUREMENT OF TRANSITION RADIATION CHARACTERISTICS IN THE PREWAVE ZONE WITH THE USE OF RESTRICTED PARABOLIC MIRROR

*(dependence of the results on  
the detector size)*

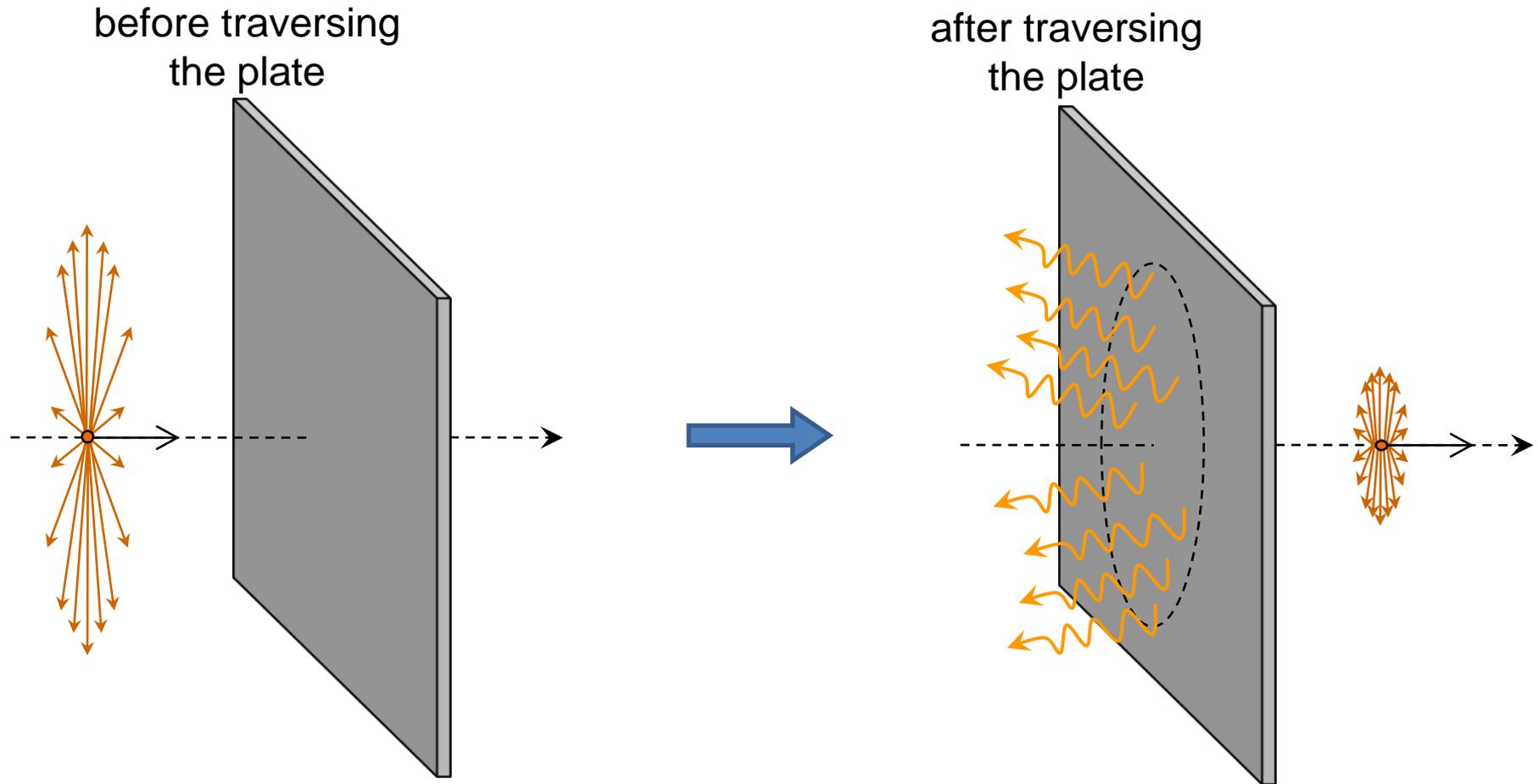
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Kharkiv, Ukraine*

*S.V. Trofymenko, N.F. Shul'ga // Phys. Rev. Accel. Beams (2016), in press*

*7<sup>th</sup> International Conference 'Channeling 2016', September 25-30, 2016, Sirmione -  
Desenzano del Garda, Italy*

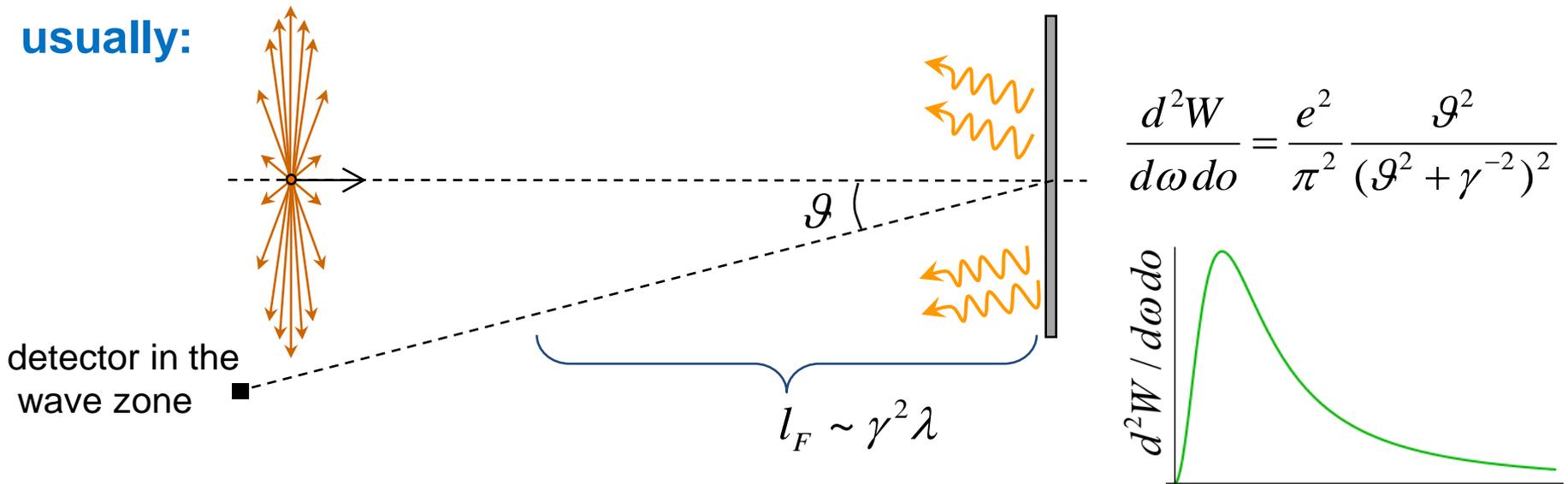
# BACKWARD TRANSITION RADIATION ON THIN METALLIC PLATE



Possibility of defining transversal and longitudinal size of the beam, its shape, divergence and particle energy via measurement of characteristics of radiation generated by the beam

# MEASUREMENT IN THE PREWAVE ZONE

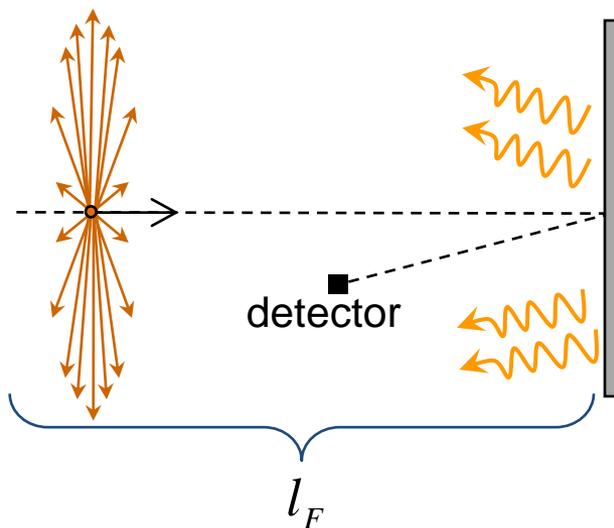
usually:



$\lambda$  – wavelength

$\gamma$  – particle Lorentz-factor

sometimes:



|   |
|---|
| $\lambda = 1\text{mm}$<br>$\mathcal{E} = 50\text{MeV}$<br>$l_F \sim 10\text{m}$ |
|---|

# PREWAVE ZONE EFFECT IN TRANSITION RADIATION

(difference between distributions over  $\vartheta$  and  $\alpha$ )

Verzilov V.A., *Phys. Lett. A* (2000)

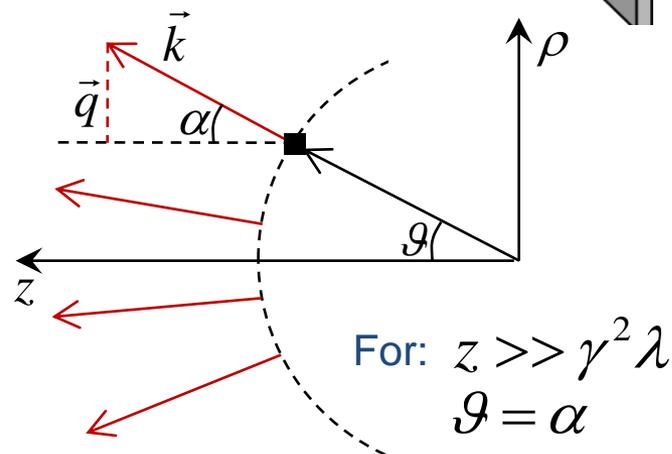
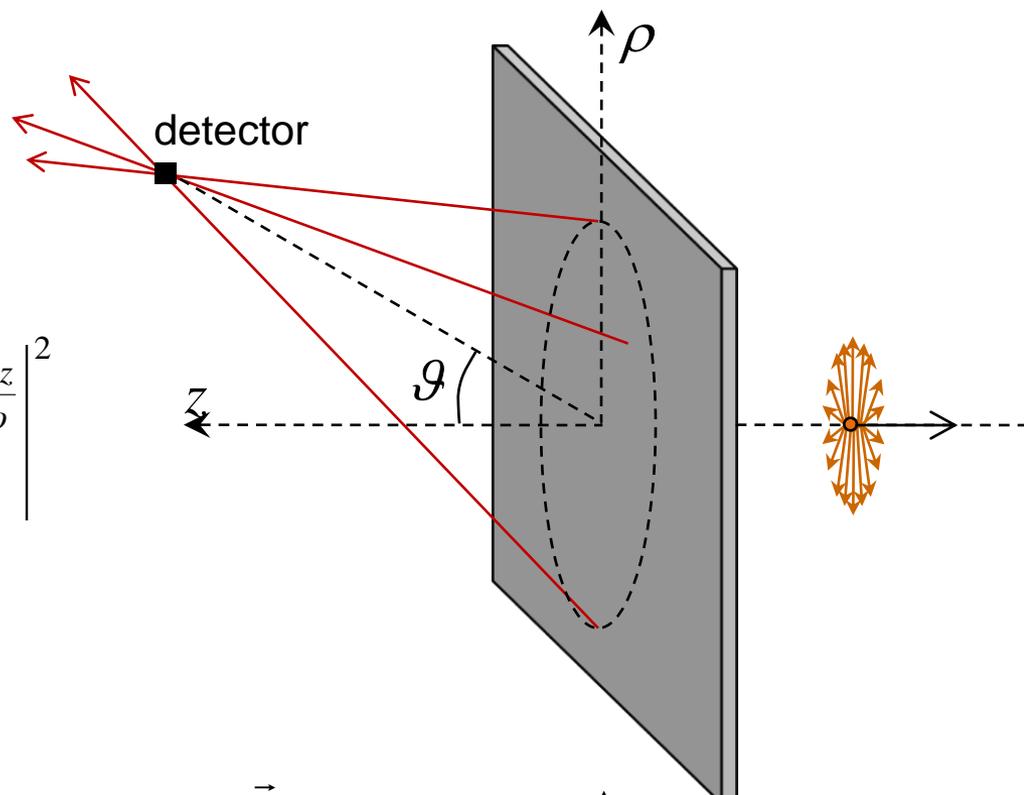
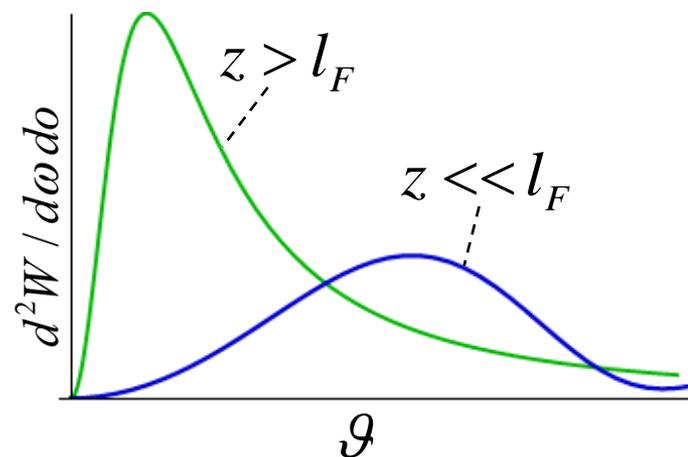
Dobrovolsky S.N., Shul'ga N.F., *NIM B* (2003)

spectral-angular density ( $z \ll \gamma^2 \lambda$ ):

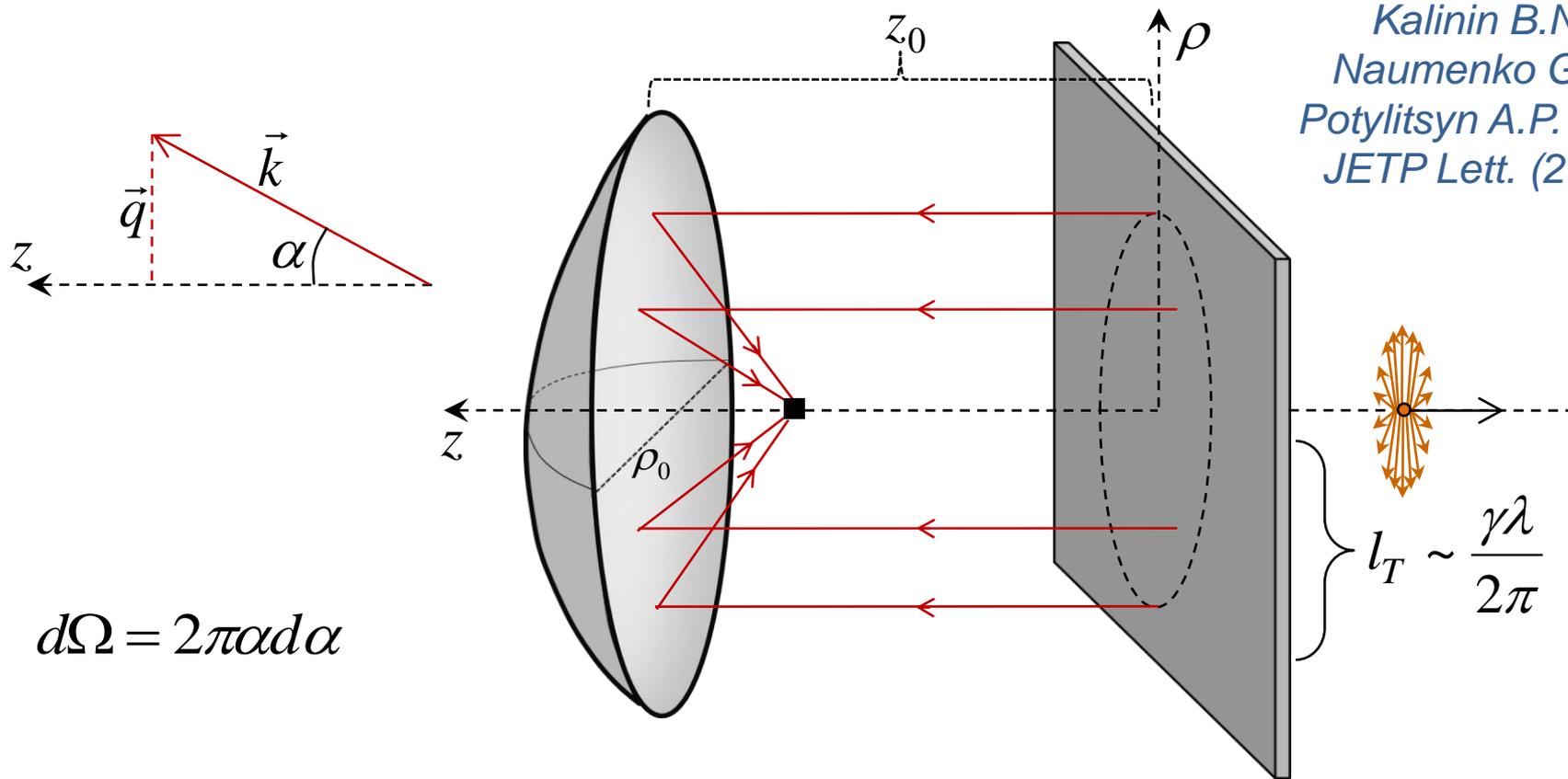
$$\frac{d^2W}{d\omega d\vartheta} = \frac{e^2 z^2}{\pi^2} \left| \int_0^\infty dq \frac{q^2 J_1(qz\vartheta)}{q^2 + \omega^2 / \gamma^2} e^{-\frac{iq^2 z}{2\omega}} \right|^2$$

$J_1(x)$  – Bessel function

$$d\vartheta = 2\pi \vartheta d\vartheta$$



# PREWAVE ZONE MEASUREMENTS WITH THE USE OF PARABOLIC MIRROR



*Kalinin B.N.,  
Naumenko G.A.,  
Potylitsyn A.P. et al.,  
JETP Lett. (2006)*

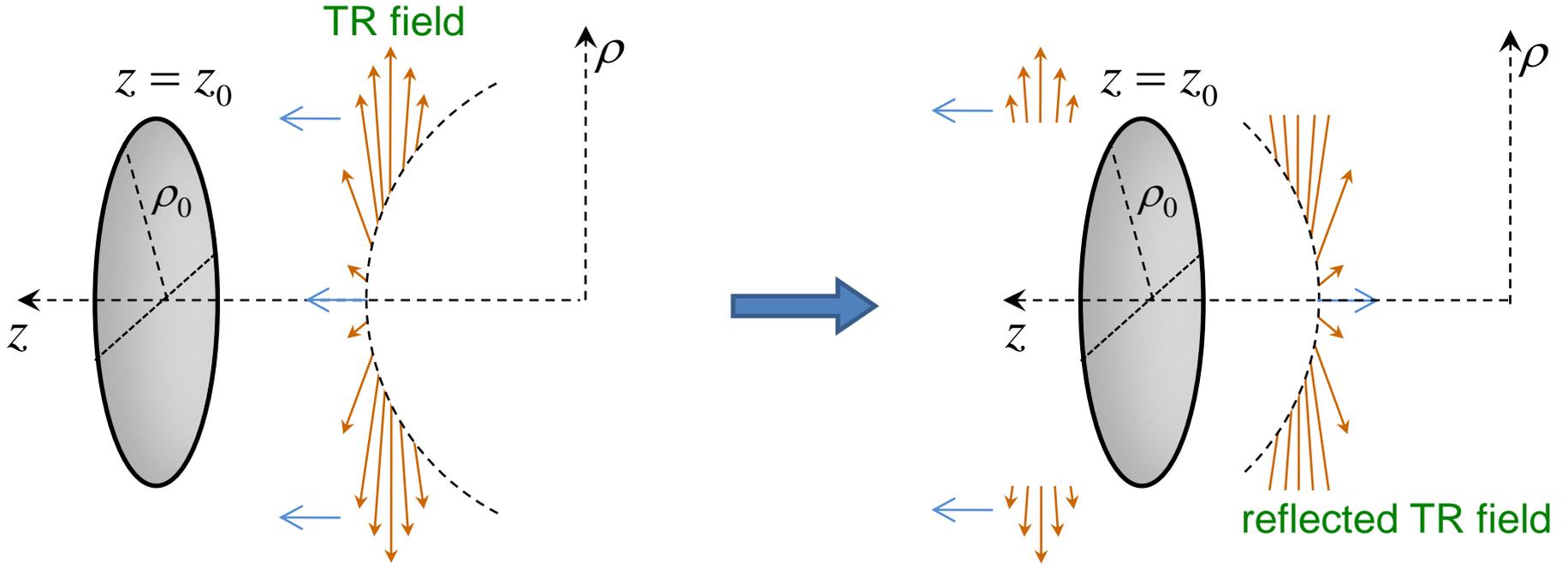
$$d\Omega = 2\pi\alpha d\alpha$$

**For:**

$$\rho_0 \gg l_T \quad \frac{d^2W}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\alpha^2}{(\alpha^2 + \gamma^{-2})^2} \quad \text{— coincides with distribution in the wave zone}$$

$\rho_0$  – transversal radius of the mirror

# CALCULATION OF TRANSITION RADIATION FIELD REFLECTED FROM THE PLANE MIRROR



**TR electric field:**

$$\vec{E}_\omega^{TR} = -\frac{ie}{\pi} e^{iz\omega} \int_0^\infty d^2q \frac{\vec{q}}{q^2 + \omega^2 / \gamma^2} e^{i\vec{q}\vec{\rho} - iq^2 z / 2\omega}$$

**boundary condition:**

$$\vec{E}_\omega^{refl}(\vec{\rho}, z_0) = -\theta(\rho_0 - \rho) \vec{E}_\omega^{TR}(\vec{\rho}, z_0)$$

# REFLECTED FIELD SPECTRAL DENSITY DISTRIBUTION OVER WAVE VECTOR DIRECTIONS

$$\vec{E}_\omega^{\text{refl}}(\rho, z) = 2e \frac{\vec{\rho}}{\rho} e^{-i\omega(z-2z_0)} \int dq q J_1(q\rho) F(q, \rho_0) e^{iq^2(z-z_0)/2\omega}$$

**where:** 
$$F(q, \rho_0) = \int_0^\infty d\eta \frac{\eta^2 e^{-iz_0\eta^2/2\omega}}{\eta^2 + \omega^2/\gamma^2} \int_0^{\rho_0} dx x J_1(qx) J_1(\eta x)$$

# REFLECTED FIELD SPECTRAL DENSITY DISTRIBUTION OVER WAVE VECTOR DIRECTIONS

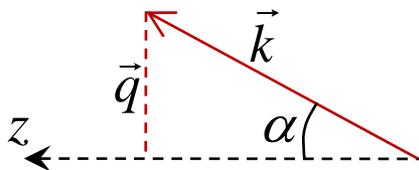
$$\vec{E}_\omega^{\text{refl}}(\rho, z) = 2e \frac{\vec{\rho}}{\rho} e^{-i\omega(z-2z_0)} \int dq q J_1(q\rho) F(q, \rho_0) e^{iq^2(z-z_0)/2\omega}$$

**where:** 
$$F(q, \rho_0) = \int_0^\infty d\eta \frac{\eta^2 e^{-iz_0\eta^2/2\omega}}{\eta^2 + \omega^2/\gamma^2} \int_0^{\rho_0} dx x J_1(qx) J_1(\eta x)$$

$$\frac{d^2W}{d\omega d\Omega} = \frac{e^2 \omega^4}{\pi^2} \left| \int_0^{\rho_0} d\rho \rho J_1(\omega\alpha\rho) \int_0^\infty d\eta \frac{\eta^2 J_1(\omega\eta\rho)}{\eta^2 + \gamma^{-2}} e^{-iz_0\omega\eta^2/2} \right|^2$$

$$q = \omega\alpha$$

$$d\Omega = 2\pi\alpha d\alpha$$



$J_i(x)$  – Bessel function

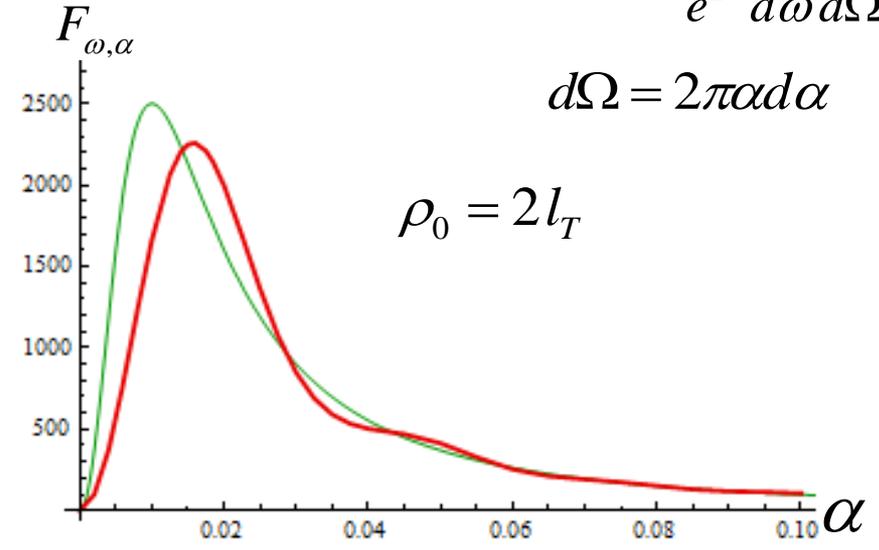
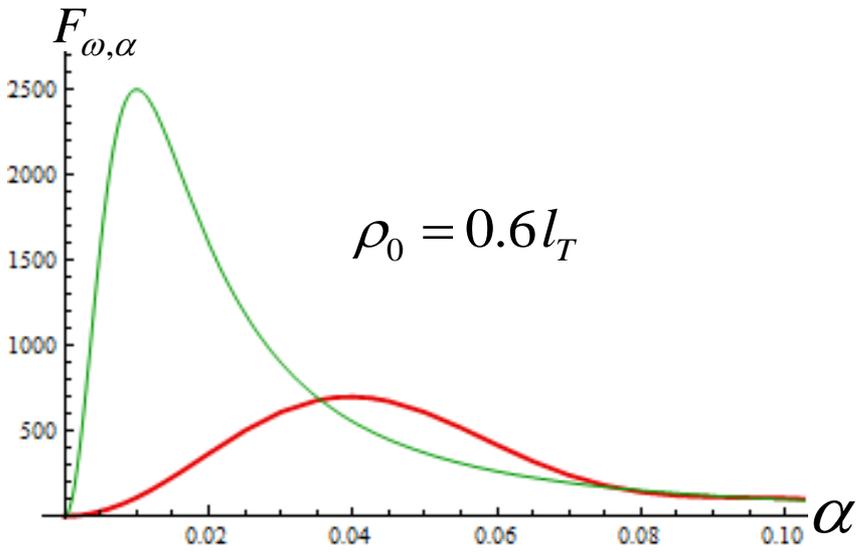
$K_i(x)$  – Macdonald function

|                                |                          |                         |                         |
|--------------------------------|--------------------------|-------------------------|-------------------------|
| $\mathcal{E} = 50 \text{ MeV}$ | $\lambda = 3 \text{ mm}$ | $l_F \sim 10 \text{ m}$ | $l_T \sim 5 \text{ cm}$ |
|--------------------------------|--------------------------|-------------------------|-------------------------|

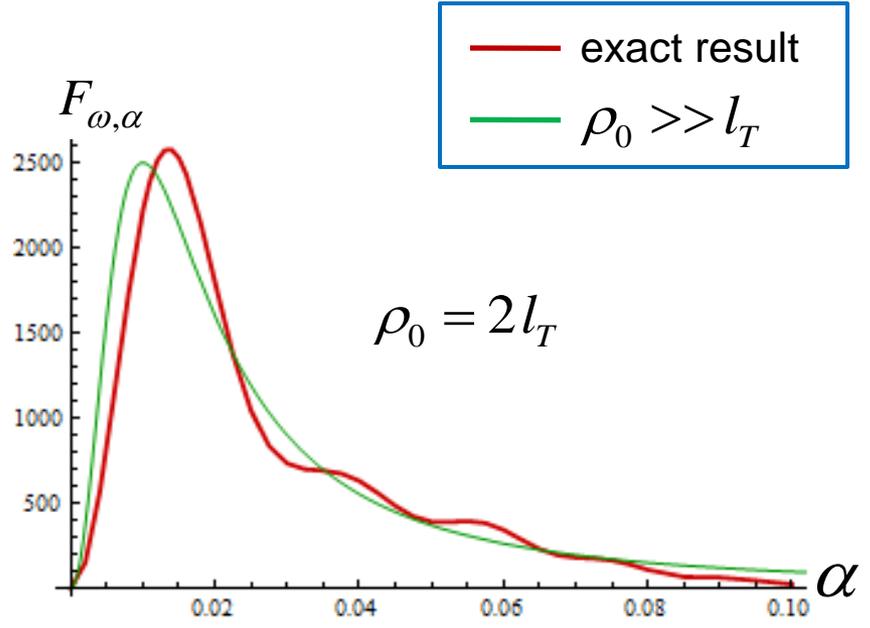
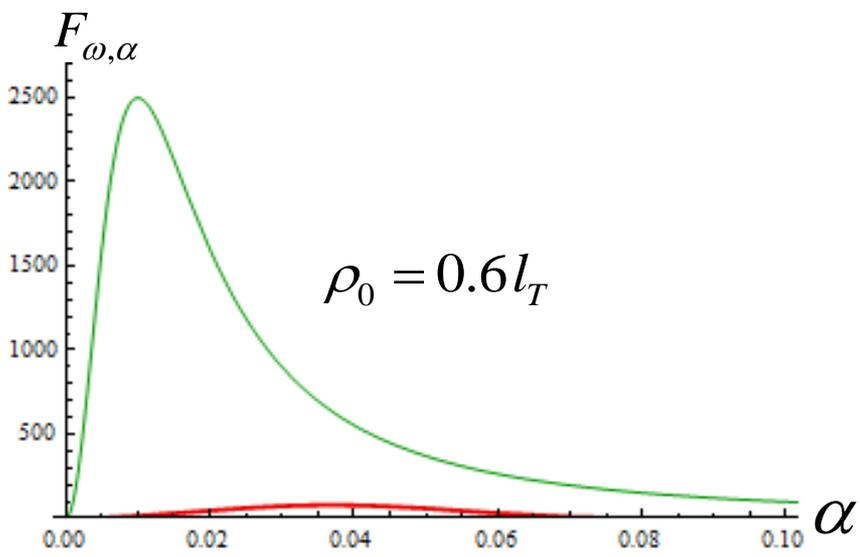
$$F_{\omega, \alpha} = \frac{\pi^2}{e^2} \frac{d^2 W}{d\omega d\Omega}$$

$$d\Omega = 2\pi \alpha d\alpha$$

For  $z_0 = 0.02 l_F$ :

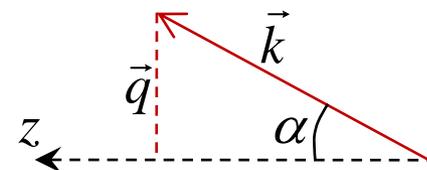


For  $z_0 = 0.1 l_F$ :



# REFLECTED FIELD ENERGY DISTRIBUTION OVER WAVE VECTOR DIRECTIONS

$$\frac{d^2W}{d\omega d\Omega} = \frac{e^2 \omega^4}{\pi^2} \left| \int_0^{\rho_0} d\rho \rho J_1(\omega \alpha \rho) \int_0^\infty d\eta \frac{\eta^2 J_1(\omega \eta \rho)}{\eta^2 + \gamma^{-2}} e^{-iz_0 \omega \eta^2 / 2} \right|^2$$



$$d\Omega = 2\pi \alpha d\alpha$$

**0-approximation** ( $z_0 \ll l_F$ ,  $\rho_0 \geq l_T$ ):

$$\frac{d^2W^{(0)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\alpha^2}{(\alpha^2 + \gamma^{-2})^2} |S^{(0)}|^2$$

where:  $S^{(0)}(\alpha, \omega, \rho_0) = 1 + \frac{\omega \rho_0}{\gamma} J_2(\omega \alpha \rho_0) K_1(\omega \rho_0 / \gamma) - \frac{\omega \rho_0}{\gamma^2 \alpha} J_1(\omega \alpha \rho_0) K_2(\omega \rho_0 / \gamma)$

coincides with distribution of transition radiation by electron on restricted target of radius  $\rho_0$  (N.F. Shul'ga, S.N. Dobrovolsky, JETP (2000))

**1-approximation** ( $z_0 \ll l_F$ , arbitrary  $\rho_0$ ):

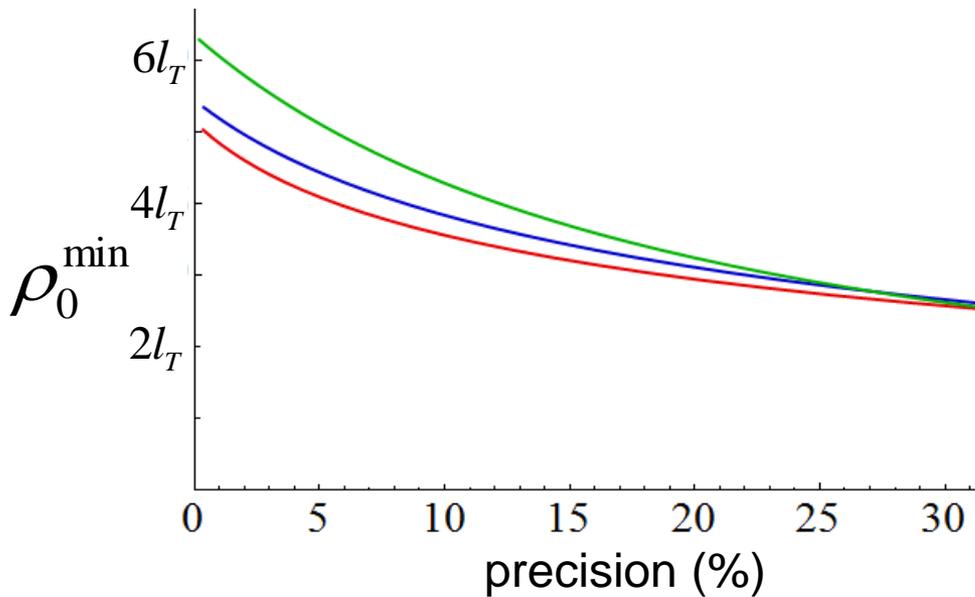
$$\frac{d^2W^{(1)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\alpha^2}{(\alpha^2 + \gamma^{-2})^2} |S^{(0)} + S^{(1)}|^2$$

$J_i(x)$  – Bessel function

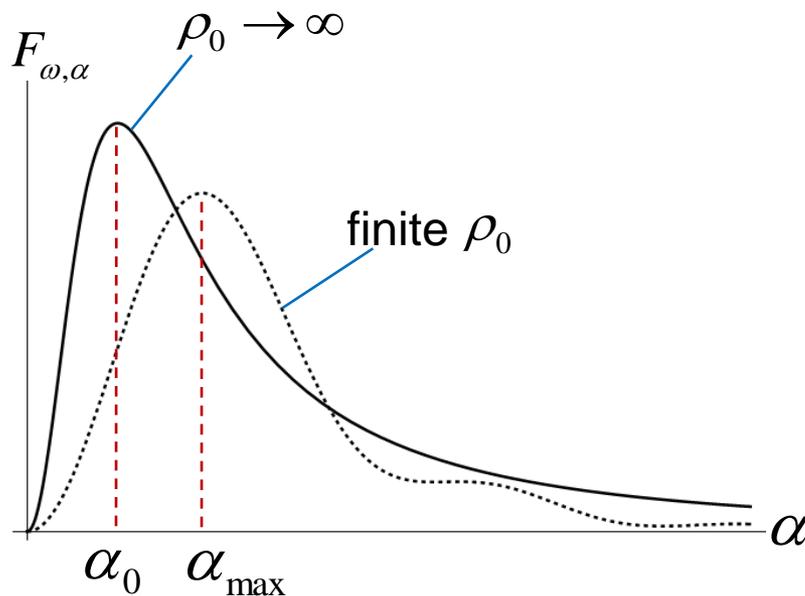
$K_i(x)$  – Macdonald function

where:  $S^{(1)}(\alpha, \omega, \rho_0) = -\frac{W}{\alpha} (\alpha^2 + \gamma^{-2}) \int_0^{\rho_0} d\rho J_1(\omega \alpha \rho) e^{i\omega \rho^2 / 2z_0}$

# THE MINIMUM SIZE OF THE MIRROR FOR THE 'WAVE ZONE RESULTS' IN THE PREWAVE ZONE



—  $z = 0.2l_F$   
 —  $z = 0.1l_F$   
 —  $z = 0.02l_F$



$$\text{precision} = \frac{\alpha_{\max} - \alpha_0}{\alpha_0}$$

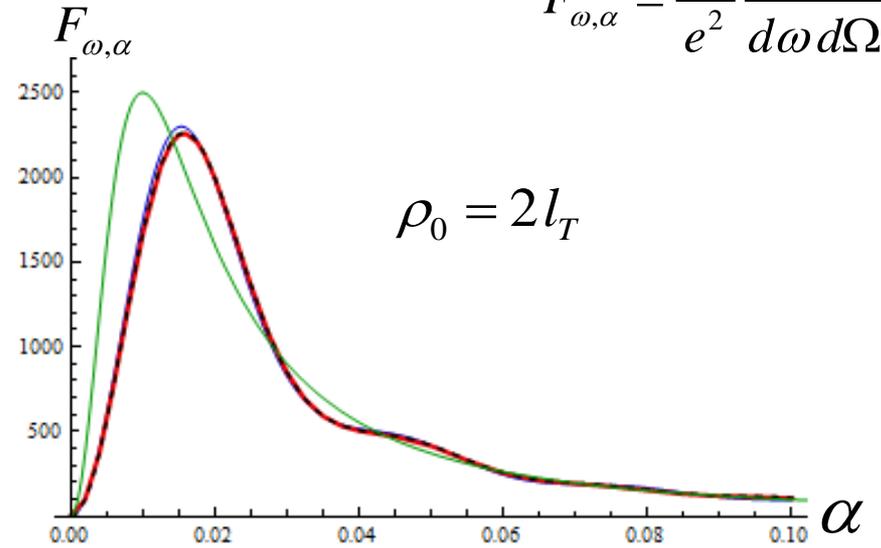
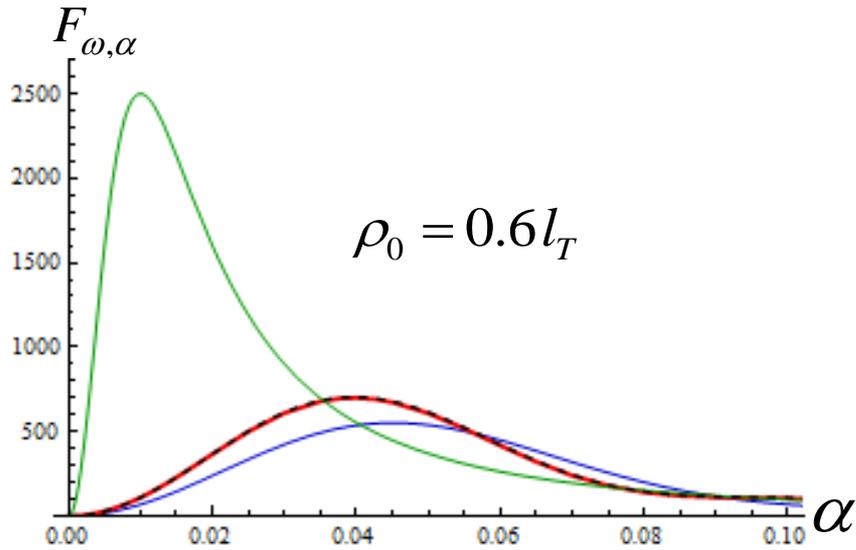
# CONCLUSIONS

- ❖ In the prewave zone there are two approaches to measurement of radiation angular distribution:
  - spatial distribution of radiated energy (over  $\mathcal{G} = \rho / z$ )
  - distribution over wave vector directions (over  $\alpha = q / k$ )
  
- ❖ In the wave zone both these distributions are identical
  
- ❖ The problem of radiation distribution measurement in the prewave zone with the use of the mirror of arbitrary radius  $\rho_0$  is considered
  
- ❖ The minimum transverse size of the detecting system for obtaining the 'wave zone results' in the prewave zone, as a function of the required accuracy, is defined

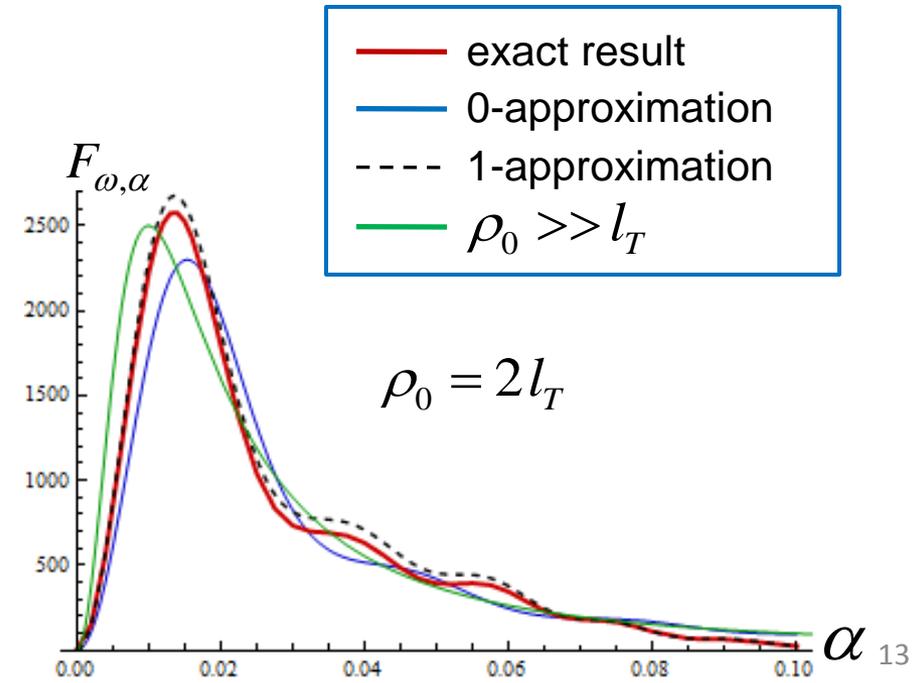
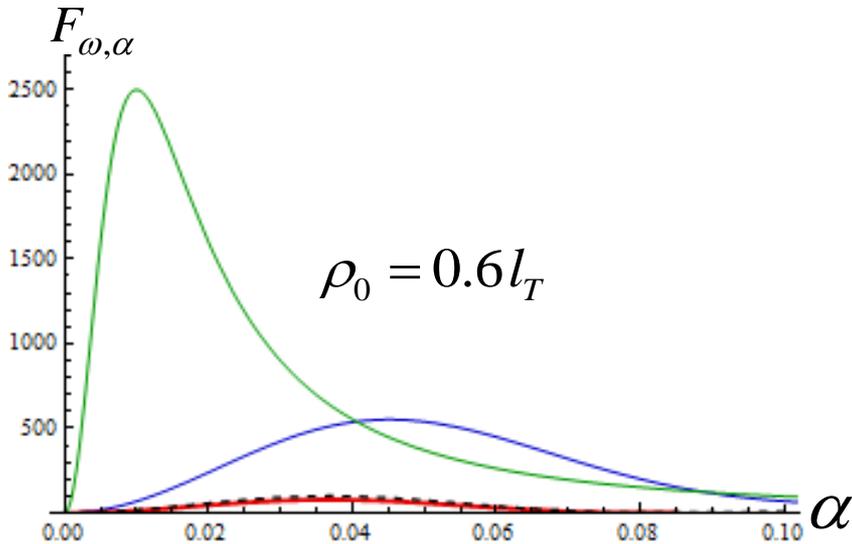
|                                |                          |                         |                         |
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| $\mathcal{E} = 50 \text{ MeV}$ | $\lambda = 3 \text{ mm}$ | $l_F \sim 10 \text{ m}$ | $l_T \sim 5 \text{ cm}$ |
|--------------------------------|--------------------------|-------------------------|-------------------------|

For  $z_0 = 0.02 l_F$ :

$$F_{\omega,\alpha} = \frac{\pi^2}{e^2} \frac{d^2W}{d\omega d\Omega}$$



For  $z_0 = 0.1 l_F$ :



|  |                  |
|--|------------------|
| <span style="color: red;">—</span>       | exact result     |
| <span style="color: blue;">—</span>      | 0-approximation  |
| <span style="color: black;">- - -</span> | 1-approximation  |
| <span style="color: green;">—</span>     | $\rho_0 \gg l_T$ |

