ON THE MEASUREMENT OF TRANSITION RADIATION CHARACTERISTICS IN THE PREWAVE ZONE WITH THE USE OF RESTRICTED PARABOLIC MIRROR

(dependence of the results on the detector size)

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Possibility of defining transversal and longitudinal size of the beam, its shape, divergence and particle energy via measurement of characteristics of radiation generated by the beam.
MEASUREMENT IN THE PREWAVE ZONE

usually:

- detector in the wave zone

sometimes:

- detector

- $l_F \sim \gamma^2 \lambda$

- $\lambda$ – wavelength

- $\gamma$ – particle Lorenz-factor

- $\frac{d^2W}{d\omega\,d\phi} = \frac{e^2}{\pi^2} \frac{\mathcal{G}^2}{(\mathcal{G}^2 + \gamma^{-2})^2}$

- $\lambda = 1\,mm$

- $\mathcal{E} = 50\,MeV$

- $l_F \sim 10\,m$
PREWAVE ZONE EFFECT IN TRANSITION RADIATION
(difference between distributions over $\vartheta$ and $\alpha$)


spectral-angular density ($z \ll \gamma^2 \lambda$):

$$ \frac{d^2W}{d\omega \, d\vartheta} = \frac{e^2 z^2}{\pi^2} \left| \int_0^\infty dq \frac{q^2 J_1(qz, \vartheta)}{q^2 + \omega^2 / \gamma^2} e^{-\frac{iqz \vartheta}{2\omega}} \right|^2 $$

$J_1(x)$ – Bessel function

$do = 2\pi \vartheta d\vartheta$

For:

$$ z \gg \gamma^2 \lambda $$

$\vartheta = \alpha$
PREWAVE ZONE MEASUREMENTS WITH THE USE OF PARABOLIC MIRROR

For:

\[ \rho_0 >> l_T \]

\[ \frac{d^2 W}{d \omega d \Omega} = \frac{e^2}{\pi^2} \frac{\alpha^2}{(\alpha^2 + \gamma^{-2})^2} \]

– coincides with distribution in the wave zone

\[ \rho_0 \] – transversal radius of the mirror

CALCULATION OF TRANSITION RADIATION FIELD REFLECTED FROM THE PLANE MIRROR

TR electric field:

\[
\vec{E}^{TR}_\omega = -\frac{ie}{\pi} e^{iz\omega} i q \int_0^\infty d^2 q \frac{\vec{q}}{q^2 + \omega^2 / \gamma^2} e^{iq\rho - iq^2 z / 2\omega}
\]

boundary condition:

\[
\vec{E}^{refl}_\omega (\rho, z_0) = -\theta(\rho_0 - \rho) \vec{E}^{TR}_\omega (\rho, z_0)
\]
REFLECTED FIELD SPECTRAL DENSITY DISTRIBUTION OVER WAVE VECTOR DIRECTIONS

\[ \vec{E}^\text{refl}_\omega (\rho, z) = 2e \frac{\vec{\rho}}{\rho} e^{-i\omega(z-2z_0)} \int dq q J_1(q \rho) F(q, \rho_0) e^{iq^2(z-z_0)/2\omega} \]

where:

\[ F(q, \rho_0) = \int_0^\infty d\eta \frac{\eta^2 e^{-iz_0\eta^2/2\omega}}{\eta^2 + \omega^2 / \gamma^2} \int_0^\infty dxx J_1(qx) J_1(\eta x) \]
REFLECTED FIELD SPECTRAL DENSITY DISTRIBUTION OVER WAVE VECTOR DIRECTIONS

\[ \vec{E}_{\omega}^{\text{refl}}(\rho, z) = 2e^{i\omega(z-2z_0)} \int dq q J_1(q\rho) F(q, \rho_0) e^{iq^2(z-z_0)/2\omega} \]

where:

\[ F(q, \rho_0) = \int_0^\infty d\eta \frac{\eta^2 e^{-iz_0\eta^2/2\omega}}{\eta^2 + \omega^2 / \gamma^2} \int_0^\infty dxx J_1(qx) J_1(\eta x) \]

\[ \frac{d^2W}{d\omega d\Omega} = \frac{e^2 \omega^4}{\pi^2} \left[ \int_0^{\rho_0} d\rho \rho J_1(\omega\alpha\rho) \int_0^\infty d\eta \frac{\eta^2 J_1(\omega\eta\rho)}{\eta^2 + \gamma^{-2}} e^{-iz_0\omega \eta^2 / 2} \right]^2 \]

\[ q = \omega\alpha \]
\[ d\Omega = 2\pi\alpha d\alpha \]

\[ J_i(x) – \text{Bessel function} \]
\[ K_i(x) – \text{Macdonald function} \]
For $z_0 = 0.02l_F$:

$\rho_0 = 0.6l_T$  

$F_{\omega,\alpha}$

For $z_0 = 0.1l_F$:

$\rho_0 = 0.6l_T$  

$\rho_0 = 2l_T$  

$F_{\omega,\alpha}$

\[ F_{\omega,\alpha} = \frac{\pi^2}{e^2} \frac{d^2W}{d\omega d\Omega} \]

\[ d\Omega = 2\pi\alpha d\alpha \]

For:

\[ \rho_0 >> l_T \]

\[ M_{eV} = 50 \]

\[ l_T = 5 cm \]

\[ l_F = 10 m \]

\[ \lambda = 3 mm \]
REFLECTED FIELD ENERGY DISTRIBUTION OVER WAVE VECTOR DIRECTIONS

\[ \frac{d^2W}{d\omega d\Omega} = \frac{e^2 \omega^4}{\pi^2} \left| \int_0^{\rho_0} d\rho \rho J_1(\omega \alpha \rho) \int_0^\infty d\eta \frac{\eta^2 J_1(\omega \eta \rho)}{\eta^2 + \gamma^{-2}} e^{-iz_0\omega \eta^2/2} \right|^2 \]

0-approximation \((z_0 << l_F, \rho_0 \geq l_T)\): \[
\frac{d^2W^{(0)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\alpha^2}{(\alpha^2 + \gamma^{-2})^2} |S^{(0)}|^2
\]

where: \(S^{(0)}(\alpha, \omega, \rho_0) = 1 + \frac{\omega \rho_0}{\gamma} J_2(\omega \alpha \rho_0)K_1(\omega \rho_0 / \gamma) - \frac{\omega \rho_0}{\gamma^2 \alpha} J_1(\omega \alpha \rho_0)K_2(\omega \rho_0 / \gamma)\)

coincides with distribution of transition radiation by electron on restricted target of radius \(\rho_0\) \((N.F. Shul'ga, S.N. Dobrovolsky, JETP (2000))\)

1-approximation \((z_0 << l_F, \text{arbitrary } \rho_0)\): \[
\frac{d^2W^{(1)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\alpha^2}{(\alpha^2 + \gamma^{-2})^2} \left| S^{(0)} + S^{(1)} \right|^2
\]

where: \(S^{(1)}(\alpha, \omega, \rho_0) = -\frac{w}{\alpha} (\alpha^2 + \gamma^{-2}) \int_0^{\rho_0} d\rho J_1(\omega \alpha \rho)e^{iw\rho^2/2z_0} \)

\(J_i(x)\) – Bessel function \(K_i(x)\) – Macdonald function

\(d\Omega = 2\pi\alpha d\alpha\)
THE MINIMUM SIZE OF THE MIRROR FOR THE ‘WAVE ZONE RESULTS’ IN THE PREWAVE ZONE

\[ \rho_0^{\text{min}} \]

precision (%)

\[ F_{\omega, \alpha} \]

finite \( \rho_0 \)

\[ \alpha_0 \rightarrow \infty \]

\[ \text{precision} = \frac{\alpha_{\text{max}} - \alpha_0}{\alpha_0} \]

Graph with curves for \( z = 0.2l_F \), \( z = 0.1l_F \), and \( z = 0.02l_F \).
CONCLUSIONS

- In the prewave zone there are two approaches to measurement of radiation angular distribution:
  - spatial distribution of radiated energy (over $\mathcal{J} = \rho/z$)
  - distribution over wave vector directions (over $\alpha = q/k$)

- In the wave zone both these distributions are identical

- The problem of radiation distribution measurement in the prewave zone with the use of the mirror of arbitrary radius $\rho_0$ is considered

- The minimum transverse size of the detecting system for obtaining the ‘wave zone results’ in the prewave zone, as a function of the required accuracy, is defined
For $z_0 = 0.02l_F$:

$$F_{\omega,\alpha}$$

$\rho_0 = 0.6l_T$

For $z_0 = 0.1l_F$:

$$F_{\omega,\alpha}$$

$\rho_0 = 0.6l_T$

$\rho_0 = 2l_T$

$\rho_0 = 2l_T$

$\rho_0 >> l_T$

$F_{\omega,\alpha} = \frac{\pi^2}{e^2} \frac{d^2W}{d\omega d\Omega}$

$E = 50\, MeV$  $\lambda = 3\, mm$  $l_F \sim 10\, m$  $l_T \sim 5\, cm$