B $\longrightarrow \pi$ SEMILEPTONIC DECAYS

Status Report from Lattice QCD

CKM2008

Rome, 9 - 13 September, 2008

presented by :

Junko Shigemitsu
The Ohio State University

Extraction of $|V_{ub}|$ from exclusive $B \to \pi$ semileptonic decay branching fractions requires knowing the form factor $f_+(q^2)$.

$$\frac{d\Gamma(B \to \pi)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3} \lambda^3 |V_{ub}|^2 |f_+(q^2)|^2$$

$$\lambda \equiv \left[(M_B^2 + m_\pi^2 - q^2)^2 - 4M_B^2 m_\pi^2\right]^{1/2}$$

Form factors parameterize nonperturbative QCD effects in hadronic matrix elements.

$$\begin{aligned} \langle \pi(p_\pi)|V^\mu|B(p_B)\rangle &= f_+(q^2) \left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - m_\pi^2}{q^2} q^\mu\right] \\ &+ f_0(q^2) \frac{M_B^2 - m_\pi^2}{q^2} q^\mu \\ &= \sqrt{2M_B}\,[v^\mu f_\parallel + p_\perp^\mu f_\perp] \end{aligned}$$

$$v^\mu = \frac{p_B^\mu}{M_B}, \quad p_\perp^\mu = p_\pi^\mu - (p_\pi \cdot v)\, v^\mu, \quad q^\mu = p_B^\mu - p_\pi^\mu$$

Lattice QCD Calculations of $f_+(q^2)$

The initial pioneering calculations were all carried out in the quenched ($N_f = 0$) approximation (UKQCD, APE, JLQCD, FNAL, HPQCD).

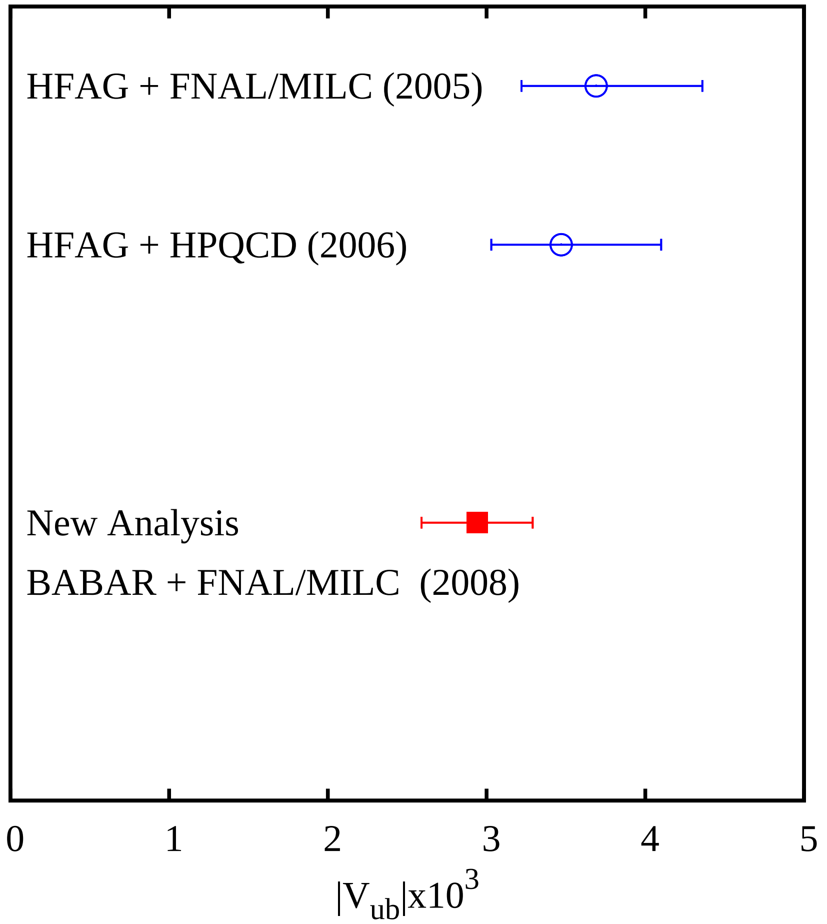
$\boxed{N_f = 2 + 1}$

FNAL/MILC : Nucl.Phys.Proc.Suppl. 140, 461 (2005)

HPQCD : PRD73:074502 (2006),
Erratum-ibid.D75:119906 (2007)

FNAL/MILC : New Analysis (2008)
talk by Ruth Van de Water at LAT08.

Recent Lattice Results for Exclusive $|V_{ub}|$



$$\boxed{3.69 \pm 0.21^{+0.64}_{-0.42} \times 10^{-3}}$$

$$\boxed{3.47 \pm 0.20^{+0.60}_{-0.39} \times 10^{-3}}$$

$\sim$ 15% errors

$\boxed{2.94 \pm 0.35 \times 10^{-3}}$ (Preliminary)

(Van de Water at LAT08)

$\sim$ 12% errors

Types of Errors

Errors in $f_+(q^2)$

- Statistical & Fitting

- Chiral and Continuum Extrapolations

- Other (operator matching, discretization errors in heavy and light quark actions, finite volume, tuning of masses ....)

Errors upon going from $f_+(q^2)$ to $|V_{ub}|$

—- most lattice data only at $q^2 > 16\text{GeV}^2$

—- how to exploit experimental data over entire q^2 range

Errors in $f_+(q^2)$

1. Statistical and Fitting Errors

For many static quantities (masses, decay constants, B_K ...) systematic errors now dominate over statistical errors.

Not true for semileptonic form factor calculations. Statistical errors are still at the $8 \sim 10\%$.

— complicated fits necessary

— need hadrons (usually just the pion) with nonzero momenta

Want : $\langle \pi(p_\pi)|V_\mu|B(p_B)\rangle$

What is actually calculated is a 3-point function

$$C^{(3)}(\vec{p}_\pi,\vec{p}_B,t,T_B) = \sum_{\vec{z}}\sum_{\vec{y}} \langle \Phi_\pi(0) V_\mu(\vec{z},t) \Phi^\dagger_B(\vec{y},T_B)\rangle \, e^{i\vec{p}_B\cdot\vec{y}}\, e^{i(\vec{p}_\pi-\vec{p}_B)\cdot\vec{z}}$$

$$C^{(3)}(\vec{p}_\pi,\vec{p}_B,t,T_B) \;\to\; \sum_{k=0}^{N_\pi-1}\sum_{j=0}^{N_B-1} (-1)^{k*t}\,(-1)^{j*(T_B-t)} \times A_{jk}\, e^{-E_\pi^{(k)}t}\, e^{-E_B^{(j)}(T_B-t)}$$

$$\boxed{A_{00} \propto \langle \pi(p_\pi)|V_\mu|B(p_B)\rangle}$$

How can we reduce statistical + fitting errors ?

- increase statistics considerably

- double ratios
  used successfully for heavy-to-heavy (Hashimoto et al., Laiho)
  and light-to-light (Becirevic)
  exploratory studies in D semileptonic (Haas et al.)

- better sources Φ_B, Φ_π
  smearings, random wall sources (K.Wong, E.Gregory et al.)

- "twisted B.C." (used by ETMC and Becirevic-Haas-Mescia in D semileptonic decays)

- better fitting methods exploiting Bayesian statistics

Fighting statistical + fitting errors :

Will require huge effort and vastly more computational resources.

Not very glamorous but crucial if we want to reduce errors in $f_+(q^2)$ and $|V_{ub}|$. Our experience with other quantities is that once statistical + fitting errors are at the $\sim$% level or less, systematic errors such as chiral & continuum extrapolations can be dealt with in a much more sophisticated way.

Errors in $f_+(q^2)$

2. Chiral and Continuum Extrapolations

Lattice simulations are done with light quark masses $m_l > m_{u/d}$ and at finite lattice spacings a. Accummulate results at several values of m_l and a and then extrapolate,

$$m_{light} \to m_{u/d} \qquad\qquad a \to 0$$

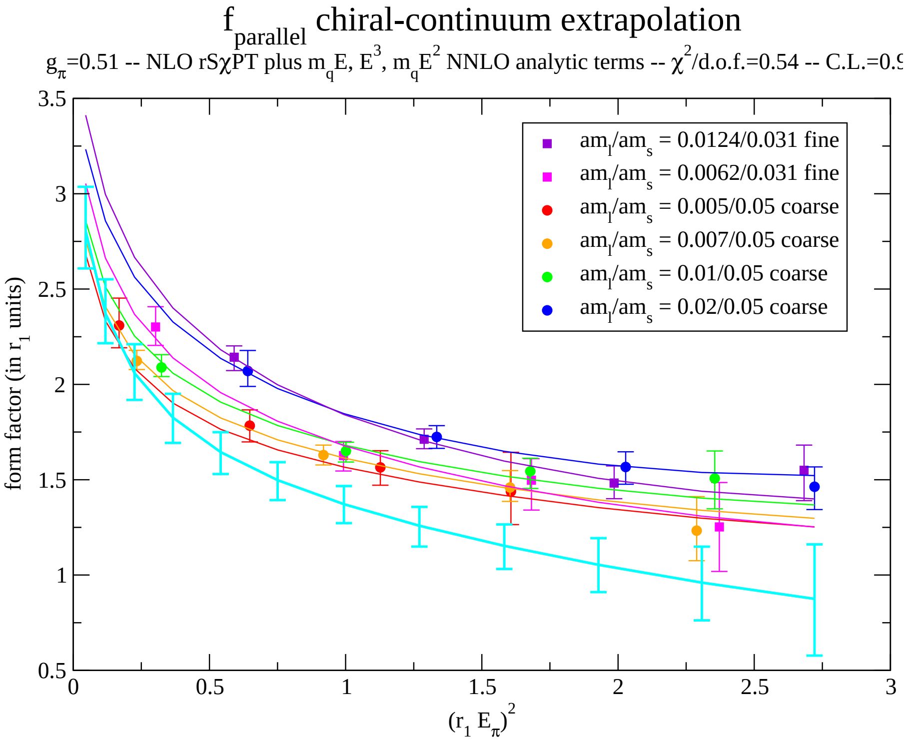
Guidance from (Staggered) Chiral Perturbaton Theory (Aubin & Bernard, Becirevic et al., Lee & Sharpe)

New Analysis (van de Water 2008) uses,

$$f_{\|,\perp} = c_0[1+(chiral-logs)+c_1 m_l + c_2 \sum m_{sea} + c_3 E_\pi + c_4 E_\pi^2 + c_5 a^2]$$
$$+ \text{ other terms} \propto m_l^p E_\pi^q$$

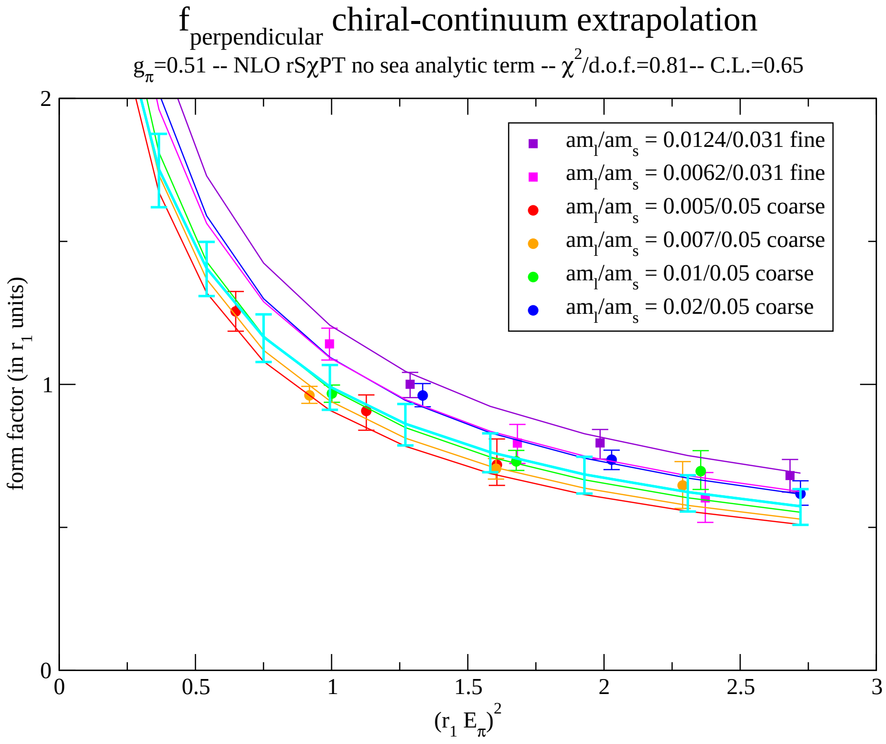
and fits to all data simultaneously ($E_\pi \equiv v \cdot p_\pi$).

Chiral-Continuum Extrapolations of $f_\parallel$ and $f_\perp$



$f_{parallel}$ chiral-continuum extrapolation

g_π=0.51 -- NLO rSχPT plus $m_q E$, E^3, $m_q E^2$ NNLO analytic terms -- χ^2/d.o.f.=0.54 -- C.L.=0.9



$f_{perpendicular}$ chiral-continuum extrapolation

g_π=0.51 -- NLO rSχPT no sea analytic term -- χ^2/d.o.f.=0.81-- C.L.=0.65

(Van de Water LAT08)

New Analysis is an improvement over past approaches

$$f_{\parallel,\perp} = c_0[1+(chiral-logs)+c_1 m_l+c_2\sum m_{sea}+c_3E_\pi+c_4E_\pi^2+c_5a^2]$$

In the past c_3 and c_4 not included. The remaining c_i taken to be unknown functions of E_π; i.e. chiral extrapolations had to be done at fixed E_π

- original data interpolated to common values of E_π using some model ansatz (BK, BZ etc.) $\rightarrow$ model dependence

- separate chiral extrapolations carried out for each value of E_π.

So New Analysis has removed a previous model dependence whose effects were hard to quantify.

However, extrapolation errors still $\sim$ 7% !

How can we reduce Chiral-Continuum Extrapolation Errors ?

Once statistical-fitting errors are significantly reduced, one should be able to introduce many more higher order terms into

$$f_{\parallel,\perp} = c_0[1 + (chiral - logs) + c_1 m_l + c_2 \sum m_{sea} + c_3 E_\pi + c_4 E_\pi^2 + c_5 a^2]$$

$a^p, E_\pi^p, m^p, m^p E_\pi^q, \alpha_s^p a^q, \ldots$ etc. (some such terms already included in previous plot)

Bayesian fits will be crucial and allow factors of 2 or 3 more fit parameters. Need as much theoretical input as possible to fix priors in such fits.

Going from $f_+(q^2)$ to $|V_{ub}|$

$$\frac{d\Gamma(B \to \pi)}{dq^2} = |V_{ub}|^2 \left\{ \frac{G_F^2}{192\pi^3 M_B^3} \lambda^3 |f_+(q^2)|^2 \right\}$$

LHS: from experiment over a wide range in q^2

RHS: $f_+(q^2)$ from Lattice QCD to date only for $q^2 > 16 \mathrm{GeV}^2$

In the past the above relation was not exploited optimally. One used

$$\int_{q^2_{min}}^{q^2_{max}} \frac{d\Gamma(B \to \pi)}{dq^2} dq^2$$

with $q^2_{min} = 16\mathrm{GeV}^2$. Lattice data was fit to various ansaetze (BK, BZ, z-expansion) to get an analytic expression one could integrate over. This introduced further model dependence and also prevented taking advantage of all experimental data.

New Analysis does a simultaneous fit to experimental and lattice data while allowing $|V_{ub}|$ to be one of the fit parameters

- 12-bin BABAR experimental data
- “z-variable” (Becher&Hill, Arnesen et al.)

$$z = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}} \qquad \text{where} \qquad t_\pm \equiv (m_B \pm m_\pi)^2,$$

physical region : $0 < q^2 < t_- \equiv q^2_{max}$.

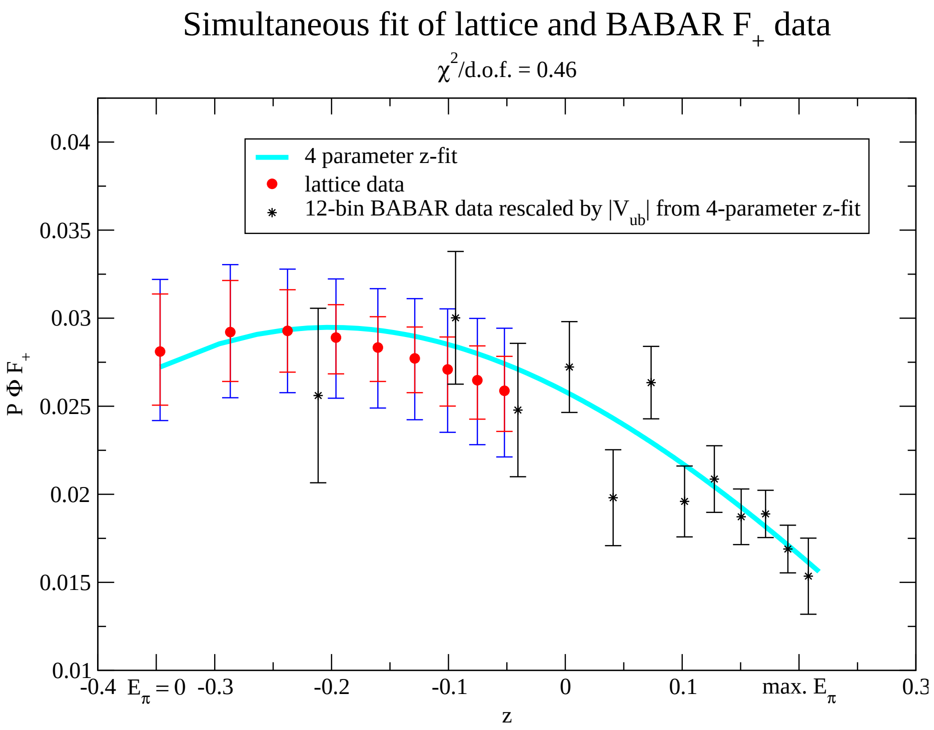
$$\Longrightarrow -0.34 < z < 0.22 \qquad (t_0 = 0.65 t_-)$$

z-Fit to the Form Factor $f_+(q^2)$

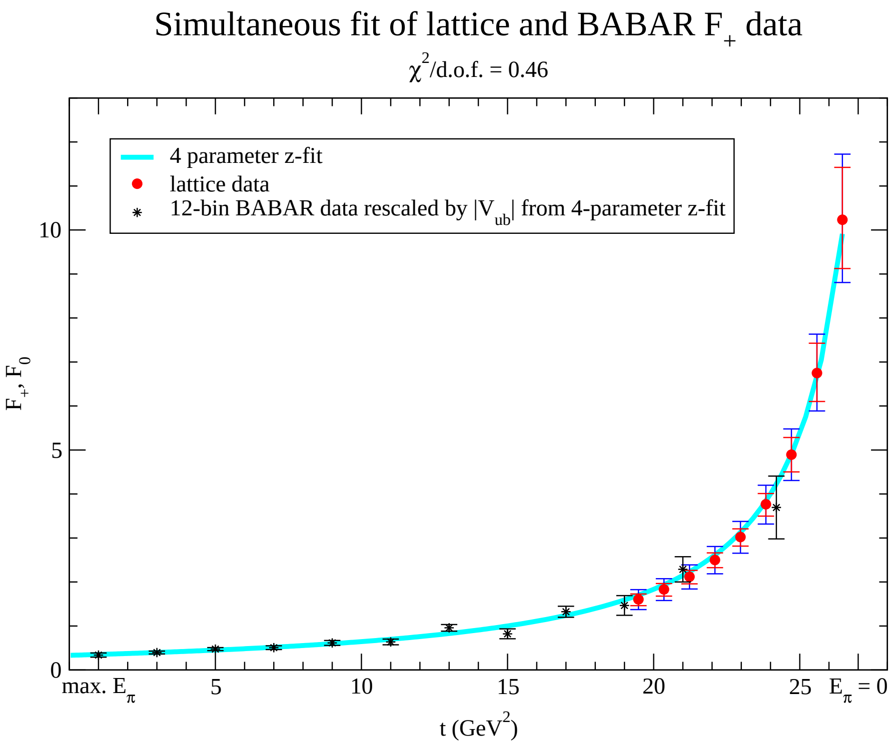
$$f_+(q^2) = \frac{1}{P(q^2)\,\Phi(q^2,t_0)} \sum_{k=0}^{k_{max}} a_k(t_0)\,[z(q^2,t_0)]^k.$$

The function $P(q^2)$ accounts for the isolated B^* pole (which happens to lie between t_- and t_+). $\Phi(q^2,t_0)$ in principle known smooth function. The coefficients a_k satisfy the constraint, $\sum_{k=0}^{k_{max}} a_k^2 \leq 1$ (Arnesen et al., Boyd et al.)

Simultaneous Fit to Experimental and Lattice Data


Simultaneous fit of lattice and BABAR F_+ data

χ^2/d.o.f. = 0.46


Simultaneous fit of lattice and BABAR F_+ data

χ^2/d.o.f. = 0.46

$|V_{ub}| = 2.94 \pm 0.35 \times 10^{-3}$ (Preliminary) (Van de Water LAT08)

Other Simultaneous Fits to Lattice, Experimental and Light Cone Sum Rules Data

Flynn & Nieves (PRD76:031302(R) (2007))

[Old FNAL/MILC + HPQCD] + [LCSR] + [CLEO + Belle + BABAR]
Omnes representation for $f_+(q^2)$

$$|V_{ub}| = [3.47 \pm 0.29 \pm 0.03] \times 10^{-3}$$

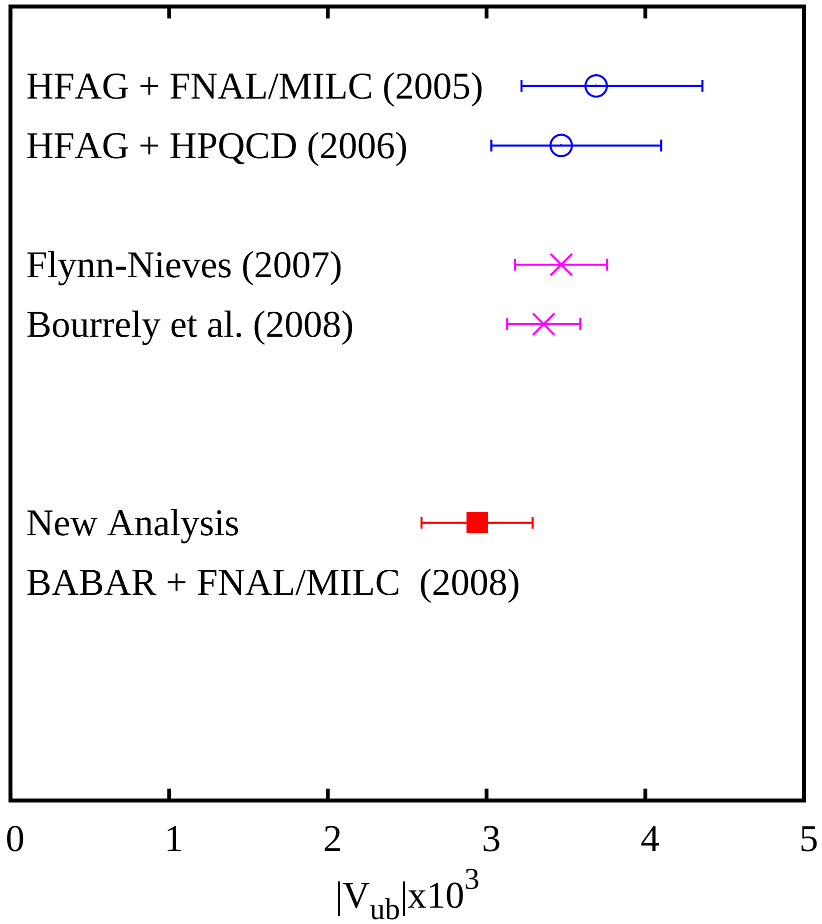
Bourrely-Caprini-Lellouch (arXiv:0807.2722 [hep-ph])

[Old FNAL/MILC + HPQCD] + [LCSR] + [CLEO + Belle + BABAR]
Modified z-expansion for $f_+(q^2)$

$$|V_{ub}| = [3.36 \pm 0.23] \times 10^{-3}$$

Authors had to guess correlations in lattice errors

Results for Exclusive $|V_{ub}|$



no simultaneous fit with experiment

simultaneous fit to
(FNAL/MILC + HPQCD) +
(all experiments) + LCSR $f_+(0)$

new chiral/cont. extrapolation
simultaneous fit Lattice + BABAR
proper correlations in lattice errors.

D Meson Semileptonic Decays

$D \rightarrow \pi$ and $D \rightarrow K$ semileptonic decays important for CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$. CKM matrix element independent ratios $\frac{1}{\Gamma(D\rightarrow l\nu)}\frac{d\Gamma(D\rightarrow\pi l\nu)}{dq^2}$ also of interest. Many issues common with B semileptonic decays.

- statistical and fitting : somewhat less challenging for D than for B decays
- chiral/continuum extrapolation : same new methods applicable
- Also moving towards the Becher-Hill parametrization of form factors
- benefits of simultaneous fits to lattice and experimental data true here as well

Recent Work on D Semileptonic Decays

$\boxed{N_f = 2+1}$

FNAL/MILC PRL95:122002 (2005)
+ CLEO-c $\Longrightarrow$ $\boxed{|V_{cs}| = 1.014 \pm 0.016 \pm 0.106}$
(F.Lodovico ICHEP08)

$\boxed{N_f = 2}$

ETMC :
all-to-all propagators computed with a stochastic method.
twisted B.C. for better control over pion momenta
BK parametrization

Becirevic, Haas, and Mescia :
Improved Wilson quarks (QCDSF configurations)
Double ratio method
also twisted B.C.

All $N_f = 2$ work still preliminary.

Summary

- Lattice QCD calculations crucial for exclusive $|V_{ub}|$ determinations (also $|V_{cs}|$ and $|V_{cd}|$)

- Improvements in analysis methods
  — chiral/continuum extrapolations
  — simultaneous fits to lattice and experiment (+ LCSR)

- much more work needed to reduce statistical/fitting errors

- lessons for D semileptonic from B semileptonic and vice versa

- $N_f = 2 + 1$ B semileptonic results to date all use staggered light quarks. Need results from other quark actions.