SM $BR(B \to X_{s,d}\gamma)$ calculations

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Theoretical framework to calculate BR$(B \to X_s\gamma)$

$B \to X_s\gamma$ is an inclusive decay. $\to$ theoretically rather clean.

HQE: $\Gamma[B \to X_s\gamma] = \Gamma[b \to s\gamma(g)] +$ corr. in Λ_{QCD}/m_b.

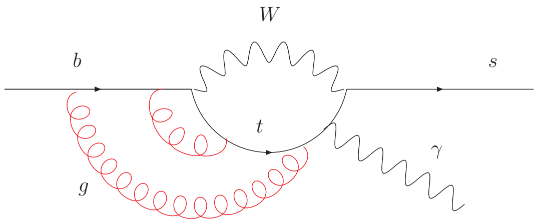
- no linear corrections in Λ_{QCD}/m_b when restricting to local operators
- Corr. start at $\mathcal{O}(\Lambda^2_{QCD}/m_b^2)$; they are related to the motion of the b-quark inside the meson

There are, however, contributions which scale like $\alpha_s(m_b)\Lambda/m_b$, induced by (non-local) light-cone operators (Lee,Neubert,Paz 2006)

But let us first discuss the main contribution: the free b-quark decay $\Gamma[b \to X_s\gamma]$.

Well-known: This partonic decay rate is significantly enhanced by QCD-effects.

When exchanging n gluons there are large logs:



$$\left(\frac{\alpha_s}{\pi}\right)^n \log^n \frac{m_b^2}{M^2} \qquad M = m_t, m_W: \text{ leading logs (LL)}$$

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n-1} \frac{m_b^2}{M^2} \qquad \text{next-to-leading logs (NLL)}$$

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n-2} \frac{m_b^2}{M^2} \qquad \text{NNLL logs}$$

To get a theoretical branching ratio which is of comparable precision as the present measurements, one has to resum LL, NLL and NNLL terms.

Useful machinery to achieve resummation: construct effective Hamiltonian and resum logs using RGE techniques.

The effective Ham.

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu) \quad .$$

contains the following 8 dim-6 operators:

$O_1 = (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})$ (current-current op.)

$O_2 = (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta})$ (current-current op.)

$O_3 - O_6$ Gluonic penguin operators (also 4-Fermi operators)

$O_7 = \frac{e}{16\pi^2} m_b(\mu)\,(\bar{s}\sigma_{\mu\nu}Rb)\,F^{\mu\nu}$ phot. dipole

$O_8 = \frac{g_s}{16\pi^2} m_b(\mu)\,(\bar{s_\alpha}\sigma_{\mu\nu}T^A_{\alpha\beta}Rb_\beta)\,G^{\mu\nu,A}$ gluonic dipole

Well-known: The calculation of BR$(B \to X_s\gamma)$ consists of three steps:

	LL	NLL	NNLL
-matching at $\mu = \mu_W$: $\to C_i(\mu_W)$	α_s^0	α_s^1	α_s^2
-RGE: $\to C_i(\mu_b)$ [with $\mu_b = O(m_b)$]	α_s^1	α_s^2	α_s^3
-calc. of matrix element $\langle X_s\gamma\|O_i(\mu_b)\|b\rangle$	α_s^0	α_s^1	α_s^2

Motivation to go to NNLL precison

The NLL calc. were completed in 1998.

In 2001, Gambino and Misiak realized that the NLL BR suffers from a rather large theor. uncertainty related to the renormalization scheme used for m_c.

To keep pace with the developments on the experimental side, it became clear that the BR has to be worked out at NNLL precision.

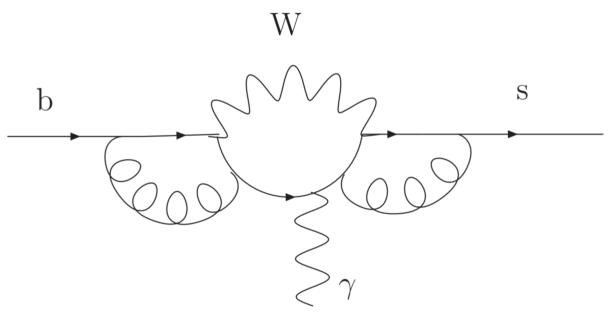
This enterprise was started about 5 years ago.

Recently, the most important contributions of such a calc. were finalized!!

Ingredients for the NNLL calculation

Matching: needed to α_s^2 precision.

This means in particular a 3-loop calculation in the full theory to fix $C_7(\mu_W)$ and $C_8(\mu_W)$ [$O(10^3)$ diagrams]:

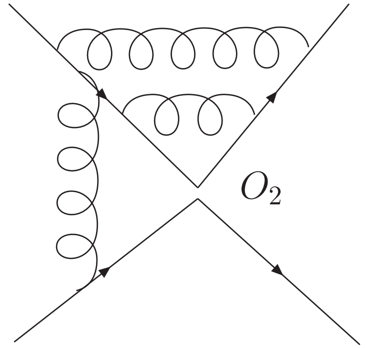


$\longrightarrow$ done by Misiak and Steinhauser, hep-ph/0401041.

For other operators $O(\alpha_s^2)$ means two-loop. Done some time ago.

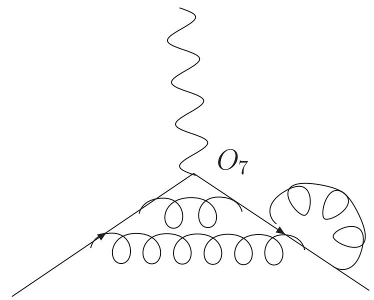
$\longrightarrow$ matching complete for NNLL $b \to s\gamma$!

Anomalous dimensions: needed up to α_s^3 precision.



• $(O_1 - O_6)$-sector

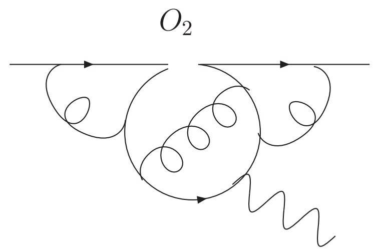
done by Gorbahn and Haisch, hep-ph/0411071.



• (O_7, O_8)-sector

was finished in 2005 by

Gorbahn, Haisch and Misiak, hep-ph/0504194.



• most difficult: mixing $O_2 \to O_7$, $O_2 \to O_8$:

about 22'000 4-loop diags!.

Done in 2006 by Czakon, Haisch and Misiak, hep-ph/0612329

$\to 8 \times 8$ anomalous dimension matrix complete for NNLL precision!

Matrix element of O_i: needed up to α_s^2

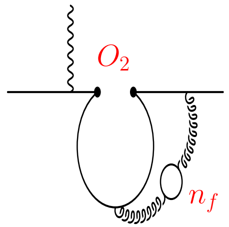
At order α_s^2 the following subprocesses are involved:

$$b \to s\gamma \; ; \; b \to s\gamma g \; ; \; b \to s\gamma gg \; ; \; b \to s\gamma q\bar{q}$$

The decay width can be decomposed into various interferences of the form (O_i, O_j). Let's look at a few of them:

- (O_2, O_7)-interference is numerically very crucial.

Only the fermionic contributions are known exactly at NNLL order:

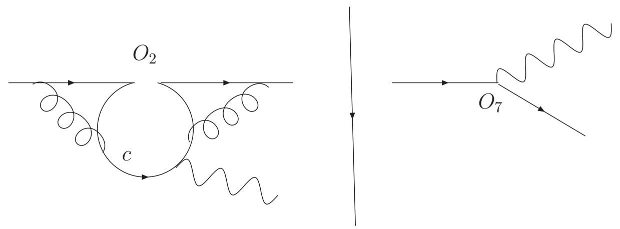


The virtual corrections were calculated by Bieri,Greub, Steinhauser, 2003 for massless quarks in the bubble.

The bremsstahlung corrections in this approx. are also available exactly (Ligeti, Luke, Manohar, Wise 1999.)

Later, also massive quarks in the bubble were taken into account (Boughezal, Czakon, Schutzmeier 2007.)

The non-fermionic corrections are extremely difficult to calculate:



m_c-dependence extremly hard to get.

Misiak and Steinhauser obtained a result for the unphys. case $m_c \gg m_b$, using HME techniques.

They then formulated an extrapolation procedure to the physical m_c, which they tested at the NLL level hep-ph/0609241.

By comparing different versions of the extrapolation procedure, they conclude that their result should be accurate within $\pm 3\%$ (at level of BR).

At the moment exact calculations are in progress for $m_c = 0$ (Czakon et al.), which will improve the extrapolation procedure.

I hope that even exact calc. will come for physical m_c and make the extrapolation obsolete!

• (O_7, O_7)-interference

There are two different calculations for this interference:

One by (Blokland et al., hep-ph/0506055), using optical theorem techniques.

And another one by my group, where we calculated the subprocesses $b \longrightarrow s\gamma$, $b \longrightarrow s\gamma g$, $b \longrightarrow s\gamma gg$ and $b \longrightarrow s\gamma q\bar{q}$ individually (Asatrian et al, 2006).

The results are identical.

• (O_7, O_8)-interference

So far, only the fermionic corrections with massless bubbles inserted in the gluon-propagator entered the prediction for the NNLL BR.

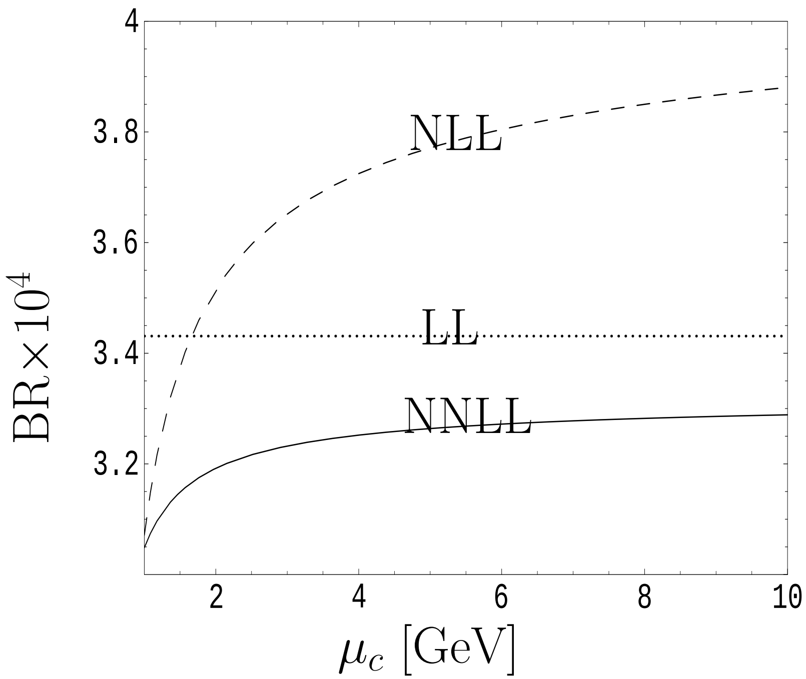
Recently T. Ewerth, hep-ph/0805.3911 published a paper on the fermionic contributions, including the mass effects in the c- and b-bubbles.

The full $O(\alpha_s^2)$ corrections are not known yet (but they are in progress [Asatrian et al.]).

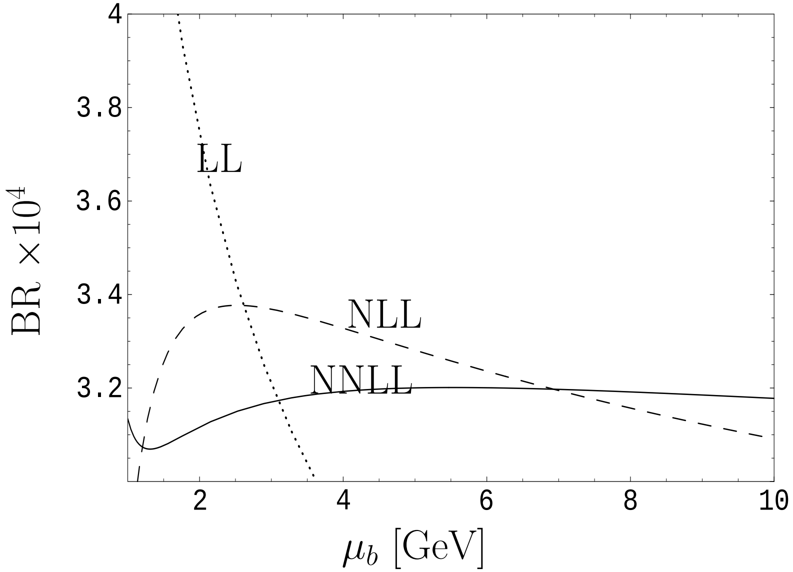
NNLL results

Recently the known individual NNLL-pieces were combined. A phenom. paper was published in PRL 98:022002, 2007, (Misiak+16 authors!).

As expected, the large uncertainty due to the renorm. scale μ_c (the scale at which m_c is renormalized), gets drastically reduced at NNLL.

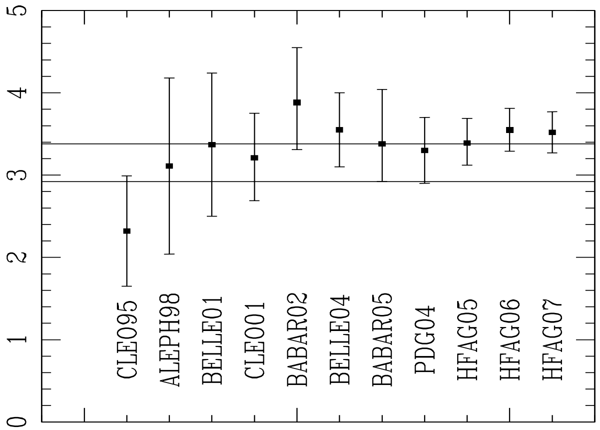


Also the dependence on the scale μ_b gets drastically reduced at NNLL.



At NNLL we obtain (PRL, 2007, (Misiak+16 authors!))

$$BR(B \to X_s\gamma)|^{\rm th}_{E_\gamma > 1.6\ {\rm GeV}} = (3.15 \pm 0.23) \cdot 10^{-4}$$



$$BR(B \to X_s\gamma)|^{\rm exp}_{E_\gamma > 1.6\ {\rm GeV}} = (3.52 \pm 0.23 \pm 0.09) \cdot 10^{-4}$$

HFAG, ArXiv:0808.1297

In the theory result various errors were added in quadrature, viz.

1. 3% higher orders (scale dependences)
2. 3% m_c extrapolation
3. 3% parametric: from $\alpha_s(m_Z)$, $m_c(m_c)$, BR_{sl} etc.
4. 5% due to a new class of non-perturbative corr. which scale like $\alpha_s \frac{\Lambda}{m_b}$
   Lee,Neubert,Paz Sept. 2006

Comment on the normalization factor C

In the result above, the branching ratio was written as

$$\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma)_{\mathrm{E_\gamma > E_0}} = \mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_c} e\bar{\nu})_{\mathrm{exp}} \left| \frac{\mathrm{V_{ts}^* V_{tb}}}{\mathrm{V_{cb}}} \right|^2 \frac{6\alpha_{\mathrm{em}}}{\pi \mathrm{C}} [\mathrm{P(E_0) + N(E_0)}] \,.$$

The perturbative part $P(E_0)$ is

$$\frac{\Gamma(b \to X_s\gamma)_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2 \, \Gamma(b \to X_u e\bar{\nu})} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\mathrm{em}}}{\pi} P(E_0) \,.$$

C is the so-called semileptonic phase-space factor:

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\bar{B} \to X_c e\bar{\nu})}{\Gamma(\bar{B} \to X_u e\bar{\nu})}$$

The expression for C is a function of m_c/m_b and of non-perturbative OPE-parameters.

All the occuring quantities in C are determined in a single global fit from the measured decay spectra of $B \to X_c e\bar{\nu}$.

For C one obtains

$$C = \begin{cases} 0.582 \pm 0.016 & \text{C. Bauer et al., hep-ph/0408002} \quad \text{1S scheme} \\ 0.546^{+0.023}_{-0.033} & \text{P. Gambino and P. Giordano, arXiv:0805.0271} \quad \text{kinetic scheme} \end{cases}$$

and m_c (after converting it to the MS-bar scheme)

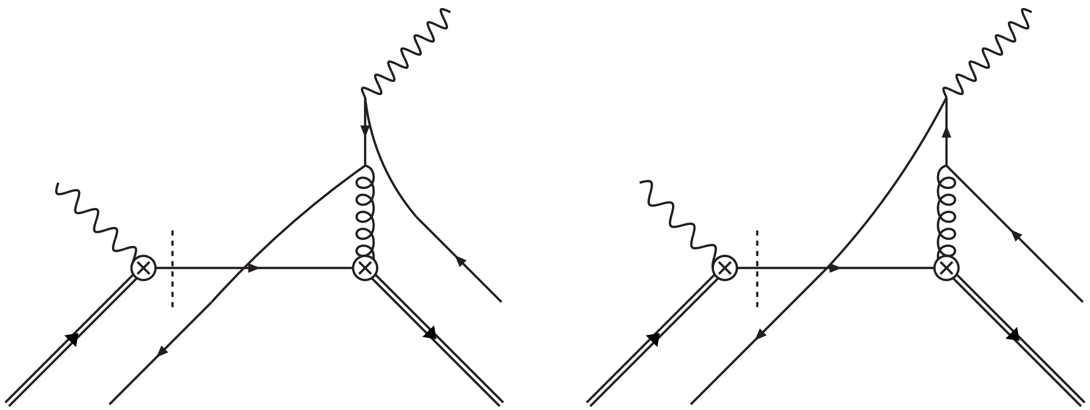
$$\overline{m}_c(\overline{m}_c) = \begin{cases} 1.224 \pm 0.057 & \text{1S scheme} \\ 1.267 \pm 0.056 & \text{kinetic scheme} \end{cases}$$

The differences cancel to some extent in the radiative BR, leading to

$$\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma)^{\mathrm{NNLL}}_{\mathrm{E_\gamma > 1.6 GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}\,, & \text{using 1S scheme hep-ph/0609232} \\ (3.25 \pm 0.24) \times 10^{-4}\,, & \text{following the kin. scheme analysis of Gambino,Giordano, see also Misiak, arXiv:0808.3134} \end{cases}$$

Comment on the non-local power corrections

There are power corrections which scale like $\alpha_s \frac{\Lambda}{m_b}$. They arise for example in the (O_7, O_8)-interference through the following mechanism (Lee,Neubert,Paz Sept. 2006):



O_7-side: hard photon is directly emitted from the operator

O_8-side: a gluon is emitted from the operator. It goes into hard photon by emitting two soft quarks.

The two 'vertical' propagators have virtualities of order $m_b\Lambda$

This mechanism leads to tri-local four-quark operators like

$$O_1 = \sum_q e_q \bar{h}_v(0)\Gamma_R q(t\bar{n})\, \bar{q}(s\bar{n})\Gamma_R h_v(0)\,.$$

The matrix elements $\langle B|O_i|B\rangle$ scale like Λ/m_b. They are, however, extremly difficult to calc. Naive model estimates point to a small red. of the BR. But it is also possible they could modify the BR up to $\sim 5\%$ (Lee,Neubert,Paz Sept. 2006).

Comment on the photon energy cut-off effects

The NNLL result given above was derived in fixed-order perturbation theory.

In 2004 Neubert pointed out that the energy cut E_0 induces an additional scale Δ

$$\Delta = m_b - 2E_0 \qquad \text{twice the width of the observed energy window}$$

which is about 1.4 GeV (for $E_0 = 1.6$ GeV).

Accounting for the photon-energy cut properly, requires to disentangle contributions associated with the hard-scale $\mu_h \sim m_b$, the soft scale $\mu_0 \sim \Delta$ and the intermediate scale $\mu_i \sim \sqrt{\Delta m_b}$, set by the typical invariant mass of the hadronic final state.

The $B \to X_s\gamma$ decay rate with a photon-energy cut obeys a factorization formula which implements this separation:

$$\Gamma(E_0) \sim |H_\gamma(\mu)|^2 \int_0^\Delta dp_+ (m_b - p_+)^3 \int_0^{p_+} d\omega m_b J(m_b(p_+ - \omega), \mu)\, S(\omega, \mu)$$

Neubert, 2004 showed that the double convolution integral can be expanded using an OPE, relating it to forward B-meson matrix elements of local HQET operators.

In 2006, Becher and Neubert used this framework to calculate the fraction of events $F(E_0)$, which passes the photon energy-cut

$$F(E_0) \doteq \Gamma(E_0)/\Gamma(0)$$

They resummed those leading and next-to-leading logs of $\delta \equiv \Delta/m_b$ which are not power-suppressed (in δ).

Power-suppressed terms on the other hand were retained in fixed order perturbation theory [$O(\alpha_s^2)$] (resummation of log's not yet available)

Combining this calculation with our fixed order results, they got a BR which somewhat smaller

$$BR(B \to X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}} = (2.98 \pm 0.26) \cdot 10^{-4} \qquad \text{Becher-Neubert 2006}$$

Very recently Misiak, arXiv:0808.3134 pointed out that this “partial resummation” is unreliable.

To illustrate this, consider the (O_7, O_7) contribution to $F(E_0)$ in fixed order perturbation theory:

$$F_{77}(E_0) = 1 + \frac{\alpha_s}{\pi}\phi^{(1)}(\delta) + \frac{\alpha_s^2}{\pi^2}\phi^{(2)}(\delta) + \dots$$

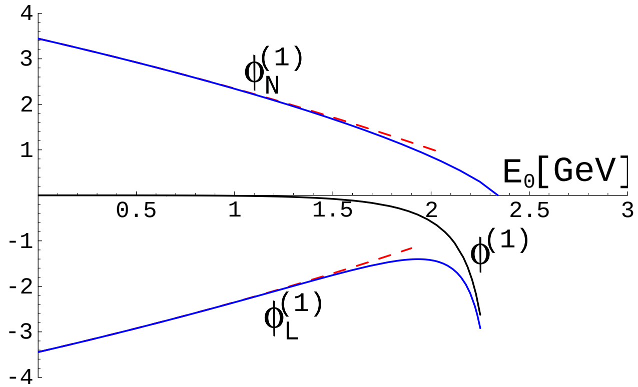
Split each $\phi^{(k)}$ into two parts:

$$\phi^{(k)} = \phi_L^{(k)} + \phi_N^{(k)}$$

$\phi_L^{(k)}$ is a polynomial in $\log(\delta)$

$\phi_N^{(k)}$ contains powers of δ (vanishes at endpoint)

Concretely, for $\phi^{(1)}$ the splitting reads:



$$\phi_L^{(1)}(\delta) = -\frac{2}{3}\ln^2\delta - \frac{7}{3}\ln\delta - \frac{31}{9}$$

$$\phi_N^{(1)}(\delta) = \frac{10}{3}\delta + \frac{1}{3}\delta^2 - \frac{2}{9}\delta^3 + \frac{1}{3}\delta(\delta-4)\ln\delta$$

Only at very large values of E_0 $\phi_L^{(1)}$ dominates.

However, at the relevant $E_0 = 1.6$ GeV large cancellations between $\phi_L^{(1)}$ and $\phi_N^{(1)}$!!

Same situation for $\phi_L^{(2)}$ and $\phi_N^{(2)}$, which are expl. known.

General arguments imply the same situation for all the other $\phi^{(k)}$.

$\Longrightarrow$ When resumming the leading power pieces and leaving the power-suppressed pieces unresummed at $O(\alpha_s^2)$-level, the necessary cancellations do not happen at the $O(\alpha_s^3)$-level.

As a consequence, the $O(\alpha_s^3)$-terms get highly overestimated.

Would be nice if one could resum logs in power-suppressed contributions as well.

Until this can be done, the fixed order result for the BR seems more reliable.

Some remarks on $B \to X_d\gamma$

Note: There was not much activity on the inclusive decay $B \to X_d\gamma$ in the last 10 years on the theory side.

This is mainly because the corresponding measurements seemed difficult.

However, very recently, an inclusive measurement of the kinematical branching ratio was presented by BABAR, ArXiv:0807.4975:

$$BR(B \to X_d\gamma)_{0.6\ \text{Gev} \le M_{X_d} \le 1.8\ \text{GeV}} = [7.2 \pm 2.7(\text{stat}) \pm 2.3(\text{syst})] \times 10^{-6}\,.$$

7 exclusive channels were reconstucted and corrected for missing final states [this correction is dominant source of syst. error]

The analogous procedure was done for $B \to X_s\gamma$, leading to

$$BR(B \to X_s\gamma)_{0.6\ \text{Gev} \le M_{X_s} \le 1.8\ \text{GeV}} = [215 \pm 14(\text{stat}) \pm 33(\text{syst})] \times 10^{-6}\,.$$

Using the framework of Kagan,Neubert 1998, it was estimated that both kinematical BR's cover $(50 \pm 4)\%$ of the total BR.

A important quantity is

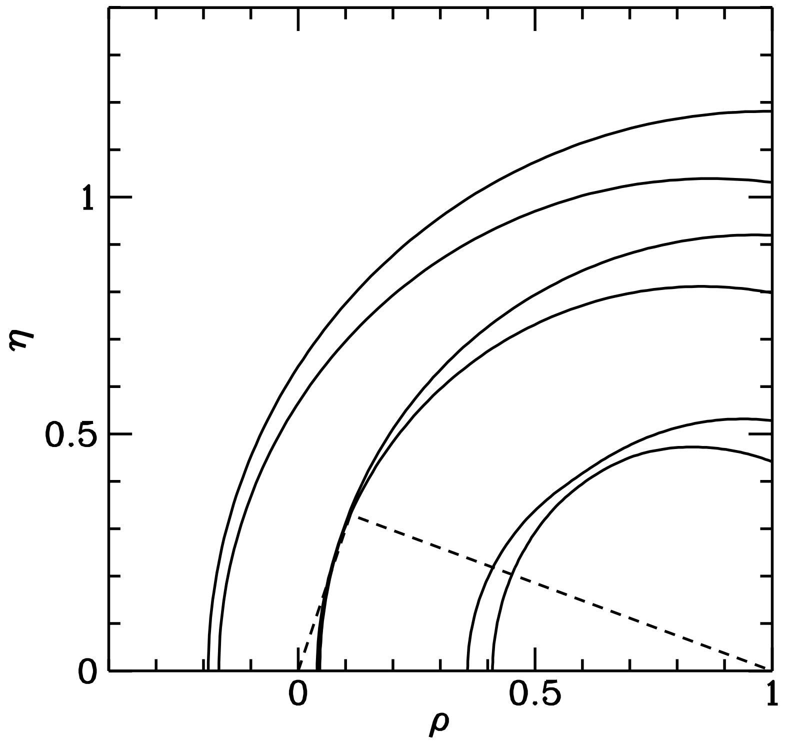
$$R(d\gamma/s\gamma) \doteq \frac{\mathrm{BR}(\mathrm{B} \to \mathrm{X_d}\gamma)}{\mathrm{BR}(\mathrm{B} \to \mathrm{X_s}\gamma)}$$

where uncertainties (both exp. and theor.) tend to cancel. The measurement of $R(d\gamma/s\gamma)$ yields

$$R(d\gamma/s\gamma) = 0.033 \pm 0.013 \pm 0.009.$$

On the theoretical side, $R(d\gamma/s\gamma)$ was calculated by Ali,Asatrian,Greub, Phys. Lett. B 429,87,(1998), using NLL predicitions for the involved BR's.

$R(d\gamma/s\gamma)$ sensitively depends on the CKM paramters (ρ, η), mainly through the combination $|V_{td}/V_{ts}|$.



A measurement of $R(d\gamma/s\gamma)$ carves out specific regions in the $(\rho, \eta)-$plane:

Plot from Asatrian, Ali, Greub 1998

3 hypoth. measurements of $R(d\gamma/s\gamma)$:

0.074 (top)

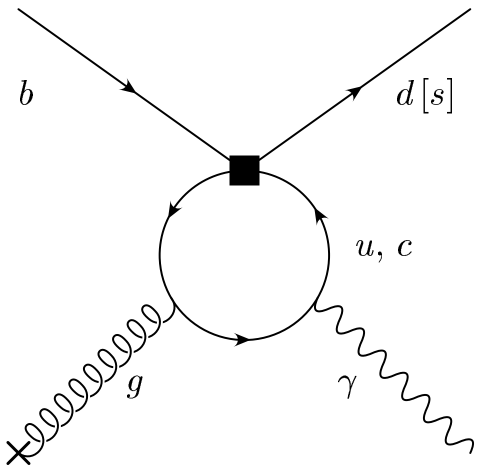
0.046 (middle)

0.017 (bottom)

BABAR used this paper to get: $|V_{td}/V_{ts}| = 0.177 \pm 0.043 \pm 0.001$.

Compatible with $|V_{td}/V_{ts}|$ from exclusive decays $B \to (\rho, \omega)\gamma$ and from B_s/B_d oscillations.

Note that there is a problem related to u-quark loops:



c- and u-quark loops can interact with soft gluons surrounding the b-quark in the B-meson.

- For c-quark loops, one can expand the loop-function in powers of $t = k_g k_\gamma / m_c^2$. This generates the so-called Voloshin terms. In $BR(B \to X_s \gamma)$ an effect of about 3%.

- There is no such expansion in the case of the u-quark loop.

  Non-perturbative effects in the u-quark loop are not well understood!

  Buchalla, Isidori and Rey,hep-ph/9705253 argue that an expansion in $1/t$ can be done $\rightarrow$ non-local operators. From naive dim. counting, they except the leading contr. to be of order $\Lambda_{\mathrm{QCD}}/m_b$.

Summary for $B \to X_s\gamma$

The matching calc. for the Wilson coefficents are complete at NNLL.

Also the anomalous dimensions are completely known.

There are missing matrix elements, e.g the (O_7, O_8) and (O_8, O_8) interferences.

The $m_c = 0$ results for the NNLL contributions from the 4-quark operators are awaited. They are expected to improve the extrapolation in m_c.

There are rather strong arguments that the fixed order BR (with $E_0 = 1.6$ GeV or lower) is more reliable than the one in the present version of MSOPE, where only the logs in the leading power terms are resummed.