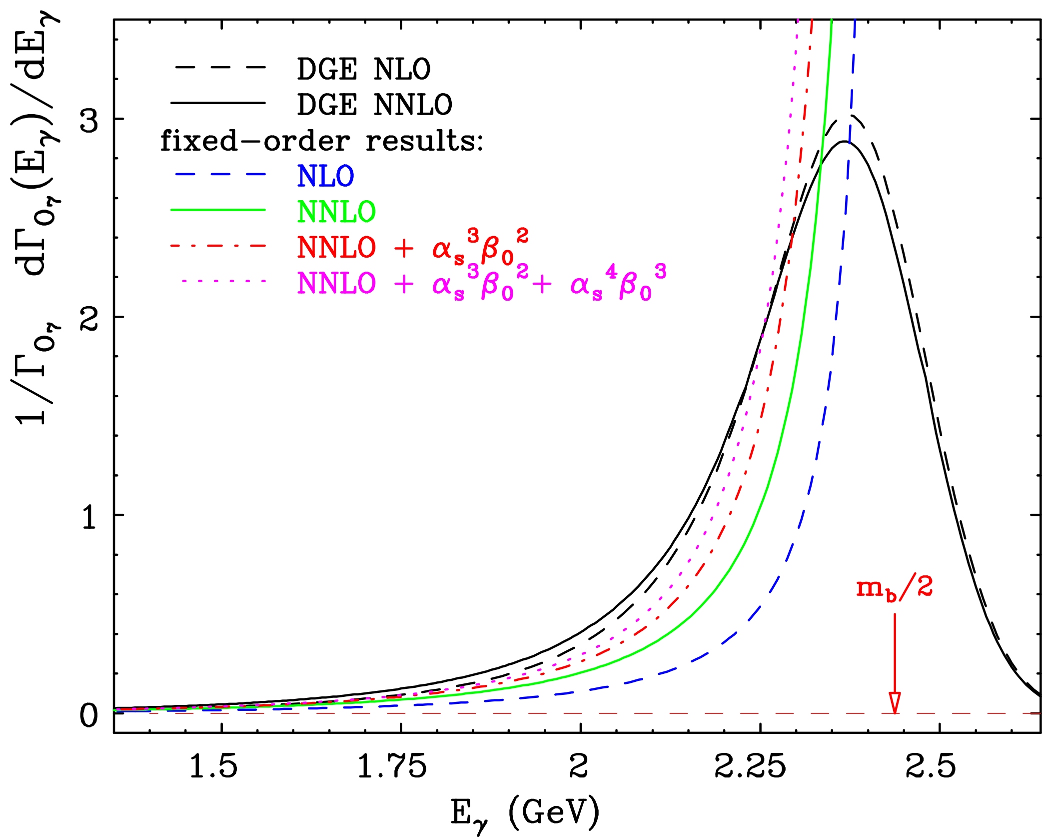
CKM08 workshop, Rome

Theoretical Review:

$B \to X_s\gamma$ Spectrum and Moments

Einan Gardi (Edinburgh)



Theoretical Review: $B \to X_s\gamma$ Spectrum and Moments

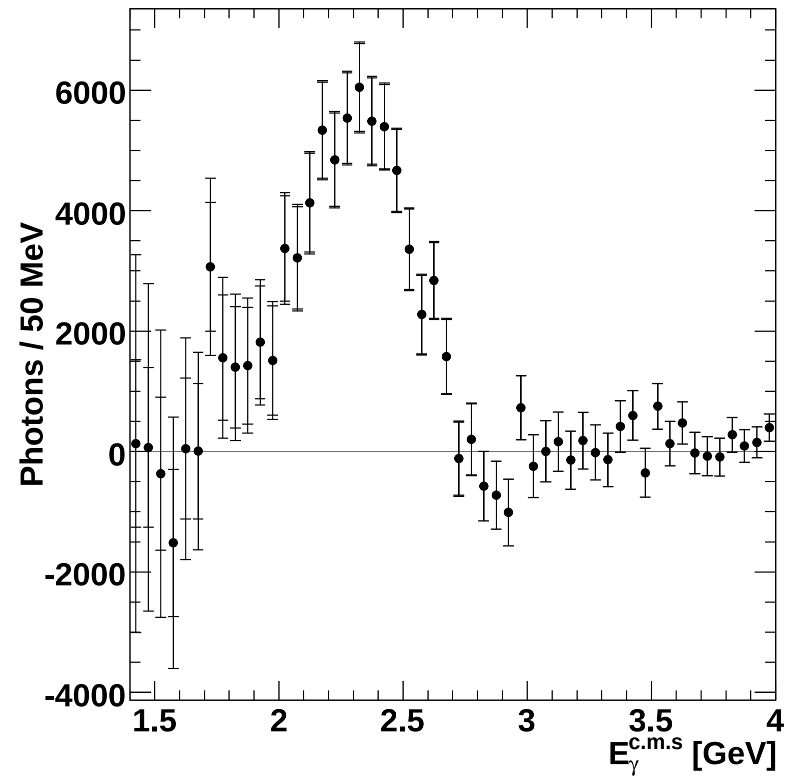
Plan of the talk:

- Why are we interested in the $B \to X_s\gamma$ spectrum?
- Experimental measurements and OPE-based fit to moments (Benson, Bigi & Uraltsev, Buchmüller & Flächer)
- Approaches to resummation in $\bar{B} \to X_s\gamma$
  - Multi Scale OPE (Becher & Neubert)
  - Dressed Gluon Exponentiation (Andersen & Gardi)
- Resum or not? (Misiak, arXiv:0808.3134)
- Operator-dependent spectrum (Andersen & Gardi)
- New non-perturbative effects (Neubert, Lee & Paz hep-ph/0609224)

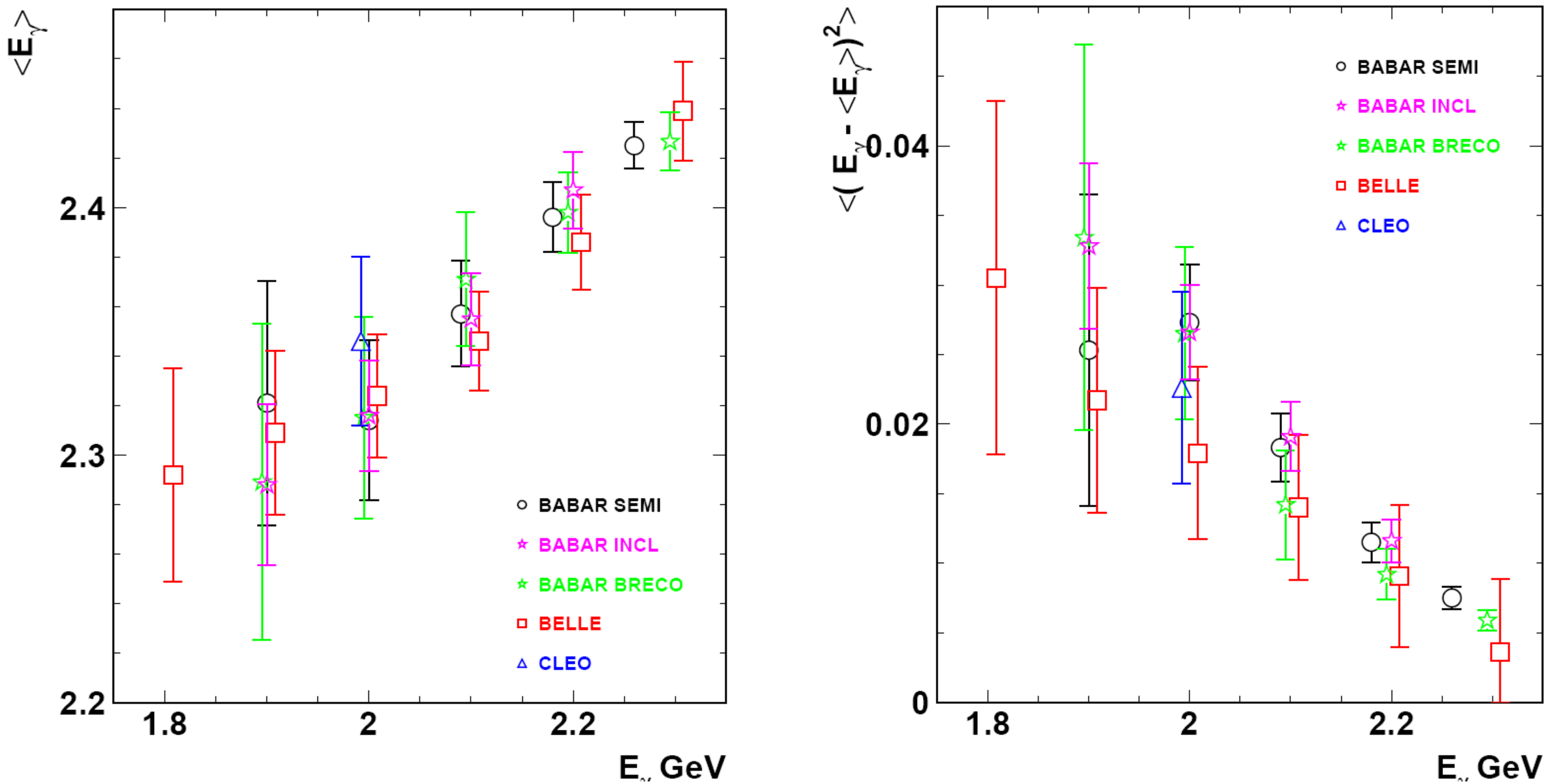
moments and spectra - why

New physics is in the total width, so why is the spectrum interesting?

- deal with kinematic cuts: comparing the measured $\mathcal{B}(E_\gamma > E_{\text{cut}})$ with $E_{\text{cut}} \gtrsim 1.8$ GeV to the computed "total" width
- use $\bar{B} \to X_s\gamma$ data to determine m_b, μ_π^2 (with $b \to c$)
- challenge and improve our understanding of QCD



The measured moments



- Agreement between independent measurements.
- Moments with high cuts are well measured!

How is the data used today?

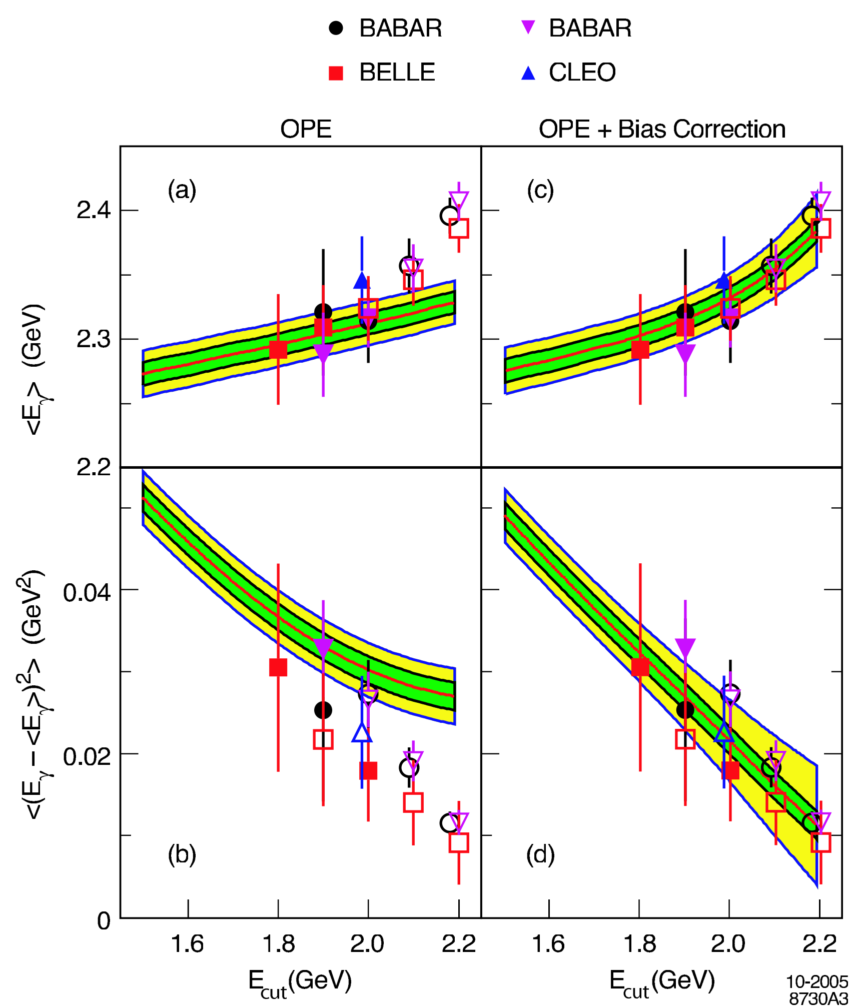
- OPE based fit in the "kinetic scheme" is used to extract m_b and μ_π^2, combined with $b \to c$ moments fit (Buchmüller and Flächer)
- Branching Fraction data is extrapolated from $\mathcal{B}(E_\gamma > E_{\text{cut}})$ with $E_{\text{cut}} \sim 1.8 - 2$ GeV to $E_{\text{cut}} = 1.6$ GeV based on this fit:
  $\sim 10\%$ effect
- The gap between $E_{\text{cut}} > 1.6$ GeV and "total" is left to theorists: only a few percent.

The "kinetic scheme" fit

What 'theory' goes into it?

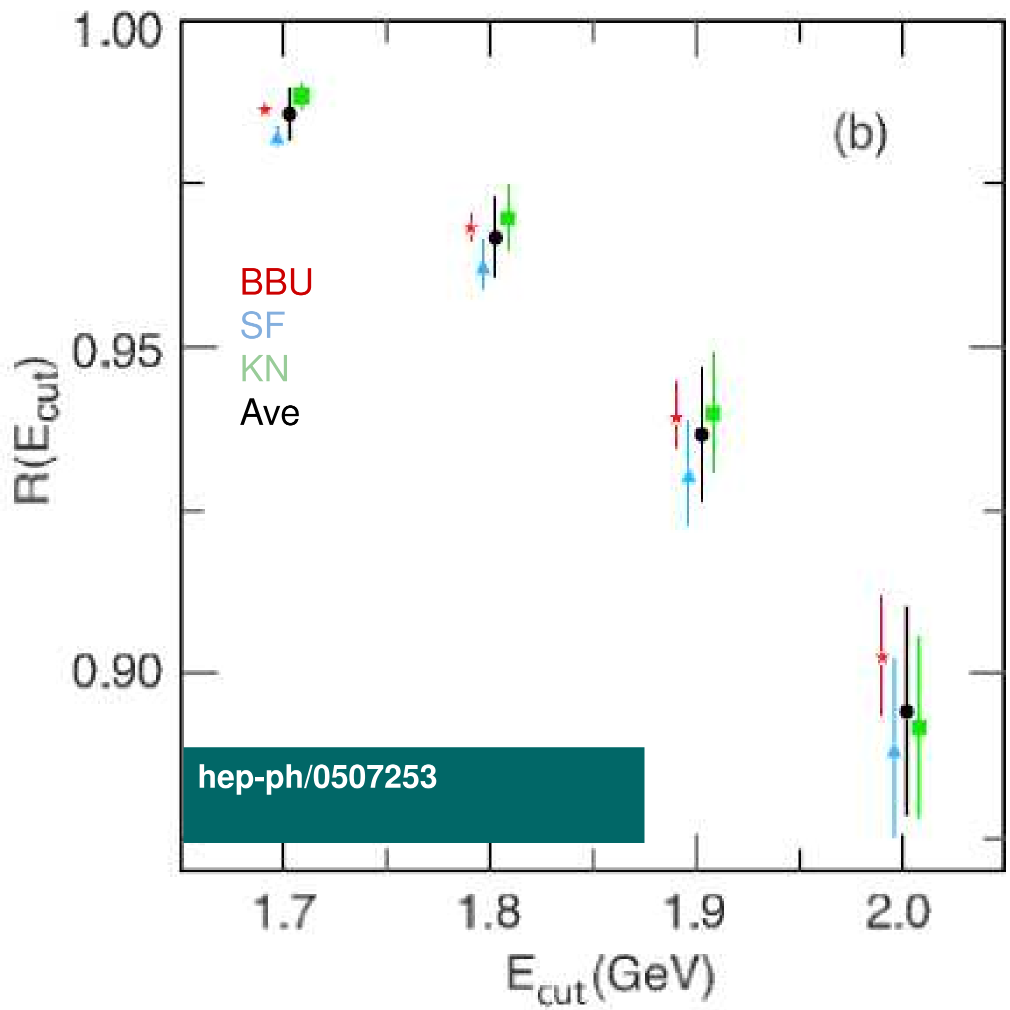
(Benson, Bigi, Uraltsev)

- Operator Product Expansion
- OPE doesn't apply at high cuts: need a "shape function". (Neubert; Bigi et al.)
- computed relation between the shape function parameters – fixed by the first two shape-function moments – and the first two E_γ moments
- perturbation theory to $\mathcal{O}(\alpha_s^2\beta_0)$



Branching fraction extrapolation factors

$$R_{\text{cut}} = \frac{\mathcal{B}(E_\gamma > E_{\text{cut}})}{\mathcal{B}(E_\gamma > 1.6\ \text{GeV})}$$ based on the "kinetic scheme" fit



Branching fraction extrapolation factors

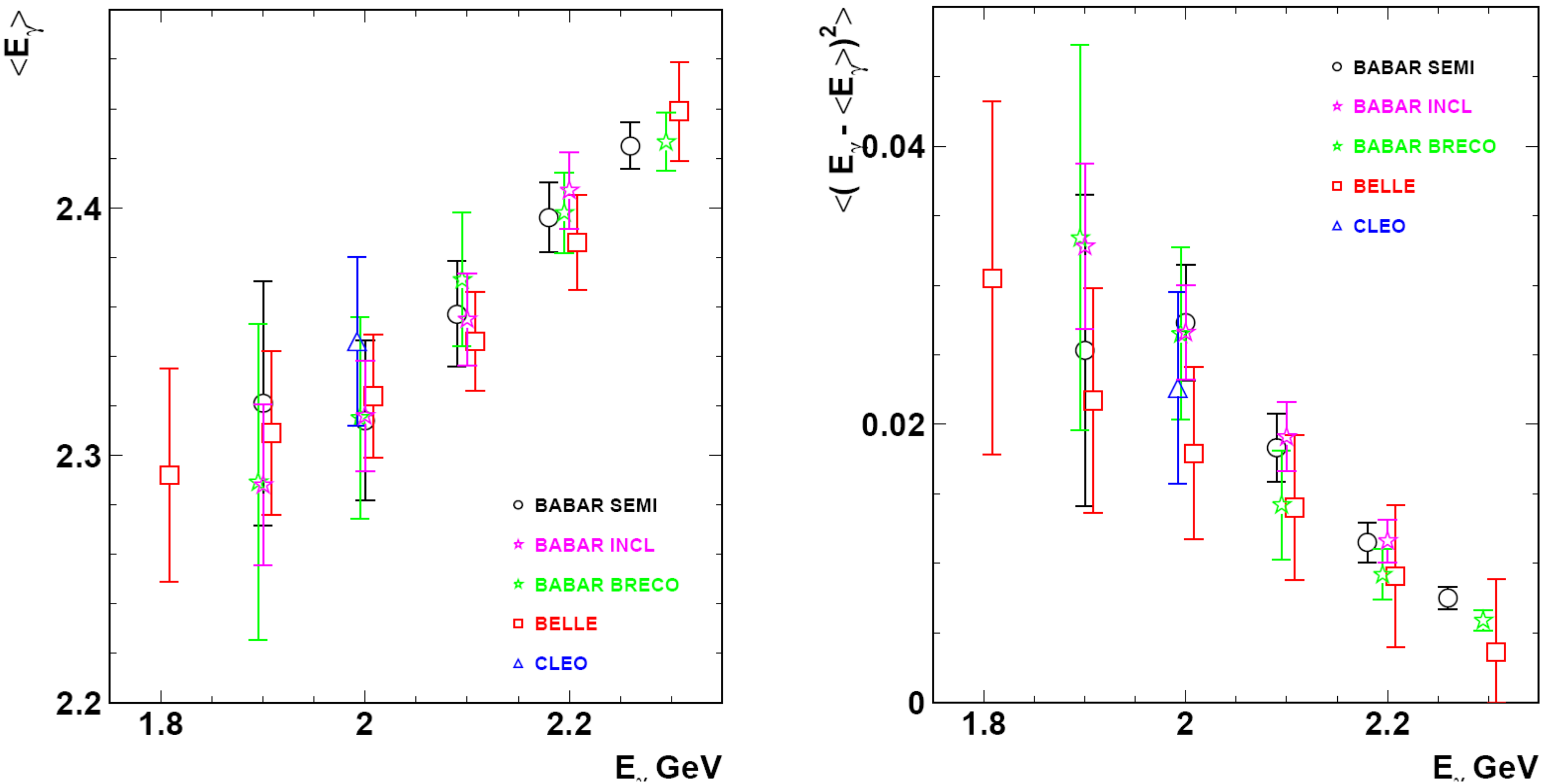
This is important, but not enough...

- We need to extrapolate to “total”, not to 1.6 GeV
- We also need relaible predictions for the spectrum above 2 GeV — the good data is there! — but there one cannot ignore the Sudakov logarithms, the dominant corrections near the endpoint.

To know the spectrum we need more theory input!

The moments with high cuts are well measured

The BF fit uses only moments with cuts $E_{\text{cut}} \leq 2.0$ GeV.



How can we *use* the well-measured high-cut moments?

What other theory input do we have?

We know a lot more (in perturbation theory!)

- Important contributions are known to NNLO, in particular the $O_7 - O_7$ spectrum
  (Melnikov & Mitov, Asatrian et al., Blokland et al., . . .)
- The dominant corrections near the endpoint, Sudakov logarithms: known *to all orders with NNLL accuracy*.
  (Gardi, Becher & Neubert)
- BLM corrections to $O_7 - O_7$ spectrum: all orders.

Yet, it is non-trivial to exploit this knowledge,

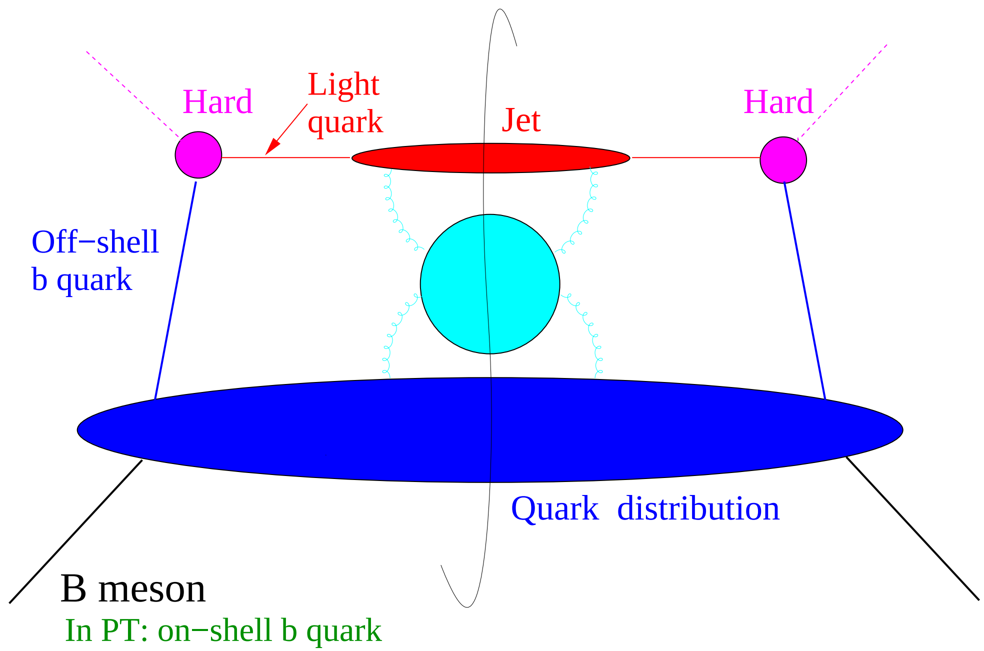
even to get a precise estimate for $T \equiv \frac{\mathcal{B}(E_\gamma > 1.6 \text{ GeV})}{\mathcal{B}(E_\gamma > 1.0 \text{ GeV})}$

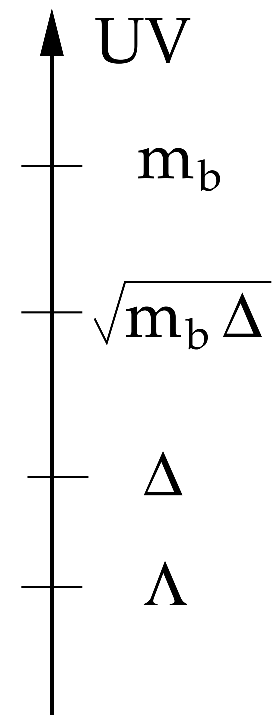
Misiak et al. - fixed order perturbation theory	$1 - T = 3.5\%$
Andersen & Gardi - resummation (DGE)	$1 - T = 1.3\% - 2\%$
Becher & Neubert - resummation (MSOPE)	$1 - T = 3\% - 13\%$

Factorization in inclusive decays (Korchemsky & Sterman '94)

Consider the region of jet kinematics: $\Delta = m_b - 2E_0 \ll m_b$

$$\Gamma^{\rm PT}(E_0) = H(m_b) J(\sqrt{m_b \Delta}) \otimes S(\Delta)$$





Hierarchy of scales $\Longrightarrow$ Factorization $\Longrightarrow$ Sudakov Resummation:

Hard: **Jet:** **Quark Distribution — Soft:**

$m_b \quad \gg \quad m_{\rm jet} = \sqrt{m_b \Delta} \quad \gg \quad \Delta$

Multi–scale OPE: Sudakov Resummation (NNLL)

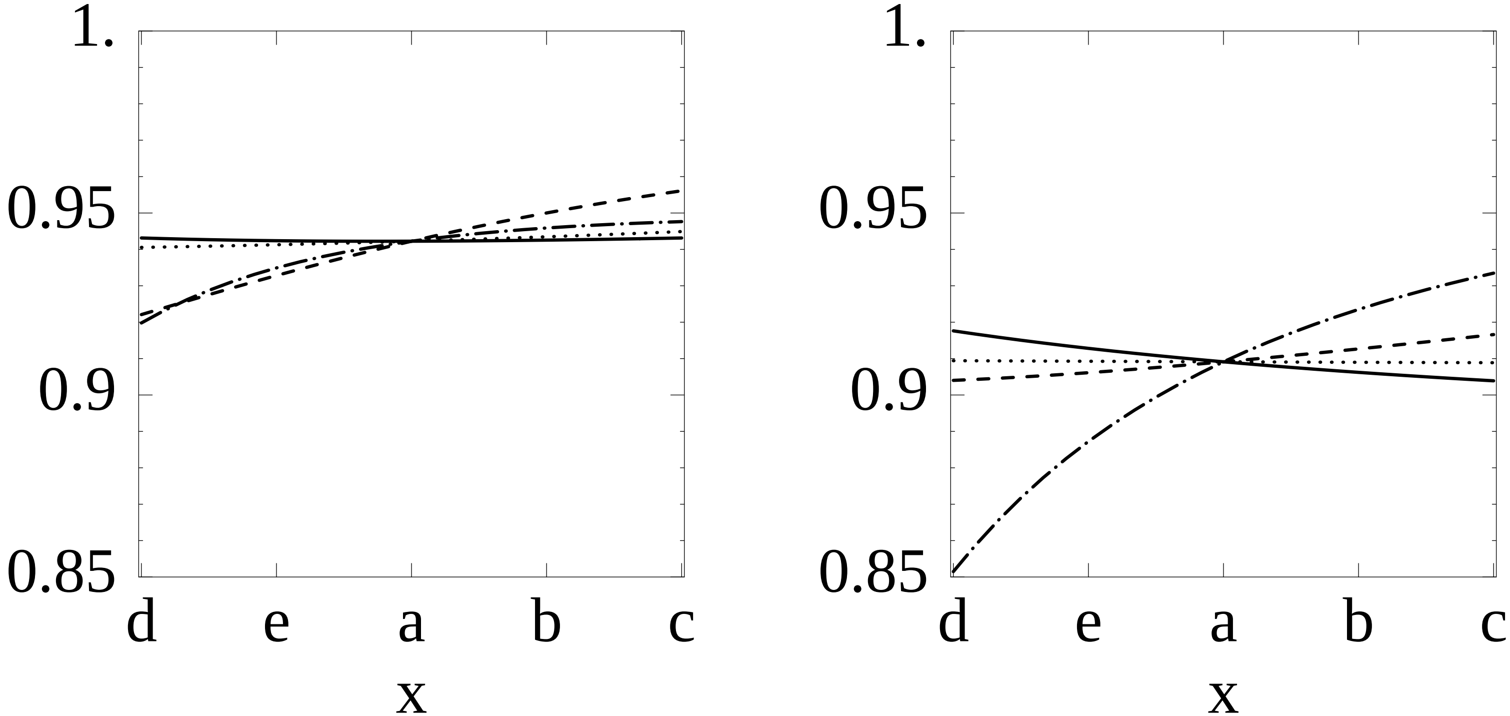
$$\Gamma(E_0) = \frac{G_F^2\alpha}{32\pi^4}\,|V_{tb}V_{ts}^*|^2\,\overline{m}_b^2(\mu)|H_\gamma(\mu)|^2\int_0^\Delta dp_+(m_b-p_+)^3$$
$$\times\int_0^{p_+} d\omega\, m_b\, J(m_b(p_+-\omega),\mu)\, S(\omega,\mu)\,,$$

$$\frac{\Gamma(E_0)}{\Gamma(0)} = U(\mu_h,\mu_i,\mu_0;\mu)$$
$$\times\left(\frac{m_b}{\mu_h}\right)^{-2a_\Gamma(\mu_h,\mu)}\left(\frac{m_b\Delta}{\mu_i^2}\right)^{2a_\Gamma(\mu_i,\mu)}\left(\frac{\Delta}{\mu_0}\right)^{-2a_\Gamma(\mu_0,\mu)}$$
$$\times\, h\left(\frac{m_b}{\mu_h}\right)\tilde{j}\left(\ln\frac{m_b\Delta}{\mu_i^2}+\partial_\eta\right)\tilde{s}\left(\ln\frac{\Delta}{\mu_0}+\partial_\eta\right)\frac{e^{-\gamma_E\eta}}{\Gamma(1+\eta)}$$
$$\times\left[p_3\left(\frac{\Delta}{m_b}\right)-\frac{\eta(1-\eta)}{6}\,\frac{\mu_\pi^2}{\Delta^2}+\dots\right]+\delta F(E_0),$$

$\eta = 2a_\Gamma(\mu_i,\mu_0) > 0$ $\qquad$ $p_3(\delta) = 1-\frac{3\delta\eta}{1+\eta}+\frac{3\delta^2\eta}{2+\eta}-\frac{\delta^3\eta}{3+\eta}$

Multi–scale OPE: results

Scale dependence in two different matching procedures (shifting $O(\Delta/m_b)$ terms between the resummation and the residual $\delta F(E_0)$):



- Significant dependence on the lowest separation scale $\mu_0 \simeq \Delta$: large subleading logarithmic corrections to the soft function.
- High sensitivity to the matching procedure, dealing with terms that are suppressed by powers of Δ/m_b near the endpoint — but are not small away from the endpoint.

Should we then give up resummation?

Misiak sais: **yes!**

fixed order perturbation theory is more reliable.

I say: **no!**

The solution to both problems encountered in Becher & Neubert analysis (hep-ph/0610067), namely

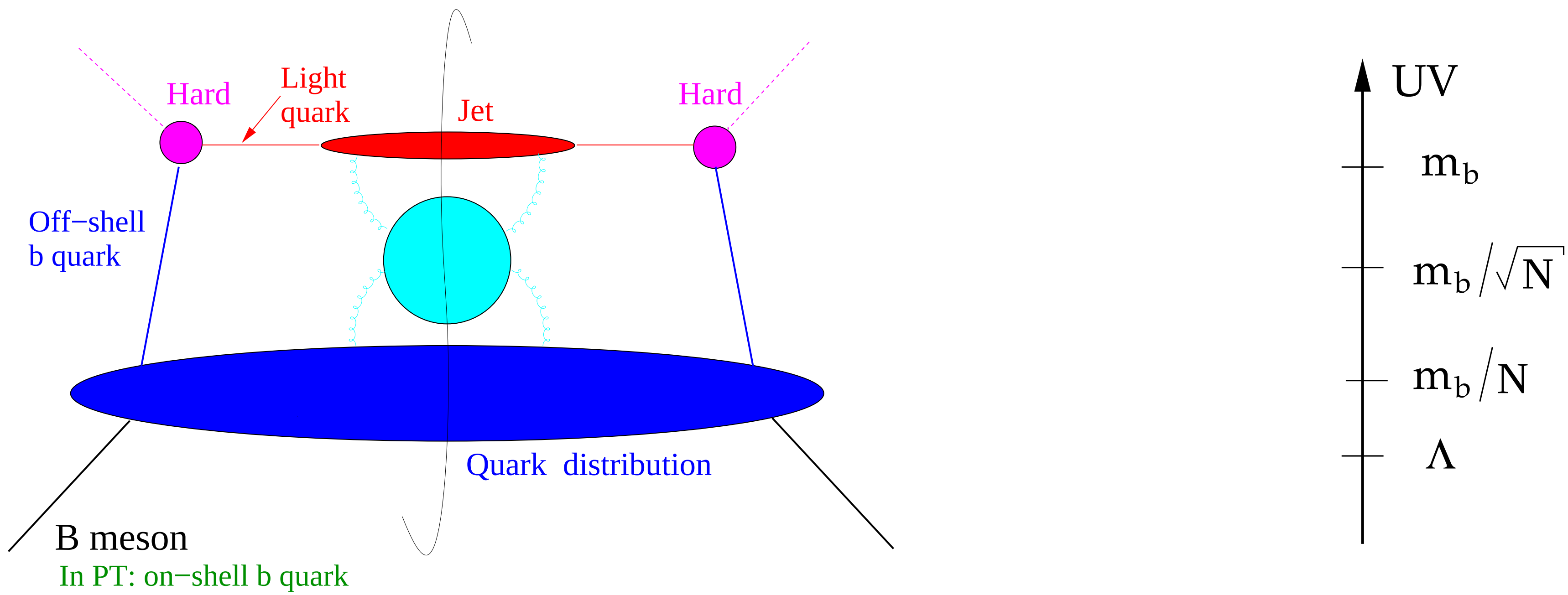
- large subleading logaritmic corrections to the soft function
- sensitivity to the matching procedure (dealing with terms that are suppressed by powers of Δ/m_b near the endpoint — but are not small away from the endpoint).

has been given in my paper with Andersen (hep-ph/0609250).

Factorization in inclusive decays: moment space

Define N such that large N probes jet kinematics $m_b - 2E_\gamma \ll m_b$:

$$\Gamma_N^{\rm PT} \equiv \int_0^{m_b/2} dE_\gamma \frac{1}{\Gamma_{\rm tot}^{\rm PT}} \frac{d\Gamma^{\rm PT}}{dE_\gamma} \left(\frac{2E_\gamma}{m_b}\right)^{N-1} = H(m_b)\, \underbrace{J(m_b^2/N;\mu)\, S_{\rm PT}(m_b/N;\mu)}_{{\rm Sud}(N, m_b)} + \mathcal{O}\left(\frac{1}{N}\right)$$



Hierarchy of scales $\Longrightarrow$ Factorization $\Longrightarrow$ Sudakov Resummation:

	Hard:		**Jet:**		**Quark Distribution — Soft:**
	m_b	$\gg$	$m_{\rm jet} = \sqrt{m_b \Delta}$	$\gg$	Δ
Moments	m_b	$\gg$	$m_b/\sqrt{N}$	$\gg$	m_b/N

DGE: going beyond NNLL Sudakov resummation

DGE = internal resummation of running–coupling corrections in the Sudakov exponent

For example, the $O_7 - O_7$ integrated spectrum

$$\frac{\Gamma(E_0)}{\Gamma(0)} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{dN}{N-1}\left(\frac{2E_0}{m_b}\right)^{1-N} H\left(\alpha_s(m_b),N\right)\times \mathrm{Sud}(N,m_b) + \int_{x=2E_0/m_b}^{1} dx\Delta R(\alpha_s(m_b),x),$$

$$\mathrm{Sud}(N,m_b)=\exp\left\{\frac{C_F}{\beta_0}\int_0^\infty\frac{du}{u}\left(\frac{\Lambda^2}{m_b^2}\right)^u\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(\frac{\Gamma(N)}{\Gamma(N-2u)}-\frac{1}{\Gamma(1-2u)}\right) - B_{\mathcal{J}}(u)\Gamma(-u)\left(\frac{\Gamma(N)}{\Gamma(N-u)}-\frac{1}{\Gamma(1-u)}\right)\right]\right\},$$

- The NNLL result can be obtained upon exapnsion to $\mathcal{O}(u^2)$
- $\mathrm{Sud}(N,m_b)$ is renormalization group invariant resumming subleading logarithms beyond NNLL.
- IR sensitivity of the soft function is reduced knowing the residues at $u=1/2$ — exact cancellation of pole mass ambiguity — and $u=1$.

DGE: Sudakov and Renormalon resummation

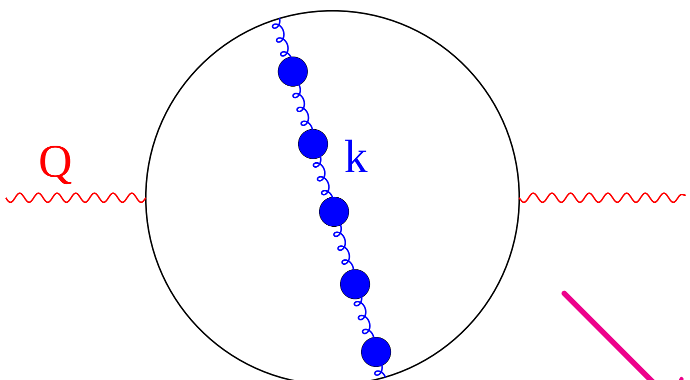
Renormalon resummation:

running–coupling corrections,

which dominate the large–order

asymptotics of the series, $n \to \infty$

$\sum_n n!\, {\alpha_s}^n \longrightarrow$ soft dynamics



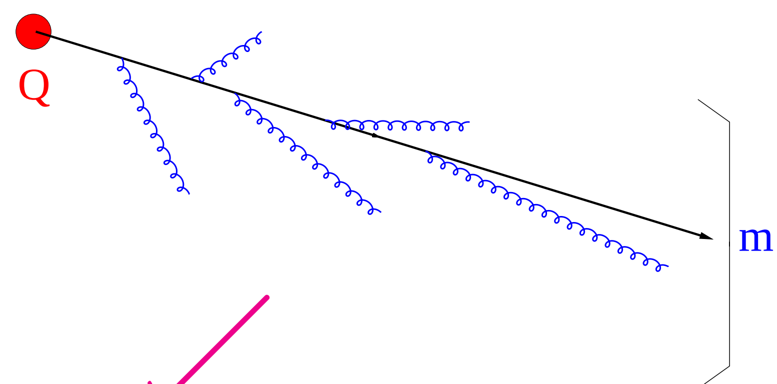
Sudakov resummation:

multiple soft and collinear radiation,

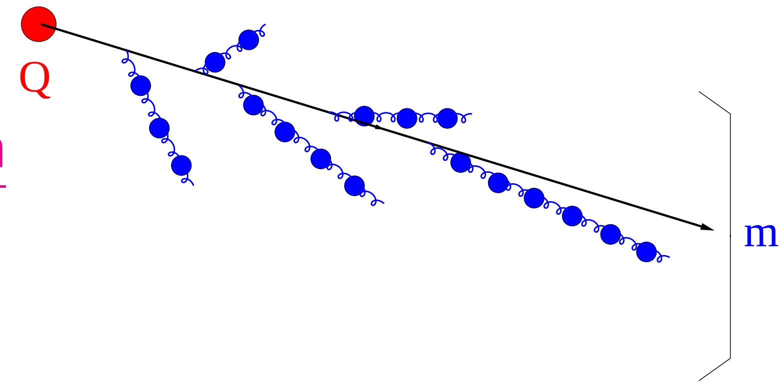
which dominate the dynamics

near threshold $\mathrm{m} \to 0$

$\sum_n {\alpha_s}^n \ln^{2n}(\mathrm{m}/\mathrm{Q})$

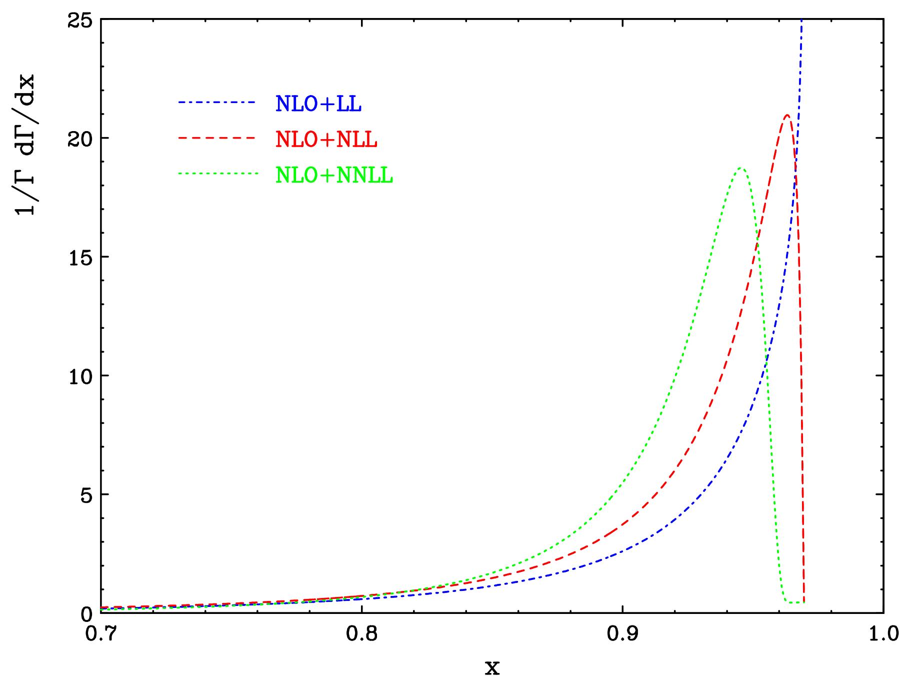


Dressed Gluon Exponentiation

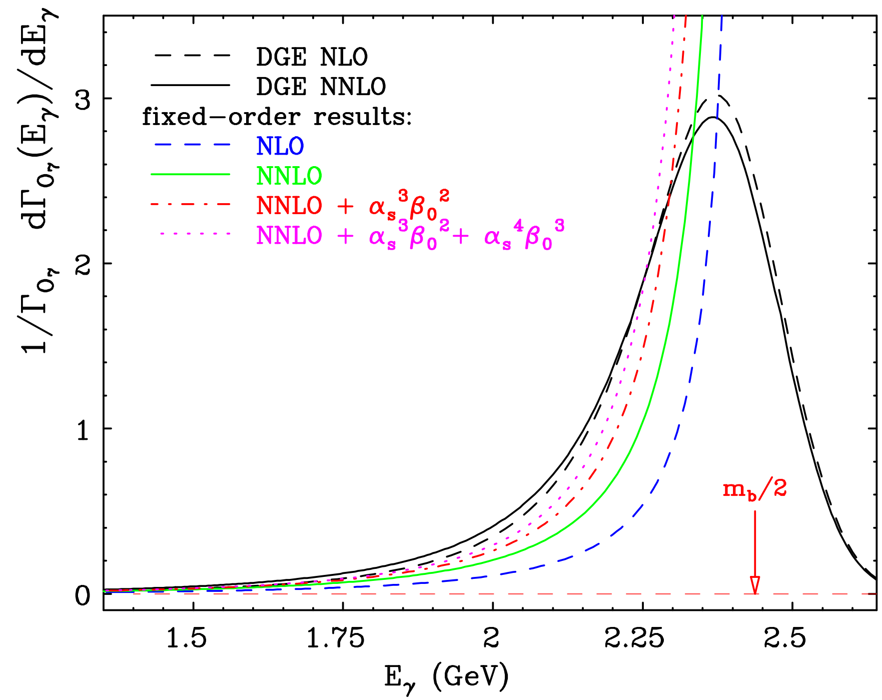


DGE: Sudakov and Renormalon resummation

Standard Sudakov resummation:



Perturbation theory *vs.* DGE

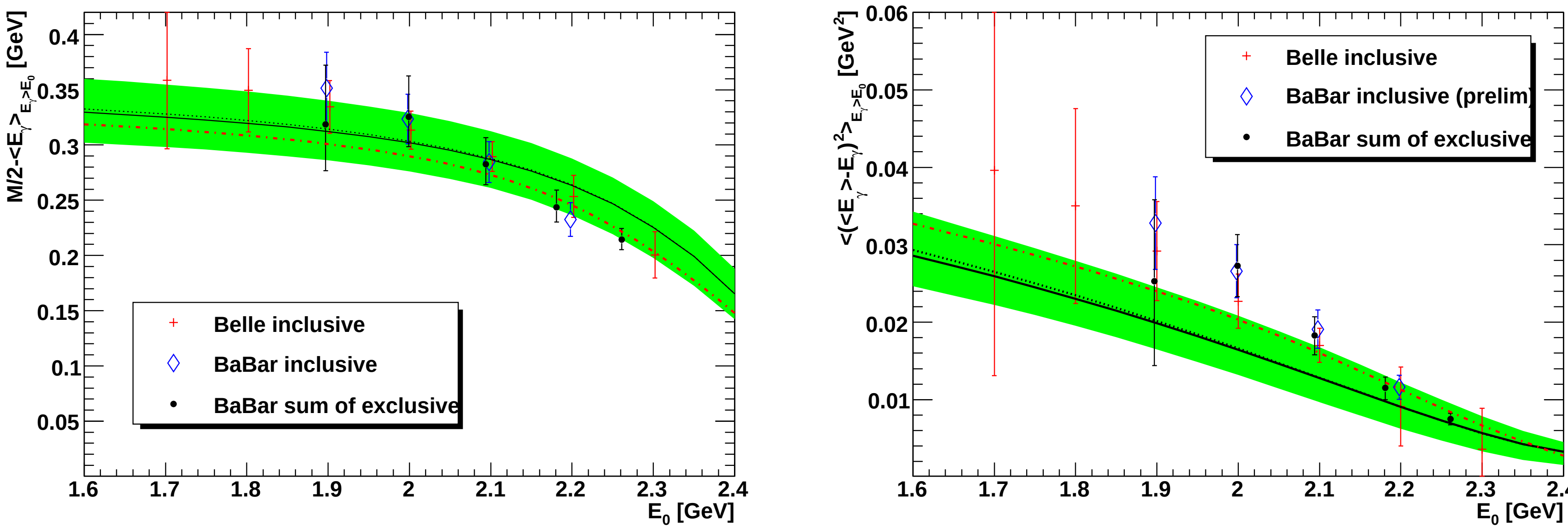


- **Left:** Fixed-logarithmic accuracy Sudakov resummation – LL, NLL, NNLL ... – does not converge well owing to large subleading logarithms (running coupling).
- **Right:** Fixed order perturbation theory does not converge well; each order diverges at $E_\gamma = m_b/2$, alternating between $\pm\infty$.
  DGE shows stability: all large corrections have been resummed.

E_γ moments as a function of the cut: theory *vs.* data

$$\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \, \frac{d\Gamma(E_\gamma)}{dE_\gamma} \, E_\gamma$$

$$\left\langle \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \, \frac{d\Gamma(E_\gamma)}{dE_\gamma} \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n$$



Andersen & Gardi

- good agreement between theory and data!
- prospects: determination of m_b and power corrections.

Misiak: why MSOPE fails below $E_\gamma = 1.6$ GeV

Define $\delta = \Delta/m_b = 1 - 2E_0/m_b$ and consider the $O_7 - O_7$ integrated spectrum:

$$\frac{\Gamma(E_0)}{\Gamma(0)} = 1 + \frac{\alpha_s}{\pi}\phi^{(1)}(\delta) + \left(\frac{\alpha_s}{\pi}\right)^2 \phi^{(2)}(\delta) + \ldots$$

Split the coefficient at each order (n) into logarithmically–enhanced and the rest:

$$\phi^{(n)}(\delta) = \phi_L^{(n)}(\delta) + \phi_N^{(n)}(\delta)$$

For example for $n = 1$: $\phi^{(1)} = \phi_L^{(1)} + \phi_N^{(1)}$

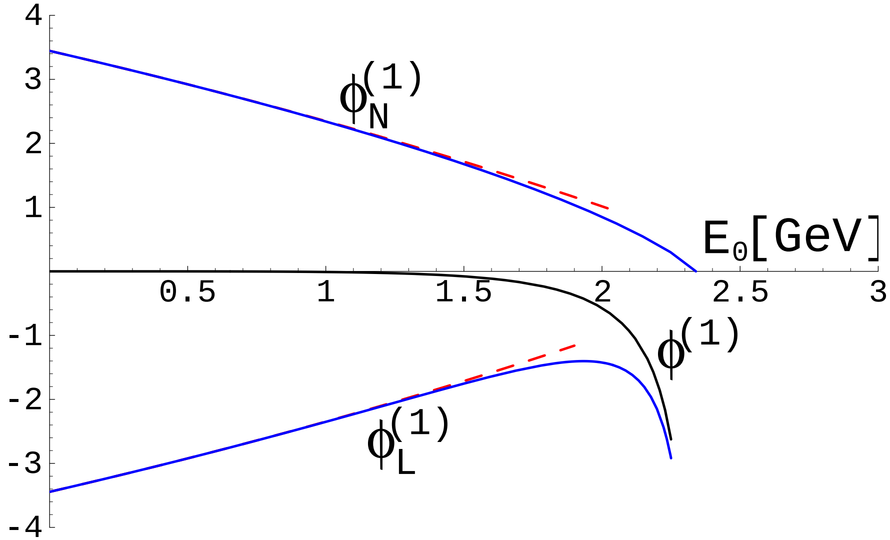
$$\phi_L^{(1)}(\delta) = -\frac{2}{3}\ln^2\delta - \frac{7}{3}\ln\delta - \frac{31}{9},$$

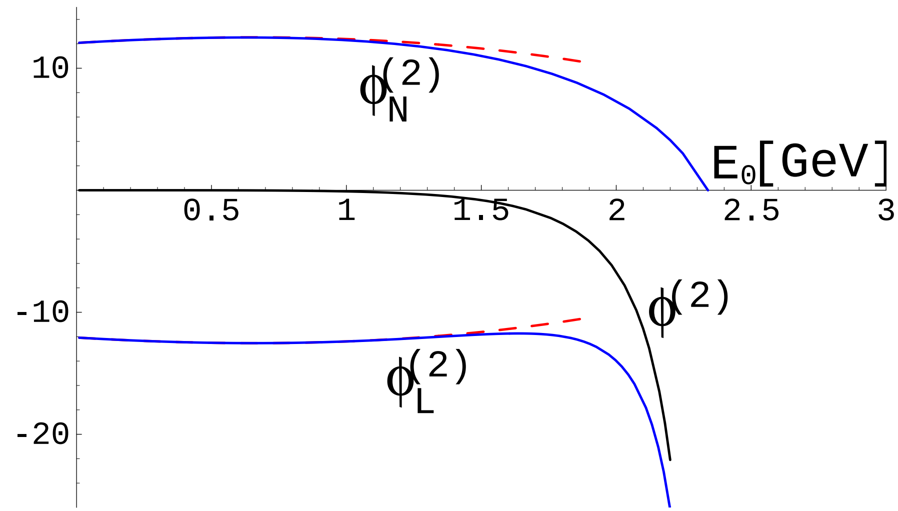
$$\phi_N^{(1)}(\delta) = \frac{10}{3}\delta + \frac{1}{3}\delta^2 - \frac{2}{9}\delta^3 + \frac{1}{3}\delta(\delta - 4)\ln\delta.$$

Misiak: why MSOPE fails below $E_\gamma = 1.6$ GeV

$$\phi^{(n)}(\delta) = \phi_L^{(n)}(\delta) + \phi_N^{(n)}(\delta)$$

The results are known for $n = 1, 2$:





- logarithmically enhanced terms dominate only for $E_0 \gtrsim 2$ GeV.
- large cancellation between $\phi_L^{(n)}(\delta)$ and $\phi_N^{(n)}(\delta)$ all the way from $E_0 = 0$ to $E_0 \simeq 1.6$ GeV
- $\Gamma(E_0) \simeq E_0^4$ for small $E_0 \Longrightarrow$ the cancellation occurs at all orders! $\Longrightarrow$ keeping only $\phi_L^{(n)}(\delta)$ is BAD below 1.6 GeV.

DGE: matching to fixed order and the small E_γ limit

In DGE we match to the fixed–order expansion in moment space. Also here there is freedom to shift $\mathcal{O}(1/N)$ terms between the resummation and the residual term:

$$\begin{aligned}\frac{\Gamma(E_0)}{\Gamma(0)} &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{dN}{N-1}\left(\frac{2E_0}{m_b}\right)^{1-N} H\left(\alpha_s(m_b),N\right)\,\times\,\mathrm{Sud}(N,m_b)\\ &+ \int_{x=2E_0/m_b}^{1} dx\Delta R(\alpha_s(m_b),x),\end{aligned}$$

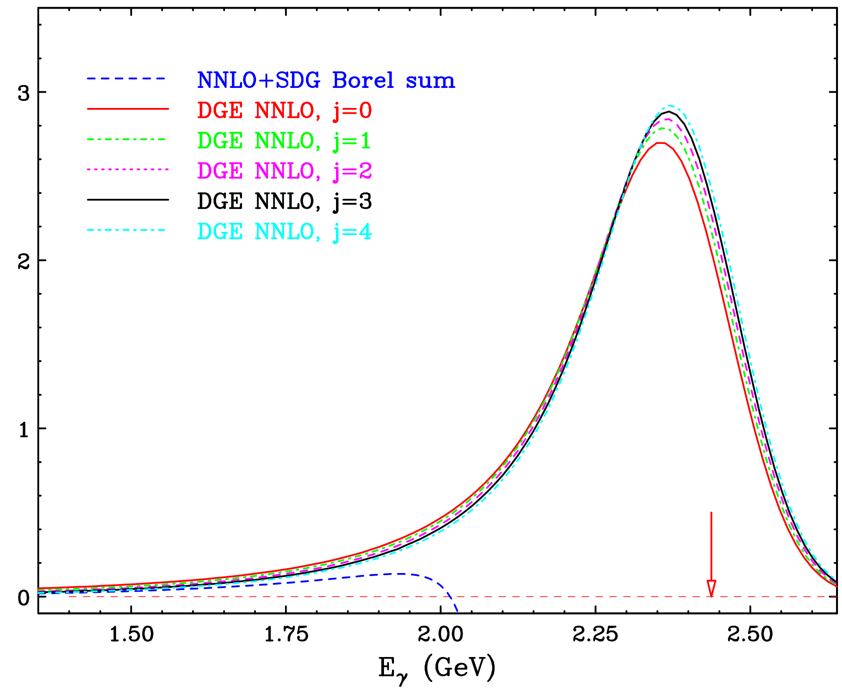
But we *use this freedom* plus additional information on the small E_γ limit where $d\Gamma/dE_\gamma \sim E_\gamma^J$ to extending the range of applicability of the resummed result. Defining:

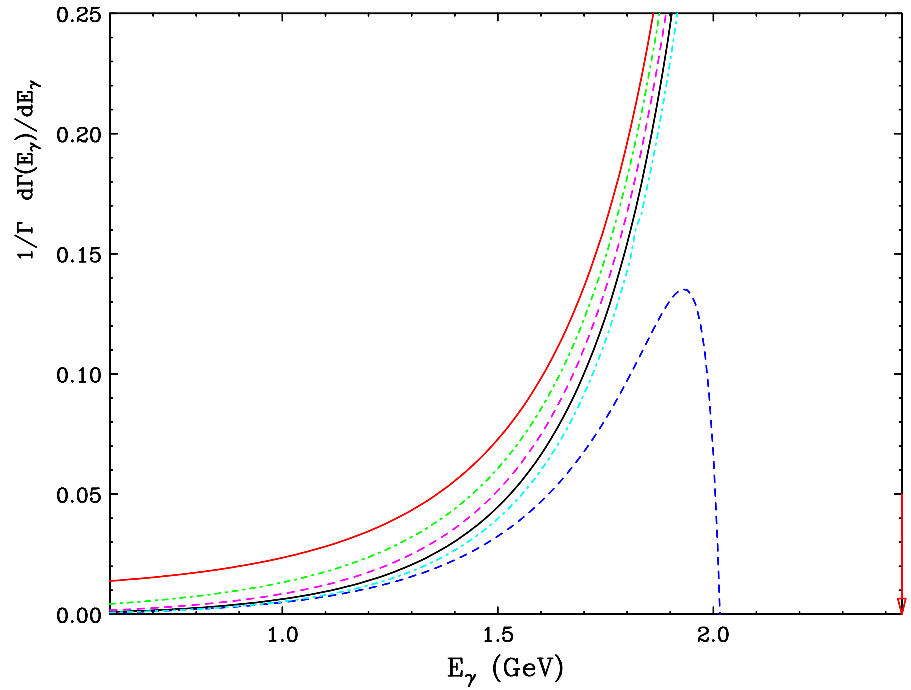
$$\widetilde{\mathrm{Sud}}^{(J)}(N,m_b) = \exp\left\{\frac{C_F}{\beta_0}\int_0^\infty\frac{du}{u}\,T(u)\left(\frac{\Lambda^2}{m_b^2}\right)^u\left[B_{\mathcal{S}}(u)\Gamma(-2u)\times\right.\right.$$
$$\left.\left.\left(\frac{\Gamma(N+J)}{\Gamma(N+J-2u)}-\frac{\Gamma(J+1)}{\Gamma(J+1-2u)}\right)-B_{\mathcal{J}}(u)\Gamma(-u)\left(\frac{\Gamma(N+J)}{\Gamma(N+J-u)}-\frac{\Gamma(J+1)}{\Gamma(J+1-u)}\right)\right]\right\}$$

there are no poles for $N>-J$, so the Sudakov factor would not give rise to a tail that falls slower than E_γ^J.

DGE: matching to fixed order and the small E_γ limit

Changing J from 0 through 4:





Between $J = 0$ (standard Sudakov resummation — constant E_γ tail) and $J = 3$ (the correct power suppression at small E_γ)

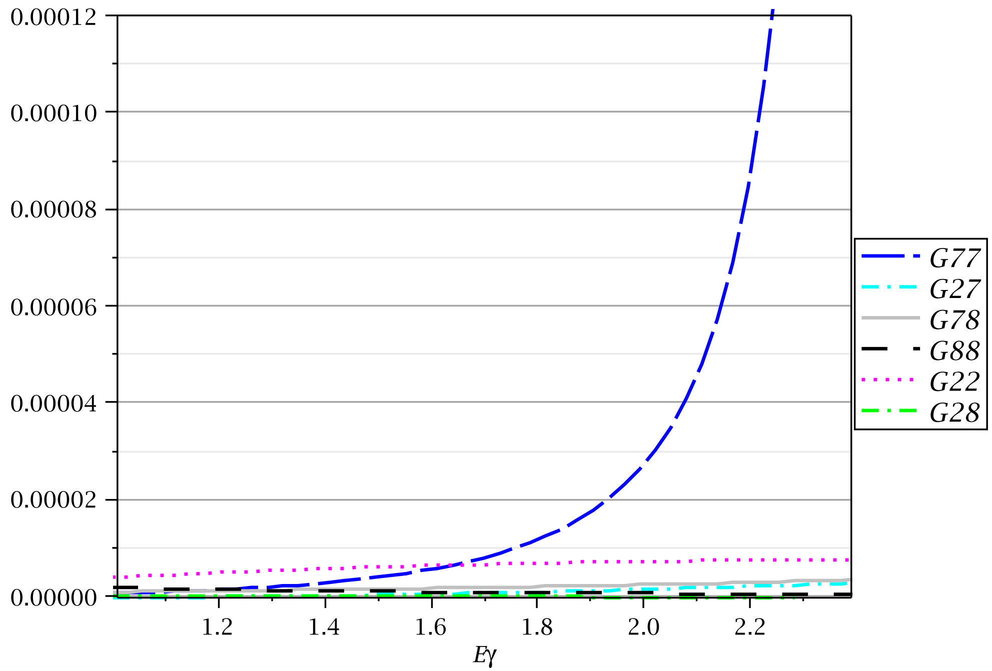
$T \equiv \dfrac{\mathcal{B}(E_\gamma > 1.6 \text{ GeV})}{\mathcal{B}(E_\gamma > 1.0 \text{ GeV})}$ gets smaller by a factor of 2!

Operator–dependent spectrum

- Common lore: the spectrum does not depend on the short–distance interaction.
- Indeed, true to a good approximation: in parturbation theory, not only $O_7 - O_7$, but all interference terms e.g. $O_2 - O_7$ have a common Sudakov factor peaking near $m_b/2$.
- But considering the tail the details do matter! E.g. $O_2 - O_2$ starts off with an extra hard gluon, generating a significant contribution at low E_γ.

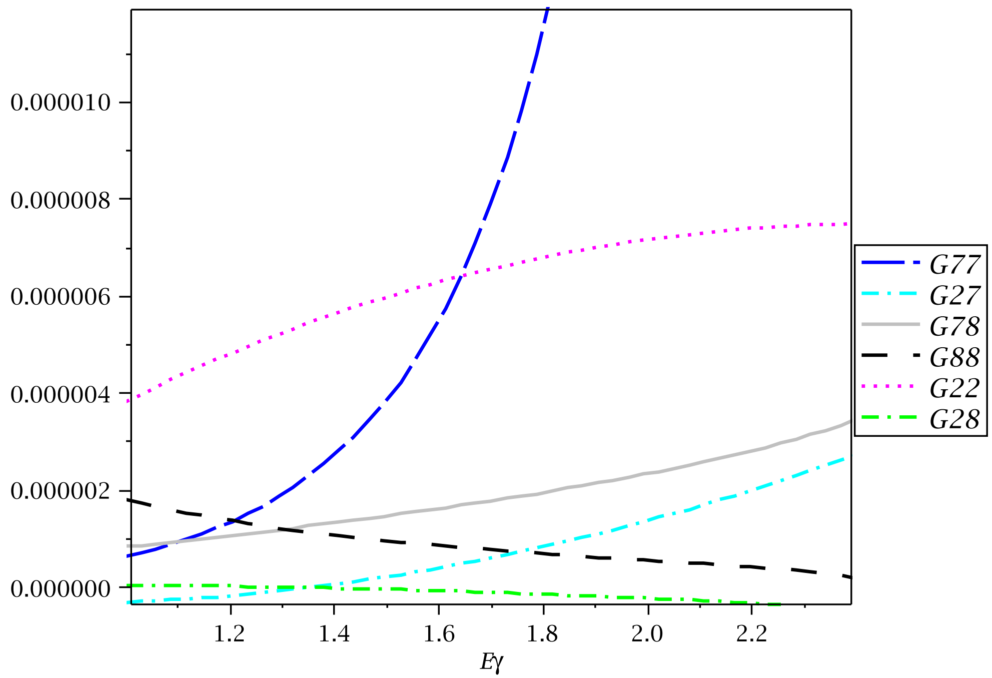
Operator–dependent spectrum

At first sight only $O_7 - O_7$ is important



Operator–dependent spectrum

But looking at the tail other operator contributions are not small!

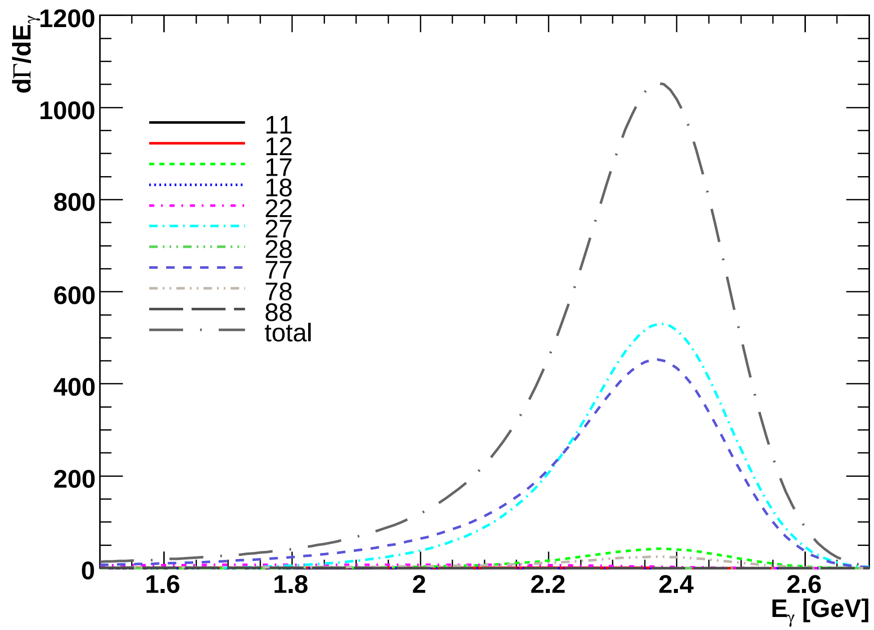


However, this is all at NLO. All but $O_7 - O_7$ start at this order, so to have any control of the renormalization scales (α_s, m_c) one needs full NNLO.

Moreover, soft and collinear radiation effects modify all spectra!

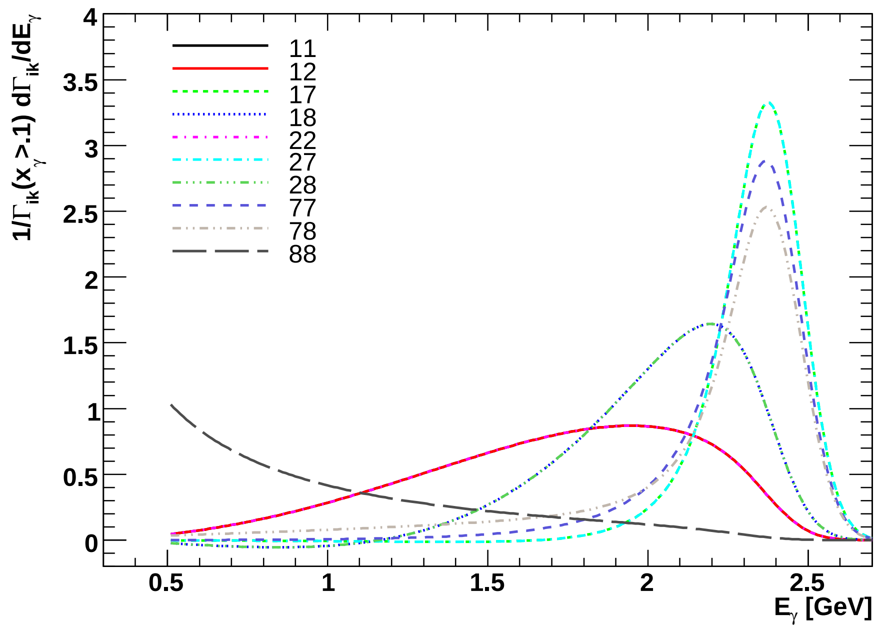
Operator–dependent spectrum: resummation

Upon including resummation we observe that $O_2 - O_7$ and $O_2 - O_7$ are roughly as important in the peak region, while all others are subdominant.



Operator–dependent spectrum: resummation

Let us normalize each of the matrix elements separately, to examine the shapes:

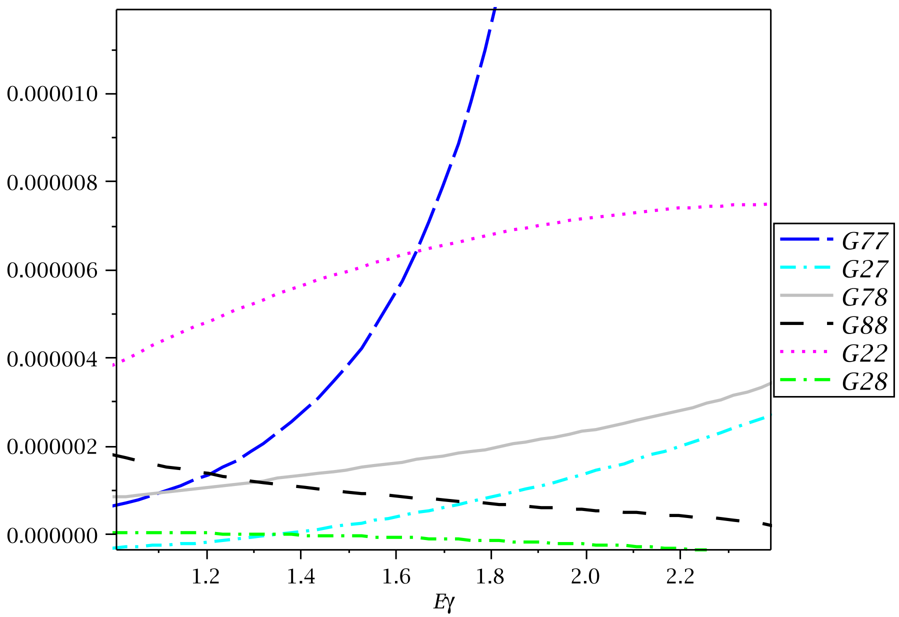


So the shapes are not universal, e.g. $O_2 - O_8$ and $O_2 - O_2$. This is important at the tail.

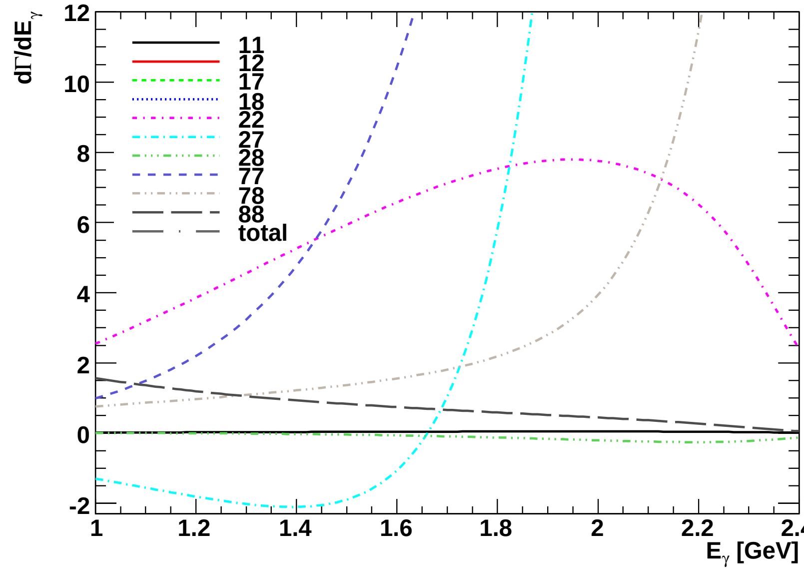
Operator–dependent spectrum: resummation

Returning to the proper relative normalization, let's look at the tail:

NLO



Resummed

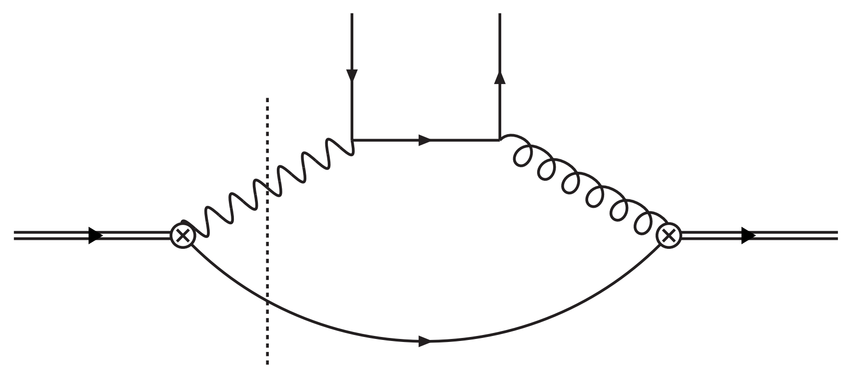


Resummation has an impact well below 1.6 GeV:

- $O_7 - O_7$ and interference terms such as $O_2 - O_7$ and $O_7 - O_8$ have a Sudakov peak — the impact extends far below the peak.
- After resummation $O_2 - O_7$ terns more nagative, and flips sign already at 1.6 GeV (while at NLO this occurs around 1.4 GeV).
- After resummation $O_2 - O_2$ becomes the largest contribution below 1.45 GeV (while at NLO this occurs around 1.65 GeV).

New non-perturbative uncertainties

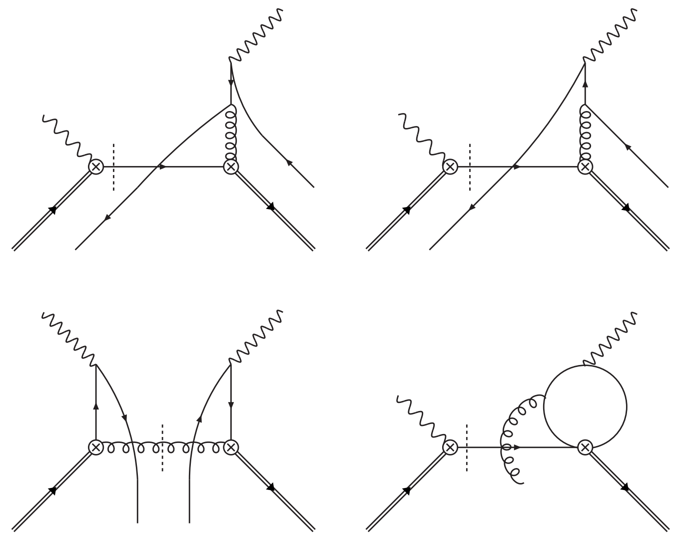
- Striking demonstration of the inapplicability of the OPE in $\bar{B} \to X_s \gamma$: "Enhanced Non-local Power Corrections to $\bar{B} \to X_s \gamma$ decay rate" by Lee, Neubert & Paz
- The standard derivation of the OPE relies on appling the optical theorem.



- However, in $\bar{B} \to X_s \gamma$, in presence of Weak operators other than O_7 the photon couples to hight quarks. In this case, not all cuts correspond to the physical process of interest where the photon is part of the final state.
- Therefore, we have to live without 'full proof' OPE even for the fully integrated rate.

New non-perturbative uncertainties

- Leading contribution: non-perturbative $O_7 - O_8$ interference, involving a constituent light quark (and therefore contributes to the asymetry between B^- and $\bar{B}^0$)



- the energetic photon emerges from an energetic light quark, thus (1) inducing a second collinear sensitive region (2) new soft function.
- Order of magnitude in total rate: $\frac{\Delta\Gamma}{\Gamma_{77}} \sim \frac{C_8}{C_7} \alpha_s \frac{\Lambda}{m_b}$
- Numerical estimate difficult. Rough model (Vacuum Insertion Approximation) gives $0.3\% \to 3\%$.

Conclusions

Resummation *is* the key to describing the $\bar{B} \to X_s\gamma$ spectrum

- For the region below 1.6 GeV completion of the NNLO calculation is important.
- Uncertainties of non-perturbative origin will remain!
  OPE violating effects should be estimated and accounted for.
- Both data and theory have improved: fits to moments/spectra should be done using several different theory approaches and over a wide range of photon energies.
- Splitting the branching fraction extrapolation procedures at 1.6 GeV is sensible if different tools apply below and above. At present it looks unnatural.
  Let's confront theory and experimental BF at $E_\gamma > 1.8$ GeV.
- Exciting field, despite its advanced age ($\sim$ 15 years)