

# $\Delta F = 2$ Observables and Fine-Tuning in a Custodially Protected Warped Extra Dimension

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based on: MB, BURAS, DULING, GORI, WEILER, 0809.1073

# $\Delta F = 2$ in Warped Extra Dimensions

many analyses in the existing literature

BURDMAN, HEP-PH/0205329, HEP-PH/0310144; AGASHE, PEREZ, SONI, HEP-PH/0408134; MOREAU, SILVA-MARCOS, HEP-PH/0602155; CHANG, KIM, SONG, HEP-PH/0607313; CSAKI, FALKOWSKI, WEILER, 0804.1954; ...

## What is new in BBDGW?

### First complete analysis of $\Delta F = 2$ processes

- within the **custodially protected RS model**
- including simultaneously **all  $\Delta F = 2$  operators**
- performing **RG-running at the NLO level**
- including both **strong and electroweak gauge boson contributions**
- considering **all interesting  $\Delta F = 2$  observables simultaneously**
- analysing **fine-tuning** in flavour physics

# Main Messages from BBDGW

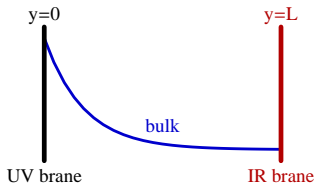
- 1 confirmation of **generic bound  $M_{KK} \gtrsim 20 \text{ TeV}$  from  $\epsilon_K$**   
CSAKI, FALKOWSKI, WEILER, 0804.1954
- 2 also for  $M_{KK} \gtrsim (2 - 3) \text{ TeV}$  **agreement with  $\epsilon_K$  possible without relevant fine-tuning**
- 3  **$\Delta M_K$  and  $\epsilon_K$  are governed by KK gluon contributions**
- 4 in  $B_{d,s} - \bar{B}_{d,s}$  **electroweak KK modes** equally important
- 5 **tree level down-type FCNCs through Z eliminated** by custodial protection of  $Z b_L \hat{b}_L$  (both  $\Delta F = 2$  and  $\Delta F = 1$ )
- 6 possible **tensions in the SM** ( $\epsilon_K, S_{\psi K_S}, \dots$ ) can be **solved**
- 7  **$S_{\psi\phi}$  and  $A_{SL}^S$  can be large**

# The basic RS Set-up

5D spacetime with **warped** metric:

RANDALL, SUNDRUM, HEP-PH/9905221

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad 0 \leq y \leq L$$



- **fermions and gauge bosons** live in the bulk
- **Higgs** localised on IR brane

CHANG ET AL., HEP-PH/9912498

GROSSMAN, NEUBERT, HEP-PH/9912408

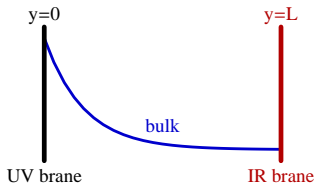
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- energy scales suppressed by **warp factor**  $e^{-ky}$   
→ **natural** explanation of **gauge hierarchy** problem
- **Kaluza-Klein (KK) excitations** live close to the IR brane

# Constraints from EW Precision Tests

**S parameter:**

$$M_{KK} \gtrsim (2-3) \text{ TeV}$$

AGASHE ET AL., HEP-PH/0308036

**T parameter:**

- without protection:  $M_{KK} \gtrsim 10 \text{ TeV}$

(may be softened by heavy Higgs)

CASAGRANDE ET AL., 0807.4937

BARBIERI ET AL., HEP-PH/0603188, ...)

- with **custodially enlarged gauge symmetry** → ✓

AGASHE ET AL., HEP-PH/0308036; CSAKI ET AL., HEP-PH/0308038

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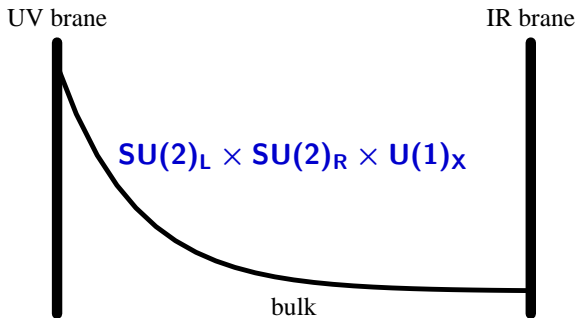
**anomalous  $Z b_L \bar{b}_L$  coupling:** (exp.:  $\lesssim 5 \cdot 10^{-3}$ )

- corrections arise naturally at the  $-(1-2)\%$  level
- protection by **discrete  $SU(2)_L \leftrightarrow SU(2)_R$  symmetry** ✓  
→ enlarged fermion representations

AGASHE ET AL., HEP-PH/0605341; CONTINO ET AL., HEP-PH/0612048

CARENA ET AL., HEP-PH/0607106

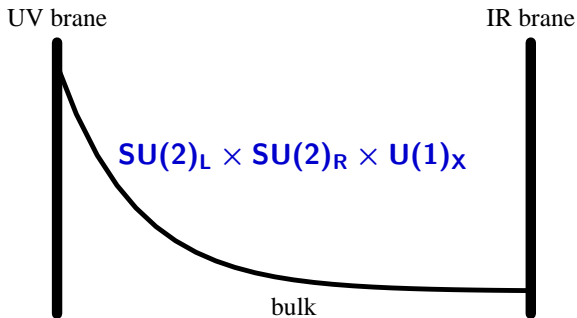
# A Realistic Model in the Reach of LHC



+ ( $L \leftrightarrow R$ )-symmetric fermion representations



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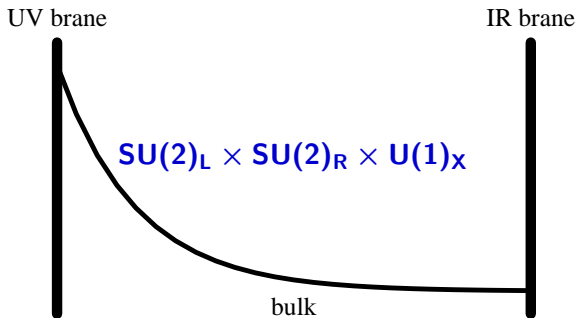
$$SU(2)_R \times U(1)_X \rightarrow U(1)_Y$$

by boundary conditions

+ ( $L \leftrightarrow R$ )-symmetric fermion representations

**low energy theory:**  $SU(2)_L \times U(1)_Y$  in the absence of EWSB

# A Realistic Model in the Reach of LHC



$SU(2)_R \times U(1)_X \rightarrow U(1)_Y$   
by boundary conditions

$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$   
by Higgs VEV

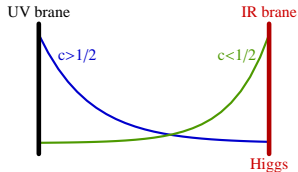
+ ( $L \leftrightarrow R$ )-symmetric fermion representations

**low energy theory:**  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

# Fermion Localisation and Yukawa Couplings

**zero mode profile** depends strongly on bulk mass parameter  $c$ :

$$f^{(0)}(y, c) \propto e^{(\frac{1}{2}-c)ky}$$

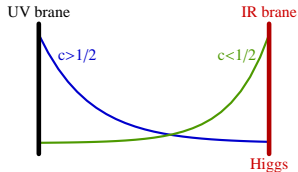


$c > \frac{1}{2}$ : localisation around **UV brane**  
 $c < \frac{1}{2}$ : localisation around **IR brane**

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$c > \frac{1}{2}$ : localisation around **UV brane**  
 $c < \frac{1}{2}$ : localisation around **IR brane**

**effective 4D Yukawa couplings:**

$$(Y_{u,d})_{ij} = (\lambda_{u,d})_{ij} f_i^Q f_j^{u,d}$$

- $\lambda_{u,d} \sim \mathcal{O}(1)$  anarchic complex  $3 \times 3$  matrix
- **hierarchical structure** can be naturally generated by exponential suppression of  $f^{Q,u,d}$  (fermion profile on IR brane)

# Flavour Violation by KK Gauge Bosons

- **KK gauge bosons** localised close to **IR brane**:  $g(y) \sim e^{ky}$
- couplings to SM fermions depend on their localisation
- **flavour eigenbasis:**

$$\bar{\psi}_i G_\mu \psi_i \sim -ig^{4D} \gamma_\mu \sqrt{kL} (\mathbf{f}_i^\psi)^2 + \text{const.}$$

flavour-diagonal, but non-universal!

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- rotation to **mass eigenbasis** via  $\mathcal{D}_{L,R}$ : (estimate for anarchic  $\lambda_{u,d}$ )

$$\begin{aligned} \bar{d}_L^i G_\mu d_L^j &\sim -ig^{4D} \gamma_\mu \sqrt{kL} \mathbf{f}_i^Q \mathbf{f}_j^Q \\ \bar{d}_R^i G_\mu d_R^j &\sim -ig^{4D} \gamma_\mu \sqrt{kL} \mathbf{f}_i^d \mathbf{f}_j^d \end{aligned}$$

- **tree level FCNCs arise!**
- **protected by RS-GIM mechanism**

AGASHE, PEREZ, SONI, HEP-PH/0408134

# Contributions to $\Delta F = 2$

- **KK gluons**

AGASHE, PEREZ, SONI, HEP-PH/0408134  
CSAKI, FALKOWSKI, WEILER, 0804.1954; BBDGW

- **KK weak gauge bosons** ( $Z_H, Z', A^{(1)}$ )

BBDGW

subdominant in  $K - \bar{K}$ , but **competitive** in  $B - \bar{B}$

- **Z boson**

BBDGW

**evaded** thanks to custodial protection mechanism  
→ extends **custodial protection** to  $Z d_L^i \bar{d}_L^j$  couplings

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- generally: **new operators** are induced:

$$\begin{aligned} Q_{LL} &= (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma_\mu P_L d) & Q_{1LR} &= (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma_\mu P_R d) \\ Q_{RR} &= (\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma_\mu P_R d) & Q_{2LR} &= (\bar{s}P_L d)(\bar{s}P_R d) \quad (*) \end{aligned}$$

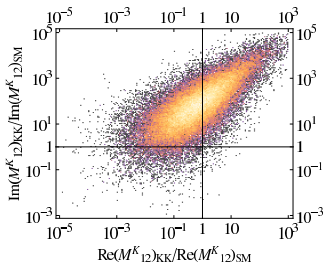
(\*) KK gluons only

$Q_{LR}$ : **QCD** ( $K$  and  $B$ ) **and chirally** (only  $K$ ) **enhanced!**

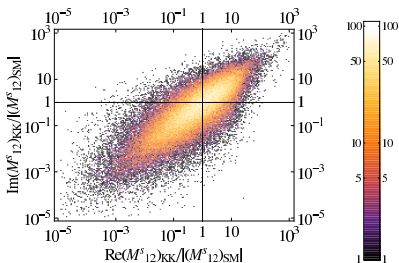


# KK Gauge Boson Contribution to $M_{12}^i$

**K –  $\bar{K}$  mixing:**



**$B_s - \bar{B}_s$  mixing:**



$$\text{Re}(M_{12}^K)_{\text{KK}} \sim \text{Re}(M_{12}^K)_{\text{SM}}$$

$$\text{Im}(M_{12}^K)_{\text{KK}} \sim 10^2 \text{Im}(M_{12}^K)_{\text{SM}}$$

**generally tension with  $\epsilon_K$**

CSAKI, FALKOWSKI, WEILER

$$|(M_{12}^S)_{\text{KK}}| \sim |(M_{12}^S)_{\text{SM}}|$$

$$\text{Arg}(M_{12}^S)_{\text{KK}} \sim \mathcal{O}(1)$$

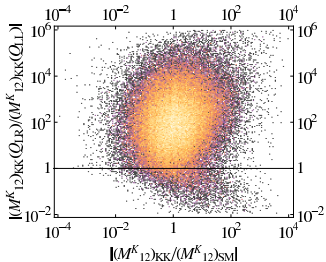
**large  $S_{\psi\phi}$  expected**

BBDGW

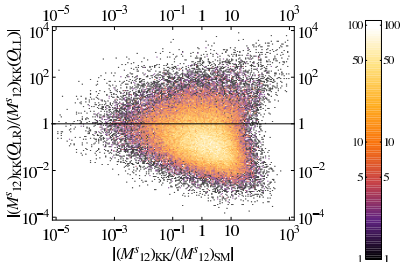
all results for  $M_{\text{KK}} \simeq 3 \text{ TeV}$

# Operator Competition in $\Delta F = 2$

**K –  $\bar{K}$  mixing:**



**$B_s - \bar{B}_s$  mixing:**



$Q_{LR}$  dominates  
by two orders of magnitude  
KK gluons dominant

$Q_{LL}$  and  $Q_{LR}$   
are competitive  
EW KK modes important

(no chiral LR enhancement in  $B$  system)

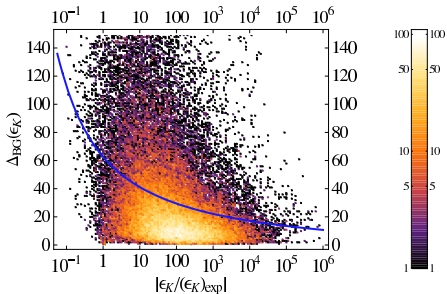
$Q_{RR}$  contribution generally small

BBDGW

# Required Fine-Tuning in $\epsilon_K$

BBDGW

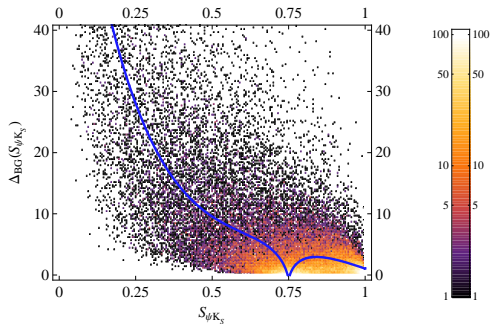
**Barbieri-Giudice measure of fine-tuning:**  
sensitivity of observable to small variation of model parameters



- generically  $\epsilon_K \sim 10^2 (\epsilon_K)_{exp}$
- required tuning generically increases with decreasing  $\epsilon_K$
- $\epsilon_K \sim (\epsilon_K)_{exp}$  **possible without significant tuning**

# Situation for other $\Delta F = 2$ Observables: $S_{\psi K_S}$

BBDGW



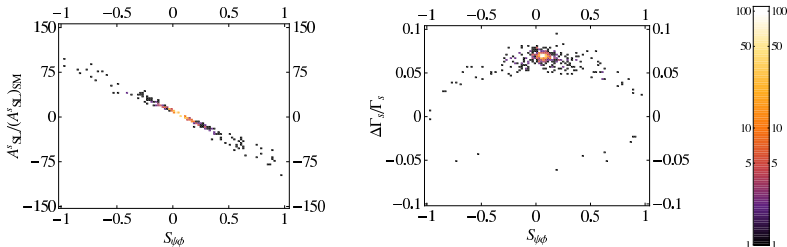
- generically  $S_{\psi K_S} \sim (S_{\psi K_S})_{\text{SM}}$  predicted
- **possible tension** between SM and data easily **resolved**

similar situation for other  $\Delta F = 2$  observables

# CP-Violation in $B_s - \bar{B}_s$ Mixing

after imposing existing  $\Delta F = 2$  constraints:

BBDGW



- full range  $-1 < \mathbf{S}_{\psi\phi} < 1$  possible  
→ can explain recent CDF and DØ data
- strong **correlation with  $A_{SL}^s$**  (see LIGETI ET AL., HEP-PH/0604112)  
→  $A_{SL}^s / (A_{SL}^s)_{SM} \sim 100$  possible
- $\Delta\Gamma_s / \Gamma_s$  can deviate significantly from SM prediction

Our complete analysis of  $\Delta F = 2$  processes in a custodially protected warped extra dimension showed:

- $K - \bar{K}$  dominated by  $\mathcal{Q}_{LR}$ , KK gluons
- $\mathcal{Q}_{LL}$  important for  $B_{d,s} - \bar{B}_{d,s}$ , sizable electroweak KK contributions
- custodial protection for  $Z b_L \bar{b}_L$  eliminates flavour violating Z coupling  $Z d_L^i \bar{d}_L^j$

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- $\epsilon_K$  constraint can be fulfilled without significant tuning
- simultaneous agreement with all  $\Delta F = 2$  data can be obtained
- large new physics effects in  $S_{\psi\phi}$ ,  $A_{SL}^S$  and  $\Delta\Gamma_s$  are possible

Implications for rare K and  $B_{d,s}$  decays:

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# Back-up slides

# Sources of Flavour Violation & Parameter Counting

AGASHE, PEREZ, SONI, HEP-PH/0408134

**Flavour is violated by:**

- **bulk mass terms**  $c_Q, c_u, c_d$ :  
3 × 3 hermitian matrices
- **Yukawa couplings**  $\lambda_u, \lambda_d$ :  
3 × 3 complex matrices

3 × 6 real parameters  
3 × 3 complex phases  
2 × 9 real parameters  
2 × 9 complex phases

---

36 real parameters  
27 complex phases

**$U(3)^3$  flavour symmetry**

can be used to remove

– 9 real parameters  
– 17 complex phases

**physical** flavour parameters:

**27 real parameters**  
**10 complex phases**

## bulk fermions in RS

$$(Y_{u,d}^{\text{WED}})_{ij} \propto (\lambda_{u,d})_{ij} e^{-kL(c_Q^i - c_{u,d}^j)}$$

self-similarity along  $y$

bulk mass parameters  $c_{Q,u,d}^i$

IR brane at  $y = L$

warp factor  $e^{-kL}$

## Froggatt-Nielsen symmetry

$$(Y_{u,d}^{\text{FN}})_{ij} \propto (\lambda_{u,d})_{ij} \epsilon^{a_i - b_j^{u,d}}$$

$U(1)_F$  symmetry

$U(1)_F$  charges  $Q_F = a_i, b_j^{u,d}$

VEV of scalar  $\Phi$  ( $Q_F = 1$ )

$$\epsilon = \langle H \rangle / \langle \Phi \rangle \ll 1$$

- **geometric interpretation of flavour symmetry**
- FN formulae for masses and flavour mixings can be applied  
→ dependence on  $\lambda_{u,d}$  and CP phases made explicit

BBDGW; CASAGRANDE ET AL., 0807.4937

quark masses:

$$\begin{aligned}
 m_b &= \frac{v}{\sqrt{2}} \lambda_{33}^d f_3^Q f_3^d \\
 m_s &= \frac{v}{\sqrt{2}} \frac{\lambda_{33}^d \lambda_{22}^d - \lambda_{23}^d \lambda_{32}^d}{\lambda_{33}^d} f_2^Q f_2^d \\
 m_d &= \frac{v}{\sqrt{2}} \frac{\det(\lambda^d)}{\lambda_{33}^d \lambda_{22}^d - \lambda_{23}^d \lambda_{32}^d} f_1^Q f_1^d
 \end{aligned}$$

flavour mixing matrices (responsible for FCNCs):

$$(\mathcal{D}_L)_{ij} = \omega_{ij}^d \frac{f_i^Q}{f_j^Q} \quad (\mathcal{D}_R)_{ij} = \rho_{ij}^d \frac{f_i^d}{f_j^d} \quad (i < j)$$

$(\omega_{ij}^d, \rho_{ij}^d)$ : functions of  $\lambda_d$

analogous formulae for the up-type quarks

# New Flavour and CP Violating Effects

- $Z^{(0)}$  coupling becomes non-universal  
⇒ **tree level FCNC mediated by  $Z^{(0)}$  boson**

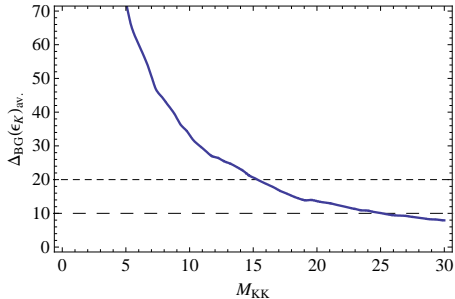
$$Z^{(0)} \bar{q}_i q_j \propto g F_i F_j \frac{v^2}{M_{KK}^2}$$

- **gauge KK modes** are localised at IR brane  
⇒ flavour universality is broken, FCNCs arise

$$Z^{(1)} \bar{q}_i q_j \propto g F_i F_j$$

$$G^{(1)} \bar{q}_i q_j \propto g_s F_i F_j$$

- **loop contributions of new heavy particles**
- **9 new CP phases** in the mixing matrices  $\mathcal{U}_{L,R}, \mathcal{D}_{L,R}$



average required tuning in  $\epsilon_K$ , depending on  $M_{KK}$

→ generic naturalness bound:  $M_{KK} \simeq 20 \text{ TeV}$