Theory of $B \to V\gamma$ decays

Ben Pecjak

DESY

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Outline

1) $B \to V\gamma$ decays in QCD factorization and SCET

- sample result: $|V_{td}/V_{ts}|$

2) Recent progress and limitations

- NNLO perturbative corrections
- power corrections and endpoint divergences

Rare radiative $B \to V\gamma$ decays

Examples: $B \to (\rho, K^*, \omega, \phi)\gamma$ decays

- all involve FCNC
- potential to constrain new physics and CKM parameters

Observables and their relevance

- branching fractions $\leftrightarrow |V_{td}/V_{ts}|$
- CP asymmetries $\leftrightarrow$ new physics, $(\bar{\rho}, \bar{\eta})$
- isospin violation $\leftrightarrow$ new physics, $(\bar{\rho}, \bar{\eta})$

Experimental measurements have continued to improve

- relevant to review theory status

Theoretical challenge: hadronic matrix elements

Amplitude for $b \to s\gamma$ transitions:

$$\mathcal{A} \sim \langle V\gamma|\mathcal{H}_{\rm eff}|\bar{B}\rangle \sim \sum_{p=u,c} V_{ps}^* V_{pb} \sum_{i=1}^{8} C_i \langle V\gamma|Q_i^p|\bar{B}\rangle$$

- **Main challenge**: evaluate $\langle V\gamma|Q_i^p|\bar{B}\rangle$ = hadronic matrix elements

Most important operators:

$$Q_1^p = (\bar{s}\,p)_{V-A}\,(\bar{p}\,b)_{V-A} \qquad Q_2^p = (\bar{s}_i\,p_j)_{V-A}\,(\bar{p}_j\,b_i)_{V-A}, \qquad (p = u, c)$$

$$Q_7 = -\frac{e\,\overline{m}_b}{8\pi^2}\,\bar{s}\,\sigma^{\mu\nu}\,[1+\gamma_5]\,bF_{\mu\nu}, \qquad Q_8 = -\frac{g\,\overline{m}_b}{8\pi^2}\,\bar{s}\,\sigma^{\mu\nu}\,[1+\gamma_5]\,T^a\,bG^a_{\mu\nu}$$

For $b \to d\gamma$ replace $s \to d$

Theoretical approaches

- QCD factorization
  (Ali, Parkhomenko; Bosch, Buchalla; Beneke, Feldmann, Seidel)
- SCET $\simeq$ QCD factorization + resummation
  (Chay, Kim; Grinstein, Grossman, Ligeti; Becher, Hill, Neubert)
- QCD factorization + QCD sum rules
  (Ball, Jones, Zwicky)
- pQCD
  (Keum, Matsumori, Sanda, Yang)

Talk will focus on QCD factorization-based approaches
(pQCD in write-up)

QCD factorization

Matrix elements of Q_i obtained as a series in $\alpha_s, \, \Lambda_{\mathrm{QCD}}/m_b \ll 1$

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = t_i^{\mathrm{I}}\,\zeta_{V_\perp} + t_i^{\mathrm{II}} \star \phi_+^B \star \phi_\perp^V + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

- $\zeta_{V_\perp}$ (soft function) and $\phi^{B,V}$ (LCDAs) are non-perturbative
- t^{I} and t^{II} are perturbative hard-scattering kernels

$$t^{\mathrm{I}} = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + \ldots \qquad t^{\mathrm{II}} = \mathcal{O}(\alpha_s) + \ldots$$

$$\text{"vertex corrections"} \qquad \text{"spectator corrections"}$$

- $1/m_b$ power corrections may or may not factorize
(but crucial for interesting observables such as isospin asymmetries)

SCET approach

SCET factorization formula:

(all-orders proof in Becher, Hill, Neubert '05)

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = t_i^{\mathrm{I}}\,\zeta_{V_\perp} + t_i^{\mathrm{II}} \star \phi_+^B \star \phi_\perp^V + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

- $\zeta_{V_\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- hard-scattering kernels = SCET matching coefficients

$$\begin{aligned} t_i^{\mathrm{I}} &= C_i^A(m_b,\mu) \\ t_i^{\mathrm{II}} &= C_i^B(m_b,\mu) \star j_\perp(m_b\Lambda,\mu) \qquad \text{(subfactorization)} \end{aligned}$$

- large logs in t_i^{II} resummed by solving RG equations
- potential to systematically treat $1/m_b$ corrections

Power corrections

Leading power corrections are naively $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b) \sim 10\%$.

However, systematic treatment lacking due to:

- divergent convolutions in QCDF and SCET (conceptual point)
- potentially large set of subleading form factors and LCDAs (practical point)

Current approaches are pragmatic

- focus on corrections important for key observables (isospin, CP asymmetries)

Will give two examples at end of talk

QCDF results

Ali, Lunghi, Parkhomenko; Bosch, Buchalla; Beneke, Feldmann, Seidel, . . .

- form factors and LCDAs from QCD sum rules
- t^{I}, t^{II} known at NLO (α_s)
- annihilation at tree level ($1/m_b$ correction)

Analysis by Ball, Jones, Zwicky in QCDF + sum rules included

- improved sum-rule predictions for form factor ratios
- estimate of $1/m_b^2$ effects related to annihilation
  (“long distance photon emission”)
- estimate of some $1/m_b$ effects beyond annihilation
  (soft gluon emission potentially important for indirect CP asymmetries)

Sample application: determination of $|V_{td}/V_{ts}|$

$$\frac{\mathcal{B}(B \to \rho\gamma)}{\mathcal{B}(B \to K^*\gamma)} \propto \left|\frac{V_{td}}{V_{ts}}\right|^2 \xi^2_{\rho/K^*} \left[1 + \Delta R(\bar{\rho}, \bar{\eta})\right]$$

- $\Delta R \sim 0.1$ can be calculated in QCDF
- many uncertainties reduced or cancel in the ratio

Result from Ball, Jones, Zwicky '07

$$\left|\frac{V_{td}}{V_{ts}}\right|^{\mathrm{HFAG}}_{B \to V\gamma} = 0.192 \pm 0.016\,(\mathrm{exp}) \pm 0.014\,(\mathrm{th})$$

- theory errors dominated by form-factor ratio ξ_{ρ/K^*}
- improved lattice results for $f_V^\perp$ will reduce error on ξ_{ρ/K^*}

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NNLO perturbative corrections

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = t_i^{\mathrm{I}}\,\zeta_{V_\perp} + t_i^{\mathrm{II}} \star \phi_+^B \star \phi_\perp^V$$

NNLO perturbative corrections

Recently, some α_s^2 (NNLO) contributions were calculated (Ali, Greub, BP '07)

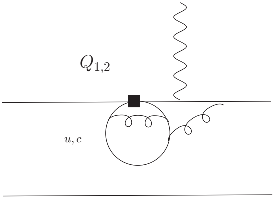
- vertex and spectator corrections from $Q_{7,8}$
- vertex corrections from $Q_{1,2}$ in large-n_f limit ($C_F n_f$ terms)

Numerical study of $\mathrm{Br}(B \to K^*\gamma)$ showed

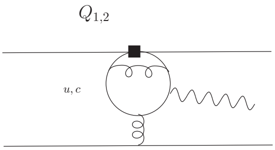
- α_s^2 vertex correction from Q_1 contributes $\sim 10\%$
- an m_c dependence $\sim 10\%$, because $C_F n_f$ terms do not fix a perturbative definition for m_c (need C_F^2 terms)
- large correction to imaginary part of amplitude ($\sim 40\%$)

Remaining: finish calculation for $Q_{1,2}$ and numerics for other observables

The missing pieces: $Q_{1,2}$



vertex



spectator

For vertex corrections:

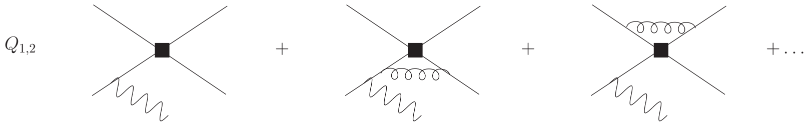
- three loops, depending on m_c^2/m_b^2 and with strong phases
- virtual corrections to $B \to X_s\gamma$, in progress

For spectator corrections:

- two loops, depending on $m_c^2/m_b^2, u$ and with strong phases
- must be calculated from scratch

Power corrections of $\mathcal{O}(\alpha_s/m_b)$: two examples

Annihilation with $Q_{1,2}$



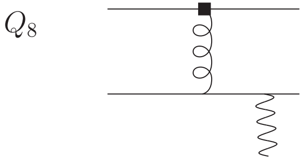
Result to $\mathcal{O}(\alpha_s)$: (Ali, Parkhomenko, BP (preliminary))

$$\mathcal{A}_{\text{ann}} \sim \phi_+^B \star t_{\text{ann}}^{\text{II}} \star \phi_\perp^{V,1/m_b}$$

Example of factorization at $\mathcal{O}(\alpha_s/m_b)$

- IR divergences absorbed in LCDAs
- convolution integral converges

"Annihilation" with Q_8



$$\text{Result at } \mathcal{O}(\alpha_s) : \qquad \frac{\mathcal{A}_{\text{ann}}^8}{\mathcal{A}_{\text{LO}}} \sim \frac{\alpha_s \lambda_B}{m_b} \int_0^1 du \, \frac{\phi_\perp^V(u)}{(1-u)^2} = \infty$$

- endpoint divergence in convolution integral breaks factorization
- small numerically (Kagan, Neubert)
- a conceptual problem

Summary

Reviewed theory status of $B \to V\gamma$ decays

Systematic studies rely on QCD factorization (or pQCD)

Current status:

- leading power terms complete to NLO (and almost to NNLO)
- some $1/m_b$ corrections factorize and have been calculated
- other $1/m_b$ corrections have been estimated with sum rules

Used $|V_{td}/V_{ts}|$ as a sample application, will include others in write-up

General theory of power corrections lacking, due to endpoint divergences.