Form factor determinations from light-cone sum rules

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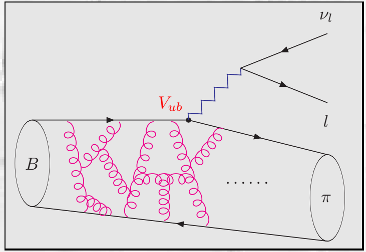
B$\longrightarrow \pi$ l ν-decay



- hadronic matrix element needed

$$\langle \pi(p)|\bar{u}\,\gamma_\mu\, b|\bar{B}(p+q)\rangle \;=\; 2p_\mu\, f^+_{B\pi}(q^2) + q_\mu\left[f^+_{B\pi}(q^2) + f^-_{B\pi}(q^2)\right]$$

- $f^+_{B\pi}(q^2)$, $f^-_{B\pi}(q^2)$: form factors

- lepton spectrum ($m_l = 0$)

$$\frac{d\Gamma}{dq^2}(B\to\pi^- l^+\nu_l) = \frac{G_F^2\,|V_{ub}|^2}{192\,\pi^3\, m_B^3}\,\lambda^{3/2}(q^2)\,|f^+_{B\pi}(q^2)|^2$$

$$\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2, \quad 0 \le q^2 \le (m_B - m_\pi)^2 \simeq 26.4\,\mathrm{GeV}^2$$

- knowledge of $f^+_{B\pi}(q^2) \longrightarrow |V_{ub}|$

Method in short

correlation function

$$F(q^2) = i \int d^4x\, e^{iqx} \langle \pi(p) | T\{ j_\mu(x) j_5^\dagger(0)\} | 0 \rangle$$

hadronic dispersion relation

operator-product-expansion

$$F^{(had)}(q^2) = \frac{2m_B^2 f_B f_{B\pi}^+(q^2)}{m_b(m_B^2 - (p+q)^2)} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - (p+q)^2} \qquad F^{(OPE)}(q^2) = \frac{1}{\pi}\int_{m_b^2}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s - (p+q)^2}$$

quark-hadron-duality

$$\int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - (p+q)^2} \approx \frac{1}{\pi}\int_{s_0^B}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s - (p+q)^2}$$

$$\frac{2m_B^2 f_B f_{B\pi}^+(q^2)}{(m_B^2 - (p+q)^2)} = \frac{1}{\pi}\int_{m_b^2}^{s_0^B} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s - (p+q)^2}$$

Borel-transformation

sum rule

$$f_{B\pi}^+(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2 f_B} \frac{1}{\pi} \int_{m_b^2}^{s_0^B} ds\, \mathrm{Im}\, F^{(OPE)}(s)\, e^{-s/M^2}$$

Applications

- $B \to \pi$-form factor topic of this talk
- wide range of other applications
- $B_{(s)} \to K$-form factors [P. Ball, R. Zwicky 04] [G. Duplančić, B. Melić 08]
- $B \to V$-form factors [P. Ball, R. Zwicky 04]
- lcsr with B-meson distribution amplitudes [A. Khodjamirian, T. Mannel, N.O. 05, 06] [F. de Fazio, T. Hurth, T. Feldmann 05, 07]
  - for $B \to D^{(*)}$-form factors [S. Faller, C. Klein, A. Khodjamirian, T. Mannel 08]
- $D \to K, \pi$-form factors [P. Ball, 06] [C. Klein, A. Khodjamirian, N.O., work in progress]

Hadronic side

- starting point: correlation function

$$F_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle\pi(p)|\mathrm{T}\left\{\bar{u}(x)\gamma_\mu b(x), \bar{b}(0) i m_b \gamma_5 d(0)\right\}|0\rangle$$
$$= F(q^2,(p+q)^2)\,p_\mu + \widetilde{F}(q^2,(p+q)^2)\,q_\mu$$

Hadronic side

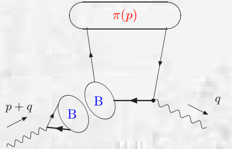
- starting point: correlation function

$$\begin{aligned} F_\mu(p,q) &= i\int d^4x\, e^{iq\cdot x}\langle\pi(p)|\mathrm{T}\{\bar u(x)\gamma_\mu b(x),\ \underbrace{\bar b(0) i m_b\gamma_5 d(0)}_{}\}|0\rangle \\ &= F(q^2,(p+q)^2)\,p_\mu + \widetilde{F}(q^2,(p+q)^2)\,q_\mu \end{aligned}$$

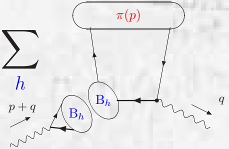
$$\overbrace{\qquad\qquad}^{\sum |B(p+q)\rangle\langle B(p+q)|}$$

- inserting complete set of B-meson states → hadronic sum

$F(q^2,(p+q)^2) =$



$$\underbrace{\qquad}_{\dfrac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2}} \quad + \sum_h$$



$$\underbrace{\qquad}_{\displaystyle\int_{s_0^h}^{\infty} ds\,\frac{\rho^h(q^2,s)}{s-(p+q)^2}\ (*)}$$

(∗) subtraction terms to render dispersion integral finite not shown

Light-cone expansion I

- $q^2 \ll m_b^2$, $(p+q)^2 < 0$, $|(p+q)^2| \gg \Lambda$ contraction of quark-fields

$$
\begin{aligned}
& i \int d^4x\, e^{iqx} \langle \pi(p) | T\{\bar{u}(x)\gamma_\mu b(x), \bar{b}(0) i m_b \gamma_5 d(0)\} | 0 \rangle \\
= \quad & i m_b \int d^4x\, e^{iqx} \langle \pi(p) | \bar{u}(x)\gamma_\mu S(x, m_b)\gamma_5 d(0) | 0 \rangle
\end{aligned}
$$

- universal input: nonlocal matrix elements, expansion near the light-cone in different twists

$$
\begin{aligned}
\langle \pi(p) | \bar{u}(x)\Gamma d(0) | 0 \rangle &\sim \varphi_\pi^{(t)}(u) \quad \text{Twist } t = 2, 3, 4, \ldots \\
\langle \pi(p) | \bar{u}(x) G_{\mu\nu}(vx)\Gamma d(0) | 0 \rangle &\sim \Phi_{3\pi}^{(t)}(\alpha_i) \quad \text{Twist } t = 3, 4, \ldots
\end{aligned}
$$

- general form : convolution

$$
F^{(OPE)}(q^2, (p+q)^2) = \sum_{t=2}^{4} \int Du_i \sum_{k=0,1} \left(\frac{\alpha_s}{\pi}\right)^k T_k^{(t)}(q^2, (p+q)^2, u_i)\varphi_\pi^{(t)}(u_i)
$$

Light-cone expansion II

- summation of local operators : distribution amplitudes

$$\langle \pi(p)|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle = \sum_n \frac{1}{n!}\langle \pi(p)|\bar{u}(\overleftarrow{D}\cdot x)^n\gamma_\mu\gamma_5 d|0\rangle$$

$$\langle \pi(p)|\bar{u}\overleftarrow{D}_{\alpha_1}\dots\overleftarrow{D}_{\alpha_n}\gamma_\mu\gamma_5 d(0)|0\rangle = p_\mu p_{\alpha_1}\dots p_{\alpha_n} M_n^{(2)}\cdots$$

$M_n^{(t)}$: local matrix elements of twist t

Light-cone expansion II

- summation of local operators : distribution amplitudes

$$\langle\pi(p)|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle = \sum_n \frac{1}{n!}\langle\pi(p)|\bar{u}(\overleftarrow{D}\cdot x)^n\gamma_\mu\gamma_5 d|0\rangle$$
$$= p_\mu \sum_n \frac{(p\cdot x)^n}{n!} M_n^{(2)} + x^2 p_\mu \sum_n \frac{(p\cdot x)^n}{n!} M_n^{(4)} + \ldots$$

$M_n^{(t)}$: local matrix elements of twist t

- for free propagator after integration $p\cdot x \rightarrow \xi$

$$\xi = \frac{2p\cdot q}{m_b^2 - q^2} = \frac{(p+q)^2 - q^2}{m_b^2 - q^2} \sim \mathcal{O}(1)$$

- infinite sum of local operators, but $x^2 \longrightarrow \frac{q^2}{(m_b^2-q^2)^2}, \frac{1}{m_b^2-q^2}$
- higher twist in lcsr suppressed by $\dfrac{\Lambda^2}{m_b^2 - uq^2 - \bar{u}(p+q)^2}$

Final steps I

- Analytic continuation in $(p+q)^2$ via dispersion integral

$$F^{(OPE)}(q^2,(p+q)^2) = \frac{1}{\pi}\int_{m_b^2}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(q^2,s)}{s-(p+q)^2}$$

(no subtraction terms shown)

- leads to following sum rule

$$\frac{2m_B^2 f_B f_{B\pi}^+(q^2)}{m_B^2-(p+q)^2} + \int_{s_0^h}^{\infty} ds \frac{\rho^h(s,q^2)}{s-(p+q)^2} = \frac{1}{\pi}\int_{m_b^2}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s,q^2)}{s-(p+q)^2}$$

- Borel transformation eliminates subtraction terms and suppresses higher states

$$B_{M^2}\left(\frac{1}{s-(p+q)^2}\right)^n = \frac{1}{(n-1)!}\frac{1}{(M^2)^{n-1}}e^{-s/M^2}, \quad B_{M^2}\left((p+q)^2\right)^n = 0$$

$\Rightarrow$ higher twist suppressed by powers of $\frac{1}{M^2}$

Final steps II

- semilocal quark-hadron duality allows to approximate hadronic spectrum

$$\int_{s_0^h}^{\infty} ds\, \rho^h(s,q^2)\, e^{-s/M^2} \simeq \frac{1}{\pi}\int_{s_0^B}^{\infty} ds\, \mathrm{Im}\, F^{(OPE)}(s,q^2)\, e^{-s/M^2}$$

- s_0^B: duality threshold
- final expression:

sum rule

$$f_B f^+_{B\pi}(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2}\frac{1}{\pi}\int_{m_b^2}^{s_0^B} ds\, \mathrm{Im}\, F^{(OPE)}(s,q^2)\, e^{-s/M^2}$$

Overview once again

correlation function

$$F(q^2) = i \int d^4x\, e^{iqx} \langle \pi(p) \,|\, T\{ j_\mu(x) j_5^\dagger(0) \} \,|\, 0 \rangle$$

hadronic dispersion relation

operator-product-expansion

$$F^{(had)}(q^2) = \frac{2m_B^2 f_B f_{B\pi}^+(q^2)}{m_B^2 - (p+q)^2} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - (p+q)^2}$$

$$F^{(OPE)}(q^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} ds \frac{\operatorname{Im} F^{(OPE)}(s)}{s - (p+q)^2}$$

quark-hadron-duality

$$\int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - (p+q)^2} \approx \frac{1}{\pi} \int_{s_0^B}^{\infty} ds \frac{\operatorname{Im} F^{(OPE)}(s)}{s - (p+q)^2}$$

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main uncertainties

- ▷ universal input

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main uncertainties

- ▷ universal input
- ▷ truncated OPE

Overview once again

correlation function

$$F(q^2) = i \int d^4x\, e^{iqx} \langle \pi(p) \,|\, T\{ j_\mu(x) j_5^\dagger(0)\} \,|\, 0 \rangle$$

hadronic dispersion relation

$$F^{(had)}(q^2) = \frac{2m_B^2 f_B f_{B\pi}^+(q^2)}{m_B^2 - (p+q)^2} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s-(p+q)^2}$$

operator-product-expansion

$$F^{(OPE)}(q^2) = \frac{1}{\pi}\int_{m_b^2}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s-(p+q)^2}$$

quark-hadron-duality

$$\int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s-(p+q)^2} \approx \frac{1}{\pi}\int_{s_0^B}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s-(p+q)^2}$$

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main uncertainties

- ▷ universal input
- ▷ truncated OPE
- ▷ quark-hadron duality

Universal input

- external input parameters
  - α_s in full QCD, $\overline{MS}$-mass $m_b(m_b) = 4.164 \pm 0.025$ GeV [Kühn, Steinhauser, Sturm 07]
  - distribution amplitudes $\varphi_\pi^{(t)}(u)$, $\Phi_{3\pi}^{(t)}(\alpha_i)$

$$\varphi_\pi^{(2)} = 6u(1-u)\sum_{n=0}^{\infty} a_{2n}^\pi(\mu) C_{2n}^{3/2}(2u-1) \qquad \text{conformal expansion}$$

$C_{2n}^{3/2}$: Gegenbauer polynomials

    - ★ twist 2 Gegenbauer moments $a_2^\pi(\mu)$, $a_4^\pi(\mu), \ldots$
    - ★ higher twist parameters $f_{3\pi}$, $\omega_{3\pi}$, δ_π, $\epsilon_\pi, \ldots$ [Ball, Braun, Lenz 06]
    - ★ normalization f_π, $\mu_\pi = \frac{m_\pi^2}{m_u+m_d}$ [PDG]
  - f_B taken from two-point sum rule in $\overline{MS}$-scheme to $O(\alpha_s)$-accuracy
    - ★ condensates $\langle q\bar{q}\rangle$, $\langle G^2\rangle, \ldots$ [Jamin, Lange 01]

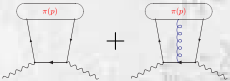
$$\langle q\bar{q}\rangle(1\ \text{GeV}) = \frac{1}{2} f_\pi^2\, \mu_\pi(1\ \text{GeV}) \qquad \text{[GMOR-relation]}$$

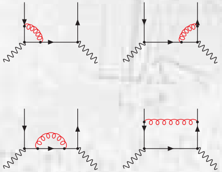
- uncertainties in principle reducible

scale dependence in general taken at one-loop order

Truncation of OPE

- twist-expansion at leading order up to twist 4
  - two- and three-particle contributions included





- α_s-corrections to scattering amplitudes

  for twist 2 and 3, two particle parts only
  - μ-dependence
- in sum rule for f_B *condensates* up to dimension 6
  - why burden with more parameters?

Truncation of OPE

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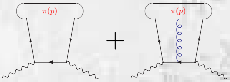
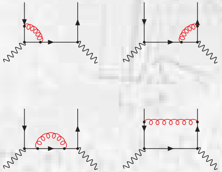


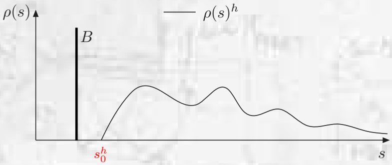
- α_s-corrections to scattering amplitudes



  for twist 2 and 3, two particle parts only
  - μ-dependence
- in sum rule for f_B *condensates* up to dimension 6
  - why burden with more parameters?
  - α_s-corrections cancel in ratio $f^+_{B\pi}(q^2) = \frac{F(q^2, M^2, s_0^B)}{f_B}$ to large amount

Quark-hadron duality

- use of quark-hadron duality amounts to approximating complex hadronic spectral function



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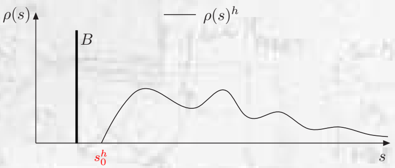
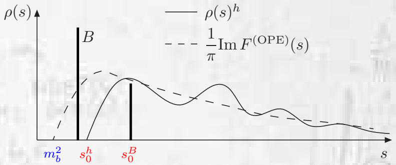
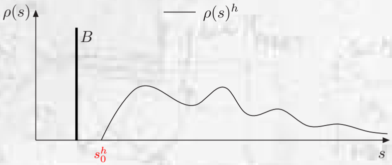


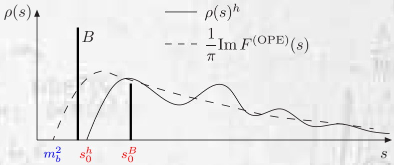


- via spectral function of OPE-result

Quark-hadron duality

- use of quark-hadron duality amounts to approximating complex hadronic spectral function





- via spectral function of OPE-result
- Borel-transformation suppresses higher states
  - sum rule not very sensitive to structure far from threshold
- approximation depends crucial on values of M^2 and s_0^B
  - criteria to fix parameters
- nevertheless uncertainty difficult to quantify

Numerics I

- sum rule parameters

	duality, Borel	scale
lcsr	s_0^B, M^2	$\mu = \mu_r = \mu_m = \mu_f$
f_B sum rule	$\bar{s}_0^B$, $\bar{M}^2$	

1. Borel-window: $15\,\mathrm{GeV}^2 \leq M^2 \leq 21\,\mathrm{GeV}^2$, $4\,\mathrm{GeV}^2 \leq \bar{M}^2 \leq 6\,\mathrm{GeV}^2$
   - higher twists/condensates suppressed e.g. Twist 4 $\sim$ 3% Twist 2
   - continuum $\leq$ 30 % of ground state
2. $\mu = 3.0 \pm 0.5\,\mathrm{GeV}$
   - NLO-terms in Twist 2,3 $\leq$ 30 % of LO in full correlator
3. s_0^B, $\bar{s}_0^B$ for every M^2, $\bar{M}^2$ from daughter sum rules, e.g.

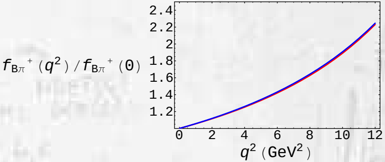
$$m_B^2 = -\frac{d}{d\left(\frac{1}{M^2}\right)} \int_{m_b^2}^{s_0^B} ds\, \mathrm{Im}\, F^{(OPE)}(s,q^2)\, e^{-s/M^2)}$$

4. further constraints from experimentally measured slope

Numerics II

- fitting simultaneously m_B^2 and slope with conditions from previous slide
  - parameters s_0^B, $a_2^\pi(\mu)$, $a_4^\pi(\mu)$ $(*)$

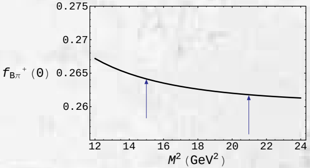
$$\frac{f_{B\pi}^+(q^2)}{f_{B\pi}^+(0)} = \frac{1}{(1-\frac{q^2}{m_{B*}^2})(1-\alpha_{BK}\frac{q^2}{m_B^2})}$$

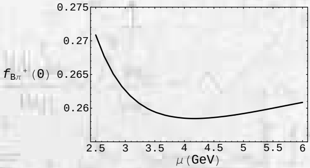


[D. Becirevic, A. B. Kaidalov 98]

$s_0^B = 36 - 35.5\,\text{GeV}^2$, $a_2^\pi(1\text{ GeV}) = 0.15 - 0.17$, $a_4^\pi(1\text{ GeV}) = 0.05 - 0.03$

- Borel-window and dependence on μ





$(*)$ trading q^2-prediction for reduced uncertainties in a_2^π, a_4^π and duality

Comparison of $\overline{MS}$- and polemass-scheme

- vector form factor

$$f^+_{B\pi}(0) = 0.263^{+0.004}_{-0.005}\Big|_{M,\bar{M}} {}^{+0.009}_{-0.004}\Big|_{\mu} \pm 0.02\Big|_{shape} {}^{+0.03}_{-0.02}\Big|_{\mu_\pi} \pm 0.001\Big|_{m_b}$$

$$f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}$$

[G. Duplančić, A. Khodjamirian, Th. Mannel, B. Melić, N. O. 08]

$$f^+_{B\pi}(0) = 0.258 \pm 0.031$$

[P. Ball, R. Zwicky 04]

- better α_s-behavior in $\overline{MS}$-scheme
- similar results for tensor currents

$$f^T_{B\pi}(0)\big|_{\overline{MS}} = 0.255 \pm 0.035$$

$$f^T_{B\pi}(0)\big|_{pole} = 0.253 \pm 0.028$$

	$\overline{MS}$-mass	pole mass
$f^+_{B\pi}(0)$	0.263	0.258
tw2 LO	50.6%	39.7%
tw2 NLO	7.5%	17.2 %
tw3 LO	46.8%	41.5 %
tw3 NLO	-4.5%	2.4 %
tw4 LO	-0.2%	-0.9%

Exclusive V_{ub} determinations

- using fit to 12 binBaBar spectrum

$$|V_{ub}|f^{+}_{B\pi}(0) = \left(9.1 \pm 0.6\big|_{shape} \pm 0.3\big|_{BR}\right) \times 10^{-4}$$

[B. Aubert et al. 06] [P. Ball 06]

Exclusive V_{ub} determinations

- using fit to 12 binBaBar spectrum

$$|V_{ub}|f^{+}_{B\pi}(0) = \left(9.1 \pm 0.6|_{shape} \pm 0.3|_{BR}\right) \times 10^{-4}$$

[B. Aubert et al. 06] [P. Ball 06]

- gives determination of $|V_{ub}|$

comparison of exclusive determinations		
Method	$\lvert V_{ub}\rvert \times 10^{-3}$	Ref.
Lattice-QCD	$3.78 \pm 0.25 \pm 0.52$	Fermilab/MILC '05
Lattice-QCD	$3.55 \pm 0.25 \pm 0.50$	HPQCD '07
Omnes-FF	$3.47 \pm 0.29 \pm 0.03$	Flynn, Nieves '07
BCL-FF	$3.36 \pm 0.23 \pm 0.01$	Bourrely et al. 08
LCSR	$3.5 \pm 0.4 \pm 0.1$	Ball '06
LCSR	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$	Duplančić et al. 08

Conclusions and outlook

- sum rules inherently approximate
- $\overline{MS}$-mass gives better control on α_s-corrections
- light-cone expansion and duality approximation seem to work
  - experimental measurement of radiative excitations for modeling hadronic spectrum highly appreciated
  - shape of leptonic spectrum helps, too
- higher accuracy in u, d-quark masses reduces uncertainty considerably
- otherwise improvement of accuracy difficult
  - full α_s-corrections to twist 3 need evolution kernel
  - α_s-corrections to three-particle contributions very tedious
    - by both no substantial numerical effect expected
  - going beyond twist 4 might be more feasible
- uncertainty around $7 - 10\%$ probably remains

Form factor parametrizations I

- pole parametriziations:

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1-\frac{q^2}{m^2_{B*}}} + \frac{f^+_{B\pi}(0) r \frac{q^2}{m^2_{B*}}}{\left(1-\frac{q^2}{m_{B*}}\right)\left(1-\alpha\frac{q^2}{m^2_{B*}}\right)}$$

[Becirevic, Kaidalov 98][Ball, Zwicky 04]

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)\left(1-\delta\frac{q^2}{m^2_{B*}}\right)}{\left(1-\frac{q^2}{m^2_{B*}}\right)\left(1-[\alpha+\delta(1-\alpha)]\frac{q^2}{m^2_{B*}}\right)}$$

[Hill 06]

- from unitarity bounds

$$f^+_{B\pi}(q^2) = \frac{1}{P(q^2)\Phi(q^2,t_0)}\sum_{k=0}^{\infty} a_k \left[z(q^2,t_0)\right]^k$$

$$\sum_{k=0}^{\infty} a_k^2 \leq 1$$

[Okubo 71][Boyd, Grinstein, Lebed 95]

- multiple-subtracted Omnes representation

$$f^+_{B\pi}(q^2) = \frac{1}{m^2_{B*}-q^2}\prod_{j=1}^{n}\left[f^+_{B\pi}(q_j^2)(m^2_{B*}-q_j^2)\right]^{\alpha_j(q^2)}$$

$$\alpha_j(q^2) = \prod_{i=o,i\neq j}^{n}\frac{q^2-q_i^2}{q_j^2-q_i^2}$$

[Albertus et al.]

- recent proposal

$$f^+_{B\pi}(q^2) = \frac{1}{1-\frac{q^2}{m^2_{B*}}}\sum_{k=0}^{k_{max}} b_k(t_0) z(q^2,t_0)$$

[Bourrely, Caprini, Lellouch]

Form factor parametrization II

- $z(q^2, t_0)$ maps complex q^2-plane on unit disc

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- multiple subtracted Omnes representation and BGL-parametrization violate scaling behavior for $|q^2| \to \infty$

$$f^+_{B\pi}(q^2 \to \infty)\big|_{Omnes} \sim \exp\left[C\left(q^2\right)^{n-1}\right] \qquad f^+_{B\pi}(q^2 \to \infty)\big|_{BGL} \sim \left(q^2\right)^{1/4}$$

- Omnes representation has no cut at $q^2 \geq t_+$
- BGL-parametrization has unphysical pole at $q^2 = t_+$ for truncation at finite k
- recent BCL-paramtrization has neither of this problems

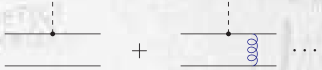
Some general remarks for B$\to \pi$ form factor

- m_b-scaling:

$$f^+_{B\pi}(q^2 \approx 0) \sim m_b^{-3/2} \qquad f^+_{B\pi}(q^2 = q^2_{max}) \sim \sqrt{m_b} \quad \text{(Isgur-Wise-limit)}$$

▷: soft and hard form factor:

$$f^+_{B\pi}(q^2) = f^{s+}_{B\pi}(q^2) + f^{h+}_{B\pi}(q^2)$$



- symmetries of soft form factor:

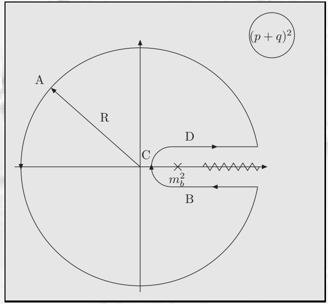
$$f^{s+}_{B\pi}(0) = f^{s0}_{B\pi}(0) = f^{sT}_{B\pi}(0) \cdots$$

[Charles et al.][Beneke, Feldmann]

- analytic function of q^2:

$$f^+_{B\pi}(q^2) = \frac{f_{B^*} g_{B^*B\pi}}{m^2_{B^*} - q^2} + \frac{1}{\pi} \int_{(m_B+m_\pi)^2}^{\infty} ds \frac{\text{Im} f^+_{B\pi}(s)}{s - q^2}$$

Analytic continuation



▷analytic continuation via Cauchy:

$$
\begin{aligned}
& F(q^2,(p+q)^2) \\
= & \frac{1}{2\pi i}\oint_C ds \frac{F(q^2,s)}{s-(p+q)^2} \\
= & \frac{1}{2\pi i}\int_0^\infty ds \frac{F(q^2,s+i\epsilon)-F(q^2,s-i\epsilon)}{s-(p+q)^2}
\end{aligned}
$$

- Schwarzsches reflectionprinciple

$$\mathrm{Im}_s F(q^2,s) = \frac{1}{2i}\left(F(q^2,s+i\epsilon)-F(q^2,s-i\epsilon)\right)$$

$$\Rightarrow \quad F(q^2,(p+q)^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\, F(q^2,s)}{s-(p+q)^2}$$