Measurement of ϕ_2 (α) from $B^0 \to \rho^+\rho^-$ and $B \to a_1\pi$ decays

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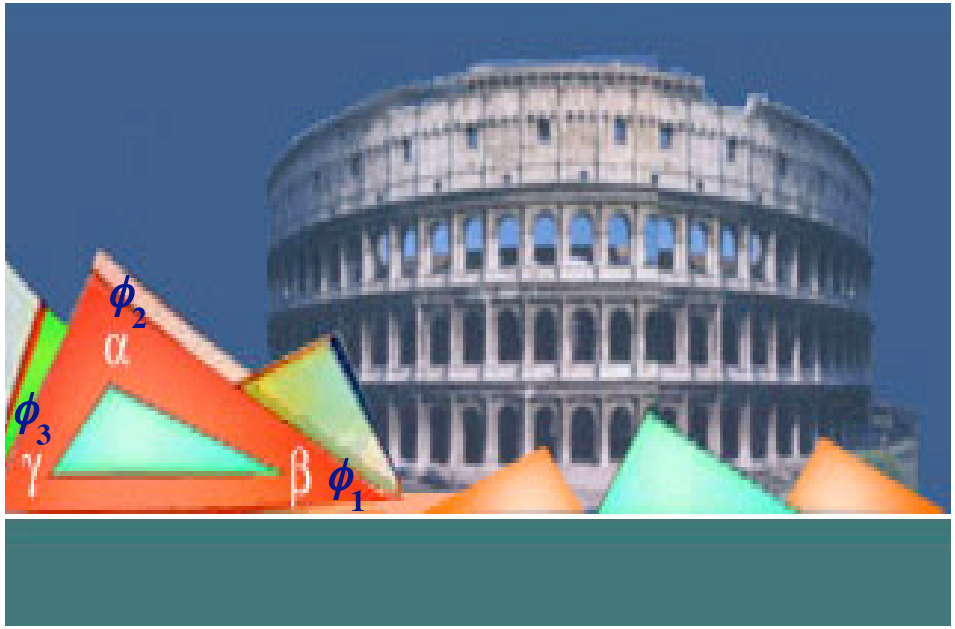


- *Isospin decomposition*
- $B^0 \to \rho^+\rho^0$
- $B^0 \to \rho^+\rho^-$
- $B^0 \to \rho^0\rho^0$
- *constraint upon* ϕ_2 (α)
- $B^0 \to a_1\pi$ *and* ϕ_2

Measurement of $\sin(2\phi_2)$ with $B^0 \to \rho\rho$

$$\lambda = \sqrt{\frac{M_{12}^*}{M_{12}}}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}$$

$$= +\left(\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right)\left(\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\right)\left[\frac{1-(\bar{A}_0/\bar{A}_2)}{1-(A_0/A_2)}\right]$$

$$= \frac{-V_{tb}^*V_{td}/(V_{ub}^*V_{ud})}{-V_{tb}V_{td}^*/(V_{ub}V_{ud}^*)} \times \left[\frac{1-(\bar{A}_0/\bar{A}_2)}{1-(A_0/A_2)}\right]$$

$$= \frac{|M|e^{i\phi_2}}{|M|e^{-i\phi_2}} \times \left[\frac{1-(\bar{A}_0/\bar{A}_2)}{1-(A_0/A_2)}\right]$$

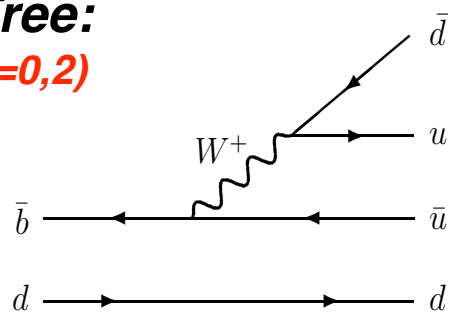
$$= e^{2i\phi_2} \times \left[\frac{1-(\bar{A}_0/\bar{A}_2)}{1-(A_0/A_2)}\right]$$

$$\Rightarrow \mathcal{A} = \frac{|\lambda|^2-1}{|\lambda|^2+1} = 0$$

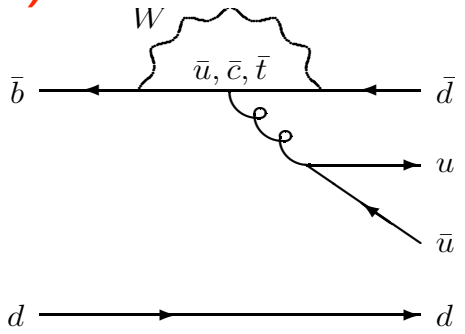
$$\mathcal{S} = \frac{2\,\mathrm{Im}\lambda}{|\lambda|^2+1} = \sin 2\phi_2$$

$$= \sqrt{1-\mathcal{A}^2}\,\sin 2(\phi_2+\delta) \quad \text{(penguin)}$$

Tree:
(I=0,2)



Penguin (gluonic):
(I=0)



For $B^0 \to \rho^+\rho^-$, there is a (small) penguin contribution
⇒ use isospin analysis to extract ϕ_2

Isospin analysis for ϕ_2

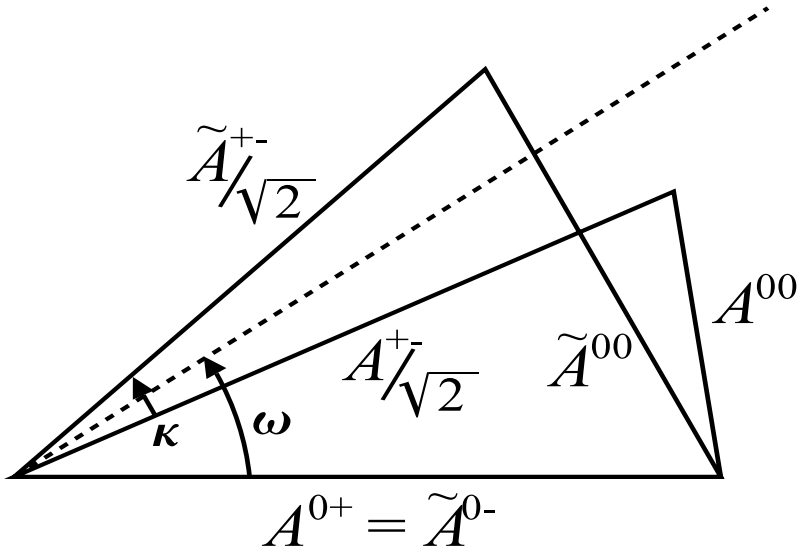
SU(2) isospin analysis:

Gronau & London, PRL 65, 3381 (1990)

$$\frac{A_{+-}}{\sqrt{2}} + A_{00} = A_{+0}$$

$$\frac{\bar{A}_{+-}}{\sqrt{2}} + \bar{A}_{00} = \bar{A}_{+0}$$

6 param. + 6 observables ⇒ all determined



$$|A_{\rm th}^{+-}| = \sqrt{a^{+-}(1-\mathcal{A}_{\pi\pi})}$$

$$|\overline{A}_{\rm th}^{+-}| = \sqrt{a^{+-}(1+\mathcal{A}_{\pi\pi})}$$

$$|A_{\rm th}^{0-}| = |A_{\rm th}^{0+}| = \sqrt{a^{0+}}$$

$$|A_{\rm th}^{00}|^2 = \frac{|A_{\rm th}^{+-}|^2}{2} + |A_{\rm th}^{0+}|^2 - \sqrt{2}\,|A_{\rm th}^{+-}|\,|A_{\rm th}^{+0}|\cos(\omega-\kappa/2)$$

$$|\overline{A}_{\rm th}^{00}|^2 = \frac{|\overline{A}_{\rm th}^{+-}|^2}{2} + |A_{\rm th}^{0+}|^2 - \sqrt{2}\,|\overline{A}_{\rm th}^{+-}|\,|A_{\rm th}^{+0}|\cos(\omega+\kappa/2)$$

$$B_{\rm th}^{\pi^+\pi^-} = \left(|A_{\rm th}^{+-}|^2 + |\overline{A}_{\rm th}^{+-}|^2\right)/2 = a^{+-}$$

$$B_{\rm th}^{\pi^0\pi^0} = \left(|A_{\rm th}^{00}|^2 + |\overline{A}_{\rm th}^{00}|^2\right)/2$$

$$B_{\rm th}^{\pi^0\pi^+} = |A_{\rm th}^{0+}|^2\,(\tau_{B^\pm}/\tau_{B^0}) = a^{+0}\cdot(\tau_{B^\pm}/\tau_{B^0})$$

$$\mathcal{A}_{\rm th}^{\pi^0\pi^0} = \frac{|\overline{A}_{\rm th}^{00}|^2 - |A_{\rm th}^{00}|^2}{|\overline{A}_{\rm th}^{00}|^2 + |A_{\rm th}^{00}|^2}$$

$$\mathcal{A}_{\rm th}^{\pi^+\pi^-} = \mathcal{A}_{\pi\pi}$$

$$\mathcal{S}_{\rm th}^{\pi^+\pi^-} = \sqrt{1-\mathcal{A}'^2_{\pi\pi}}\,\sin(2\phi_2+\kappa)$$

Complications for $\sin(2\phi_2)$ with $B^0 \to \rho\rho$

- **If $m_{\rho 1} \neq m_{\rho 2}$ wave function can be antisymmetric** Falk et al., PRD 69, 011502 (2004)

  ⇒ l=1 allowed, isospin relations do not hold

  But measurements are stable when decreasing allowed Δm region

- **electroweak penguin can have l=2**

  ⇒ isospin relations do not hold

  But no sign of direct CP asymmetry in $B^+ \to \rho^+\rho^0$ decays

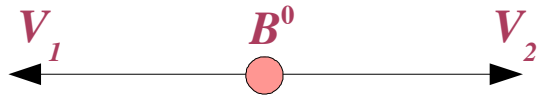
- **final state is VV, L=1 possible and has opposite CP (-1)**

  ⇒ uncertainty in measuring S, must also measure polarization

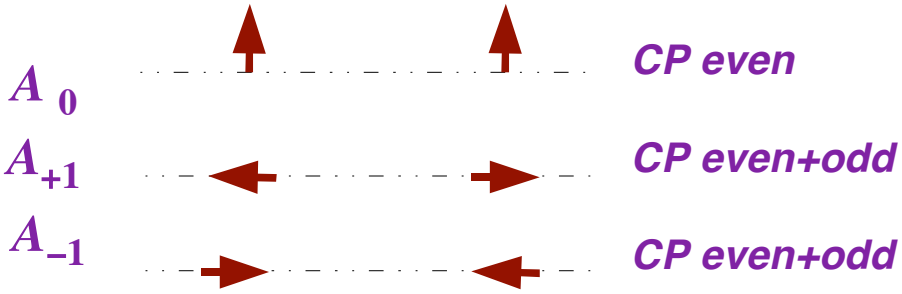
  (⇒ isospin relations hold separately for long., trans. perp., trans. par. states)

  Polarization has been measured: $f_L \approx 1$ (consistent with factorization)
  Kagan, PLB 601, 151 (2004)

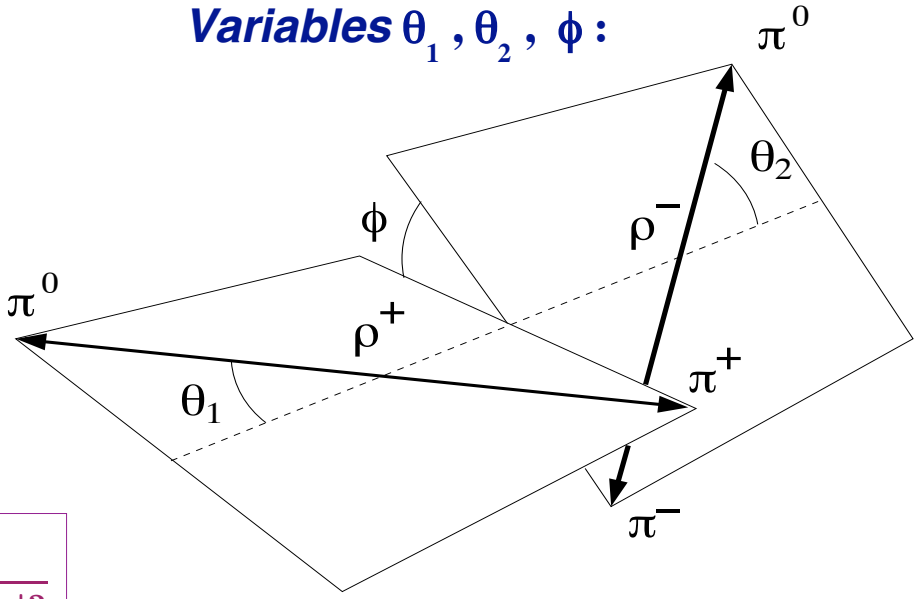
$B^0 \to \rho^+\rho^-$ polarization



Helicity basis:



Variables θ_1, θ_2, ϕ:



$$f_L \equiv \frac{|A_0|^2}{|A_0|^2 + |A_{+1}|^2 + |A_{-1}|^2}$$

After integrating over ϕ :

$$\frac{d^2N}{d\cos\theta_1\, d\cos\theta_2} \propto 4 f_L \cos^2\theta_1 \cos^2\theta_2 + (1 - f_L)\sin^2\theta_1 \sin^2\theta_2$$

ML fits (binned and unbinned) use this PDF

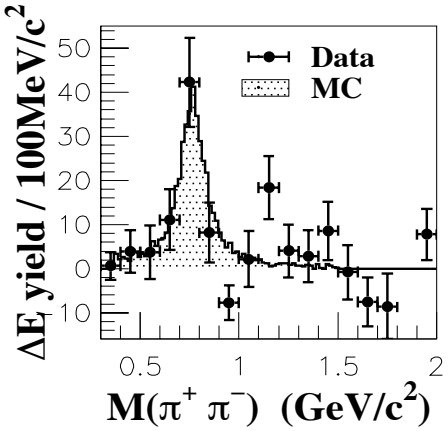
Measurement of $B^0 \to \rho^+\rho^0$

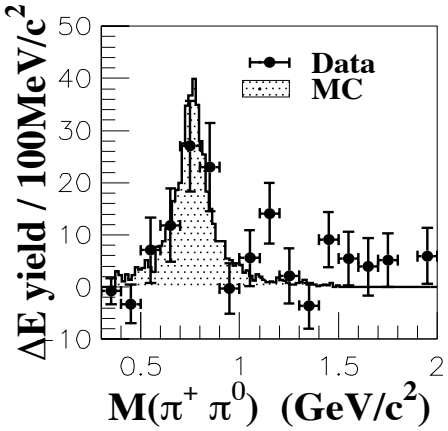
PRL 91, 221801 (2003)

78 fb^{-1} (old, not updated yet)

$B_{\rho+\rho0} = (31.7 \pm 7.1 \,^{+3.8}_{-6.7}) \times 10^{-6}$

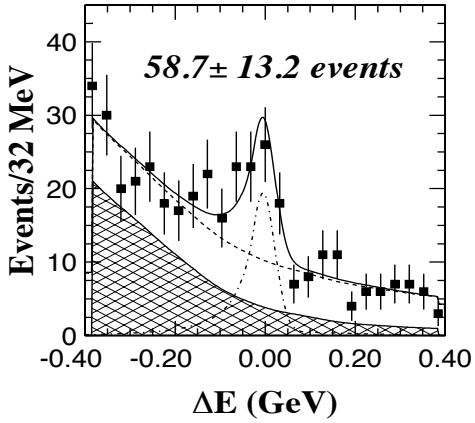
(asymmetric due to non-resonant + fraction of long. polarization)

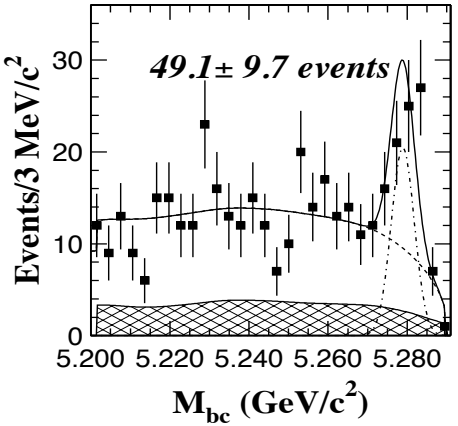




$A_{CP} = (N_- - N_+)/(N_- + N_+)$
$= -0.12 \pm 0.13 \pm 0.10$

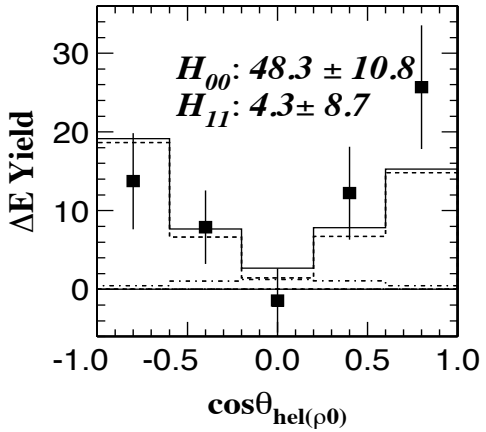
⇒ no large electroweak penguin

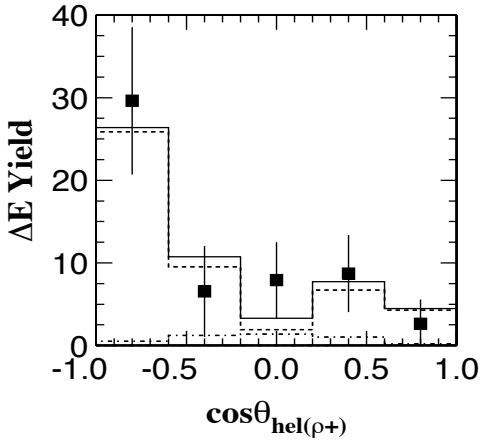




non-resonant contribution < 17%

$f_L = 0.95 \pm 0.11 \pm 0.02$





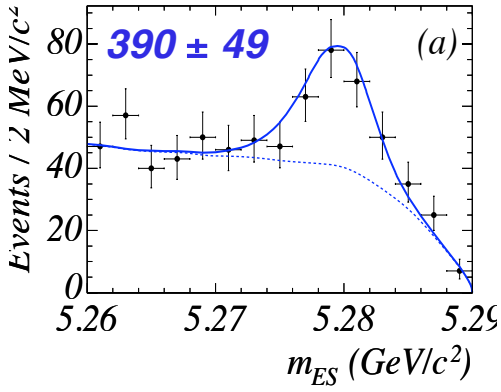
Measurement of $B^0 \to \rho^+\rho^0$

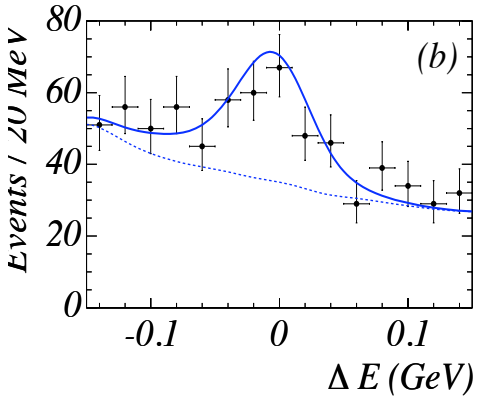
PRL 97, 261801 (2006)

211 fb^{-1}

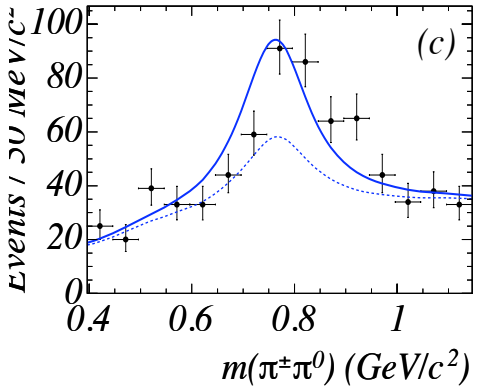
$B_{\rho+\rho0} = (16.8 \pm 2.2 \pm 2.3) \times 10^{-6}$

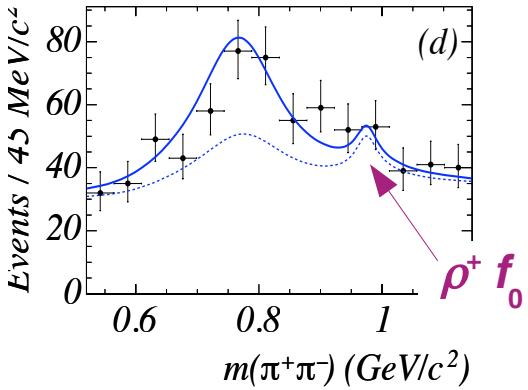
(unbinned extended ML fit to M_{bc}, ΔE, $m_{\pi\pi+}$, $\cos\theta_+$, $m_{\pi\pi 0}$, $\cos\theta_0$, x_{NN})





no PDF for non-resonant $B \to \rho\pi\pi$





$f_L = 0.905 \pm 0.042 \,^{+\,0.023}_{-\,0.027}$

$A_{CP} = 0.00 \pm 0.22 \pm 0.03$

⇒ no large electroweak penguin

Measurement of $B^0 \to \rho^+\rho^-$

PRL 96, 171801 (2006)

BELLE

253 fb^{-1} (492 fb^{-1})

Fit #1:

select ρρ region in $m_{\pi\pi}$, fit m_{bc}-ΔE

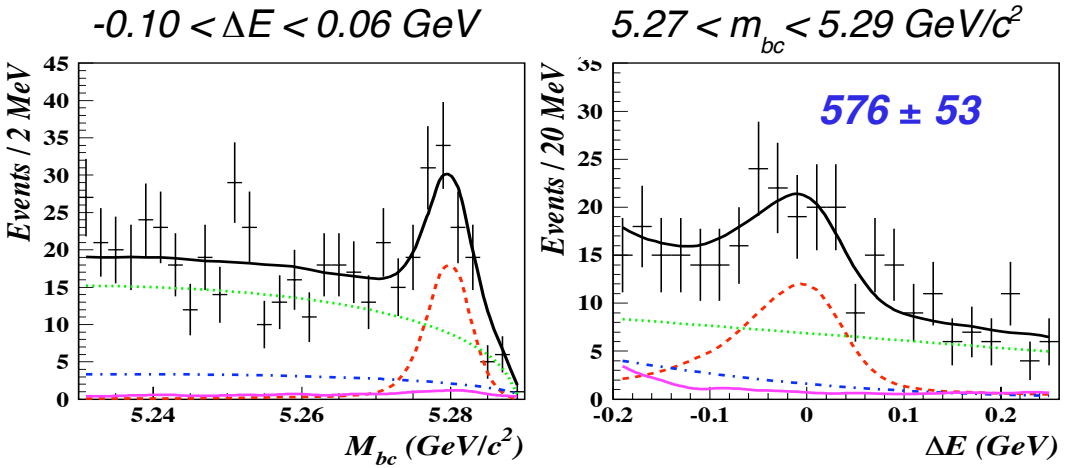
Fit #2:

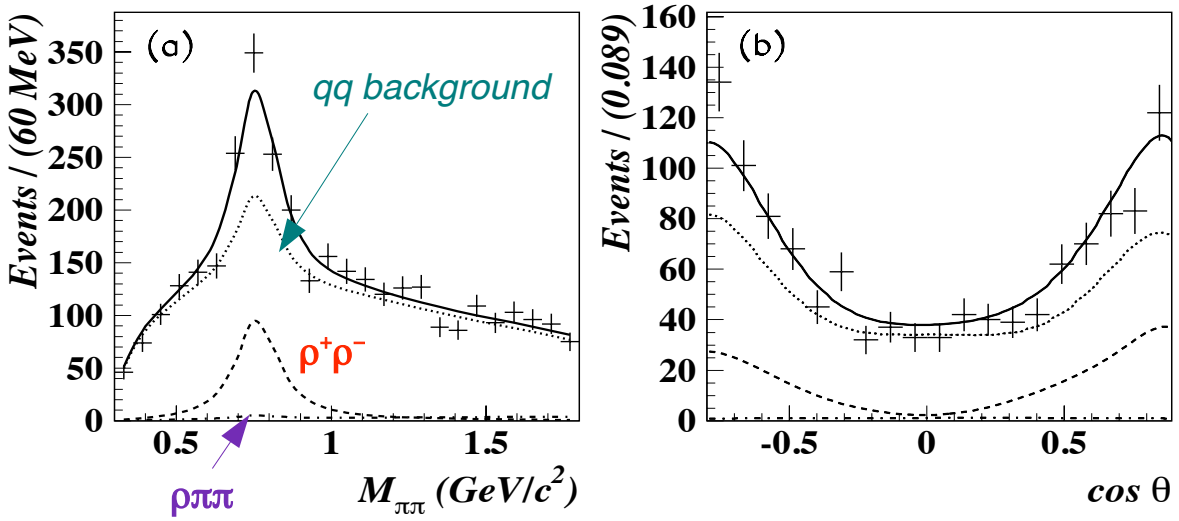
select 4π region in (m_{bc}, ΔE), fit $m_{\pi\pi}$

$B_{\rho^+\rho^-} = (22.8 \pm 3.8 \,^{+2.3}_{-2.6}) \times 10^{-6}$

$f_{\rho\pi\pi} = (6.3 \pm 6.7)$ %

$f_L = 0.941 \,^{+0.034}_{-0.040} \pm 0.030$





Measurement of $B^0 \to \rho^+\rho^-$

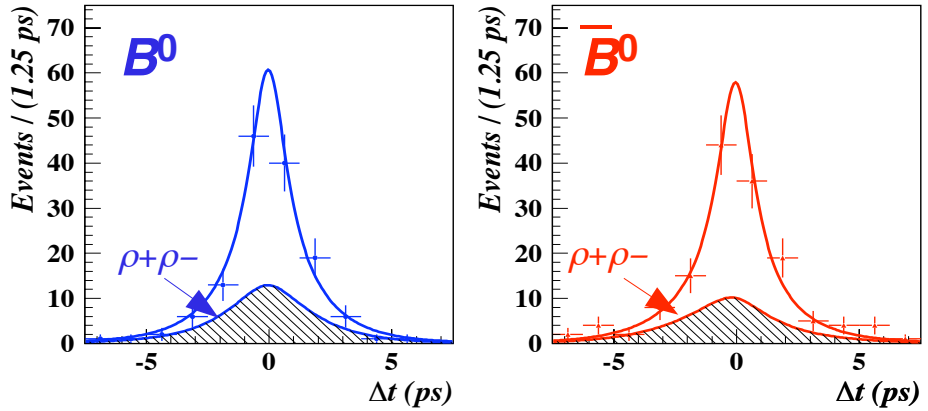
PRD 76, 011104(R) (2007)

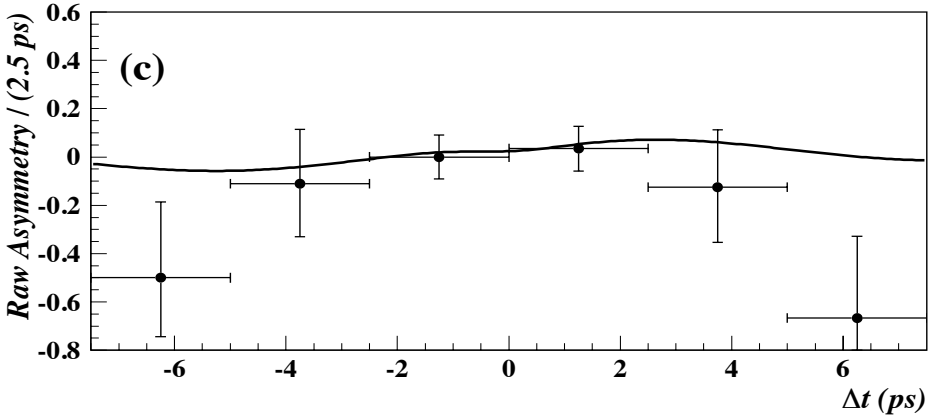
492 fb^{-1}

Also: reduce qq suppression cut, include qq PDF into ML fit
(70% increase in ε, 12% decrease in errors)

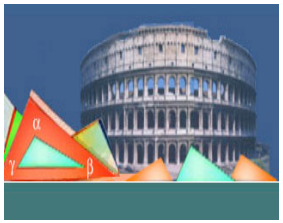
$A_{\rho\rho} = 0.16 \pm 0.21 \pm 0.08$

$S_{\rho\rho} = 0.19 \pm 0.30 \pm 0.08$





Systematics	$\delta\mathcal{A}$ ($\times 10^{-2}$)		$\delta\mathcal{S}$ ($\times 10^{-2}$)	
	$+\sigma$	$-\sigma$	$+\sigma$	$-\sigma$
Component fractions	1.5	1.9	3.9	3.7
Wrong tag fractions	0.5	0.5	0.8	0.8
$\rho\pi\pi$ nonresonant fraction	1.2	1.0	1.5	1.2
SCF fraction, Δt PDF	0.2	0.2	0.1	0.1
$\mathcal{R}$ PDF ($q\bar{q}$ suppression)	0.8	0.7	1.2	1.3
Vertexing	2.1	2.1	1.0	1.3
Resolution function	1.4	1.5	1.0	1.7
Background Δt distributions	0.5	0.5	1.0	1.1
Background asymmetry	1.1	0.0	0.0	0.4
$b \to u$ asymmetry	2.4	2.9	2.4	3.2
$\rho\pi\pi$ asymmetry	4.6	4.6	4.6	4.6
Possible fitting bias	0.3	0.0	0.3	0.0
Parameters Δm, τ_{B^0}	0.2	0.3	0.6	0.7
Transverse polarization	3.8	2.8	4.6	2.7
Tag-side interference	3.7	3.7	0.1	0.1
Total	**+8.3**	**−8.0**	**+8.4**	**−7.9**



Measurement of $B^0 \to \rho^+\rho^-$

PRD 76, 052007 (2007)

349 fb^{-1}

unbinned extended ML fit to M_{bc}, ΔE, $m_{\pi\pi+}$, cos θ_+, $m_{\pi\pi-}$, cos θ_0, x_{NN}, Δt :

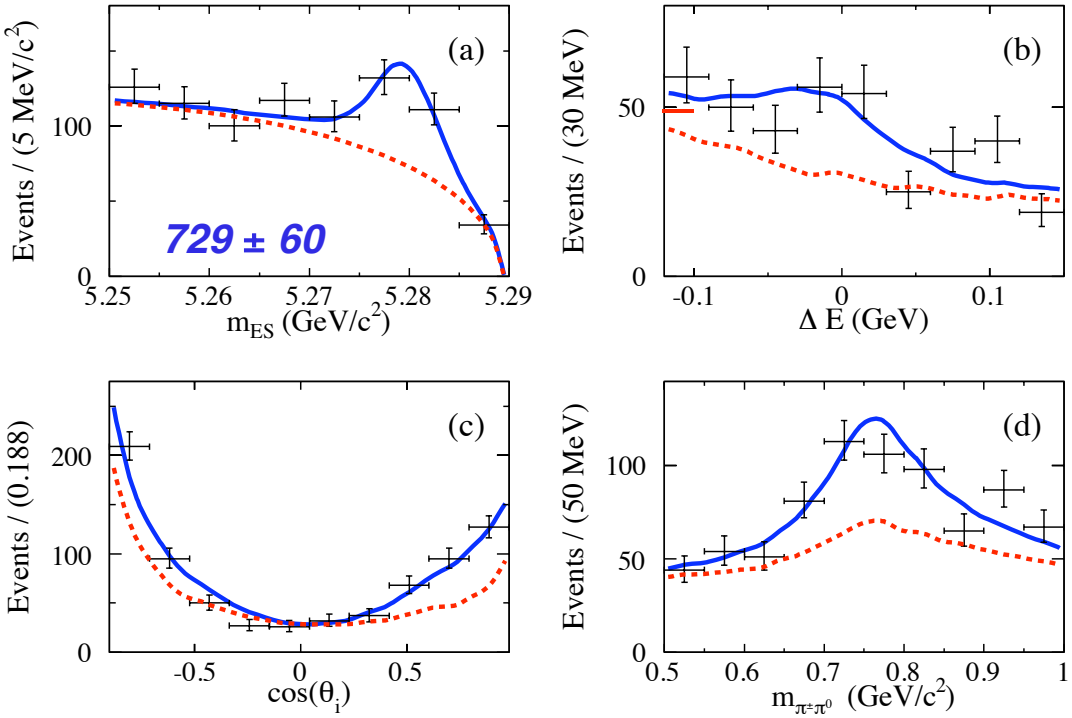
$$B_{\rho+\rho0} = (25.5 \pm 2.1 \,^{+\,3.6}_{-\,3.9}) \times 10^{-6}$$

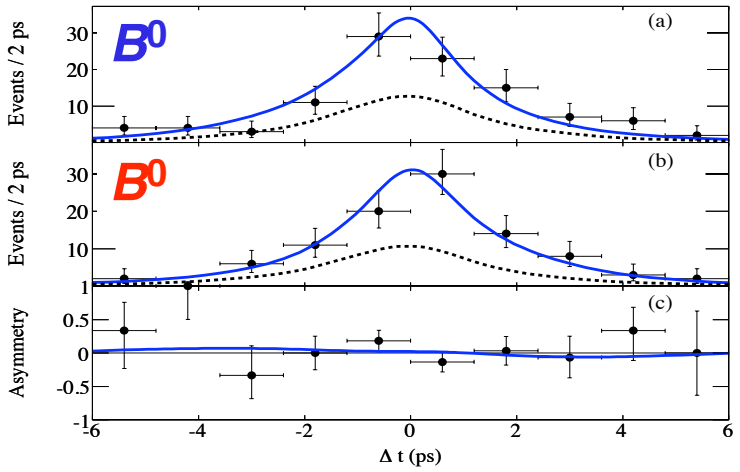
no PDF for non-resonant $B \to \rho\pi\pi$

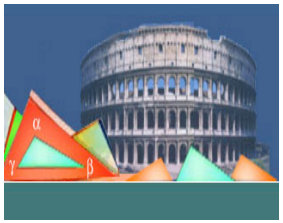
$$f_L = 0.992 \pm 0.024 \,^{+\,0.026}_{-\,0.013}$$

$$A_{\rho\rho} = -0.01 \pm 0.15 \pm 0.06$$

$$S_{\rho\rho} = -0.17 \pm 0.20 \,^{+\,0.05}_{-\,0.06}$$







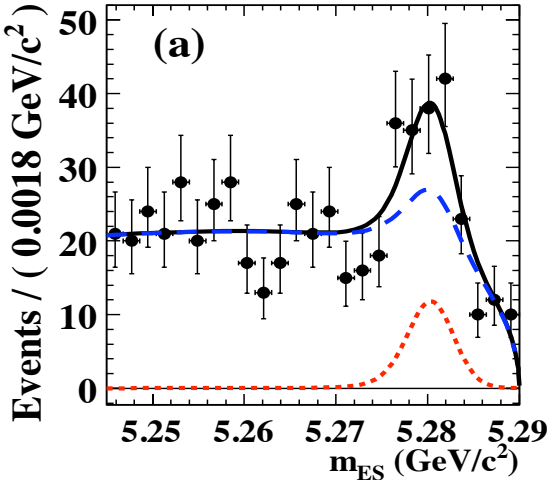
Measurement of $B^0 \to \rho^0\rho^0$

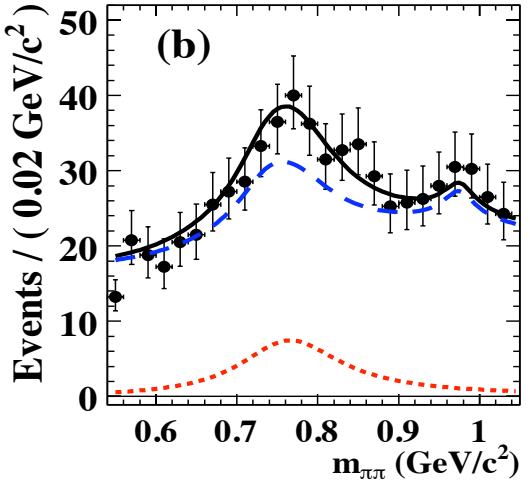
arXiv:0807.4977,
submitted to PRL

423 fb^{-1}

$(0.55 < m_{\pi\pi-1} < 1.05) \times (0.55 < m_{\pi\pi-2} < 1.05)$

unbinned extended ML fit to M_{bc}, ΔE,
$m_{\pi\pi}$, $\cos\theta_1$, $m_{\pi\pi}$, $\cos\theta_2$, x_{NN}, Δt, $\sigma_{\Delta t}$:





Mode	yield	significance (σ)	branching fraction (10^{-6})
$\rho^0\rho^0$	99^{+35}_{-34}	3.1	$0.92 \pm 0.32 \pm 0.14$
$\rho^0\pi^+\pi^-$	-12^{+39}_{-35}	-	< 8.7
$\pi^+\pi^-\pi^+\pi^-$	8^{+30}_{-25}	-	< 21.1

$f_L = 0.75^{+0.11}_{-0.14} \pm 0.04$

$A_{\rho\rho} = -0.2 \pm 0.8 \pm 0.3$

$S_{\rho\rho} = 0.3 \pm 0.7 \pm 0.2$

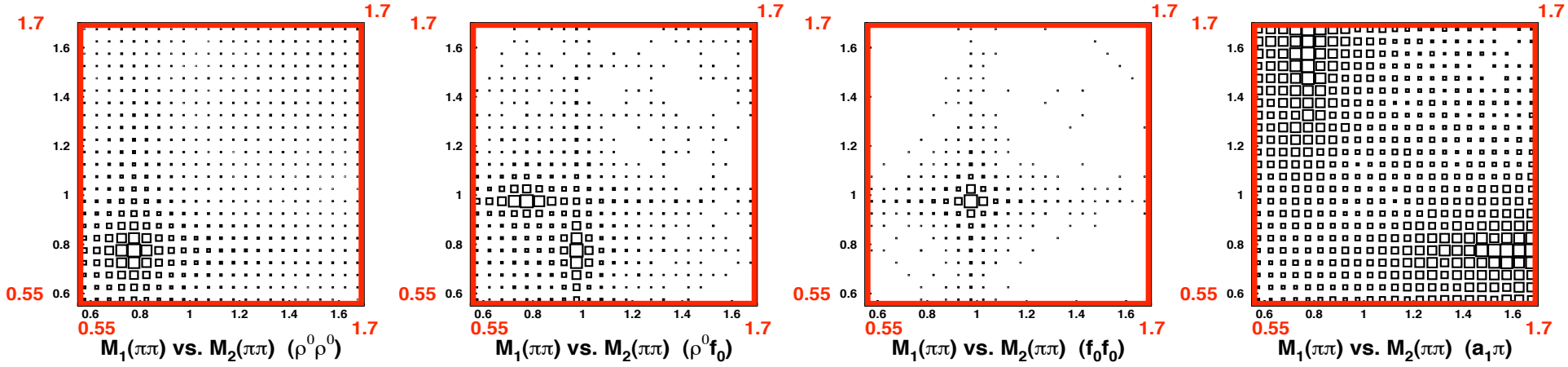
Measurement of $B^0 \to \rho^0\rho^0$

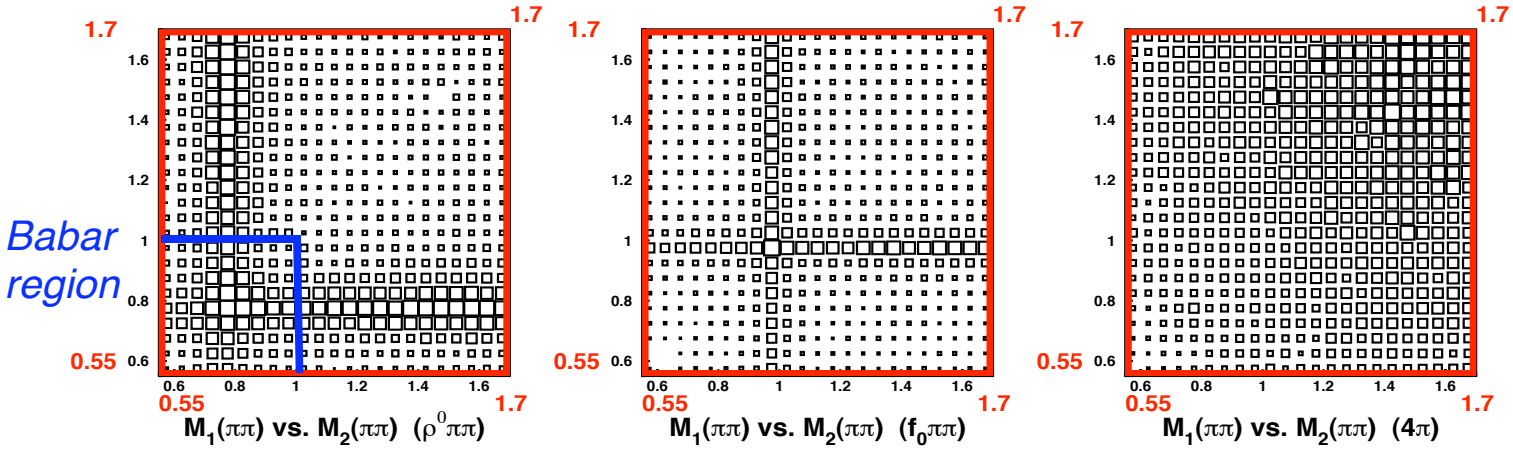
arXiv:0808.2576, submitted to PRD(RC)

605 fb^{-1}

To distinguish ρρ, ρππ, πππ, do an extended unbinned ML fit to grand canonical area (units are GeV/c^2) : $(0.55 < m_{\pi\pi-1} < 1.7) \times (0.55 < m_{\pi\pi-2} < 1.7)$

Monte Carlo:



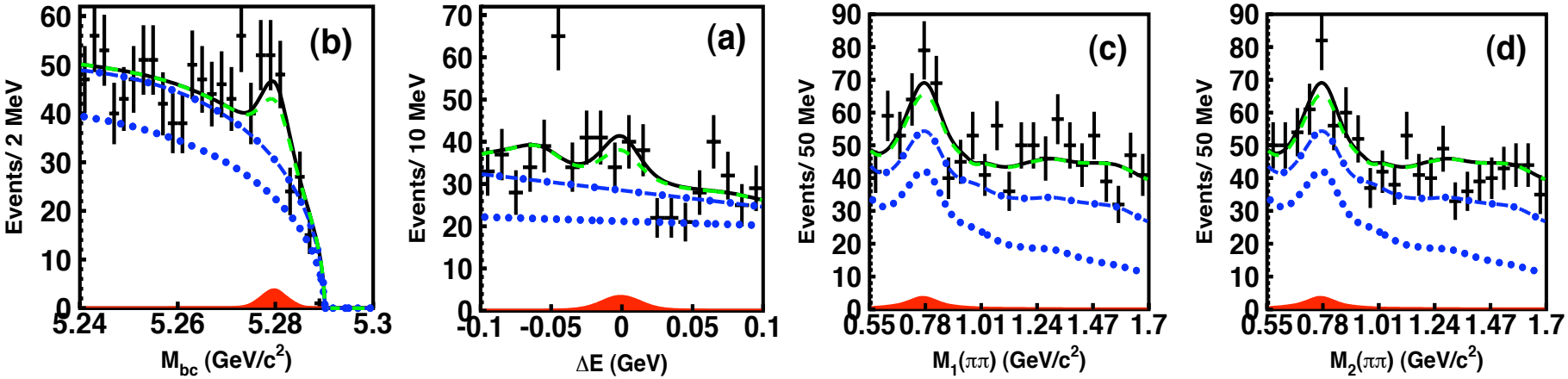


Measurement of $B^0 \to \rho^0\rho^0$

arXiv:0808.2576, submitted to PRD(RC)

BELLE

605 fb^{-1}



Mode	yield	significance (σ)	branching fraction (10^{-6})
$\rho^0\rho^0$	24.5^{+24}_{-22}	1.0	$0.4 \pm 0.4^{+0.2}_{-0.3}$
$\rho^0\pi^+\pi^-$	112.5^{+67}_{-66}	1.3	$5.9^{+3.5}_{-3.4} \pm 2.7$
$\pi^+\pi^-\pi^+\pi^-$	161.2^{+61}_{-59}	2.5	$12.4^{+4.7}_{-4.6}{}^{+2.1}_{-1.9}$

Constraint on ϕ_2 (from $B^0 \to \rho^0\rho^0$)

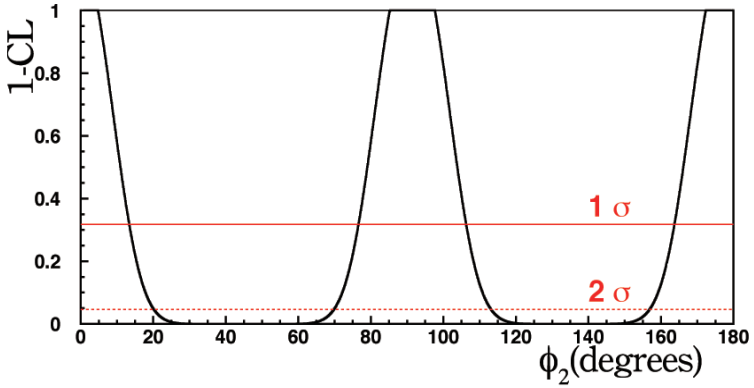
arXiv:0807.4977
arXiv:0808.2576

Construct χ^2, take CL from cumulative χ^2 distribution:

BELLE

605 fb^{-1}

[Using WA (HFAG) values for $B(\rho^+\rho^0)$, $B(\rho^+\rho^-)$, and $A_{\rho^0\rho^0}$ missing]

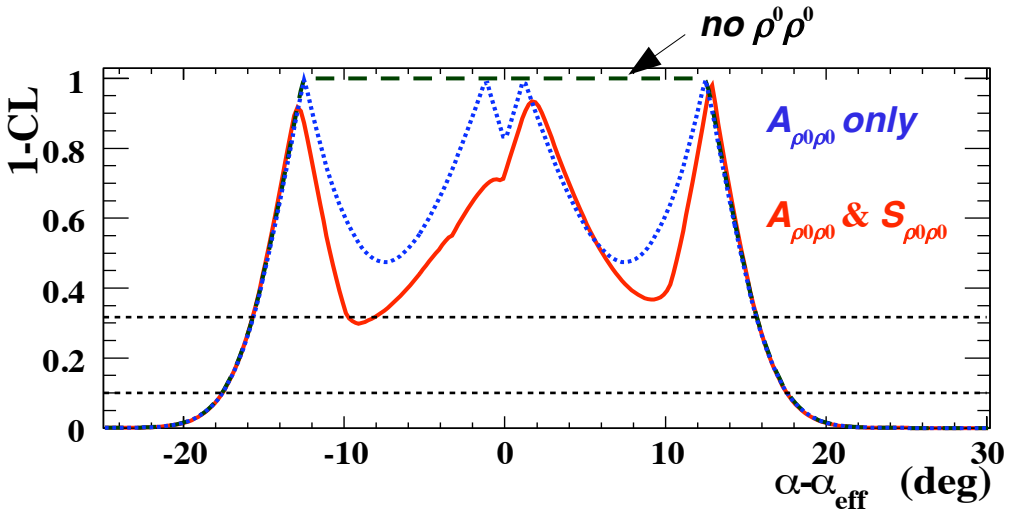


$\phi_2 = (91.7 \pm 14.9)°$

(solution consistent with other constraints on unitarity triangle)

423 fb^{-1}

[all Babar values]



$|\phi_2 - \phi_2(eff)| < 17.6°$ (90% CL)

Measurement of $B^0 \to a_1\pi^-$

PRL 98, 181803 (2007)

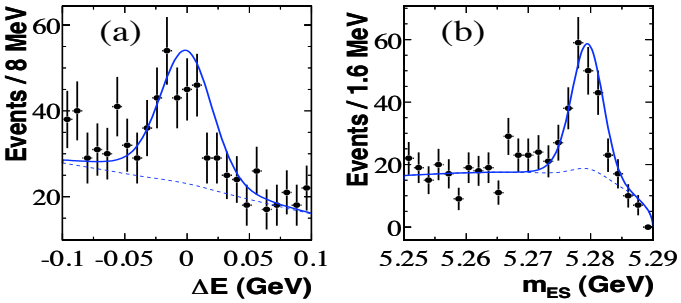
Decay time dependence: *[Gronau & Zupan, PRD 73, 057502 (2006)]*

$$\frac{dN(a_1^{\pm}\pi^{\mp})}{d\Delta t} = (1 \pm \mathcal{A}_{CP})\frac{e^{-|\Delta t|/\tau}}{4\tau}\left\{1 - q\left[(C \pm \Delta C)\cos(\Delta m\,\Delta t) - (S \pm \Delta S)\sin(\Delta m\,\Delta t)\right]\right\}$$

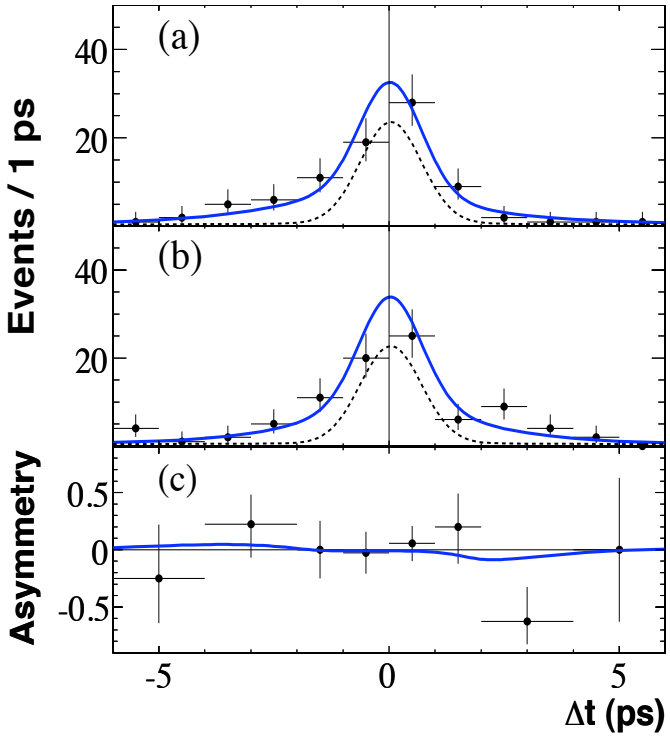
CP violating CP conserving

⇒ need to measure decay times of all four modes to unravel

349 fb^{-1}



608 ± 52



$\mathcal{A}_{CP}$ = -0.07 ± 0.07 ± 0.02

C = -0.10 ± 0.15 ± 0.09

S = 0.37 ± 0.21 ± 0.07

ΔC = 0.26 ± 0.15 ± 0.07

ΔS = -0.14 ± 0.21 ± 0.06

4-fold ambiguity for α_{eff}: solution closest to SM is $(78.6 \pm 7.3)^o$

$\phi_2\,(\alpha)$ from $B^0 \to a_1\pi^-$

Gronau, Zupan, PRD 73, 057502 (2006)

Babar, PRL 98, 181803 (2007): **4-fold ambiguity for α_{eff}: solution closest to SM is $(78.6 \pm 7.3)^o$**

To extract α from α_{eff} one can use SU(3) symmetry and $B \to a_1 K$, $B^0 \to K_{1A}\pi$ decays:

$$\mathcal{R}^+_- \equiv \frac{\lambda^2 f_\pi^2 \overline{\Gamma}(a_1^+ K^0)}{f_K^2 \overline{\Gamma}(a_1^- \pi^+)} \qquad \mathcal{R}^0_- \equiv \frac{\lambda^2 f_\pi^2 \overline{\Gamma}(a_1^- K^+)}{f_K^2 \overline{\Gamma}(a_1^- \pi^+)}$$

$$|\sin(\alpha - \alpha^-_{\rm eff})| \leq \sqrt{\mathcal{R}^+_-}\sin\gamma$$

$$|\sin(\alpha - \alpha^-_{\rm eff})| \leq \sqrt{\mathcal{R}^0_-}$$

$$\mathcal{R}^+_{+A} \equiv \frac{\lambda^2 f_{a_1}^2 \overline{\Gamma}(K^0_{1A}\pi^+)}{f_{K_1}^2 \overline{\Gamma}(a_1^+ \pi^-)} \qquad \mathcal{R}^0_{+A} \equiv \frac{\lambda^2 f_{a_1}^2 \overline{\Gamma}(K^+_{1A}\pi^-)}{f_{K_1}^2 \overline{\Gamma}(a_1^+ \pi^-)}$$

$$|\sin(\alpha - \alpha^+_{\rm eff})| \leq \sqrt{\mathcal{R}^+_{+A}}\sin\gamma$$

$$|\sin(\alpha - \alpha^+_{\rm eff})| \leq \sqrt{\mathcal{R}^0_{+A}}$$

Recent results for $B \to a_1^+ K^0$, $a_1^- K^+$ and $B^0 \to K_{1A}^+\pi^-$:

BELLE	arXiv:0706.3279 (2007)	492 fb^{-1}
BABAR	PRL 100, 051803 (2008)	347 fb^{-1}
	arXiv:0807.4760 (2008)	423 fb^{-1}

But: must know f_{a1} and size of SU(3)-breaking corrections

Summary

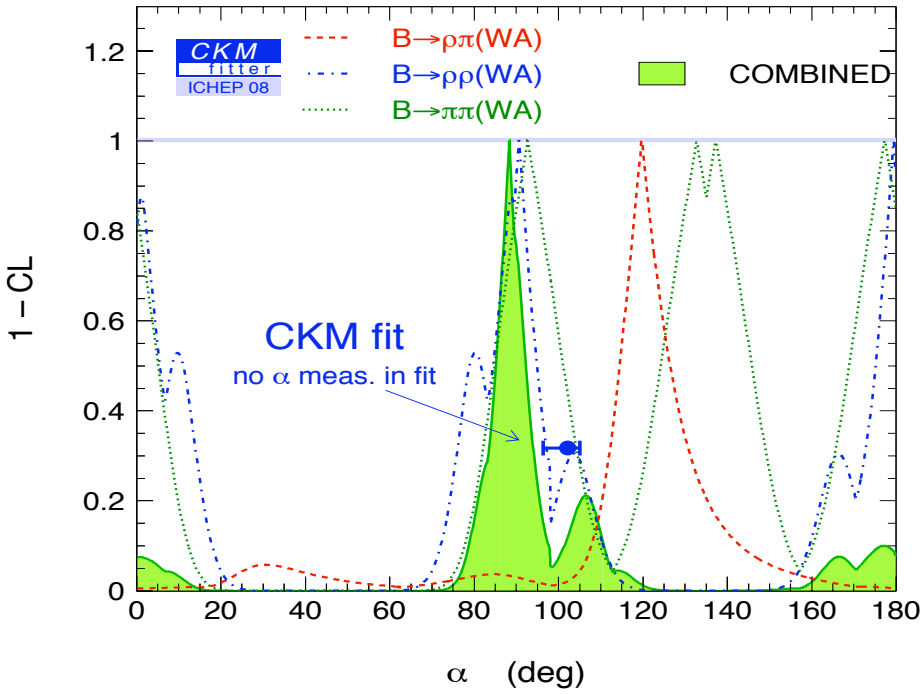
$\rho\rho$ *system has turned out to be very favorable for measuring* ϕ_2

$\rho^0\rho^0$ *just now being measured, but Belle/Babar results differ somewhat*

Final updates with full Babar and Belle data sets could make a substantial difference

$a_1\pi$ *system may be useful with SU(3) symmetry; SU(3)-breaking corrections needed*

uncertainty in ϕ_2 *from* $\rho\rho$ *penguin pollution is now* $< 15°$



CKMfitter: $\phi_2 = (88.3\ ^{+5.7}_{-4.8})°$

$< 6.5°$
$(77, 112)°$ } 2σ
$> 163°$