SU(2) [K]ChPT and RBC–UKQCD Lattice Results

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CKM2008 Rome 9 September 2008

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RBC–UKQCD lattice simulations

- We use domain wall fermions (DWF) for good chiral symmetry properties with Iwasaki gauge action
- Recent results from two $2+1$ flavour datasets with lattice spacing around 0.114 fm

$$24^3 \times 64 \times 16 \qquad L \approx 2.75\,\text{fm}$$
$$16^3 \times 32 \times 16 \qquad L \approx 1.83\,\text{fm}$$

- On the 24^3 lattice:
  - 4 values for light dynamical mass am_l corresponding to pion masses 330, 415, 555 and 670 MeV
  - lightest $m_l \sim m_s/5$
  - $am_h = 0.04$ for sea strange quark: *a posteriori* a little too large
  - lightest valence quark mass about 11% of m_s, corresponds to pion mass 240 MeV (partial quenching)
- Currently generating and analysing $32^3 \times 64 \times 16$ with $a \approx 0.09\,\text{fm}$

RBC–UKQCD lattice simulations (cont)

- Simulate with fixed bare input parameters $g(a)$, am_l (isospin limit) and am_h
- Use three physical quantities (m_π, m_K and m_{Ω^-}) to fix lattice spacing a and masses m_{ud}, m_s
- Simulate with m_l larger than m_{ud} and extrapolate to physical point
- Can work at m_s (after tuning)
- Chiral perturbation theory (ChPT) is used for the extrapolation $m_l \to m_{ud}$
  - How reliable?
  - What are the values of the Low Energy Constants (LECs)?
  - $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$ or $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$?
- We do partially quenched simulations with distinct valence and sea masses $\Longrightarrow$ use PQChPT Sharpe & Shoresh PRD62 094503 2000

SU(2) vs SU(3) ChPT

	SU(3)	SU(2)
degrees of freedom	π, K, η	π
	K and η pGBs	K and η integrated out
expand in (with $\Lambda_\chi \sim 4\pi f_\pi$)	$\left(\frac{M_{\pi,K,\eta}}{\Lambda_\chi}\right)^2$	$\left(\frac{M_\pi}{\Lambda_\chi}\right)^2, \left(\frac{M_\pi}{M_K}\right)^2$
LECs	$f(m_{c,b,t}, \Lambda_{QCD})$	$f(m_s, m_{c,b,t}, \Lambda_{QCD})$
NLO accuracy at phys masses	$\left(\frac{M_\eta}{4\pi f_\pi}\right)^4 \sim 5\%$	$\left(\frac{M_\pi}{M_K}\right)^4 < 1\%$

cf. Lellouch Lattice2008

$SU(3)_L \times SU(3)_R$ ChPT

- EFT of approximate chiral symmetry of QCD; expansion in powers of $M^2_{\pi,K,\eta}/\Lambda^2_\chi$ up to chiral logarithms
- Examples at one-loop:

$$m_\pi^2 = \chi_{ud}\left\{1 + \frac{48}{f_0{}^2}(2L_6 - L_4)\bar{\chi} + \frac{16}{f_0{}^2}(2L_8 - L_5)\chi_{ud} + \frac{1}{24\pi^2 f_0{}^2}\left(\frac{3}{2}\chi_{ud}\log\frac{\chi_{ud}}{\Lambda_\chi^2} - \frac{1}{2}\chi_\eta \log\frac{\chi_\eta}{\Lambda_\chi^2}\right)\right\}$$

$$f_\pi = f_0\left\{1 + \frac{24}{f_0{}^2}L_4\bar{\chi} + \frac{8}{f_0{}^2}L_5\chi_{ud} - \frac{1}{16\pi^2 f_0{}^2}\left(2\chi_{ud}\log\frac{\chi_{ud}}{\Lambda_\chi^2} + \frac{\chi_{ud}+\chi_s}{2}\log\frac{\chi_{ud}+\chi_s}{2\Lambda_\chi^2}\right)\right\}$$

where $\chi_i = 2B_0 m_i$ for $i = ud, s$, $\chi_\eta = (\chi_{ud} + 2\chi_s)/3$ and $\bar{\chi} = (2\chi_{ud} + \chi_s)/3$

- Do such formulae represent our data?

$SU(2)_L \times SU(2)_R$ ChPT

- One-loop examples:

$$m_\pi^2 = \chi_{ud}\left\{1 + \frac{\chi_{ud}}{16\pi^2 f^2}\left(64\pi^2 l_3^r + \log\frac{\chi_{ud}}{\Lambda_\chi^2}\right)\right\}$$

$$\equiv \chi_{ud}\left(1 - \frac{\chi_{ud}}{16\pi^2 f^2}\bar{l}_3\right)$$

$$f_\pi = f\left\{1 + \frac{m_\pi^2}{8\pi^2 f^2}\left(16\pi^2 l_4^r - \log\frac{m_\pi^2}{\Lambda_\chi^2}\right)\right\} \equiv f\left(1 + \frac{m_\pi^2}{8\pi^2 f}\bar{l}_4\right)$$

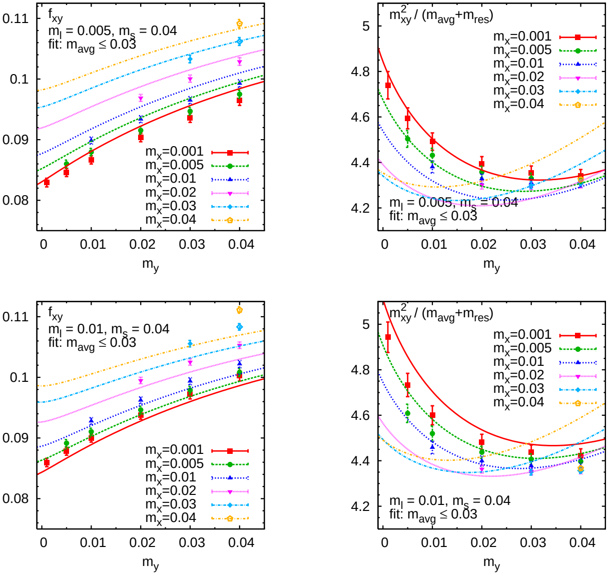
  now with $\chi_{ud} = 2Bm_{ud}$

- New set of m_s-dependent LECs
- Can "convert" from SU(3) to SU(2) for small M_π^2/M_K^2. Example:

$$l_4^r = 8L_4 + 4L_5 - \frac{1}{64\pi^2}\left(1 + \log\frac{\chi_s}{2\Lambda_\chi^2}\right)$$

- Do such formulae represent our data?

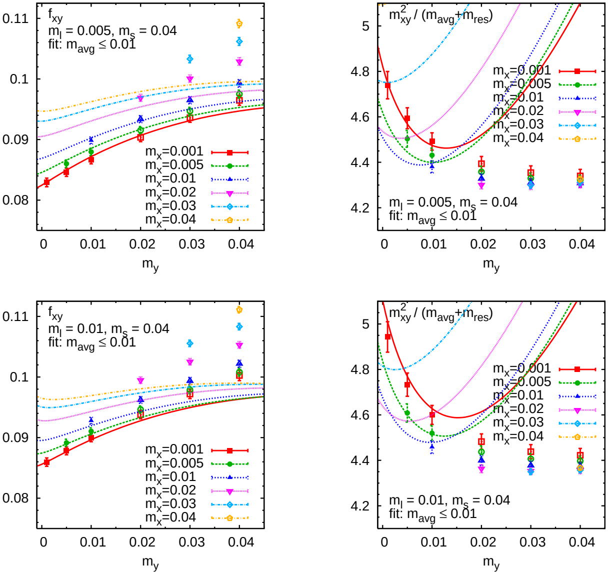
NLO $SU(3)_L \times SU(3)_R$ fit bad for $am_{avg} < 0.03$



Left: f_P

Right: m_P^2

NLO $SU(3)_L \times SU(3)_R$ fit good for $am_{avg} < 0.01$



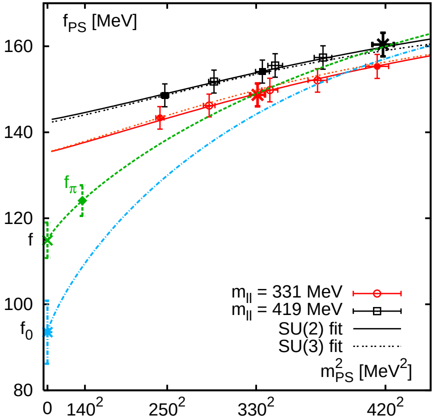
Left: f_P

Right: m_P^2

Chiral fits

- NLO SU3 chiral fits to pseudoscalar masses and decay constants work well but only at light masses, below about 400 MeV
  - NLO corrections are very large, up to 50% of LO term for decay constants
  - Fitted decay constant in chiral limit, f_0, very small
- going to NNLO would increase range of good fits
  - number of new LECs too large for the data we have
  - others use at least the analytical terms
  - work in progress (Mawhinney Lattice2008)
- Fits can also be done using $SU(2)_L \times SU(2)_R$ ChPT at light masses
  - NLO corrections smaller
  - Inclusion of analytic NNLO terms extends range of fit with small NNLO contributions

Results using SU(2) and SU(3) ChPT



$$f_\pi / f = 1.08$$

$$f / f_0 = 1.23(6)$$

Large value of f_π / f_0 leads us to present results based on $SU(2)_L \times SU(2)_R$ ChPT

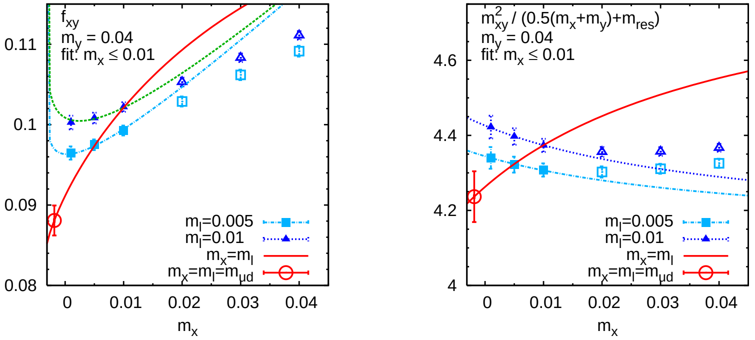
$$f_\pi = 124.1\,(3.6)_{\mathrm{stat}}(6.9)_{\mathrm{sys}}\ \mathrm{MeV}$$

SU(3) curve is for three degenerate quarks so does not show f_π

Kaon $SU(2)_L \times SU(2)_R$ ChPT

- Only u and d transform $\Longrightarrow$ include kaon as a matter field
- KChPT introduced by Roessl to study $K\pi$ scattering near threshold NPB555 507 1999
- Parallels to Heavy Meson ChPT, but $m_{K^*} \neq m_K$, whereas $m_{B^*} = m_B$ in heavy quark limit
- We derived the chiral behaviour of m_K^2, f_K and B_K in unitary and partially quenched theories and used the results in our phenomenological studies RBC–UKQCD arXiv:0804.0473 and 0809.1229
- m_s taken as $O(\Lambda_{\mathrm{QCD}})$ so expand in m_π^2/m_K^2 as well as m_π^2/Λ_χ^2
- m_K^2/Λ_χ^2 effects are fully absorbed into the LECs
- Kaon decay constant in SU(2) chiral limit, $f^{(K)}$, distinct from pion decay constant in same limit, f

Chiral behaviour of m_K^2 and f_K



- Use PQ $SU(2)_L \times SU(2)_R$ KChPT with light valence quark $am_{ud} < 0.01$ and $am_s = 0.04$
- Result arXiv:0804.0473

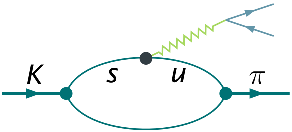
$$f_K/f_\pi = 1.205\,(0.018)_{\text{stat}}(0.062)_{\text{sys}}$$

Decay constants and LECS from SU(2) fits

$$
\begin{aligned}
f &= 114.8\,(4.1)_{\text{stat}}(8.1)_{\text{sys}}\ \text{MeV} \\
\bar{l}_3 &= 3.13\,(0.33)_{\text{stat}}(0.24)_{\text{sys}} \\
\bar{l}_4 &= 4.43\,(0.14)_{\text{stat}}(0.77)_{\text{sys}} \\
f_\pi &= 124.1\,(3.6)_{\text{stat}}(6.9)_{\text{sys}}\ \text{MeV} \\
f_K &= 149.6\,(3.6)_{\text{stat}}(6.3)_{\text{sys}}\ \text{MeV} \\
f_K/f_\pi &= 1.205\,(0.018)_{\text{stat}}(0.062)_{\text{sys}}
\end{aligned}
$$

RBC–UKQCD arXiv:0804.0473

K_{l3} decay



$$\langle \pi^+(k) | \bar{u}\gamma_\mu s | \bar{K}^0(p) \rangle$$
$$= f_+(q^2)\left(p_\mu + k_\mu - \frac{m_K^2 - m_\pi^2}{q^2} q_\mu\right) + f_0(q^2)\frac{m_K^2 - m_\pi^2}{q^2} q_\mu$$

- Require $f_0(0) = f_+(0)$ to better than 1% precision
- ChPT $\Longrightarrow$

$$f_+(0) = 1 + f_2 + f_4 + \cdots \qquad \text{where } f_n = O(M^n_{\pi,K,\eta})$$

- Reference value is $f_+(0) = 0.961(8)$ where $f_2 = -0.023$ relatively well known from ChPT and $f_4, f_6, \ldots$ are found from models Leutwyler & Roos 1984

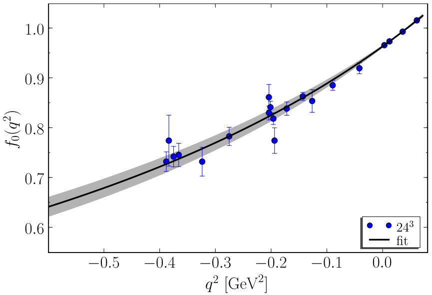
K_{l3} "standard method"

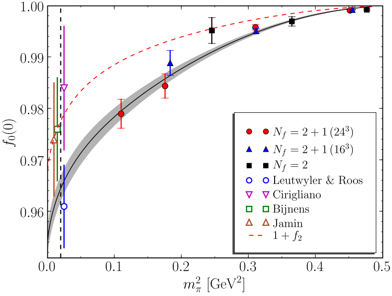
- 1% precision attainable because $1 - f_+(0)$ is calculated Becirevic et al NPB705 339 2005 based on Okamoto et al PRD61 014502 2000
- Evaluate $f_0(q^2_{\text{max}})$ with excellent precision for varying m_l using a double ratio

$$\frac{\langle\pi|\bar{s}\gamma_4 u|K\rangle\langle K|\bar{u}\gamma_4|\pi\rangle}{\langle\pi|\bar{u}\gamma_4 u|\pi\rangle\langle K|\bar{s}\gamma_4 s|K\rangle} = \left[f_0(q^2_{\text{max}})\right]^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}$$

- Evaluate somewhat less precisely at other values of q^2 for varying m_l
- Extrapolate in q^2 and do chiral extrapolation $m_l \to m_{ud}$

K_{l3}: q^2 and chiral extrapolations





- Central values from combined pole fit to q^2 and $m_l \to m_{ud}$ dependence
- $f_0(0)$ as function of pion mass

Final answer Boyle et al PRL100 141601 2008

$$f_+(0) = 0.964(5)$$

K_{l3}: chiral extrapolation at q^2_{max}

- The Callan–Treiman relation becomes in the SU(2) chiral limit (for our simulated m_s)

$$f_0(q^2_{max}) \xrightarrow{m_\pi^2 \to 0} \frac{f^{(K)}}{f} \approx 1.28$$

- Lattice results at the simulated masses are about 25% below $f^{(K)}/f$ and are increasing very slowly

m_π / MeV	q^2_{max} / GeV^2	$f_0(q^2_{max})$
671(11)	0.00235(4)	1.00029(6)
556(9)	0.01252(20)	1.00192(34)
416(7)	0.03524(62)	1.00887(89)
329(5)	0.06070(107)	1.02143(132)

- Chiral behaviour in KChPT

$$f_0(q^2_{\rm max}) = \frac{f^{(K)}}{f}\left[1 - \frac{11}{4}L + \frac{\lambda_1}{4\pi f}m_\pi + \frac{\lambda_2}{(4\pi f)^2}m_\pi^2 + \cdots\right]$$

where

$$L = \frac{m_\pi^2}{(4\pi f)^2}\log\frac{m_\pi^2}{\mu^2}$$

- Linear dependence on m_π
- Log term large but wrong sign
- Estimate linear term by converting from SU(3) (Gasser & Leutwyler 1985): also large and changes overall sign
- ... stability of chiral expansion questionable
- SU(3) description, fixing L_5 from f_K/f_π, provides semi-quantitative agreement
- Similar issues for $f_0^{B\to\pi}(q^2_{\rm max})$ vs f_B/f_π

JMF & Sachrajda arXiv:0809.1229

Eliminating the q^2 interpolation

- Momentum resolution with conventional methods is poor on the lattice:

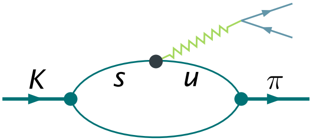
$$L = 24a \quad \text{with} \quad a^{-1} = 1.73\,\text{GeV} \quad \Rightarrow \quad 2\pi/L = 0.45\,\text{GeV}$$

- Modify momentum spectrum (relative to periodic BCs) using twisted BCs

$$q(x_i + L) = e^{i\theta_i} q(x_i) \quad \longrightarrow \quad p_i = \frac{2\pi}{L} n_i + \frac{\theta_i}{L}$$

- FV corrections remain exponentially small with twisted BCs for quantities without FSI (eg masses, decay constants, form factors) Sachrajda & Villadoro 2004
- FV corrections also exponentially small for partial twisting: periodic BCs for sea quarks, twisted for valence Sachrajda & Villadoro 2004, Bedaque & Chen 2004
- No need to generate new ensembles for every choice of twists

K_{l3} directly at $q^2 = 0$



- Tune twists to calculate matrix element at $q^2 = 0$ (or any chosen value of q^2) Boyle et al JHEP 0705:016 2007

$$\langle \pi(\vec{0}) | V_4 | K(\vec{\theta}_K) \rangle \quad \text{with} \quad |\vec{\theta}_K| = L\sqrt{\left(\frac{m_K^2 + m_\pi^2}{2m_\pi}\right)^2 - m_K^2}$$

$$\langle \pi(\vec{\theta}_\pi) | V_4 | K(\vec{0}) \rangle \quad \text{with} \quad |\vec{\theta}_\pi| = L\sqrt{\left(\frac{m_K^2 + m_\pi^2}{2m_\pi}\right)^2 - m_\pi^2}$$

- Twisted BC previously applied to K_{l3} (though not directly at $q^2 = 0$) in a quenched simulation Guadagnoli et al PRD73 114504 2006

K_{l3} directly at $q^2 = 0$

- Feasibility demonstrated on a $16^3 \times 32$ lattice at two values of m_{ud} Boyle et al JHEP 0705:016 2007
- Currently using partial twisting to get $f_0(0)$ at our lightest quark mass ($am = 0.005$) on $24^3 \times 64$ lattice
- Preliminary results suggest we can get the same accuracy with this direct method as with the "traditional" one
- We have also used partial twisting to study the electromagnetic form factor of a pion with mass 330 MeV at small momentum transfers, using NLO ChPT to determine results for a physical pion

Conclusions/Outlook

- Presented selected phenomenological results in kaon physics from $2+1$ flavour dynamical lattice simulations using action with good chiral properties
- Lattice community beginning to make strong contact with ChPT community and to determine LECs with precision
- RBC–UKQCD now moving on to finer lattices (will gain information on continuum extrapolation)
- Will continue to extend range of quantities calculated (eg $K \to \pi\pi$ decays)
- Medium term aim is target simulation with $a = 0.06\,\mathrm{fm}$, $L = 4\,\mathrm{fm}$, $m_\pi = 195\,\mathrm{MeV}$