

Inference of α in $B \rightarrow \pi\pi, \rho\rho$ decays

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collaboration

Summary:

- Ingredients & Setup (Hamiltonian, Weak contractions , theoretical assumptions, ...)
- Bayesian inference at work
- Numerical results
- Conclusions

09/09/08



The relevant Weak Hamiltonian

$$H_{\Delta C=0, \Delta S=0}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \left\{ V_{ud} V_{ub}^* \left[C_1 (Q_1^u - Q_1^c) + C_2 (Q_2^u - Q_2^c) \right] - V_{td} V_{tb}^* \left[C_1 Q_1^c + C_2 Q_2^c + \sum_{i=3,10} C_i Q_i \right] \right\}$$

$$Q_1^u = (\bar{b}u)_{(V-A)} \times (\bar{u}d)_{(V-A)}$$

$$Q_1^c = (\bar{b}c)_{(V-A)} \times (\bar{c}d)_{(V-A)}$$

$$Q_2^u = (\bar{b}d)_{(V-A)} \times (\bar{u}u)_{(V-A)}$$

$$Q_2^c = (\bar{b}d)_{(V-A)} \times (\bar{c}c)_{(V-A)}$$

$$Q_3 = \sum_{q=u,d,s,c} (\bar{b}d)_{(V-A)} \times (\bar{q}q)_{(V-A)} \quad Q_7 = \frac{3}{2} \sum_{q=u,d,s,c} e_q (\bar{b}d)_{(V-A)} \times (\bar{q}q)_{(V+A)}$$

$$Q_4 = \sum_{q=u,d,s,c} (\bar{b}q)_{(V-A)} \times (\bar{q}d)_{(V-A)} \quad Q_8 = \sum_{q=u,d,s,c} e_q (\bar{b}q)_{(V-A)} \times (\bar{q}d)_{(V-A)}$$

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$$Q_6 = -2 \sum_{q=u,d,s,c} (\bar{b}q)_{(S+P)} \times (\bar{q}d)_{(S-P)} \quad Q_{10} = \frac{3}{2} \sum_{q=u,d,s,c} e_q (\bar{b}q)_{(V-A)} \times (\bar{q}d)_{(V-A)}$$

“Current” Operator

“Penguin” and “EW Penguin” Operator

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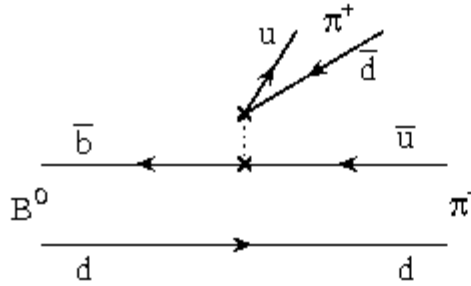
“Current” Operator

“Penguin” and “EW Penguin” Operator

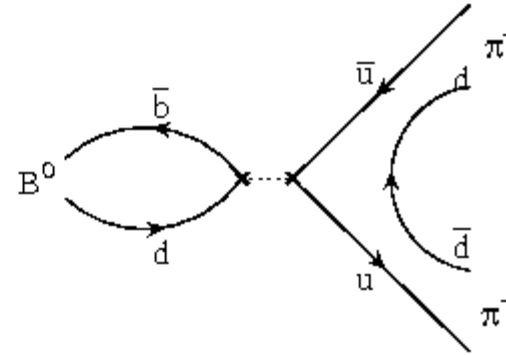
$$A(B \rightarrow \pi\pi) \propto \langle B | H_{\Delta C=0, \Delta S=0}^{\Delta B=1} | \pi\pi \rangle$$

The Wick Contractions

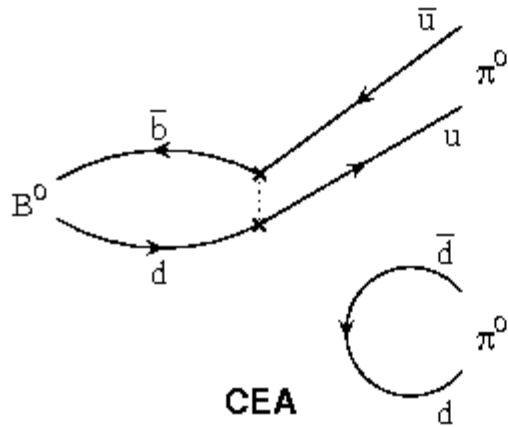
Example of topologies:



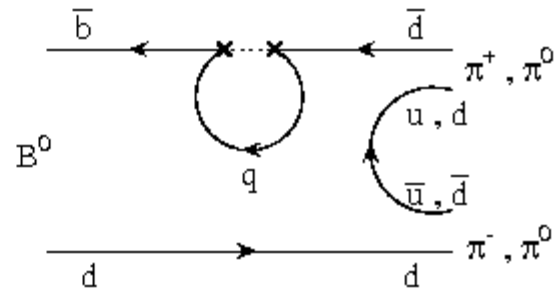
DE



DA



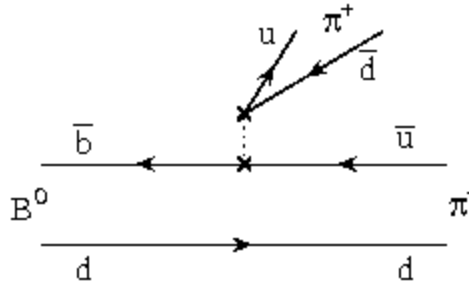
CEA



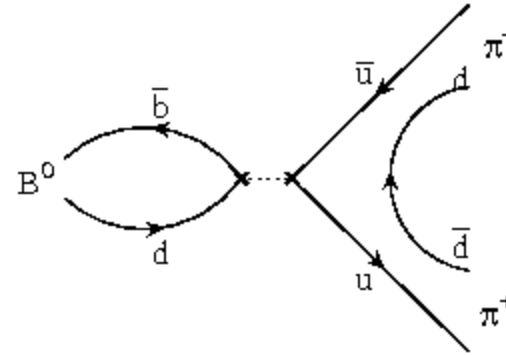
CP

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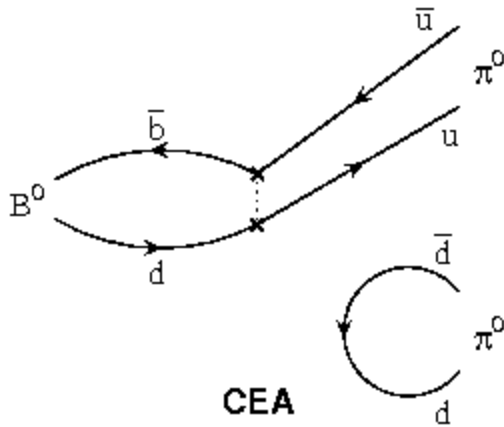
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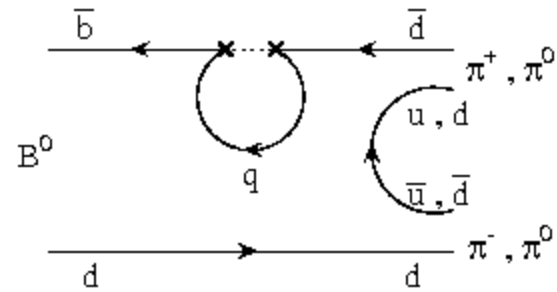
DE



DA



CEA



CP

$$P \propto C_1 \langle Q_1^c \rangle_{CP} + C_2 \langle Q_2^c \rangle_{DP} + \sum_{i=2}^5 C_{2i} \langle Q_{2i} \rangle_{DE} + \sum_{i=3}^{10} C_i \langle Q_i \rangle_{CP} + \dots$$

Assuming exact SU(2) symmetry and neglecting EWP, the amplitude can be written as:

$$A^{+-} = A(B^0 \rightarrow \pi^+ \pi^-) = e^{-i\alpha} T^{+-} + P$$

$$A^{00} = A(B^0 \rightarrow \pi^0 \pi^0) = \frac{1}{\sqrt{2}} (e^{-i\alpha} T^{00} - P)$$

$$A^{+0} = A(B^+ \rightarrow \pi^+ \pi^0) = \frac{1}{\sqrt{2}} e^{-i\alpha} (T^{00} + T^{+-})$$

In terms of this amplitude, the observables are

$$C_{\pi\pi}^{+,-,00} = \frac{|A^{+,-,00}|^2 - |\bar{A}^{+,-,00}|^2}{|A^{+,-,00}|^2 + |\bar{A}^{+,-,00}|^2}, \quad S_{\pi\pi}^{+-} = \frac{-2 \operatorname{Im} |A^{+-} \bar{A}^{+-*}|}{|A^{+,-,00}|^2 + |\bar{A}^{+,-,00}|^2}$$

$$B_{\pi\pi}^{+,-,00} = \frac{|A^{+,-,00}|^2 + |\bar{A}^{+,-,00}|^2}{2}, \quad B_{\pi\pi}^{+0} = \frac{\tau_{B^+}}{\tau_{B^0}} \frac{|A^{+0}|^2 + |\bar{A}^{+0}|^2}{2}$$

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6 parameters

$$|T^{+-}|, |T^{00}|, |P|, \varphi_{00}, \varphi_P, \alpha$$

6 experimental quantities

$$B_{\pi\pi}^{+-}, B_{\pi\pi}^{00}, B_{\pi\pi}^{+0}, C_{\pi\pi}^{+-}, C_{\pi\pi}^{00}, S_{\pi\pi}^{+-}$$

α can be determined without assumptions on the value of the QCD parameters
(Gronau-London '90)

In the Bayesian inference the inputs are the **experimental likelihoods** and the **priors** for all the parameters.

The result of the inference is a p.d.f for the parameters (eventually integrated on some of them). For example the p.d.f for α is

$$P(\alpha) \propto \int \prod_{i=1}^6 L(E_i, |T^{+-}|, \dots, \alpha) P_0(|T^{+-}|) \dots P_0(\alpha) d|T^{+-}| \dots$$

$$\text{Log } L(B^{+-}, |T^{+-}|, \dots, \alpha) = \frac{-\left(B_{ex}^{+-} - \frac{|A^{+-}|^2 + |\bar{A}^{+-}|^2}{2}\right)^2}{2\sigma_{B^{+-}}^2}$$

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Possible issue:

- Given one sextuple of exp.values, there are 8 (or 0) possible sextuple of parameters
- Some exp. value have a quite large error (C^{00})
- Same number of experimental and fit parameters

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The experimental information is not guaranteed to dominate the information contained in the priors. One must explicitly check if and how the resulting p.d.f. depends on the choice of the priors.

What information we have on the hadronic parameters ?

The parameters $|T^{+-}|, |T^{00}|, |P|$ are not arbitrary mathematical variable which can take any values between 0 and infinity, but they have a **natural scale**.

Not considering this point is the main reason of the difference of our analysis with that presented in J.Charles et al. hep-ph/0607246.

For example a value of $\alpha \approx 0$ is possible only for **infinitely large** $|T^{+-}|, |T^{00}|, |P|$

Hints to set the scale of the hadronic parameter:

1) Rough estimates using factorized amplitude of current operators, or using the Heavy quark effective theory to scale from $D \rightarrow \pi\pi$ to $B \rightarrow \pi\pi$ gives

$$|T^{+-}| \approx 1.3 \div 3.2$$

2) Assuming SU(3) symmetry one can estimate P:

$$|P|^2 = BR(B_s \rightarrow K^+ K^-) \times 10^6 \frac{\tau_{B_d^0} |V_{td}|^2}{\tau_{B_s^0} |V_{ts}|^2} \approx 1.1^2$$

Priors:

MA $|T^{+-}|$ and $|T^{00}|$ flat in the range $[0,10]$, $|P|$ flat in the range $[0,2.5]$,
 ϕ_{00} and ϕ_p flat in the range $[0,360^\circ]$, α flat in the range $[0,180^\circ]$

RI $\text{Re } T^{+-}, \text{Re } T^{00}, \text{Im } T^{00}, \text{Re } P, \text{Im } P$ flat with the condition
 $|T^{+-}|, |T^{00}| < 10$ and $|P| < 2.5$, α flat in the range $[0,180^\circ]$

ES (Explicit solution) Flat distribution in the combination of parameters which
appear in the gaussian Likelihood.
This is the **Jeffreys Prior**, with added the condition that
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We observe that once the upper limits on $|P|$ is set, the upper limits on $|T|$ are completely irrelevant because they are automatically induced by the experimental BR

Experimental inputs (HFAG)

$B_{\pi\pi}^{+-}$ 5.2 ± 0.2	$B_{\pi\pi}^{00}$ 1.31 ± 0.21	$B_{\pi\pi}^{+0}$ 5.6 ± 0.4
$C_{\pi\pi}^{+-}$ -0.38 ± 0.7	$S_{\pi\pi}^{+-}$ -0.61 ± 0.8	$C_{\pi\pi}^{00}$ -0.49 ± 0.31

$B_{\rho\rho}^{+-}$ 23.1 ± 3.3	$B_{\rho\rho}^{00}$ 1.16 ± 0.46	$B_{\rho\rho}^{+0}$ 18.2 ± 3.0
$C_{\rho\rho}^{+-}$ -0.06 ± 0.13	$S_{\rho\rho}^{+-}$ -0.05 ± 0.17	$C_{\rho\rho}^{00}$ -0.39 ± 0.93

All B are in 10^6 units

(Efficient) Montecarlo Evaluation of the p.d.f.

- 1) Generate the 6 observables with the experimental likelihood
- 2) Solve for the sextuple of paramerts (event). You get 8 or 0 event.
- 3) Weight each event with the appropriate Jacobian in order to implement the choosen prior. For example, in the “ES No cut” case, simply $J=1$
- 4) Sum over the 8 event and go to step 1.

$$c = \frac{r B_{\pi\pi}^{+0} + B_{\pi\pi}^{+-} (1 + C_{\pi\pi}^{+-}) / 2 - B_{\pi\pi}^{00} (1 + C_{\pi\pi}^{00})}{\sqrt{2r B_{\pi\pi}^{+-} B_{\pi\pi}^{+0} (1 + C_{\pi\pi}^{+-})}} , \quad \bar{c} = \frac{r B_{\pi\pi}^{+0} + B_{\pi\pi}^{+-} (1 - C_{\pi\pi}^{+-}) / 2 - B_{\pi\pi}^{00} (1 - C_{\pi\pi}^{00})}{\sqrt{2r B_{\pi\pi}^{+-} B_{\pi\pi}^{+0} (1 - C_{\pi\pi}^{+-})}} , \quad s = \pm\sqrt{1 - c^2} , \quad \bar{s} = \pm\sqrt{1 - \bar{c}^2} , \quad r = \frac{\tau_{B^+}}{\tau_{B^0}}$$

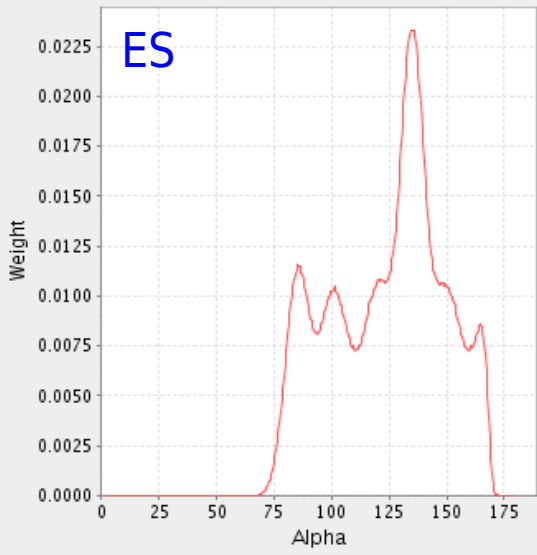
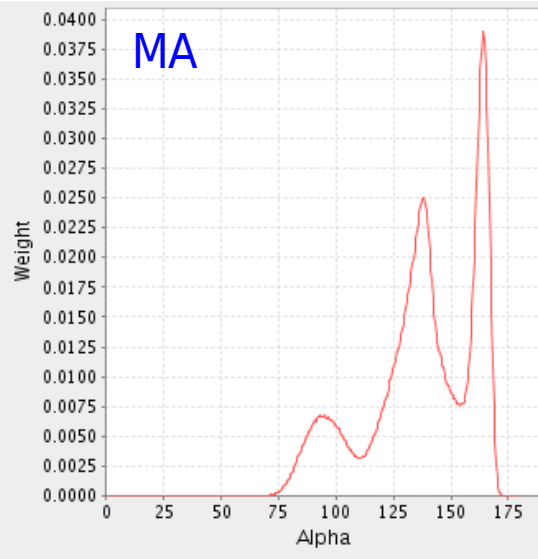
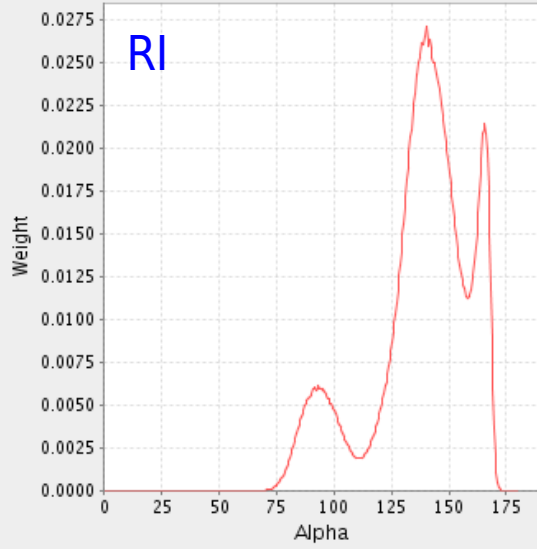
$$\sin(2\alpha_{\text{eff}}) = \frac{S_{\pi\pi}^{+-}}{\sqrt{1 - C_{\pi\pi}^{+-2}}} , \quad \cos(2\alpha_{\text{eff}}) = \pm\sqrt{1 - \sin^2(2\alpha_{\text{eff}})} , \quad \tan(\alpha) = \frac{\sin(2\alpha_{\text{eff}}) \bar{c} + \cos(2\alpha_{\text{eff}}) \bar{s} + s}{\cos(2\alpha_{\text{eff}}) \bar{c} - \sin(2\alpha_{\text{eff}}) \bar{s} + c}$$

$$|T^{+-}| = \left[\frac{B_{\pi\pi}^{+-}}{2 \sin^2(\alpha)} \left(1 \pm \sqrt{1 - C_{\pi\pi}^{+-2} - S_{\pi\pi}^{+-2}} \right) \right]^{\frac{1}{2}} , \quad |P| = \left[|T^{+-}|^2 (2 \cos^2(\alpha) - 1) + B_{\pi\pi}^{+-} \left(1 - \frac{S_{\pi\pi}^{+-}}{\tan(\alpha)} \right) \right]^{\frac{1}{2}}$$

$$|T^{00}| = \left[|P|^2 \cos(2\alpha) + 2B_{\pi\pi}^{00} \pm 2 \cos^2(\alpha) \sqrt{|P|^4 + \frac{|P|^2}{\cos^2(\alpha)} (2B_{\pi\pi}^{00} - |P|^2) - 4 \frac{B_{\pi\pi}^{002} C_{\pi\pi}^{002}}{\sin^2(2\alpha)}} \right]^{\frac{1}{2}} , \quad x_p = -\frac{|P|^2 + |T^{+-}|^2 - B_{\pi\pi}^{+-}}{\cos(\alpha)} , \quad y_p = -\frac{B_{\pi\pi}^{+-} C_{\pi\pi}^{+-}}{\sin(\alpha)}$$

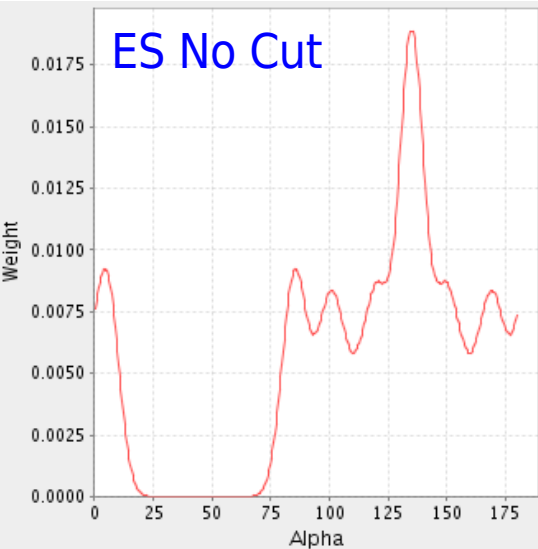
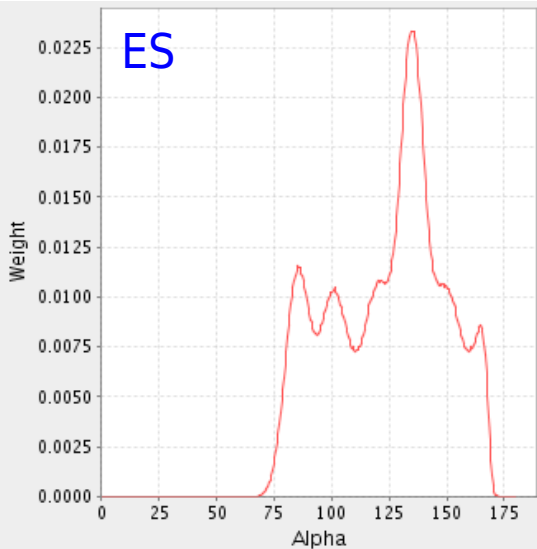
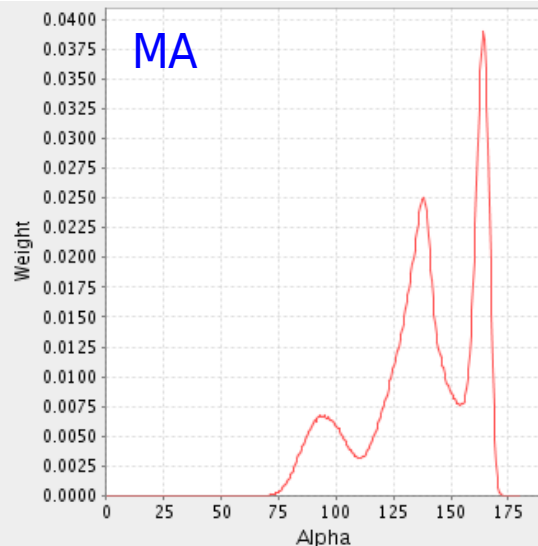
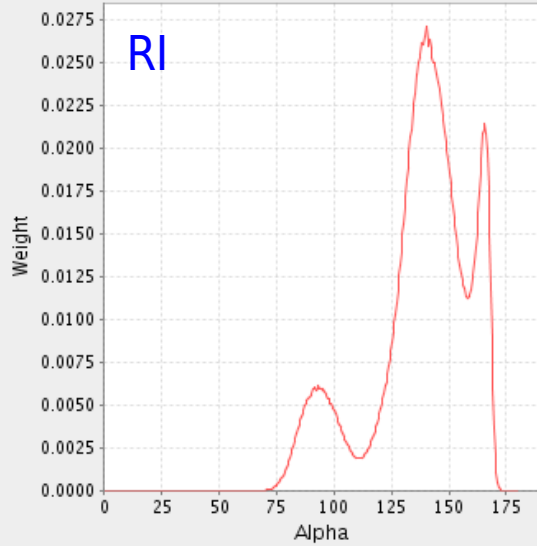
$$x_{00} = \frac{|P|^2 + |T^{00}|^2 - B_{\pi\pi}^{00}}{\cos(\alpha)} , \quad y_{00} = -\frac{B_{\pi\pi}^{00} C_{\pi\pi}^{00}}{\sin(\alpha)} , \quad \varphi_p = \text{Arg} \left(\frac{P}{T^{+-}} \right) = \arctan(x_p, y_p) , \quad \varphi_{00} = \text{Arg} \left(\frac{T^{00}}{T^{+-}} \right) = \arctan(x_{00}, y_{00})$$

$B \rightarrow \pi\pi$: results for α



- 1) In all cases the two regions $0 \leq \alpha \leq 75$ and $170 \leq \alpha \leq 180$ are excluded.
- 2) The shape in the allowed region is not significant and depends on the prior

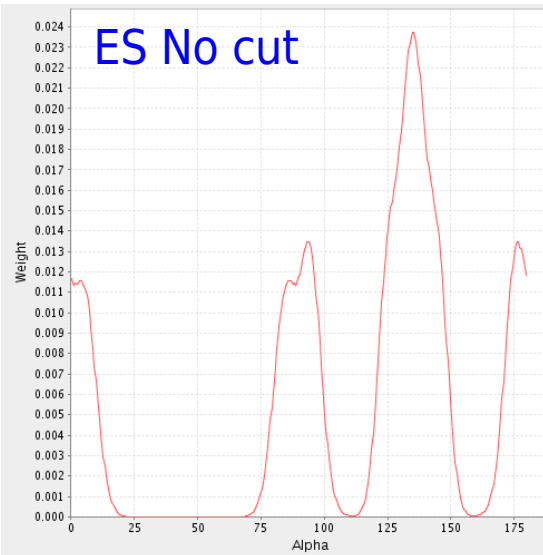
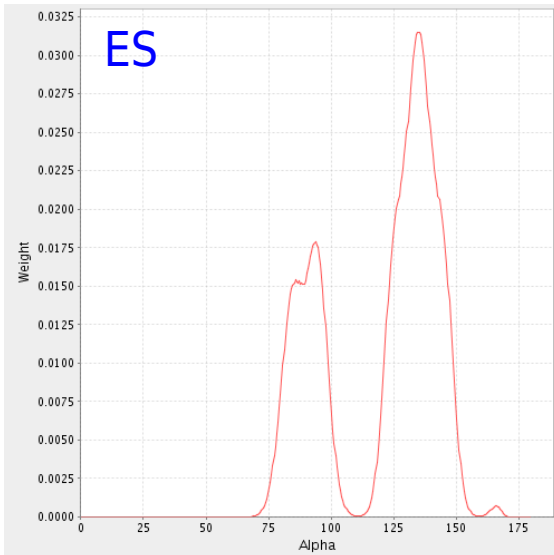
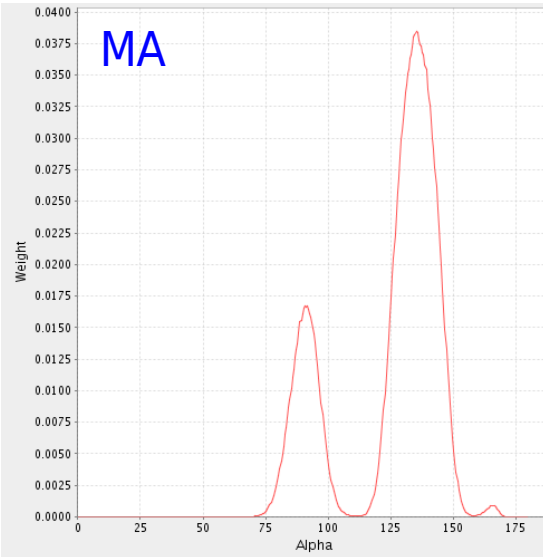
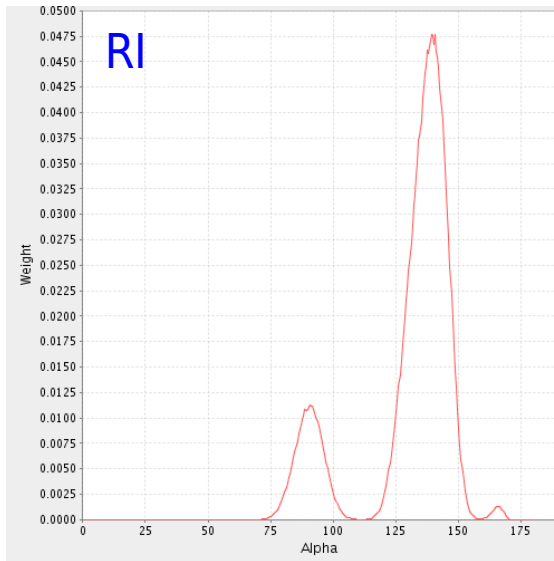
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- 1) In all cases the two regions $0 \leq \alpha \leq 75$ and $170 \leq \alpha \leq 180$ are excluded.
- 2) The shape in the allowed region is not significant and depends on the prior
- 3) The upper bound on the hadronic parameters play an important role in excluding $\alpha \approx 0$

Is this prior dependence an unavoidable feature of Bayesian approach ?

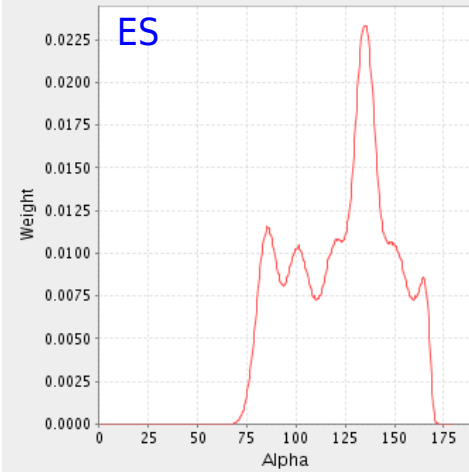
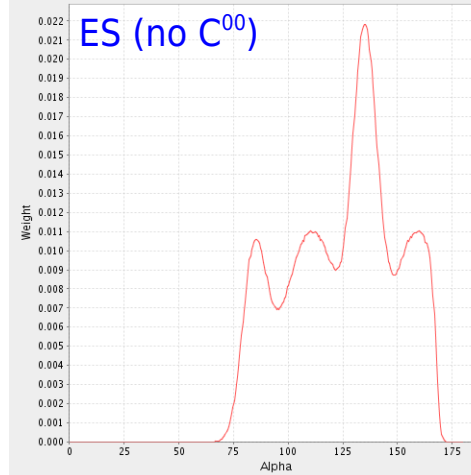
Try simply to reduce a factor 4 the error on C^{00} (so that the error is at the same level as the other asymmetries)



Increasing the experimental information decrease the dependence on the prior of the shapes.

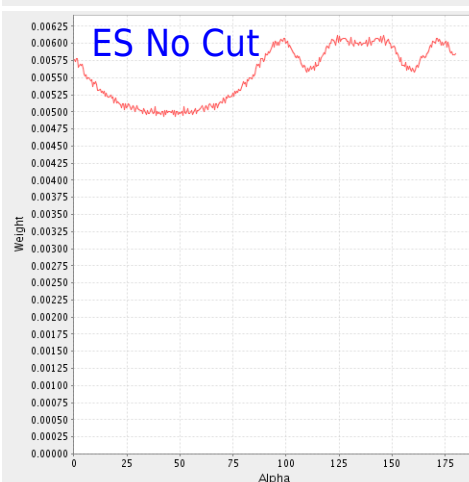
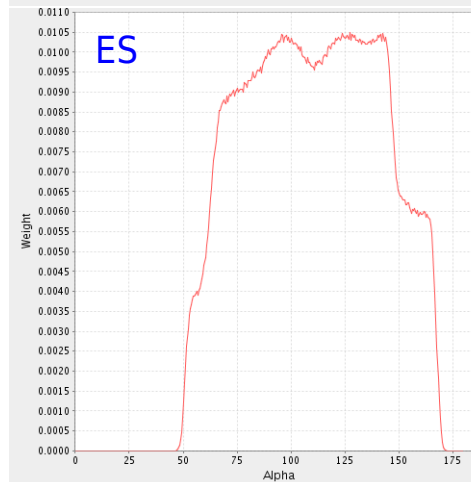
Try removing experimental information on C^{00} :

Given the present uncertainty, the measure of C^{00} does not play any role



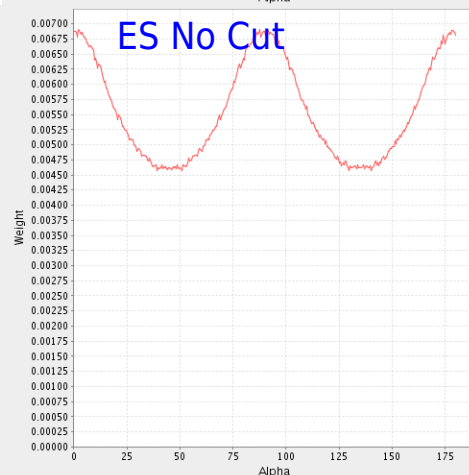
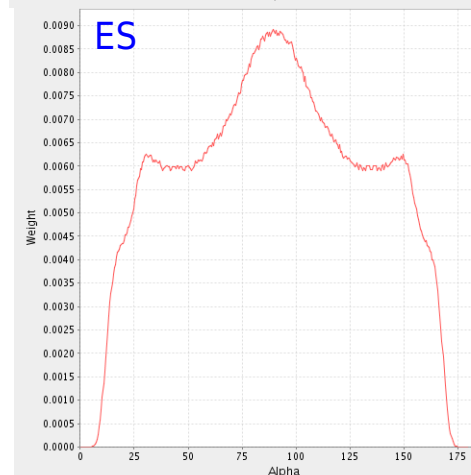
Try removing experimental information on B^{00} :

Without the cut on the hadronic parameter, we don't have any information on α

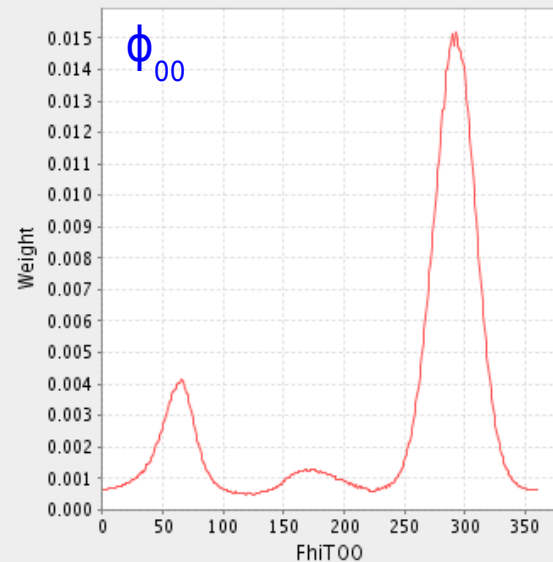
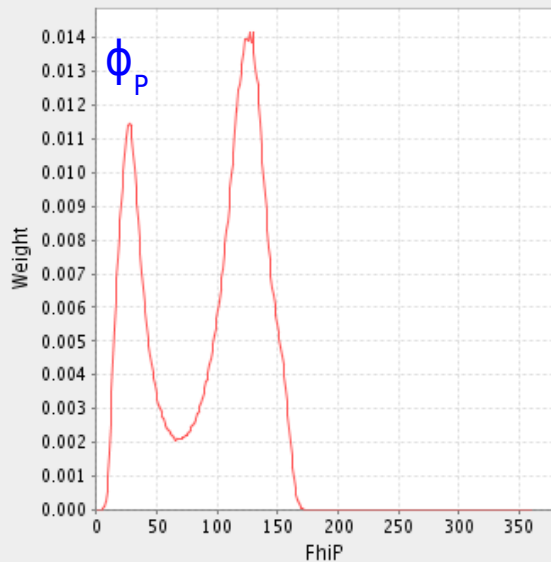
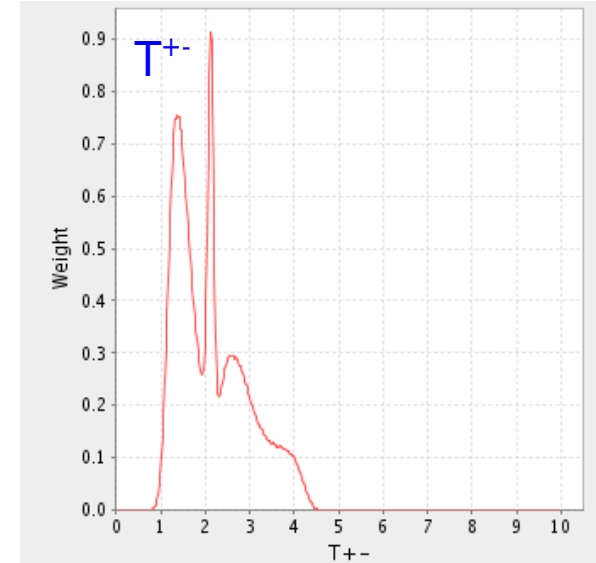
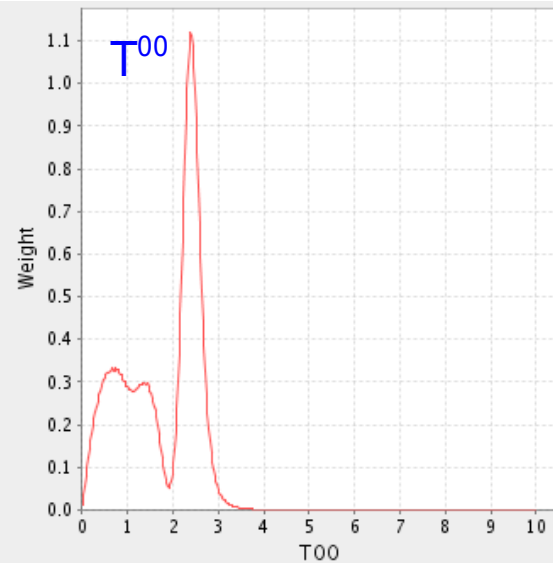
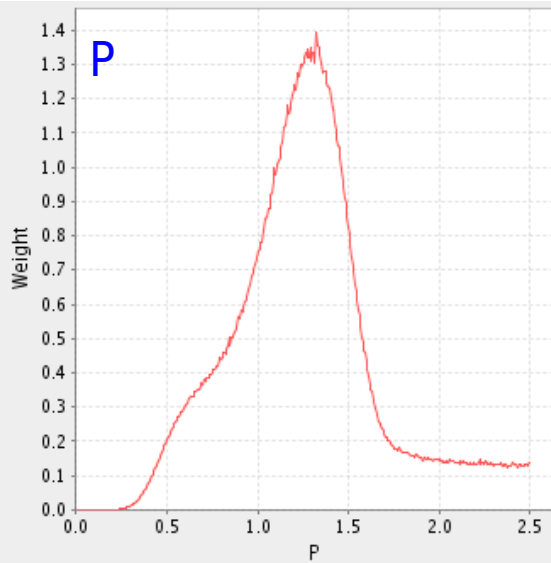


Try removing experimental information on S^{+-} :

The Result change completely, the cut on the hadronic parameter is less effective.



Results for the Hadronic Parameters (RI Prior)



- 1) Upper bound on T^{+-} and T^{00} in the prior is completely irrelevant
- 2) Data prefer a value of $P \approx 1.3$ a value 20% higher than SU(3) predictions.

Releasing SU(2) Assumption

We have not performed yet a detailed inference releasing this assumption.

There are 3 possible source of SU(2) violation:

1) EWP contribution.

The relation $A^{+0} = A^{+}/\sqrt{2} + A^{00}$ still hold, but now $|A^{+0}| \neq |\bar{A}^{+0}|$.

Estimating the impact of EWP (neglecting $Q_{7,8}$ and using SU(3), Neubert & Rosner '98)

gives: $\delta\alpha \approx 1^\circ - 2^\circ$

2) Mixing π^0 - η - η'

$$\pi^0 = |\pi_3\rangle + \varepsilon |\eta\rangle + \varepsilon' |\eta'\rangle ; \quad \varepsilon = 0.017 \pm 0.003 ; \quad \varepsilon' = 0.004 \pm 0.001$$

Using experimental upper bound on the amplitudes and varying the phases one gets

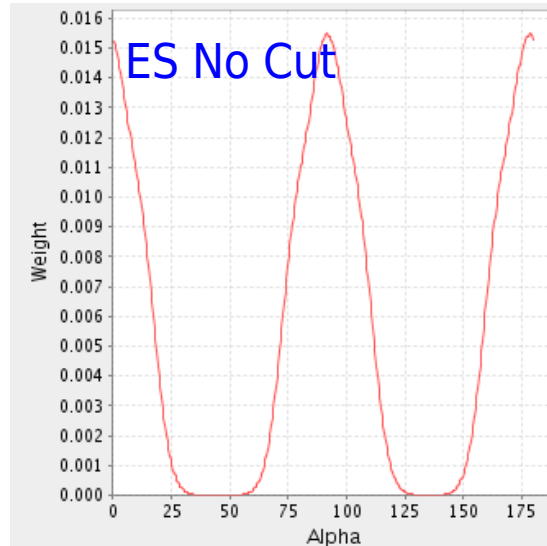
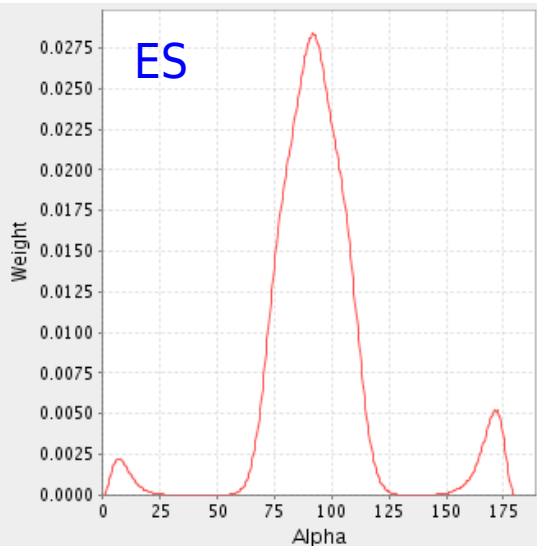
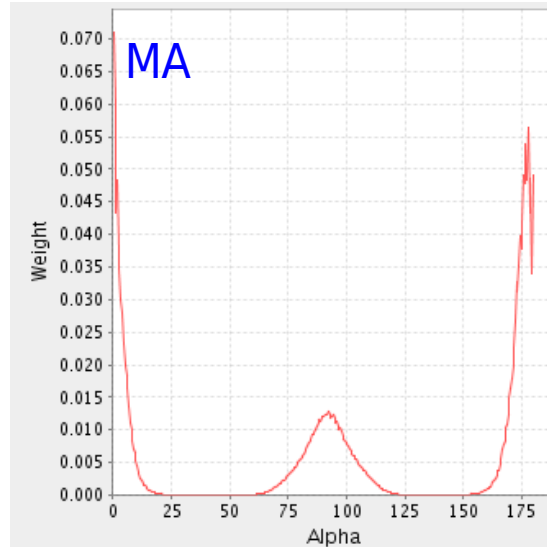
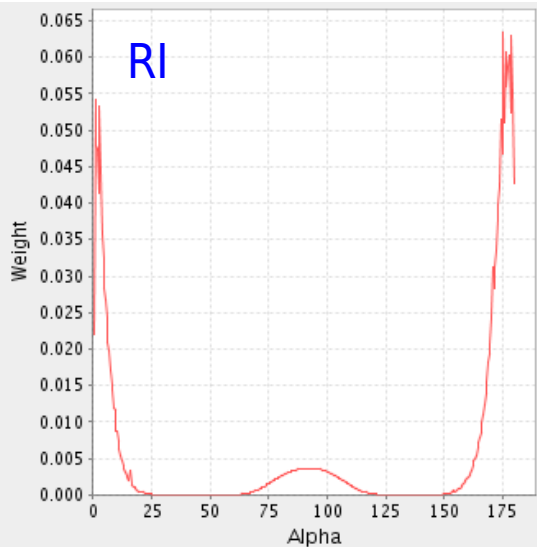
$$\delta\alpha \leq 1.6^\circ \quad (\text{J. Zupan CKM 2006})$$

3) SU(2) violation in the low-energy matrix-elements.

Topologies which differ for the exchange $u \leftrightarrow d$ can give contributions whose difference is $(m_u - m_d)/\Lambda_{\text{QCD}} \approx 1\%$ giving a similar impact on α .

The error due to SU(2) violation is few degrees

$B \rightarrow \rho\rho$



Three regions selected.
The relative heights is not significant because depend strongly on the prior.

$\alpha \approx 0$ and $\alpha \approx 180$ are not excluded because there is not a clear experimental signal of CP violation.

$B \rightarrow \rho \pi$

This is a **completely different** analysis:

The time-dependent Dalitz plot analysis of the decays of the neutral B allows one to infer the value of α without any dependence on the hadronic parameter.

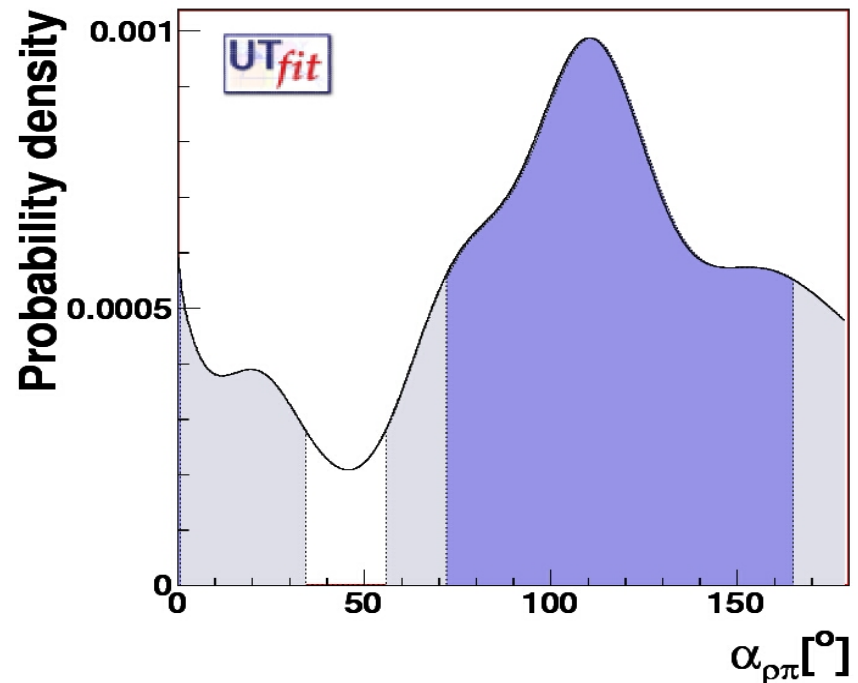
In fact if you consider the amplitude combination (assuming SU(2) exact symmetry)

$$A = A(B^0 \rightarrow \rho^+ \pi^-) + A(B^0 \rightarrow \rho^- \pi^+) + 2 A(B^0 \rightarrow \rho^0 \pi^0)$$

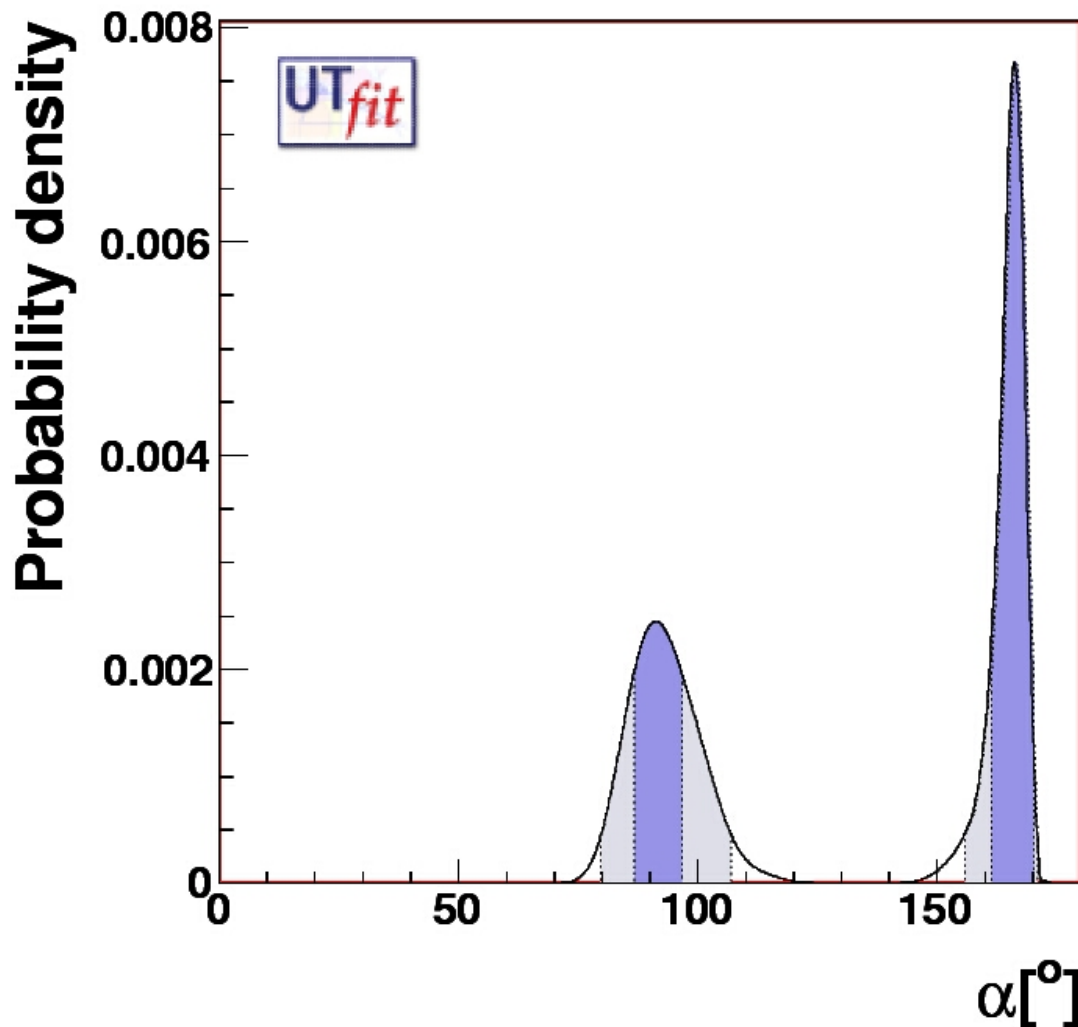
you can see that in the ratio

$$R = \frac{\bar{A}}{A} = e^{2i\alpha}$$

the hadronic parameters cancel out.



All Combined Together



Two region selected.

A clear measurement of the asymmetry in $B \rightarrow \rho\rho$ should reduce the region around 160° .

Conclusions

- 1) The Gronau London analysis applied to the current exp. data for $B \rightarrow \pi\pi$, clearly select the region $75 \lesssim \alpha \lesssim 170$.
Inside this region there are not preferred values.
Reducing the error on C^{00} strongly improve the inference.
- 2) The same analysis applied to $B \rightarrow \rho\rho$ exclude the two region $25 \lesssim \alpha \lesssim 60$ and $125 \lesssim \alpha \lesssim 150$
In this channel there is a stronger dependence on the prior because the experimental asymmetries have larger errors.
- 3) Neglecting Isospin Breaking and EWP operators introduce an error of 2~4 degrees on α . Not important in this moment, but deserving more investigation with improved experimental data.