RADIATIVE CORRECTIONS TO $K_{\ell 2}$ AND $K_{\ell 3}$ DECAYS

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Electromagnetic contributions in semileptonic weak decays

Low-energy effective field theory:

Chiral perturbation theory with virtual photons and leptons

dynamical degrees of freedom:

- **pseudoscalar octet ($\pi^\pm$, π^0, $K^\pm$, K^0, $\overline{K^0}$, η)**
- **photon (γ)**
- **light leptons ($e^\pm$, ν_e, $\mu^\pm$, ν_μ)**

Effective Lagrangian

LO ($L = 0$)

$\mathcal{L}_{p^2}$ F, B, e, G_F

$\mathcal{L}_{e^2p^0}$ Z $\longrightarrow \pi^+\pi^0$ mass difference

NLO ($L \leq 1$)

$\mathcal{L}_{p^4}$ $L_1, L_2, \ldots L_{12}$

$\mathcal{L}_{e^2p^2}$ $K_1, K_2, \ldots K_{14}$ **Urech 1995**

$\mathcal{L}_{\text{lept}}, \mathcal{L}_{\gamma}$ $X_1, X_2, \ldots X_8$ **Knecht, Neufeld, Rupertsberger, Talavera 2000**

NNLO ($L \leq 2$)

$\mathcal{L}_{p^6}$

$\mathcal{L}_{e^2p^4}$

Short distance enhancement

universal factor

$$1 - \frac{e^2}{2}(X_6^r - 4K_{12}^r) \equiv 1 - \frac{e^2}{2} X_6^{\rm phys}$$

appears in front of all semileptonic amplitudes (process independent)

$X_6^{\rm phys}$ **contains the large short-distance contribution**

$$e^2 X_6^{\rm phys}(M_\rho) = S_{\rm EW} - 1 + e^2 \tilde{X}_6^{\rm phys}(M_\rho)$$

$$S_{\rm EW} = 1 + \frac{2\alpha}{\pi} \log \frac{M_Z}{M_\rho} + \ldots = 1.0223 \pm 0.0005$$ **Sirlin 1978; 1982**

Determination of electromagnetic low-energy constants (LECs)

general approach:

isolate Green functions sensitive to specific LECs , then match to QCD

observations:

- **LECs sensitive to “heavy” degrees of freedom**
- $N_C \to \infty$: **Green functions determined by exchange of (stable) particles**
- **few-resonance approximation usually sufficient**

$\mathcal{L}_{e^2p^2}$ K_i **Ananthanarayan, Moussallam 2004**

$\mathcal{L}_{\text{lept}}$ X_i **Descotes-Genon, Moussallam 2005**

example:

determination of X_1 $\longrightarrow$ two-step matching procedure

SM $\longrightarrow$ Fermi theory $\longrightarrow$ CHPT **Descotes-Genon, Moussallam 2005**

representation for X_1

$$X_1 = \frac{3i}{8} \int \frac{d^4k}{(2\pi)^4} \left(\Gamma_{VV}(k^2) - \Gamma_{AA}(k^2)\right) / k^2$$

$$\Gamma_{VV}(k^2) \sim \lim_{p \to 0} \int d^4x e^{ikx} \langle 0|V_\mu^a(x) V_\nu^b(0)|\phi^c(p)\rangle \,, \quad V \to A : \Gamma_{AA}(k^2)$$

integral converges well $\longrightarrow$ saturate with lowest-lying V, A meson resonances

final result: $X_1 = -0.0037$

$P_{\ell 2(\gamma)} \quad (P = \pi, K)$

$$\Gamma_{P_{\ell 2(\gamma)}} = \Gamma^{(0)}_{P_{\ell 2}} \times S_{\rm EW} \times \left\{ 1 + \frac{\alpha}{\pi} F(m_\ell^2/M_P^2) \right\}$$
$$\times \left\{ 1 - \frac{\alpha}{\pi} \left[\frac{3}{2} \log \frac{M_\rho}{M_P} + c_1^{(P)} \right.\right.$$
$$+ \frac{m_\ell^2}{M_\rho^2} \left(c_2^{(P)} \log \frac{M_\rho^2}{m_\ell^2} + c_3^{(P)} + c_4^{(P)} (m_\ell/M_P) \right)$$
$$\left.\left. - \frac{M_P^2}{M_\rho^2} \tilde{c}_2^{(P)} \log \frac{M_\rho^2}{m_\ell^2} \right] \right\}$$

$$\Gamma^{(0)}_{P_{\ell 2}} = \frac{G_F^2 |V_P|^2 F_P^2}{4\pi} M_P \, m_\ell^2 \left(1 - \frac{m_\ell^2}{M_P^2} \right)^2, \qquad V_\pi = V_{ud}, \quad V_K = V_{us}$$

structure independent corrections Kinoshita 1959; Marciano, Sirlin 1993

$c_1^{(P)}$ at $\mathcal{O}(e^2p^2)$ Knecht, Neufeld, Rupertsberger, Talavera 2000

$$c_1^{(\pi)} = -4\pi^2 E^r(M_\rho) - \frac{1}{2} + \frac{Z}{4}\left(3 + 2\log\frac{M_\pi^2}{M_\rho^2} + \log\frac{M_K^2}{M_\rho^2}\right)$$

$$c_1^{(K)} = -4\pi^2 E^r(M_\rho) - \frac{1}{2} + \frac{Z}{4}\left(3 + 2\log\frac{M_K^2}{M_\rho^2} + \log\frac{M_\pi^2}{M_\rho^2}\right)$$

$$E^r = \frac{8}{3}K_1^r + \frac{8}{3}K_2^r + \frac{20}{9}K_5^r + \frac{20}{9}K_6^r - \frac{4}{3}X_1^r - 4X_2^r + 4X_3^r - \tilde{X}_6^{\mathrm{phys}}$$

$$\Rightarrow \quad c_1^{(K)} - c_1^{(\pi)} = \frac{Z}{4}\log\frac{M_K^2}{M_\pi^2}$$ independent of E^r

Determination of V_{us}/V_{ud}

$\Gamma_{K\ell 2(\gamma)}/\Gamma_{\pi\ell 2(\gamma)}$ **(exp.),** F_K/F_π **(lattice)** $\longrightarrow V_{us}/V_{ud}$ **Marciano 2004**

$\mathrm{BR}_{K_{\mu 2(\gamma)}}, \tau_{K^\pm}$ **FLAVIAnet Kaon Working Group 2008**

$\Gamma_{\pi_{\mu 2(\gamma)}}$ **PDG 2008**

$$\longrightarrow \quad \frac{V_{us}}{V_{ud}} \times \frac{F_K}{F_\pi} = 0.2760 \pm 0.0003_{\mathrm{exp}} \pm 0.0002_{\mathrm{EM}}$$

$F_K/F_\pi = 1.189 \pm 0.007$ **Follana et al. (HPQCD and UKQCD) 2008**

$$\longrightarrow \quad V_{us}/V_{ud} = 0.2321 \pm 0.0014_{\mathrm{lattice}} \pm 0.0002_{\mathrm{exp}} \pm 0.0001_{\mathrm{EM}}$$

$F_K/F_\pi = 1.205 \pm 0.065$ **Allton et al. (RBC and UKQCD) 2008**

$$\longrightarrow \quad V_{us}/V_{ud} = 0.2290 \pm 0.0124_{\mathrm{lattice}} \pm 0.0002_{\mathrm{exp}} \pm 0.0001_{\mathrm{EM}}$$

$c_2^{(P)}$, $c_3^{(P)}$, $c_4^{(P)}$, $\tilde{c}_2^{(P)}$ at $\mathcal{O}(e^2p^4)$ **Cirigliano, Rosell 2007**

$$R_{e/\mu}^{(P)} = \Gamma_{P_{e2(\gamma)}} / \Gamma_{P_{\mu2(\gamma)}} \qquad (P = \pi, K)$$

$V - A$ structure of charged currents $\longrightarrow$ $R_{e/\mu}^{(P)}$ helicity suppressed

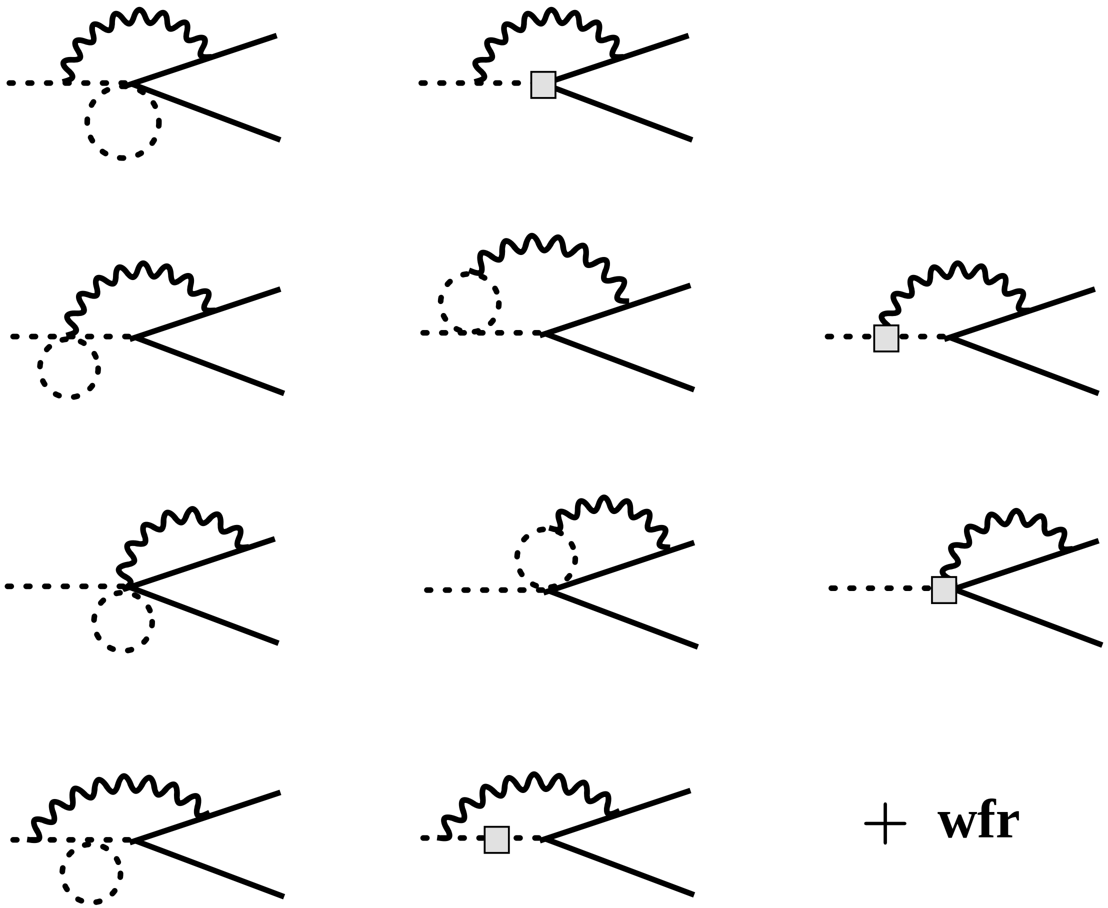
$\longrightarrow$ sensitive probe for new physics

(pseudoscalar currents, violation of lepton universality, . . .)

- **first systematic calculation to $\mathcal{O}(e^2p^4)$**
- **only diagrams with photon connected to lepton line contribute to ratio**
- **relevant counterterm determined by matching with large-N_c QCD**
- **inclusion of real photon corrections**
- **summation of leading logs $\alpha^n \log^n(m_\mu/m_e)$ (Marciano, Sirlin 1993)**

amplitudes of $\mathcal{O}(e^2p^4)$

Cirigliano, Rosell 2007



$+$ **wfr**

dashed lines: pseudoscalars, wavy lines: photons,
shaded squares: vertices from $\mathcal{L}_{p^4}$

	Cirigliano, Rosell	Marciano, Sirlin	Finkemeier
$R_{e/\mu}^{(\pi)} \cdot 10^4$	1.2352 ± 0.0001	1.2352 ± 0.0005	1.2354 ± 0.0002
$R_{e/\mu}^{(K)} \cdot 10^5$	2.477 ± 0.001		2.472 ± 0.001

experiment:

$R_{e/\mu}^{(\pi)} \cdot 10^4 = 1.230 \pm 0.004$ **PDG 2008**

$R_{e/\mu}^{(K)} \cdot 10^5 = 2.457 \pm 0.032$ **FLAVIAnet Kaon Working Group 2008**

- $R_{e/\mu}^{(\pi)}$ **confirmed with better precision**
- **discrepancy with previous calculation of** $R_{e/\mu}^{(K)}$

  main reason: form factors of Finkemeier incompatible with asymptotic behaviour of QCD

$K_{\ell 3}$ decays

$$\Gamma_{K_{\ell 3(\gamma)}} = \frac{C_K^2 G_F^2 M_K^5}{128\pi^3} |V_{us} f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)}(\lambda_i) S_{\mathrm{EW}} (1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi})$$

$$C_K = \begin{cases} 1 & \text{for } K_{e3}^0 \\ \frac{1}{\sqrt{2}} & \text{for } K_{e3}^+ \end{cases}$$

$$\delta_{\mathrm{EM}}^{K\ell} = \delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_3) + \delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_{4-3}), \qquad \delta_{\mathrm{SU(2)}}^{K\pi} = \left(\frac{f_+^{K\pi}(0)}{f_+^{K^0\pi^-}(0)}\right)^2 - 1$$

electromagnetic corrections for $K_{\ell 3}$ decay rates to $\mathcal{O}(e^2p^2)$

older analysis **Ginsberg 1967-1970**

general formulae within effective quantum field theory
Cirigliano, Knecht, Neufeld, Rupertsberger, Talavera 2002

numerics of EM corrections for K_{e3} **Cirigliano, Neufeld, Pichl 2004**

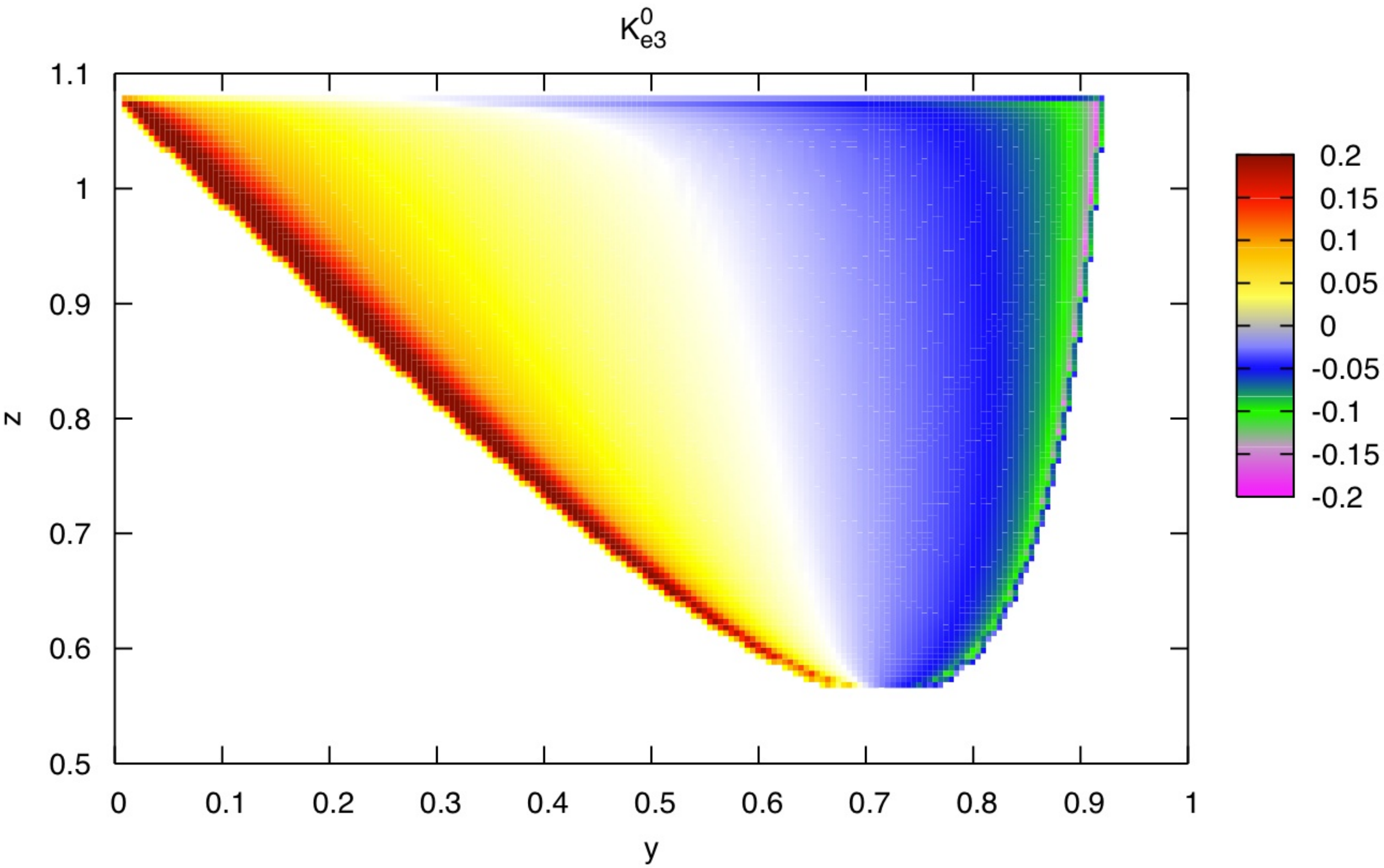
numerics of EM corrections for K_{e3} (update) and $K_{\mu 3}$ (new)
Cirigliano, Giannotti, Neufeld 2008

- **analysis at fixed chiral order $\mathcal{O}(e^2p^2)$**
- **fully inclusive prescription of real photon emission**
- **update of structure-dependent EM contributions (K_i^r, X_i^r from Ananthanarayan, Moussallam 2004; Descotes-Genon, Moussallam 2005)**

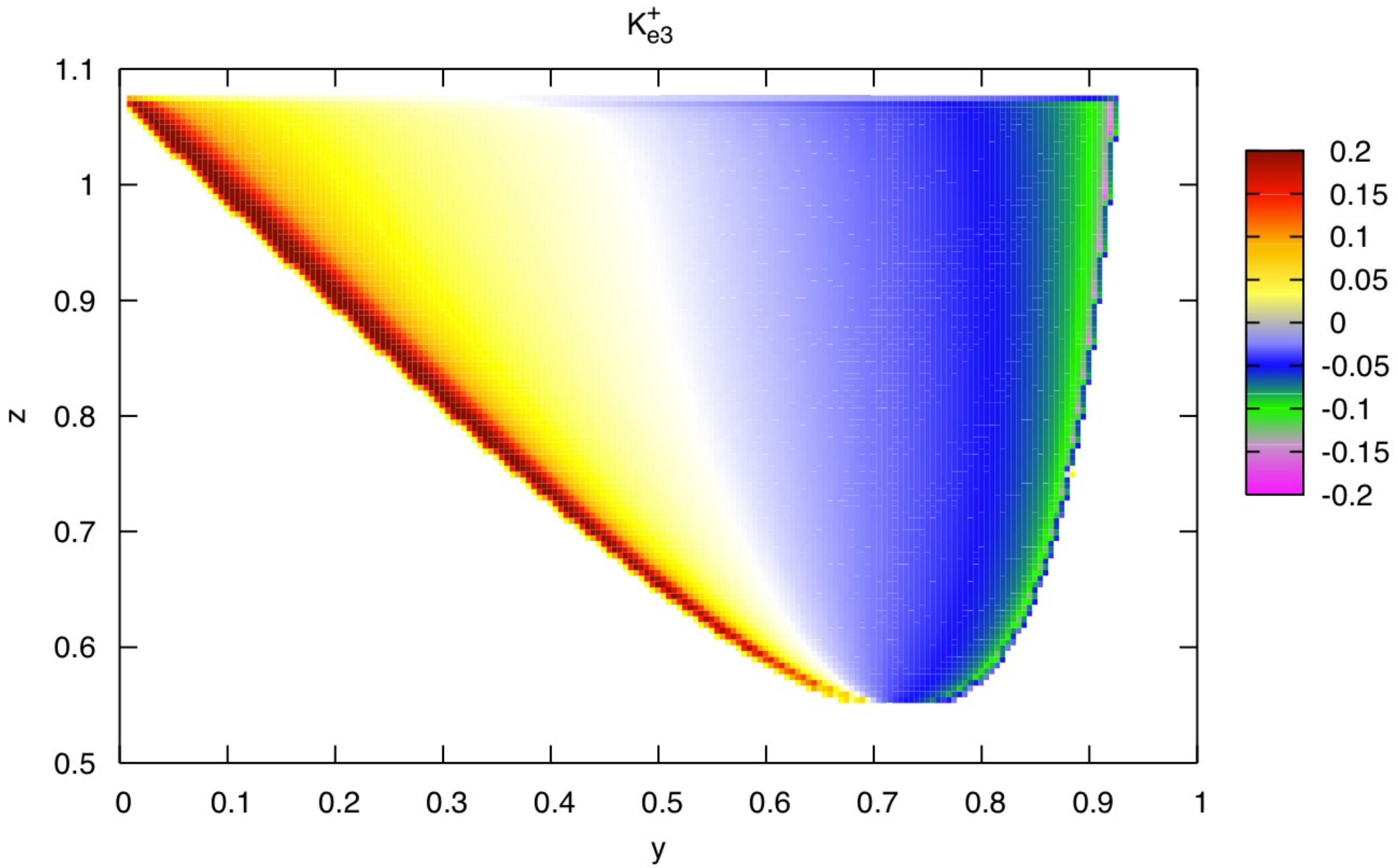
results **Cirigliano, Giannotti, Neufeld 2008**

	$I_{K\ell}^{(0)}(\lambda_i)$	$\delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_3)(\%)$	$\delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_{4-3})(\%)$	$\delta_{\mathrm{EM}}^{K\ell}(\%)$
K_{e3}^0	0.103070	0.50	0.49	0.99 ± 0.22
$K_{e3}^\pm$	0.105972	-0.35	0.45	0.10 ± 0.25
$K_{\mu 3}^0$	0.068467	1.38	0.02	1.40 ± 0.22
$K_{\mu 3}^\pm$	0.070324	0.007	0.009	0.016 ± 0.25

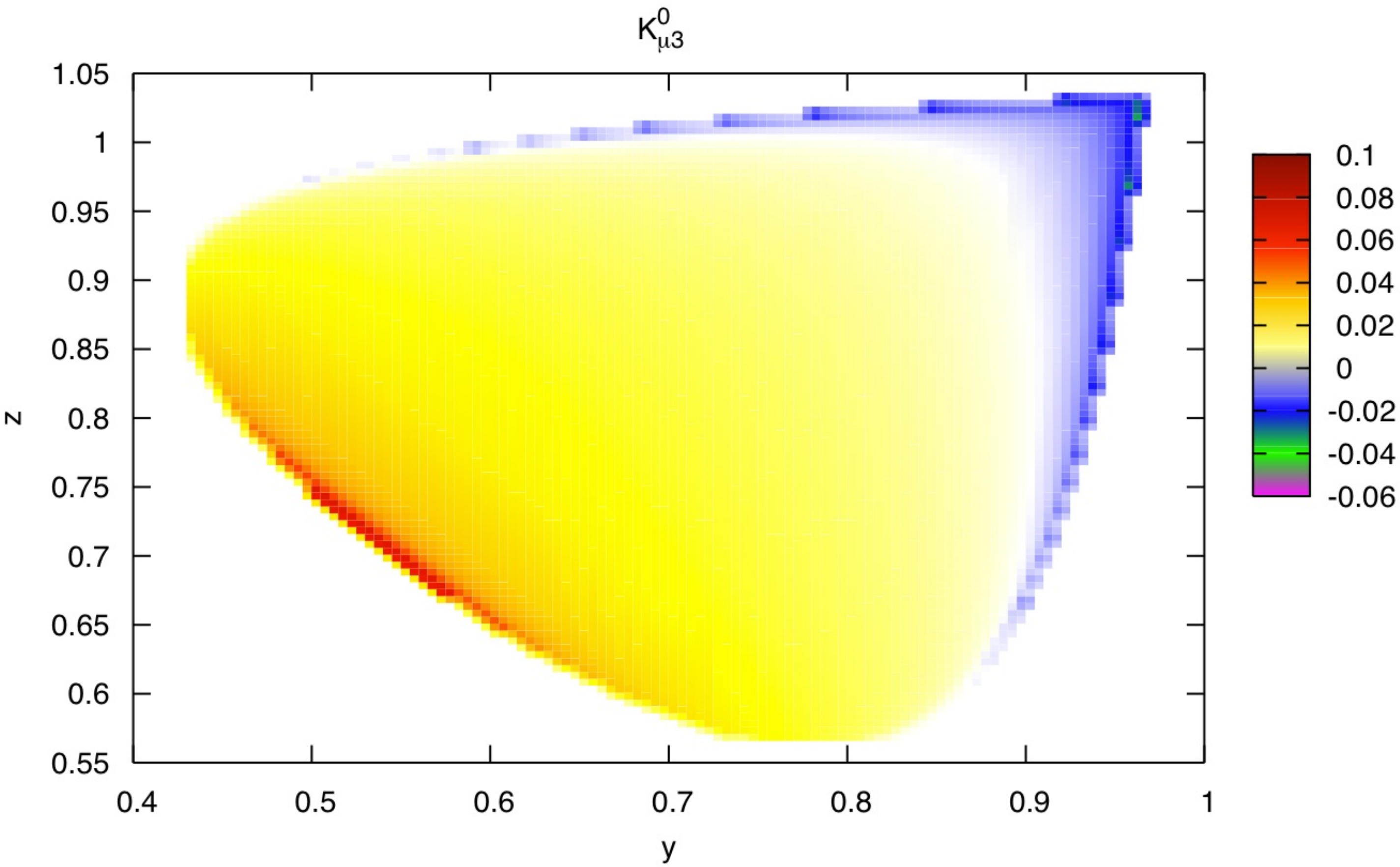
errors: estimates of higher-order contributions



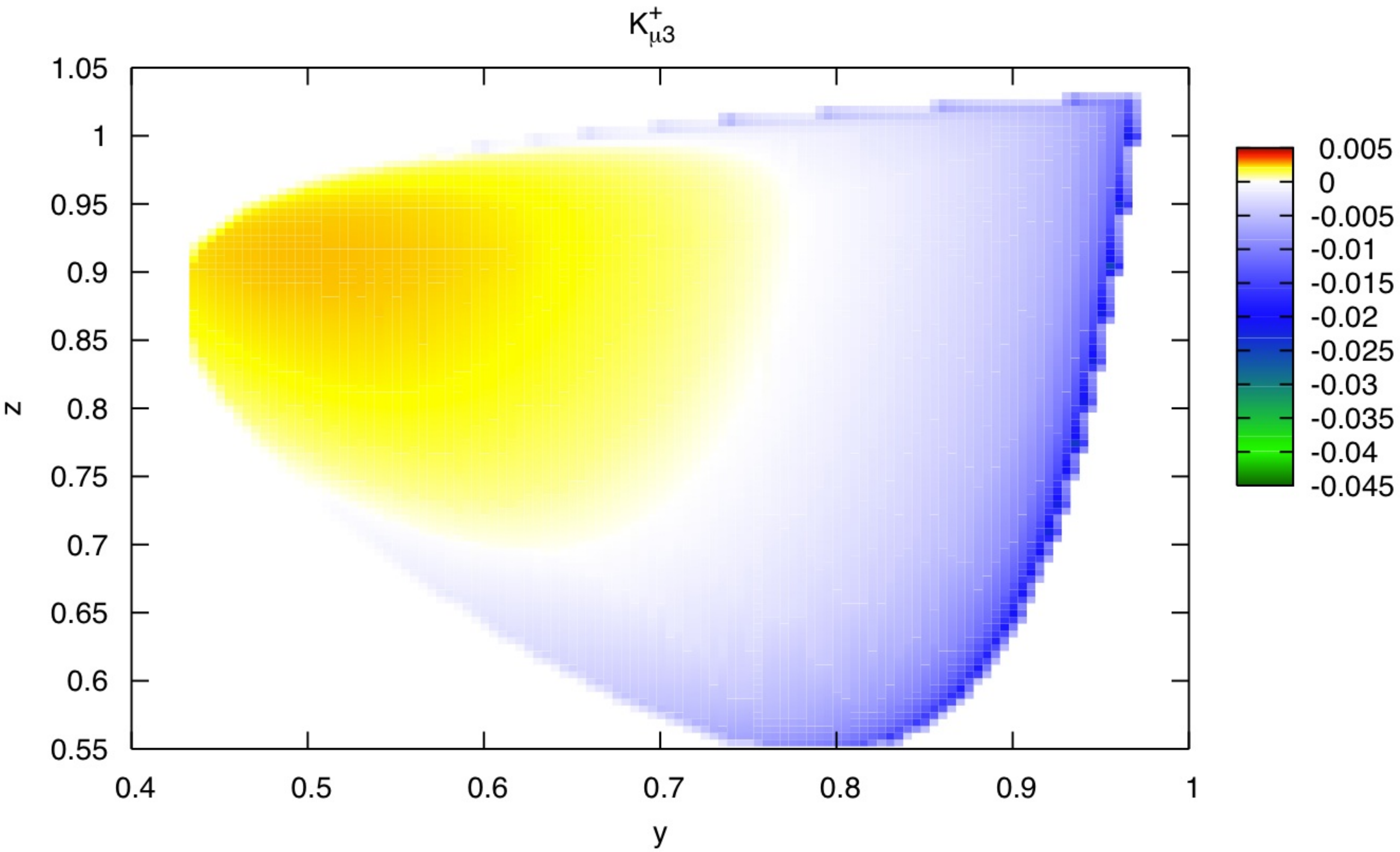
EM correction to differential distribution of K^0_{e3} $\quad (y = 2E_\ell/M_K\,,\ z = 2E_\pi/M_K)$



EM correction to differential distribution of K^+_{e3} $\quad (y = 2E_\ell/M_K\,,\ z = 2E_\pi/M_K)$



EM correction to differential distribution of $K^0_{\mu 3}$ $\quad (y = 2E_\ell/M_K\,,\ z = 2E_\pi/M_K)$



EM correction to differential distribution of $K^+_{\mu 3} \quad (y = 2E_\ell/M_K\,,\ z = 2E_\pi/M_K)$

Determination of $\delta^{K\pi}_{\mathrm{SU(2)}}$

$$\delta^{K\pi}_{\mathrm{SU(2)}} = \begin{cases} 0 & \text{for } K^0_{\ell 3} \\ 2\sqrt{3}\left(\varepsilon^{(2)} + \varepsilon^{(4)}_{\mathrm{S}} + \varepsilon^{(4)}_{\mathrm{EM}} + \ldots\right) & \text{for } K^+_{\ell 3} \end{cases}$$

$$\varepsilon^{(2)} = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \widehat{m}} \qquad \widehat{m} = \frac{m_u + m_d}{2}$$

$\longrightarrow$ **need determination of quark mass ratio**

$$R := \frac{m_s - \widehat{m}}{m_d - m_u}$$

double ratio

$$Q^2 := \frac{m_s^2 - \widehat{m}^2}{m_d^2 - m_u^2} = R\,\frac{m_s/\widehat{m} + 1}{2}$$

can be expressed in terms of meson masses and a purely EM contribution
Gasser, Leutwyler 1985

$$Q^2 = \frac{\Delta_{K\pi} M_K^2 \left(1 + \mathcal{O}(m_q^2)\right)}{M_\pi^2 \left[\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0} - (\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\rm EM}\right]}, \quad \Delta_{PQ} = M_P^2 - M_Q^2$$

$(\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\rm EM}$ **vanishes to lowest order** e^2p^0 **Dashen 1969**

$$(\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\rm EM} = e^2 M_K^2 \left[\frac{1}{4\pi^2}\left(3\ln\frac{M_K^2}{\mu^2} - 4 + 2\ln\frac{M_K^2}{\mu^2}\right) + \frac{4}{3}(K_5 + K_6)^r(\mu) - 8(K_{10} + K_{11})^r(\mu) + 16 Z L_5^r(\mu)\right] + \mathcal{O}(e^2 M_\pi^2)$$

Urech 1995; Neufeld, Rupertsberger 1995

Ananthanarayan, Moussallam 2004: large deviation from Dashen's limit

$$(\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\rm EM} = -1.5\,\Delta_{\pi^+\pi^0} \quad \longrightarrow \quad Q = 20.7 \pm 1.2$$

$Q = 22.7 \pm 0.8$ **Leutwyler 1996**

$Q = 22.0 \pm 0.6$ **Bijnens, Prades 1997**

$Q \simeq 20$ **Amoros, Bijnens, Talavera 2001**

however: $Q = 23.2$ **($\eta \to 3\pi$ at two loops)** **Bijnens, Ghorbani 2007**

determinations of second input parameter $m_s/\widehat{m} \sim 24$ rather stable

$$\left.\begin{array}{l} Q = 20.7 \pm 1.2 \\ m_s/\widehat{m} = 24.7 \pm 1.1 \end{array}\right\} \longrightarrow R = 33.5 \pm 4.3 \longrightarrow \delta_{\rm SU(2)} = 0.058(8)$$

Kastner, Neufeld 2008

$\delta_{\rm SU(2)} = 0.047(4)$ **used by FLAVIAnet Working Group 2008**

Summary

- CHPT suitable framework for EM corrections in semileptonic decays
- theoretical estimates for all electromagnetic LECs K_i^r, X_i^r
- first calculation of $R_{e/\mu}^{(\pi,K)}$ at $\mathcal{O}(e^2p^4)$ $\longrightarrow$ small uncertainties challenge for experiment
- EM corrections for all K_{l3} decay modes
- proper treatment of EM corrections mandatory in analysis of $K_{\ell 3}$ data
- (probably) large deviation from Dashen's limit $\longrightarrow$ influence on $\delta_{\mathrm{SU(2)}}^{K\pi}$