Status and prospects of inclusive b→ulν theory

Paolo Gambino
Università di Torino

After the latest lattice results on V_{ub} we feel a bit...

...under attack

Outline

- Inclusive decays, inputs, pert corrections
- Shape Function(s)
- High q^2 tail and WA
- Existing approaches to V_{ub}
- A trial comparison
- Conclusions

Many thanks to F.Di Lodovico, G.Ferrera, E.Gardi, P.Giordano

Inclusive semileptonic B decays: basic features

- ***Simple idea:*** inclusive decay do not depend on final state, factorize long distance dynamics of the meson. OPE allows to express it in terms of matrix elements of local operators
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in*** α_s***,*** Λ/m_b
- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle_\mu$$

The total s.l. width in the OPE

$$\Gamma[\bar{B} \to X_u e\bar{\nu}] \;=\; \frac{G_F^2\, m_b^5}{192\pi^3}|V_{ub}|^2 \left[1 + \frac{\alpha_s}{\pi} p_u^{(1)}(\mu) + \frac{\alpha_s^2}{\pi^2} p_u^{(2)}(r,\mu) - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2} \right.$$
$$\left. + \left(\frac{77}{6} + 8\ln\frac{\mu_{\rm WA}^2}{m_b^2}\right)\frac{\rho_D^3}{m_b^3} + \frac{3\rho_{LS}^3}{2m_b^3} + \frac{32\pi^2}{m_b^3} B_{\rm WA}(\mu_{\rm WA}) \right]$$

m_b dependence is up to twice stronger in the cut rate

OPE valid for inclusive enough measurements, away from perturbative singularities.

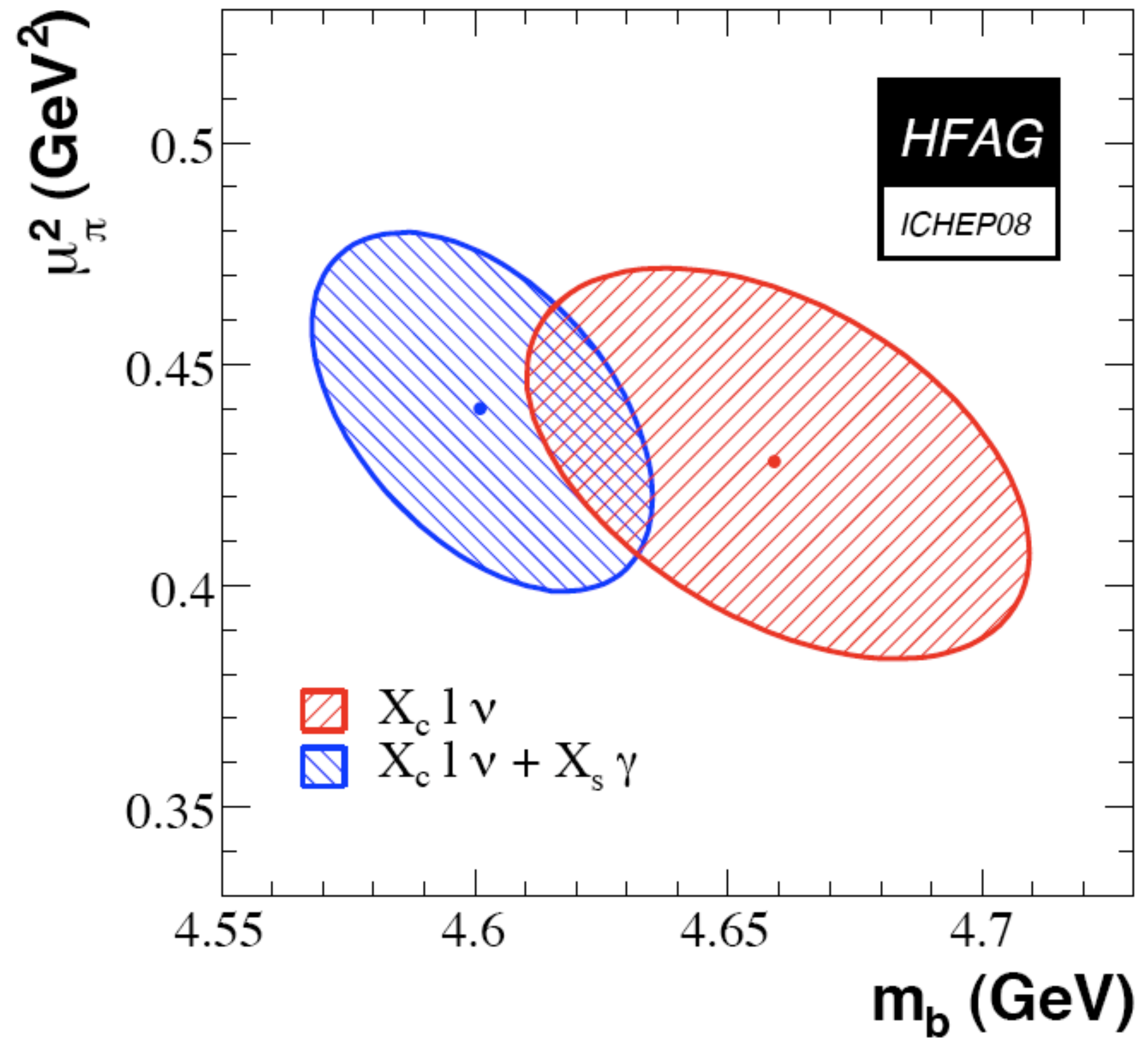
Most OPE parameters (quark masses etc) from sl decays into charm ➠talk by Schwanda

Global fit (kinetic scheme)

Inputs	$\|V_{cb}\|\ 10^3$	m_b^{kin}	χ^2/ndf
b→c & b→sγ	41.67(44)(58)	4.601(34)	29.7/57
b→c only	41.48(48)(58)	4.659(49)	24.1/46

NB here "scheme" means also a number of different assumptions and a recipe for theory errors

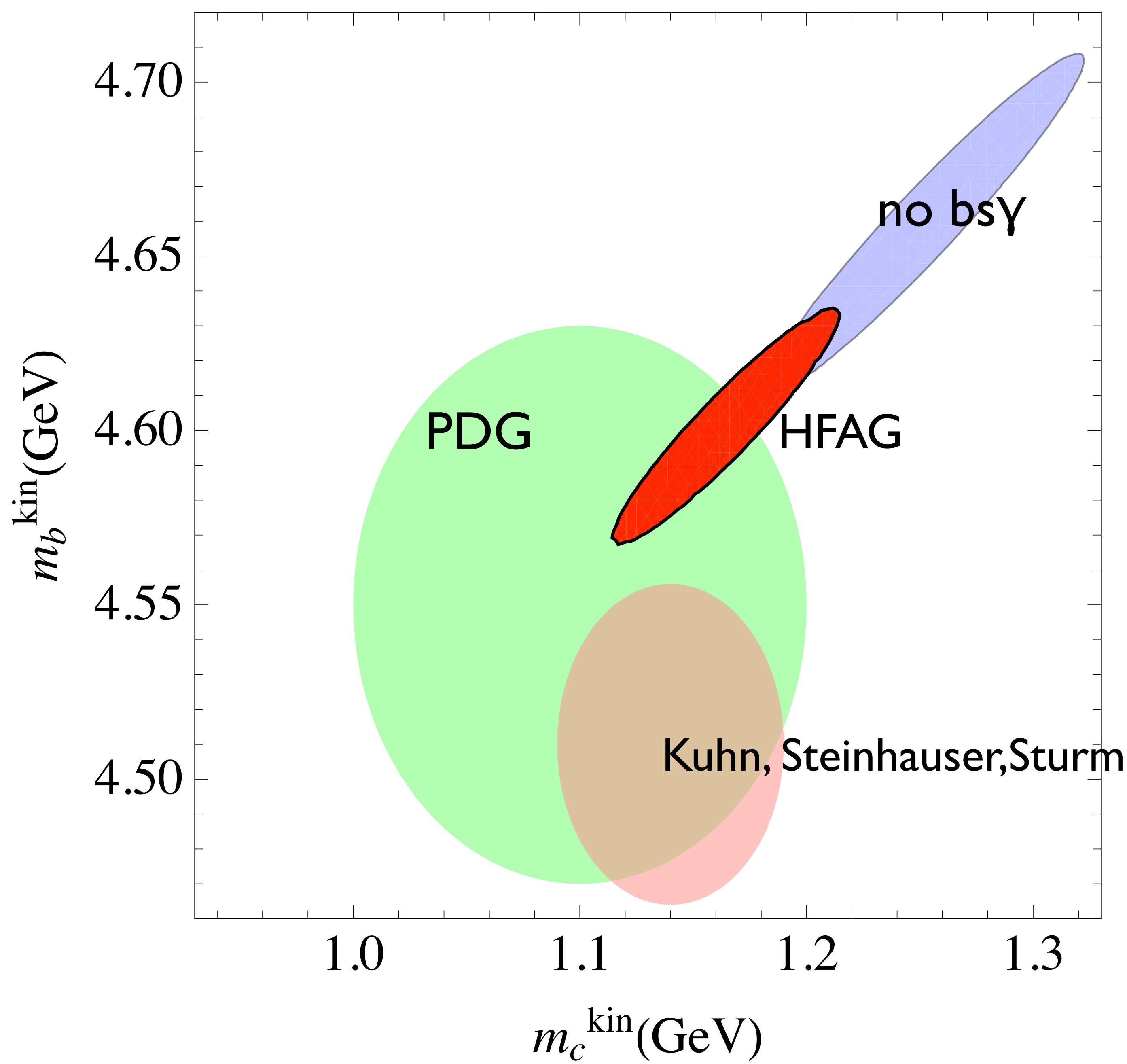
Experiment	Hadron moments	Lepton moments	Photon moments	References
BaBar	n=2 c=0.9,1.1,1.3,1.5 n=4	n=0 c=0.6,1.2,1.5 n=1 c=0.6,0.8,1.0,1.2,1.5	n=1 c=1.9,2.0 n=2 c=1.9 [3,4]	[1] arXiv:0707.2670 [2] Phys.Rev. D69 (2004) 111104
	c=0.8,1.0,1.2,1.4 [1]	n=2 c=0.6,1.0,1.5 n=3 c=0.8,1.2 [2]		[3] Phys.Rev. D72 (2005) 052004 [4] Phys. Rev. Lett. 97, 171803 (2006)
Belle	n=2 c=0.7,1.1,1.3,1.5 n=4 c=0.7,0.9,1.3 [5]	n=0 c=0.6,1.0,1.4 n=1 c=0.6,0.8,1.0,1.2,1.4 n=2 c=0.6,1.0,1.4 n=3 c=0.8,1.0, 1.2 [6]	n=1 c=1.8,1.9 n=2 c=1.8,2.0 [7]	[5] Phys.Rev. D75 (2007) 032005 [6] Phys.Rev. D75 (2007) 032001 [7] arXiv:0804.1580
CDF	n=2 c=0.7 n=4 c=0.7 [8]	.	.	[8] Phys.Rev. D71 (2005) 051103
CLEO	n=2 c=1.0,1.5 n=4 c=1.0,1.5 [9]	.	n=1 c=2.0 [10]	[9] Phys.Rev. D70 (2004) 032002 [10] Phys.Rev.Lett. 87 (2001) 251807
DELPHI	n=2 c=0.0 n=4 c=0.0 [11]	n=1 c=0.0 n=2 c=0.0 n=3 c=0.0 [11]	.	[11] Eur.Phys.J. C45 (2006) 35-59



C.Schwanda for HFAG   New: Belle bsg moments

Fits & Quark Masses

- Assumes duality but it self-consistently checks it
- Very close result for $|V_{cb}|$ in 1S scheme (Bauer et al)
- **Higher order** power corr. under control Mannel et al
- new pert $O(\alpha_s^2)$ $\Rightarrow$ **-0.5% in $|V_{cb}|$** Melnikov, Czarnecki, Pak
- part of $O(\alpha_s/m_b^2)$ Becher et al
- **Fitted $|V_{cb}|$ stable, not so the masses**
- In the global HFAG fit the $B\to X_s\gamma$ moments **change significantly** $\mathbf{m_{b,c}}$ determination. Without radiative moments the masses are too high!



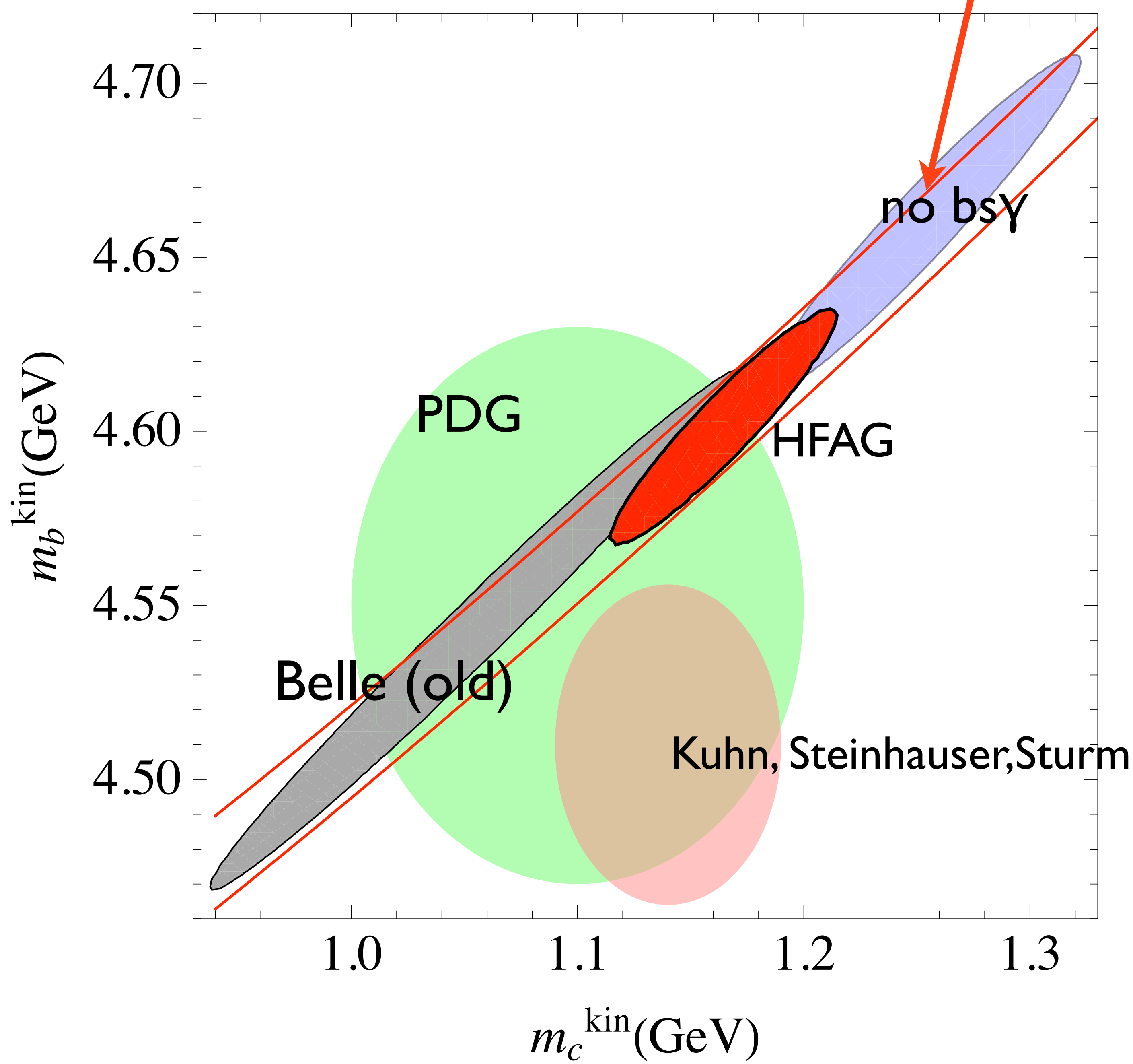
Fits & Quark Masses

OPE fails for bsg, but only at $O(\alpha_s \Lambda/m_b)$ with operators $\neq O_7$. I doubt these contributions can be relevant to normalized moments, but it must be studied

At the moment the role of radiative moments in the fits is similar to using PDG bound on m_b.

Inclusion of additional constraints? see next talk

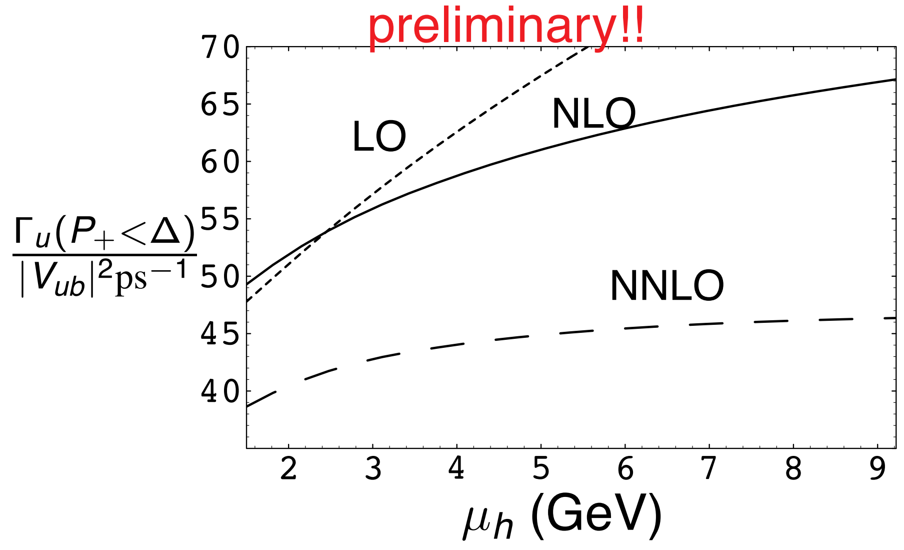
Constant values of s.l. width at fixed V_{cb}



Upgrade of moments analysis under way

Perturbative calculations

Partial rate for $P_+ < \Delta = M_D^2/M_B$



- NNLO result is smaller and less dependent on μ_h than NLO
- would lead to higher $|V_{ub}|$ compared to NLO (preliminary)

Some of the shift is due to different S at LO, NLO, NNLO

Ben Pecjak, ICHEP08

Complete $O(\alpha_s)$ implemented by all groups De Fazio-Neubert

Running coupling NNLO $O(\alpha_s^2\beta_0)$ in GGOU & DGE lead to -5% & +2%, resp. in $|V_{ub}|$
Gardi,Ridolfi, PG

NEW
Asatrian,Greub,Neubert,Pecjak in SCET-HQET corresponds to fixed order $O(\alpha_s^2)$ in the SF region

The problems with cuts

$|V_{ub}|$ from total BR($b\to ul\nu$) like incl $|V_{cb}|$ but we need kinematic cuts to avoid the ~100x larger $b\to cl\nu$ background:

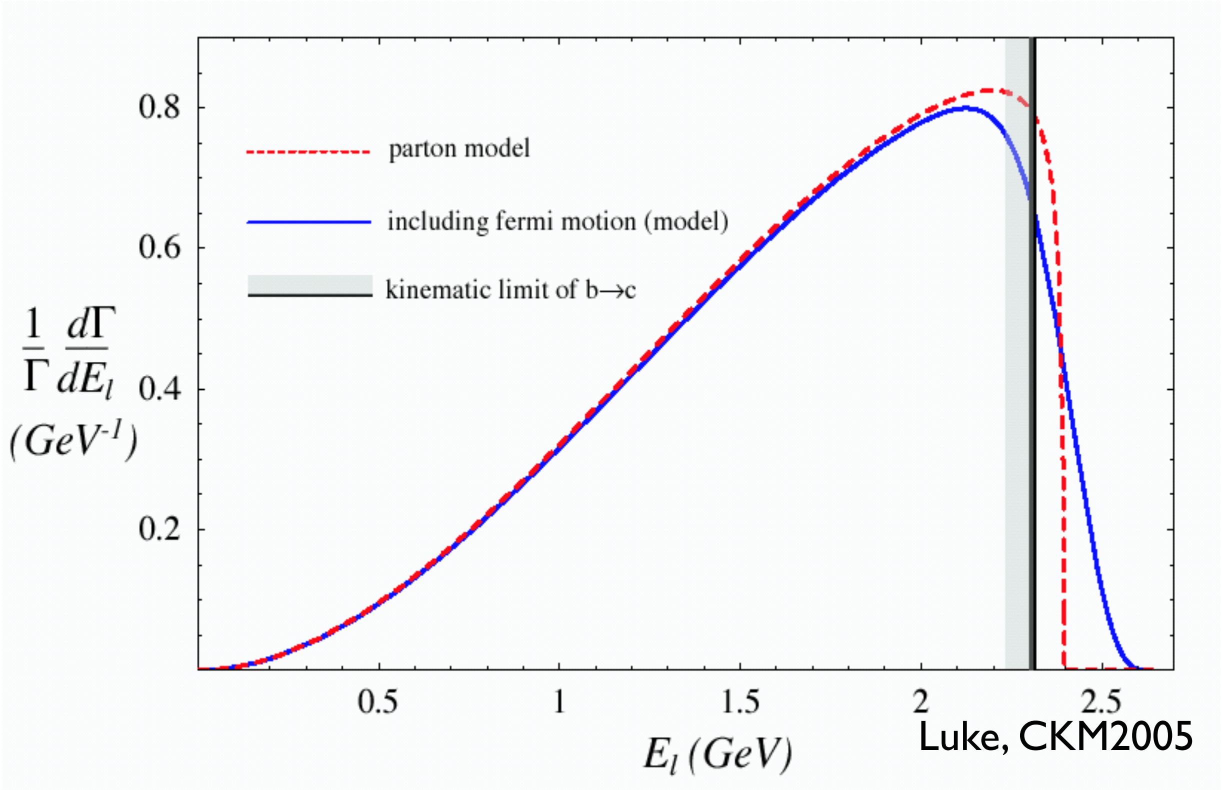
$m_X < M_D$ $\quad E_l > (M_B^2 - M_D^2)/2M_B$ $\quad q^2 > (M_B - M_D)^2$ ...

or combined (m_X, q^2) cuts

The cuts destroy convergence of the OPE that works so well in $b\to c$.

OPE expected to work only away from pert singularities

Rate becomes sensitive to "local" b-quark wave function properties like Fermi motion   Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** $f(k+)$

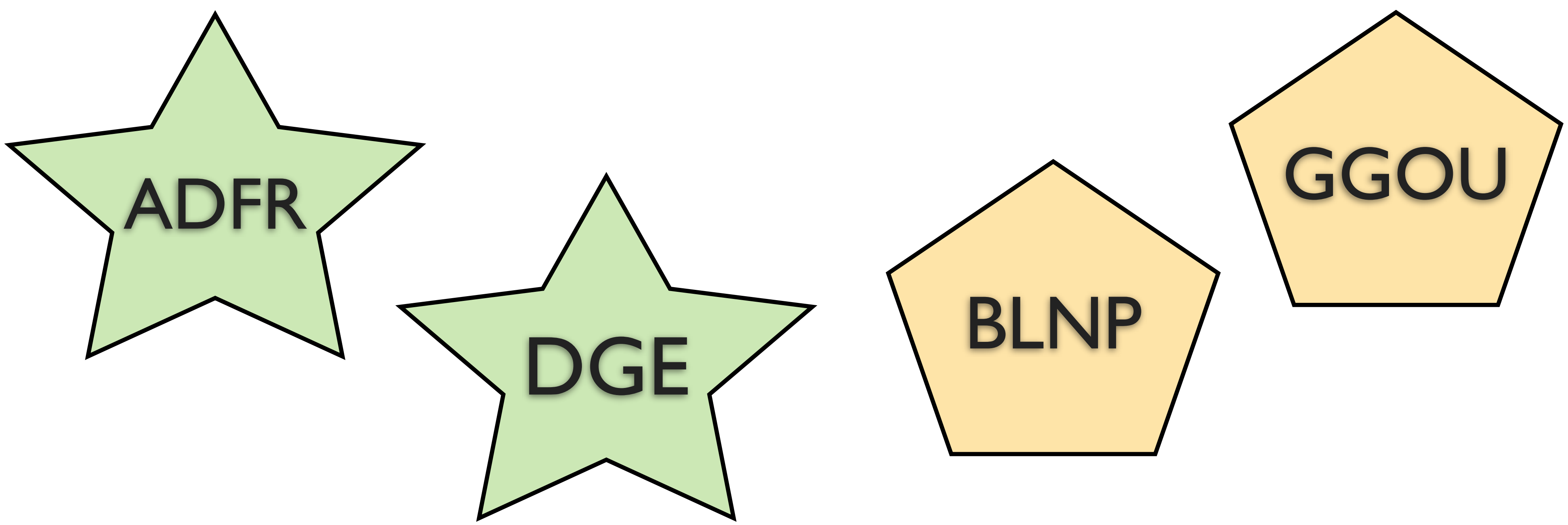


Luke, CKM2005

How to access the SF?

How to access the SF?

Prediction *based* on resummed pQCD	OPE constraints + parameterization



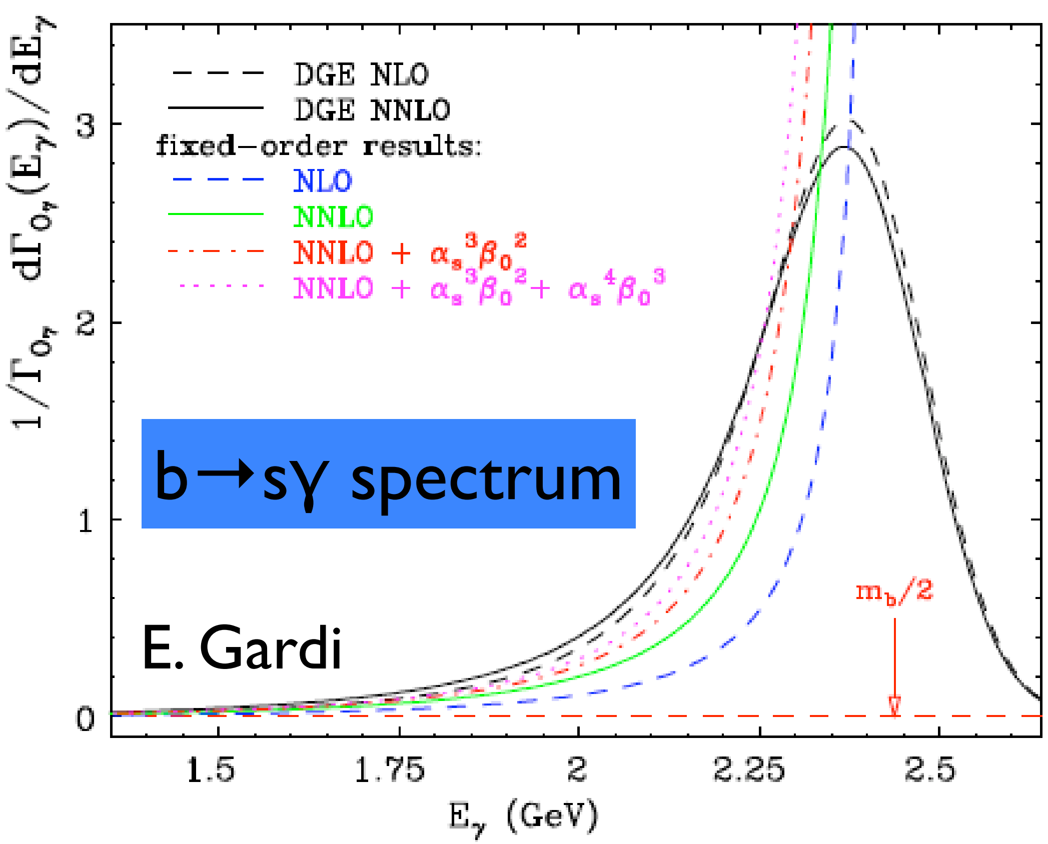
SF from perturbation theory

Resummed perturbation theory is qualitatively different: Support properties; stability! *(E. Gardi)*

b *quark* SF emerges from resummed pQCD but needs an IR prescription and power corrections for b →B

Dress Gluon Exponentiation (DGE) by Gardi et al employs renormalon resummation to define Fermi motion. Power corrections can be partly accomodated.

Aglietti et al (ADFR) use Analytic Coupling in the IR



The SF in the OPE

Local OPE has also threshold singularities and SF can be equivalently introduced resumming dominant singularities Bigi et al, Neubert

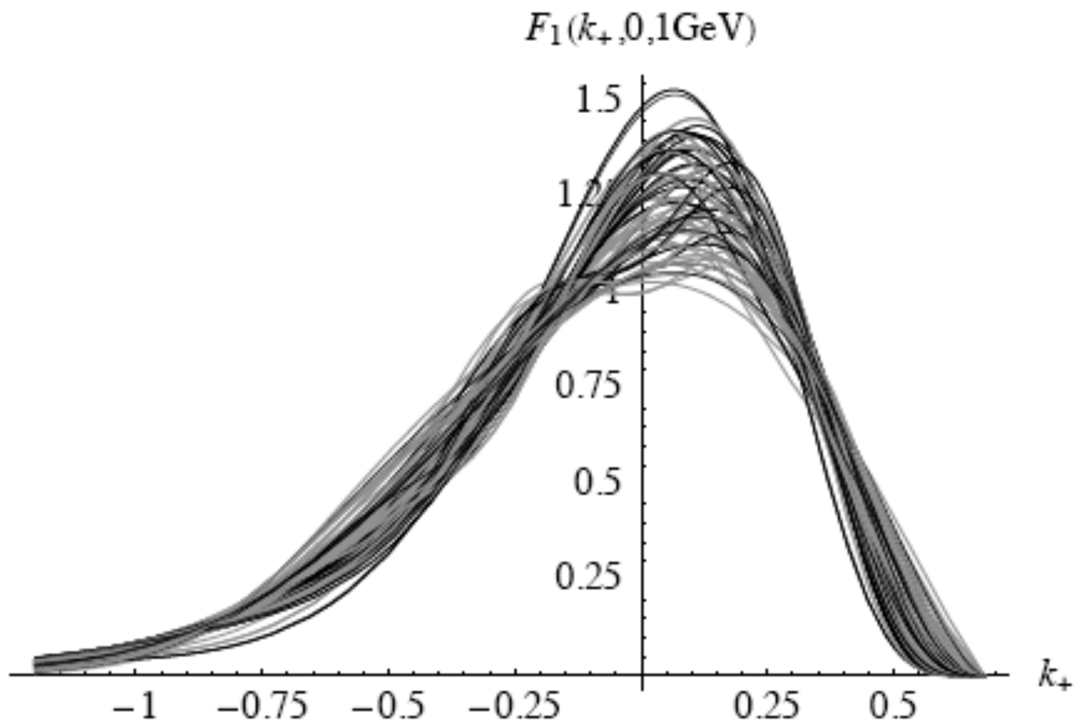
$$\frac{d\Gamma}{dE_\gamma} \propto \delta\left(E_\gamma - \frac{m_b}{2}\right) + \frac{\Lambda^2}{m_b^2}\left[\delta'\left(E_\gamma - \frac{m_b}{2}\right), \delta''\left(E_\gamma - \frac{m_b}{2}\right)\right] + \ldots$$

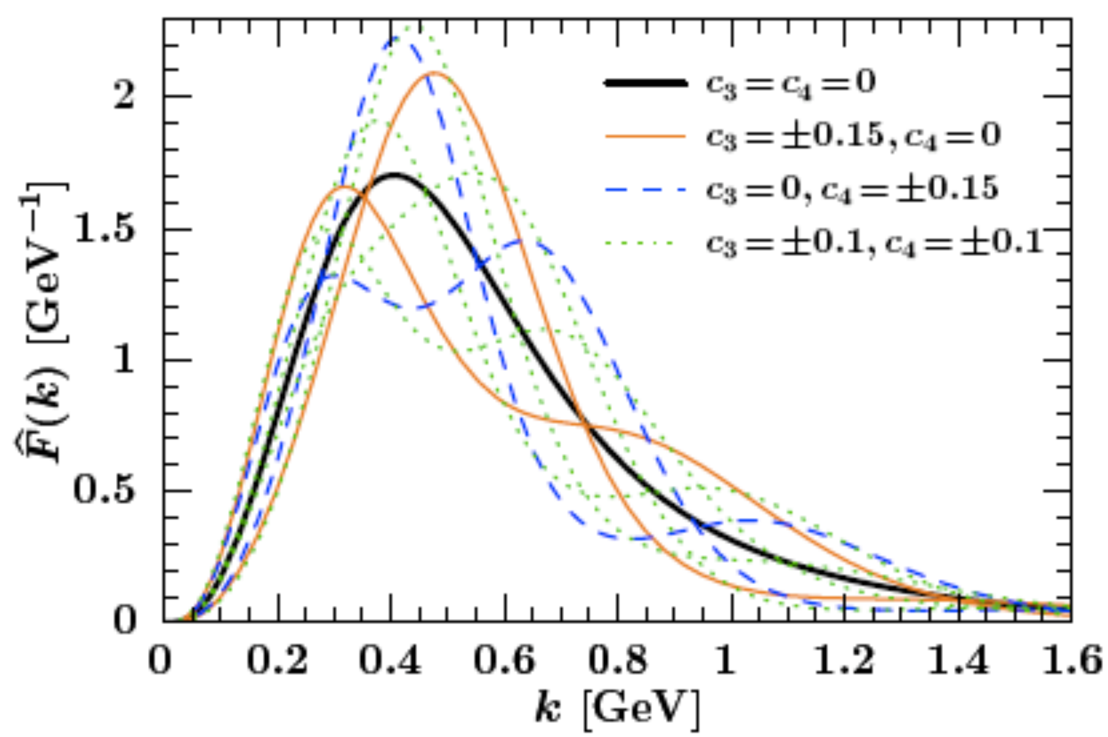
Fermi motion can be parameterized within the OPE like PDFs in DIS. At leading order in m_b only a single universal function of one parameter enters (SF).

Unlike resummed pQCD, ***the OPE does not predict the SF****, only its first few moments. One then needs an ansatz for its* ***functional form****.*

$$\int dk_+ \, k_+^n \, F_i(k_+, q^2) =$$ local OPE prediction ⇐ moments fits

Functional forms





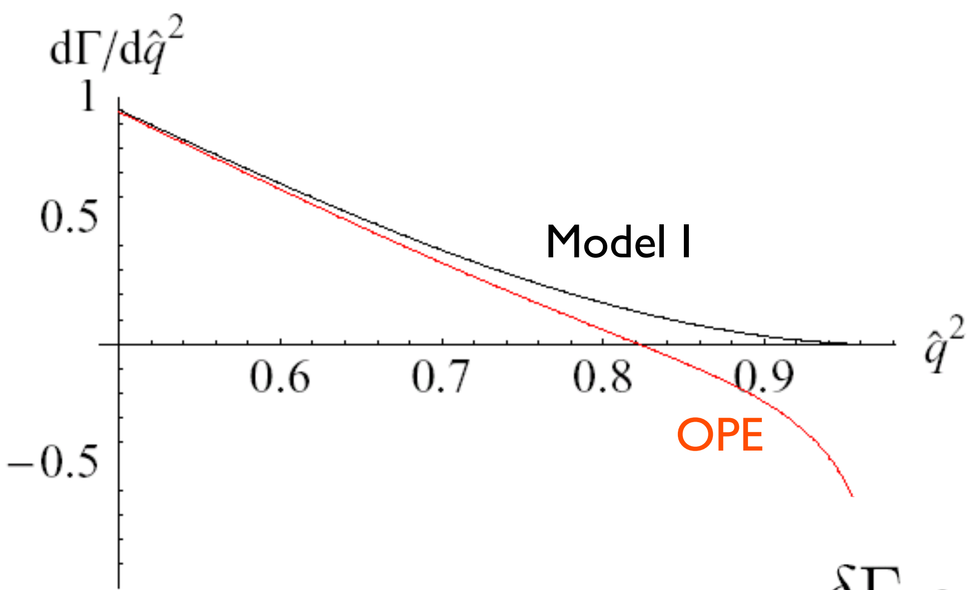
About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty (1-2%) on V_{ub}

Recent more systematic method by Ligeti et al. arXiv:0807.1926 Plot shows 9 SFs that satisfy all the first three moments

The high q^2 tail

At high q^2 higher dimensional operators are not suppressed leading to pathological features. Origin in the non-analytic square root

$$\frac{d\Gamma}{dq_0\, dq^2} \propto \sqrt{q_0^2 - q^2} \quad \Rightarrow \quad \frac{d\Gamma}{dq^2} \sim -\sum_{n=1}^{\infty} \frac{(-1)^n\, b_n(\hat{q}^2)}{(1-\hat{q}^2)^{n-2}} \left(\frac{\bar{\Lambda}}{m_b}\right)^n$$

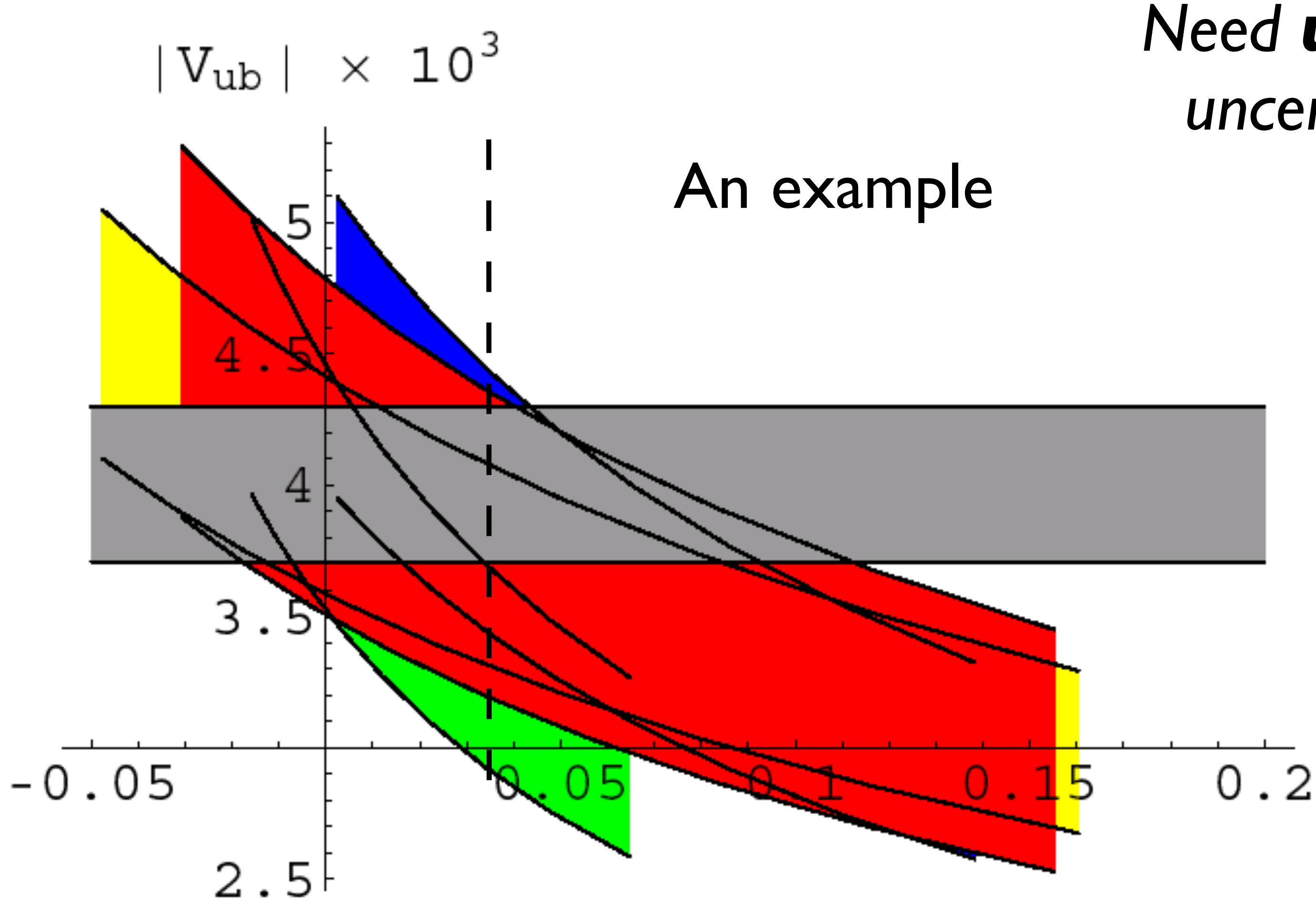


In the integrated rate the $1/m_b^3$ singularity is removed by the WA operator: needs modelling for q^2 spectrum

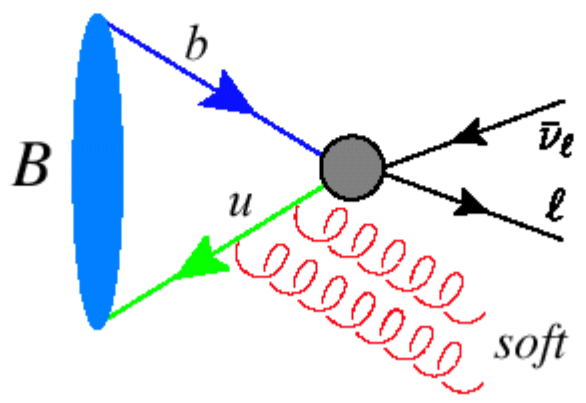
$$\delta\Gamma \sim \left[C_{\rm WA}\, B_{\rm WA}(\mu_{\rm WA}) - \left(8 \ln \frac{m_b^2}{\mu_{\rm WA}^2} - \frac{77}{6}\right) \frac{\rho_D^3}{m_b^3} + \mathcal{O}(\alpha_s) \right]$$

WA matrix element B_{WA} parameterizes global properties of the tail, affects V_{ub} determinations depending on cuts, tends to decrease V_{ub}

Constraining Weak Annihilations



WA may pollute all present estimates, and tend to ***decrease*** *the extracted* V_{ub}*. Need* ***upper cut on*** q^2 *to remove this uncertainty and/or constrain WA from* $B^{0/+}$ *and* q^2 *spectrum*



Comparing the existing approaches at common m_b

(HFAG ichep08)

$|V_{ub}|$ from DGE

Gardi & Andersen
see Gardi talk

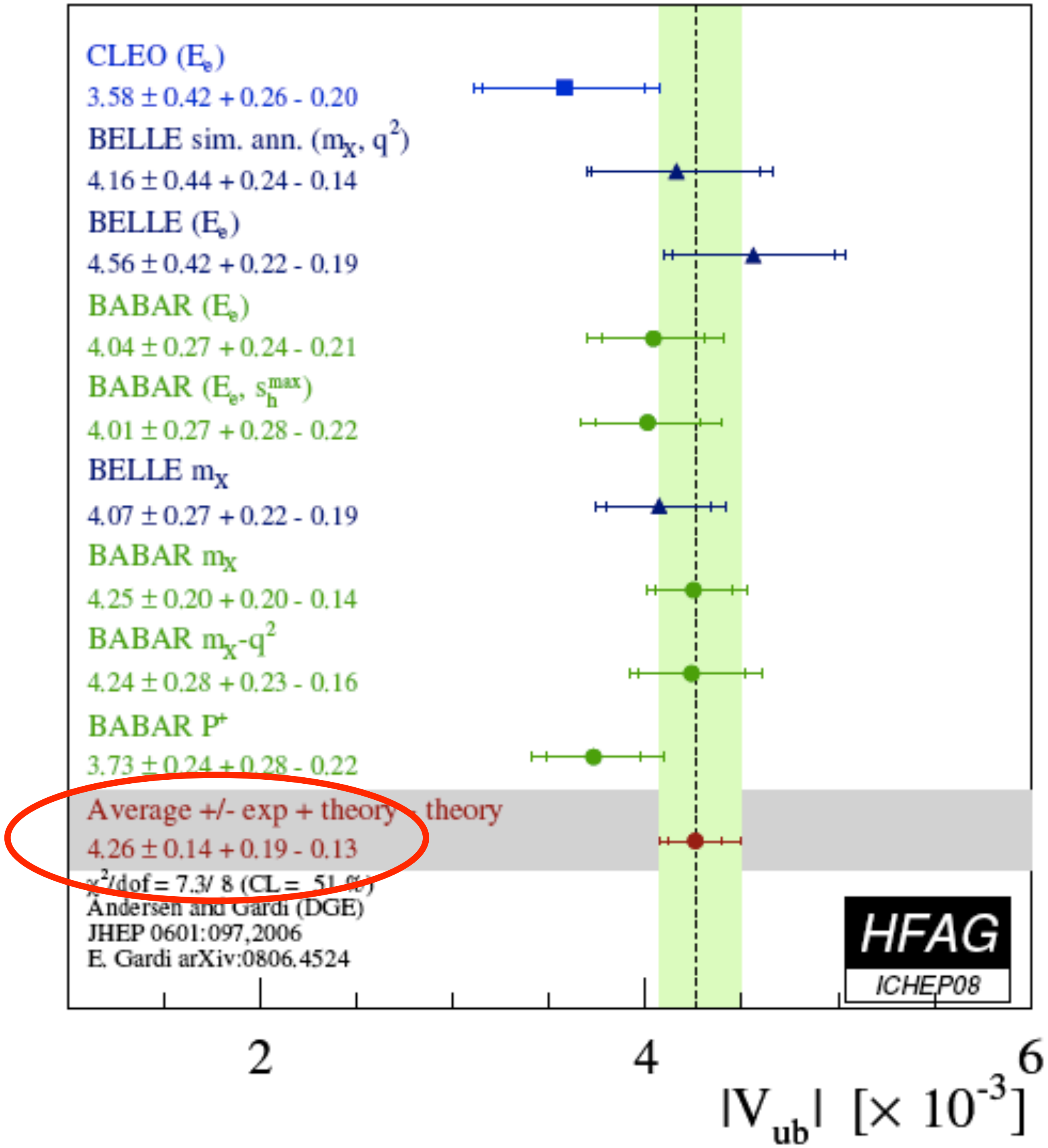
Main features of the spectra are reproduced ➠ $|V_{ub}|$ stable, small errors and good χ^2

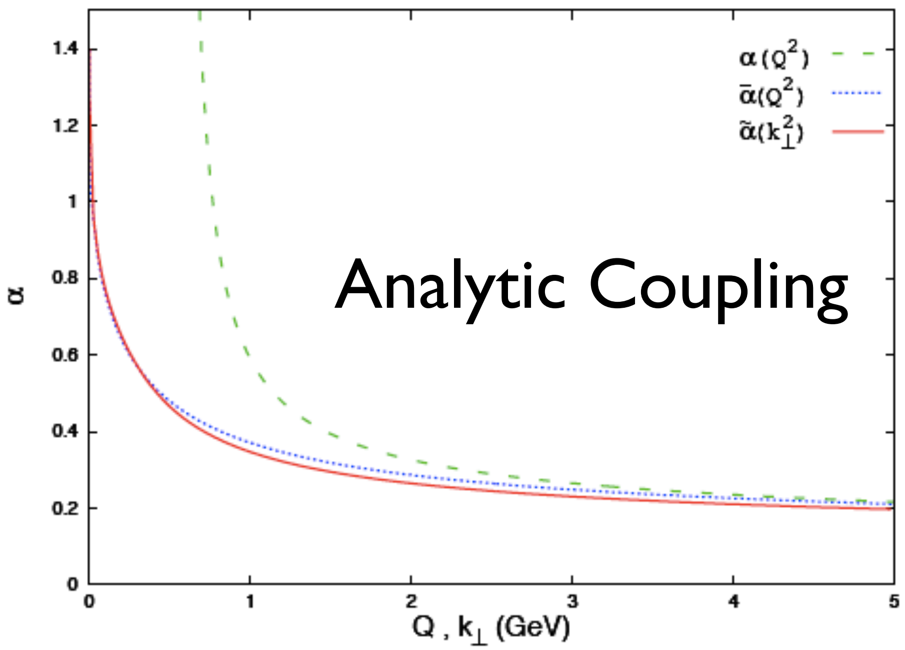
NNLL and $O(\alpha_s^2\beta_0)$ implemented

Power corrections in the SF region are included here only in theor. err.
No subleading SF.
WA error equal for all cuts.
Matches to local OPE.

Only input other than α_s
$m_b(m_b)$=4.24(4) from global fit

5-6% total error, mostly m_b





$|V_{ub}|$ from ADFR

Aglietti,Di Lodovico,Ferrera,Ricciardi

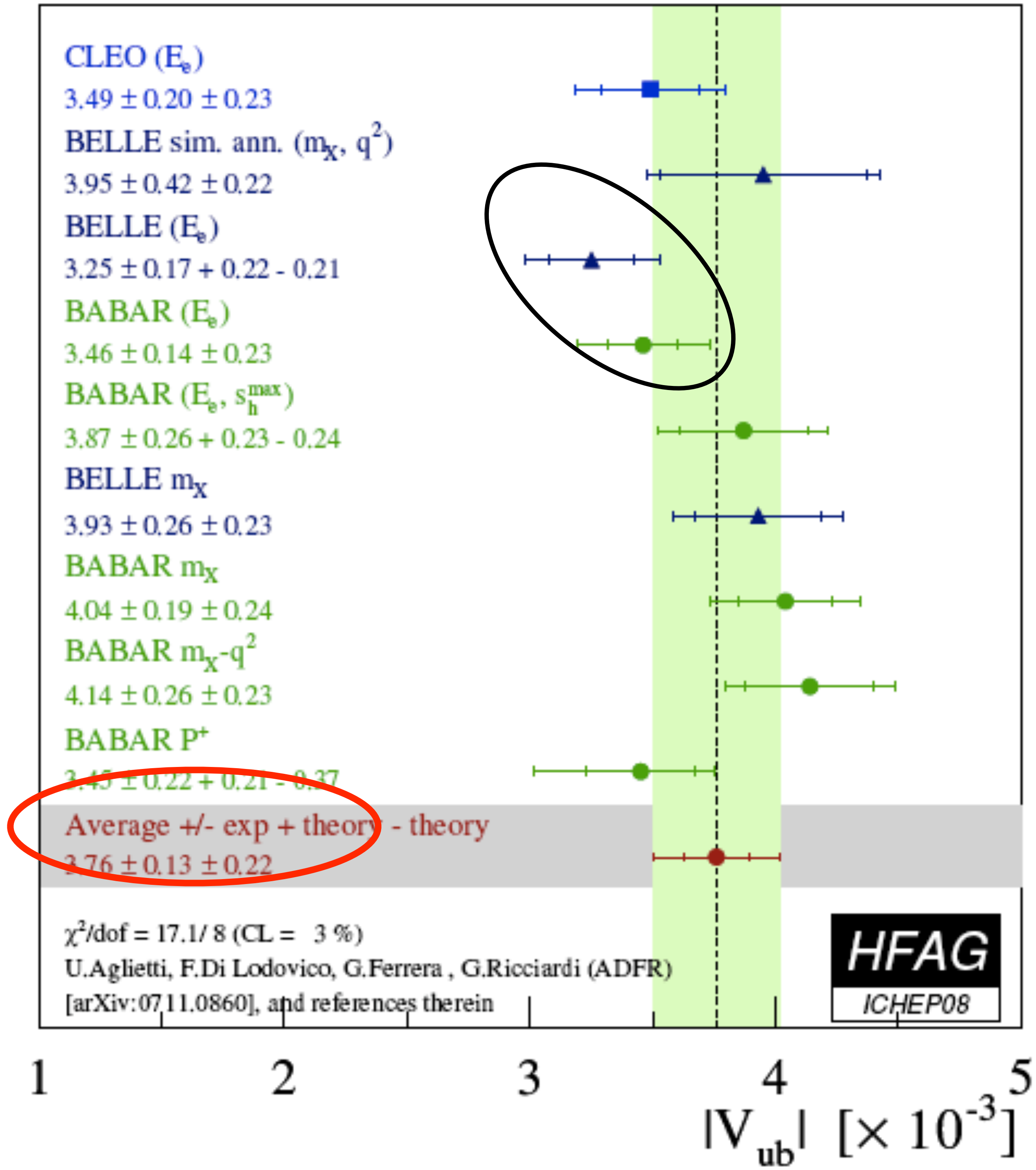
Worse consistency here.

NNLO resummation, NLO constants

Consider E_l cuts higher than 2.3GeV because their E_l apparently does not reproduce data (see later)

employs M_B in on-shell calculation of spectra: no renormalon cancellation, no convergence to OPE.
no model error

~7% total error, mostly m_c



$|V_{ub}|$ in BLNP

Bosch,Lange,Neubert,Paz

$$\tilde{W}_1^{(0)}(P_+,y) = U_y(\mu_h,\mu_i)\, H(y,\mu_h) \int_0^{P_+} d\hat{\omega}\; m_b J(y, m_b(P_+ - \hat{\omega}),\mu_i)\, \hat{S}(\hat{\omega},\mu_i)$$

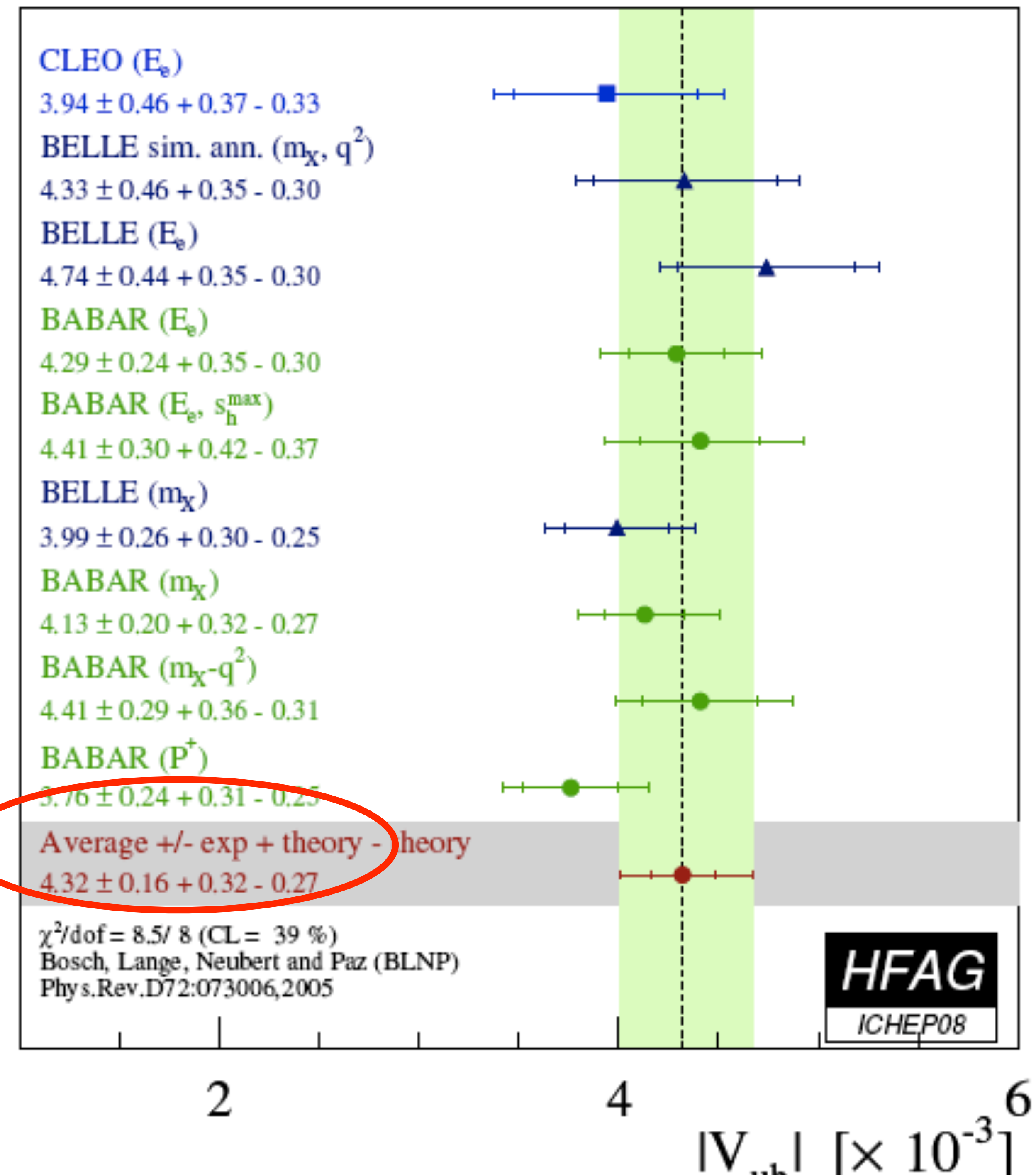
$$d\Gamma = HJ \otimes \hat{S} + \frac{1}{m_b} H_i^{\prime} J_i^{\prime} \otimes \hat{S}_i^{\prime} + \ldots$$

Good consistency. Uses elegant multiscale OPE that resums soft-collinear logs, but many largely unconstrained subleading SFs

NNLL resummation, only $O(\alpha_s, \Lambda^2/m_b^2)$ matching to OPE, 3 ffs for leading SF, extensive modelling of SSF.

m_b and μ_π^2 in SF scheme obtained from global fit in the kin scheme

~7-8% total error, main error HQE parameters



SF in GGOU

Leading SF resums leading twist effects, $m_b \to \infty$
universal, q^2 indep

→

Finite m_b distribution functions
include all $1/m_b$ effects, *non-universal*
no need for subleading SFs

$$F(k_+) \longrightarrow F_i(k_+, q^2, \mu)$$

Structure function (i = 1,2,3)

q^2 dependence

cutoff dependence (gluons with $E_g<\mu$)

$$\frac{d^3\Gamma}{dq^2\, dq_0\, dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{8\pi^3} \left\{ q^2 W_1 - \left[2E_\ell^2 - 2q_0 E_\ell + \frac{q^2}{2} \right] W_2 + q^2 (2E_\ell - q_0) W_3 \right\}$$

$$W_i(q_0, q^2) = m_b^{n_i}(\mu) \int dk_+ \; F_i(k_+, q^2, \mu) \; W_i^{pert} \left[q_0 - \frac{k_+}{2} \left(1 - \frac{q^2}{m_b M_B} \right), q^2, \mu \right]$$

This factorization formula perturbatively defines the distribution functions

see also Benson, Bigi, Uraltsev for bsγ

$$\int dk_+ \; k_+^n \; F_i(k_+, q^2) = \quad \text{local OPE}$$

Importance of subleading effects

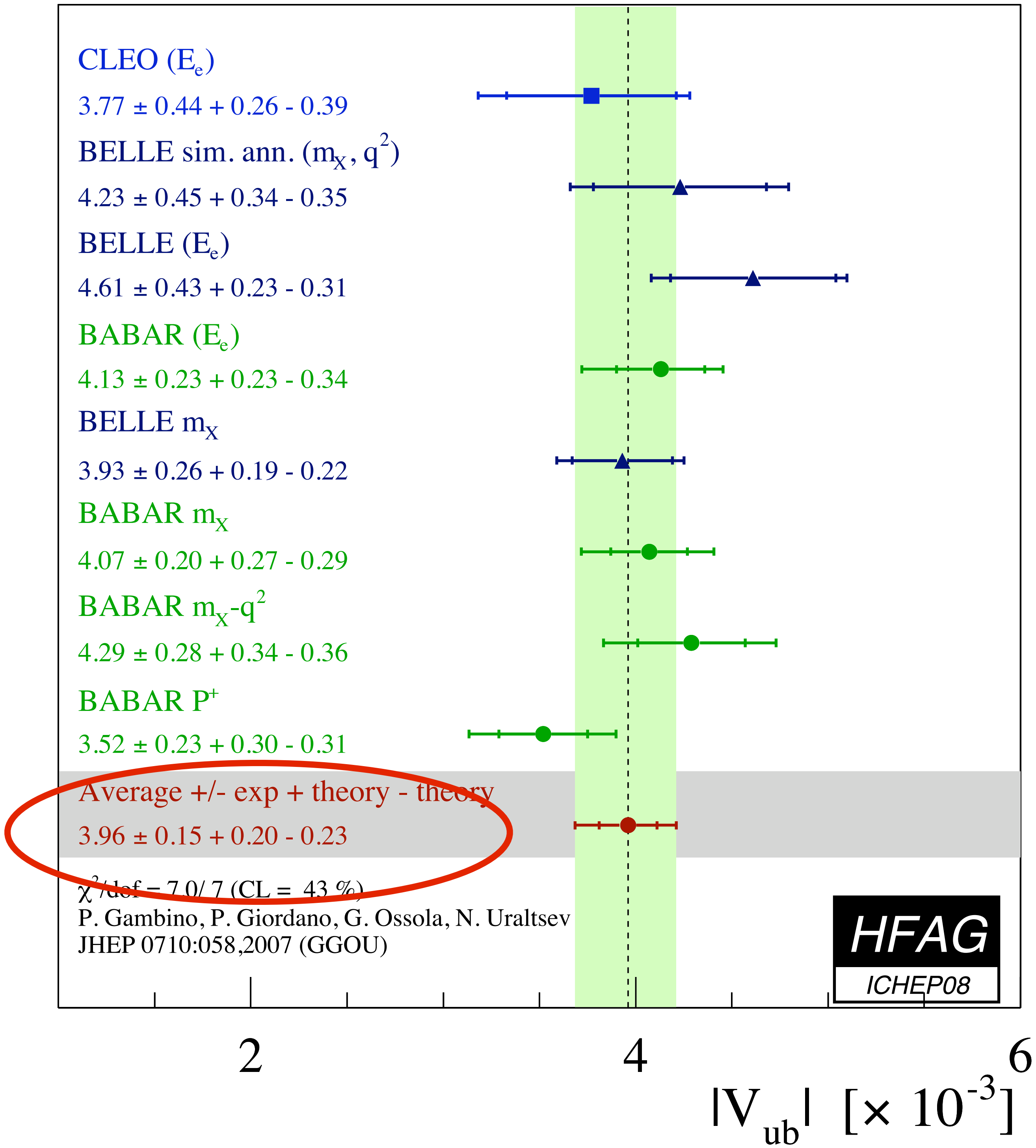
$|V_{ub}|$ in the kinetic scheme -GGOU

PG,Giordano,Ossola,Uraltsev

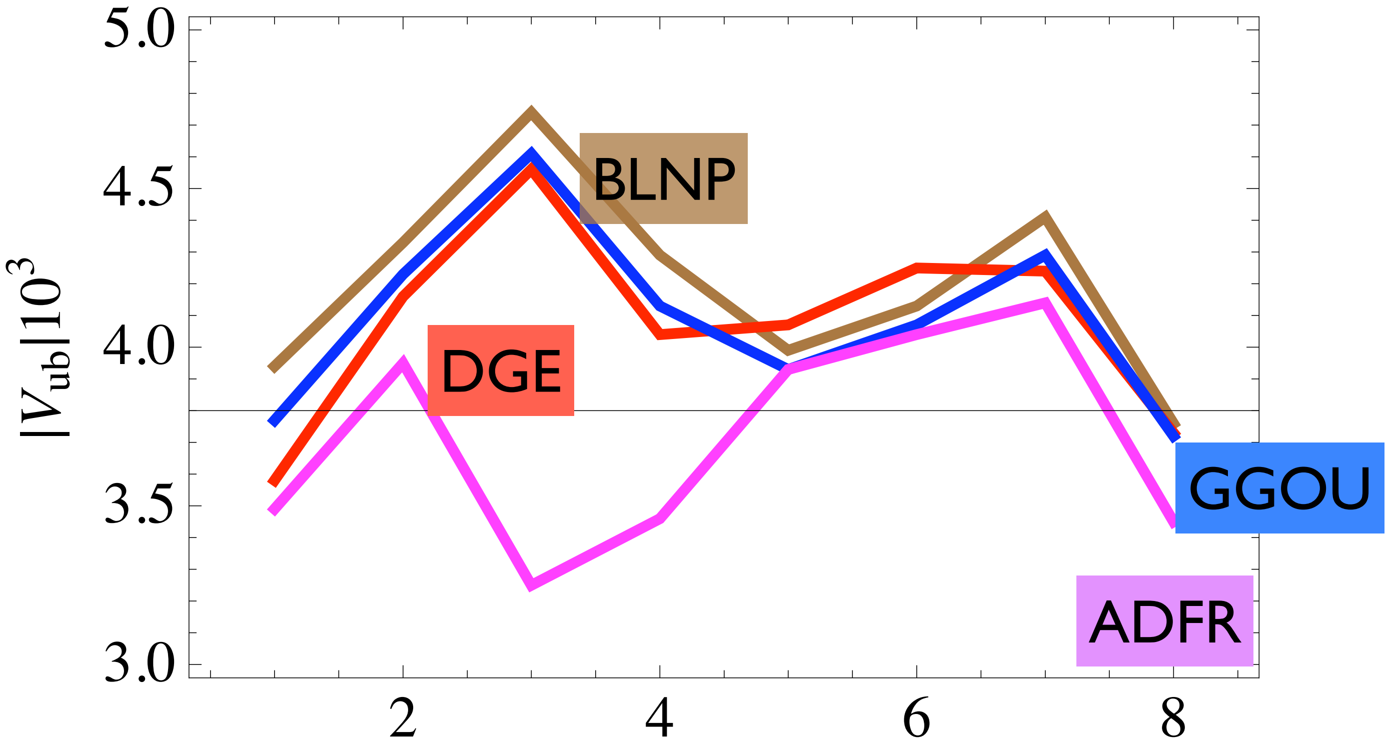
Good consistency & small th error.
OPE in a scheme with Wilsonian IR cutoff ~1GeV, all subleading $1/m_b$ and $O(\alpha_s^2\beta_0)$ terms consistently included,
careful treatment of high q^2 tail.

Inputs from global fit to the moments

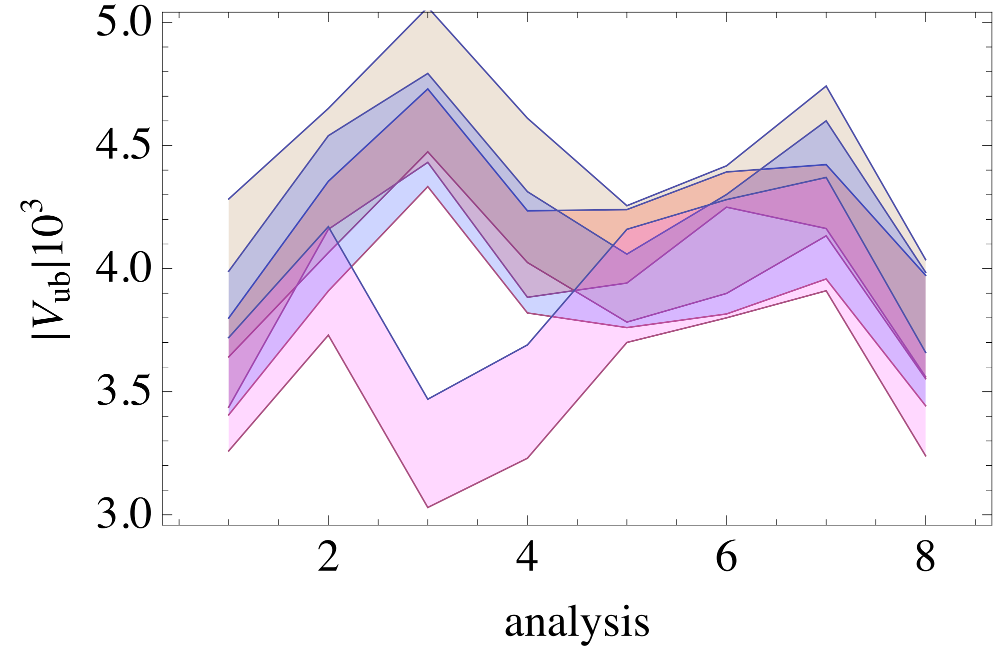
+6.3-7.0% total error

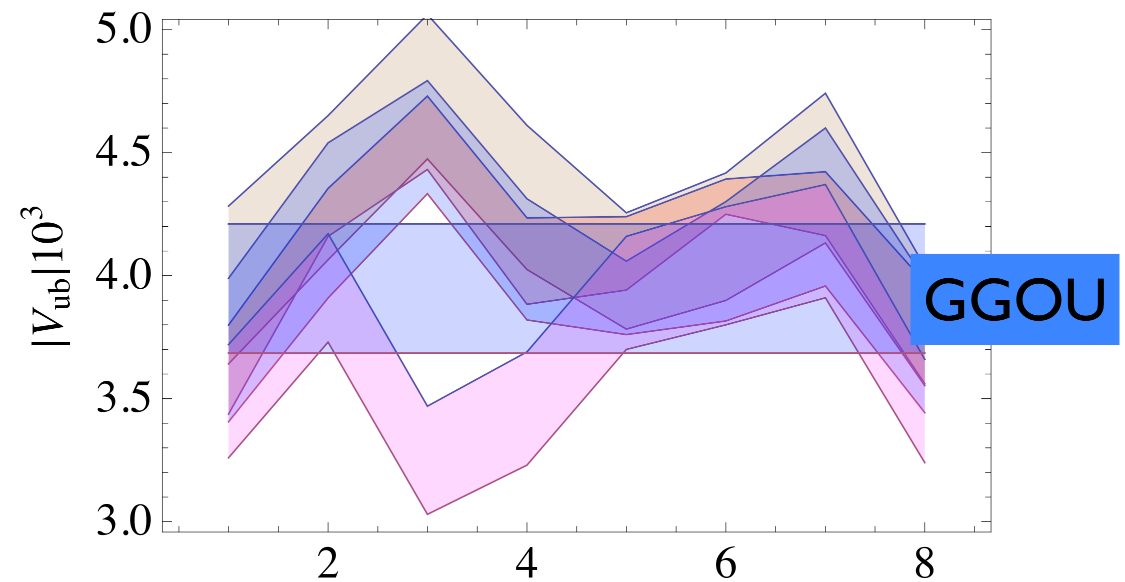


A global comparison



only theory errors
(without common parametric)

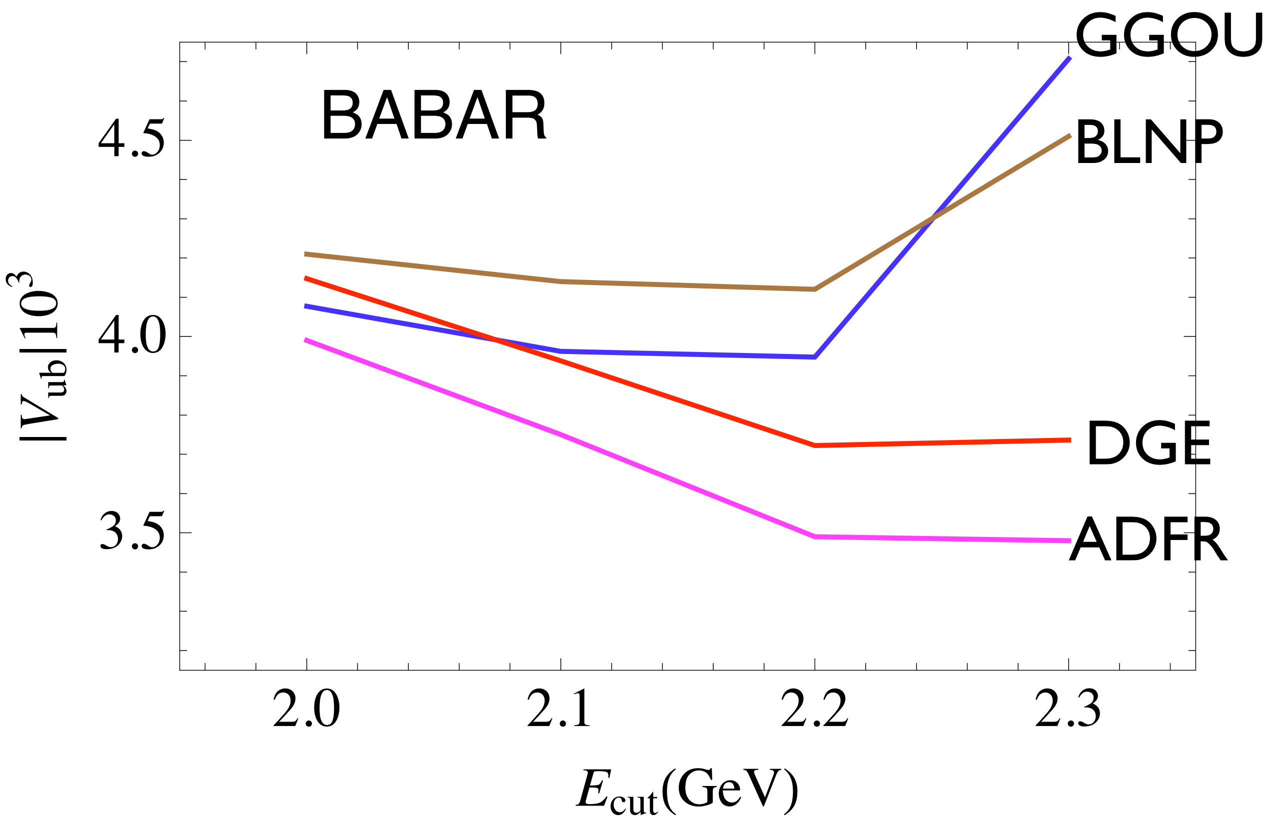


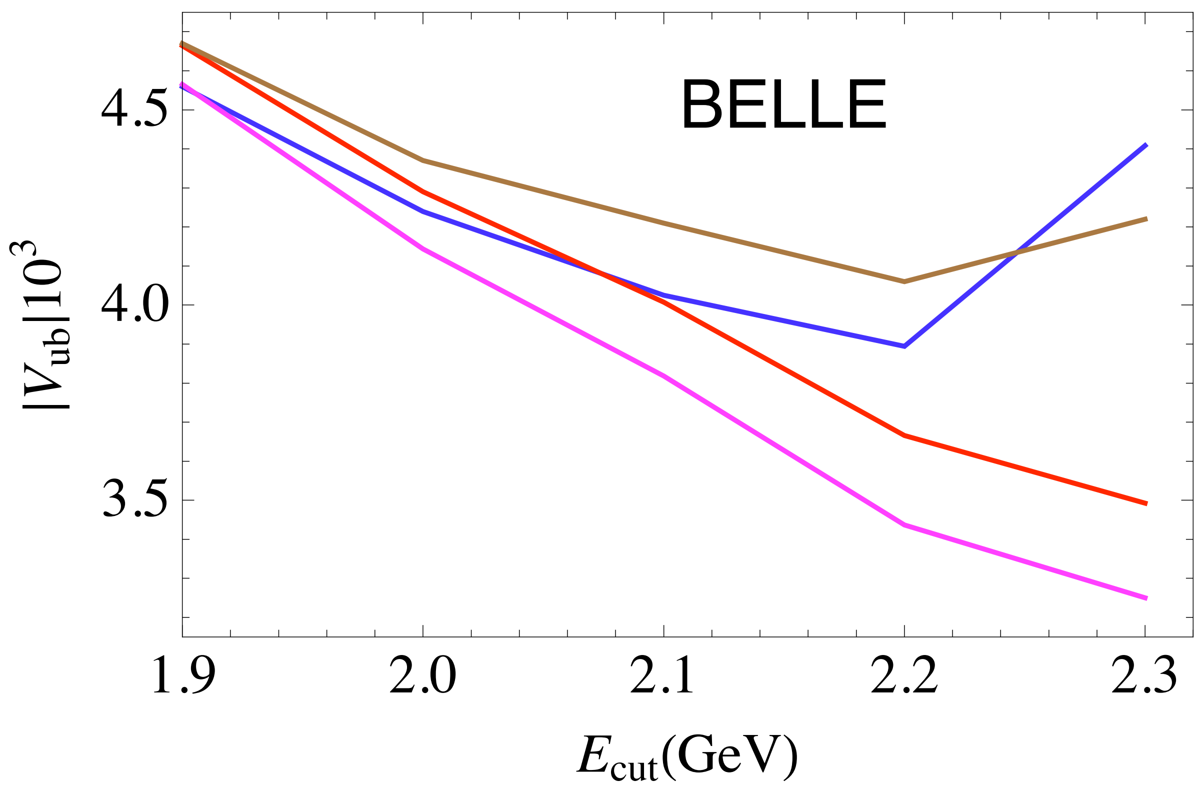


- ✶ common inputs (except ADFR)
- ✶ Overall good agreement with one exception
  SPREAD WITHIN TH ERRORS!
- ✶ Systematic offset of central values: normalization? WA? to be investigated
- ✶ Very different methods, common systematics? WA, inputs, pert corrections

Why do central values differ by 9-10%?

The lepton spectrum

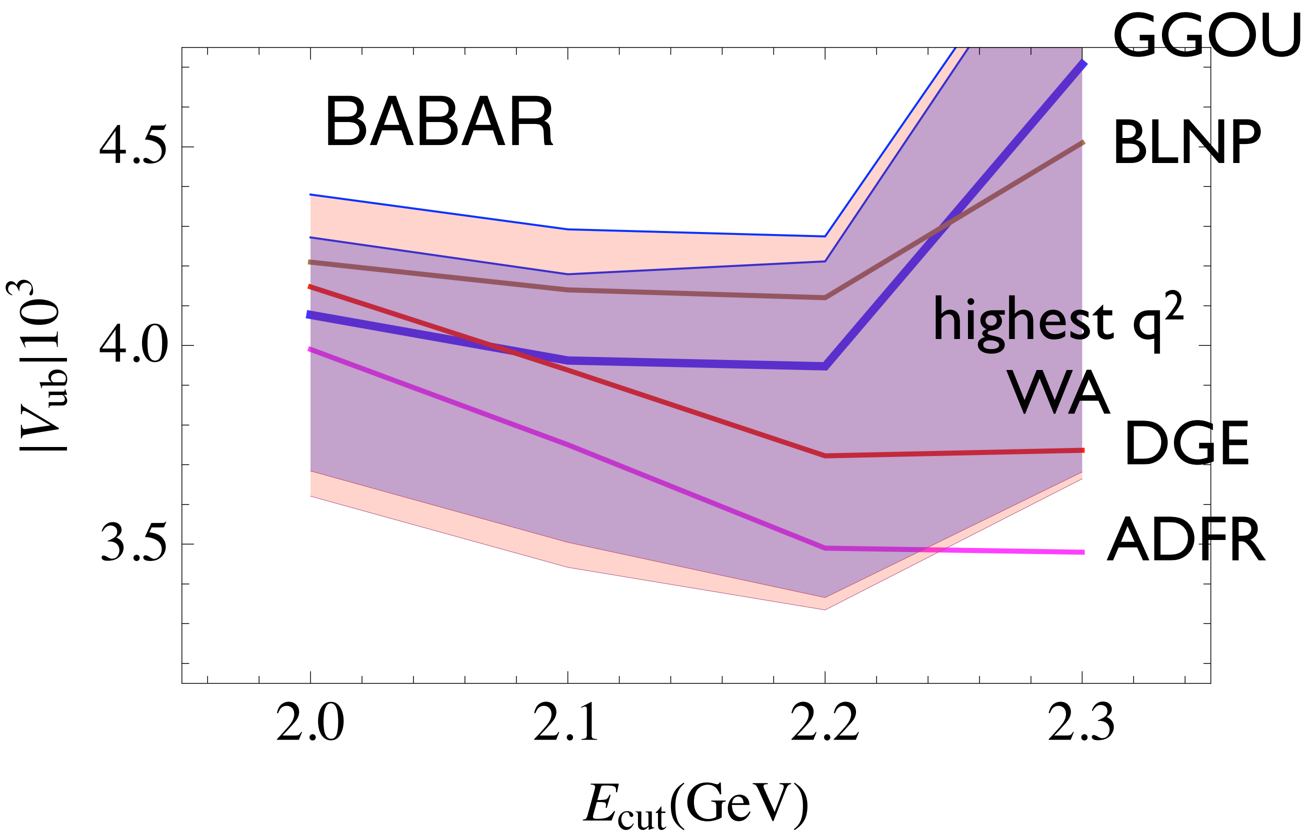




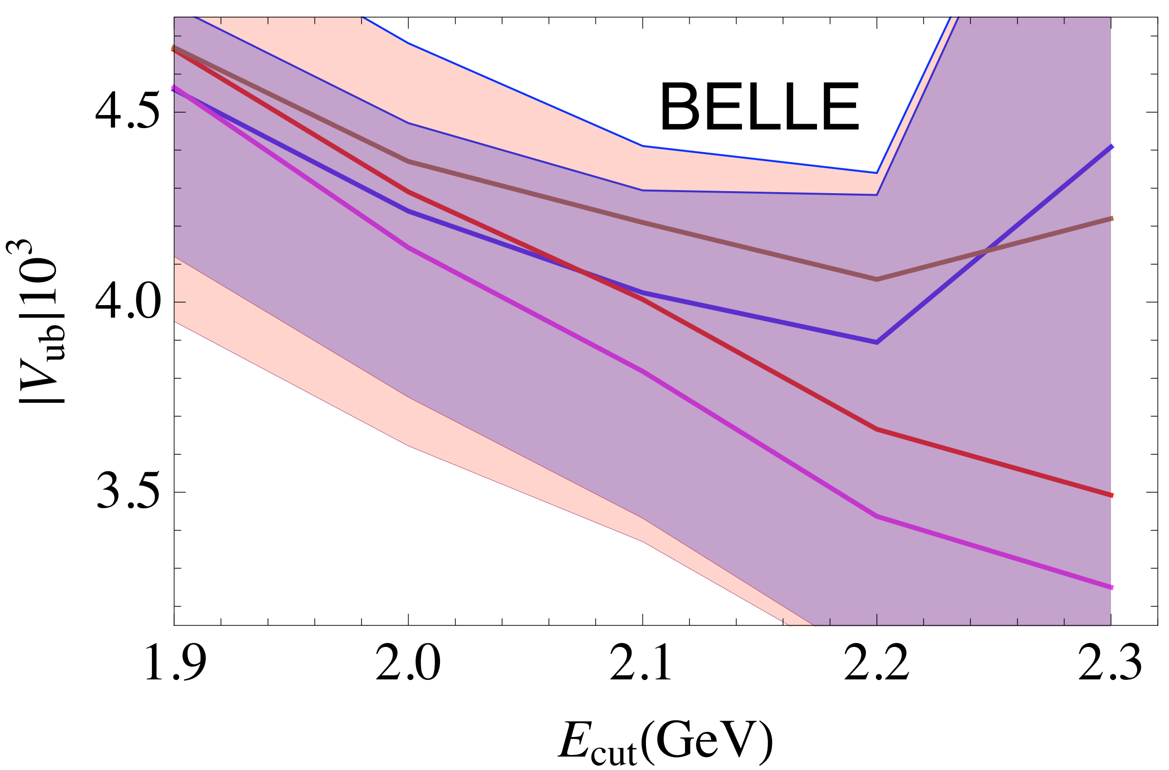
Babar *E_l determination* ***Belle*** *E_l determination*

Common inputs, m_b^{kin}=4.60GeV or $m_b(m_b)$=4.24GeV.
Exp analyses depend strongly on generator (slight inconsistency here...)

The lepton spectrum



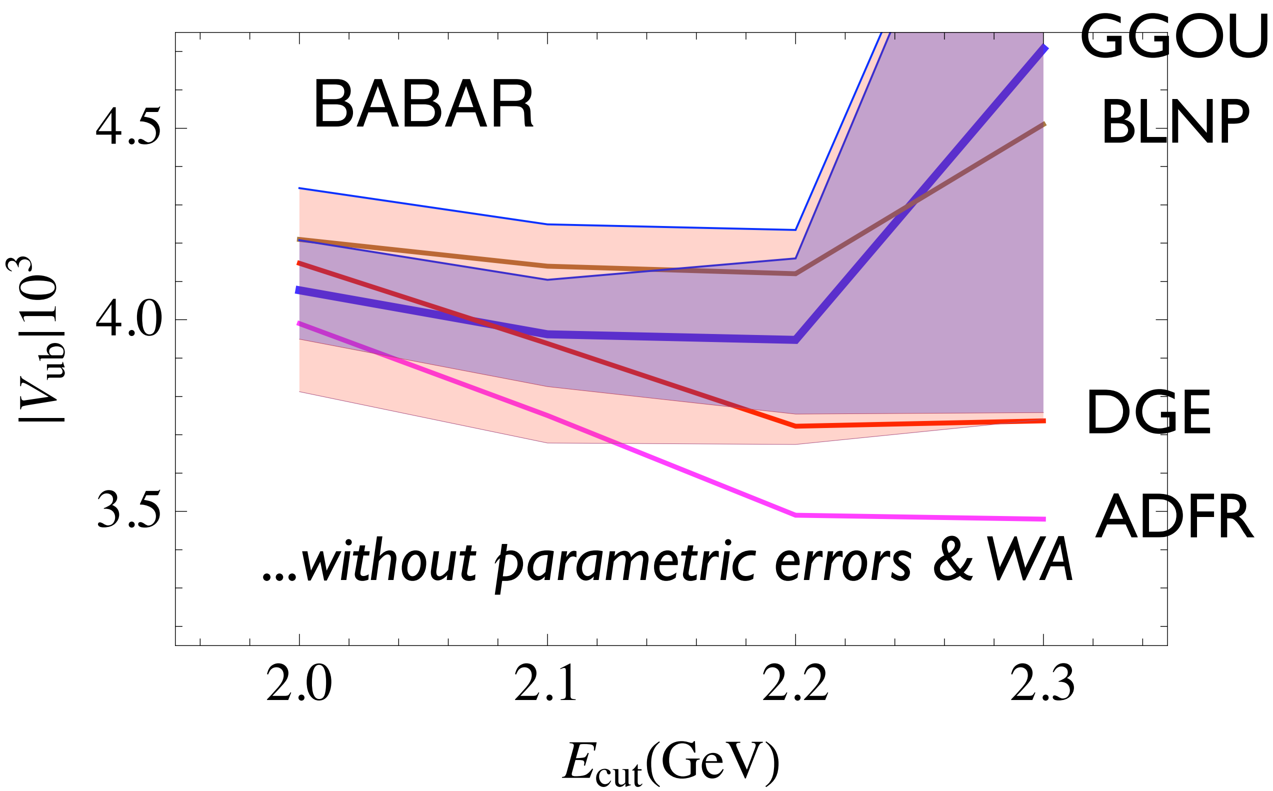
Babar *E_l determination*

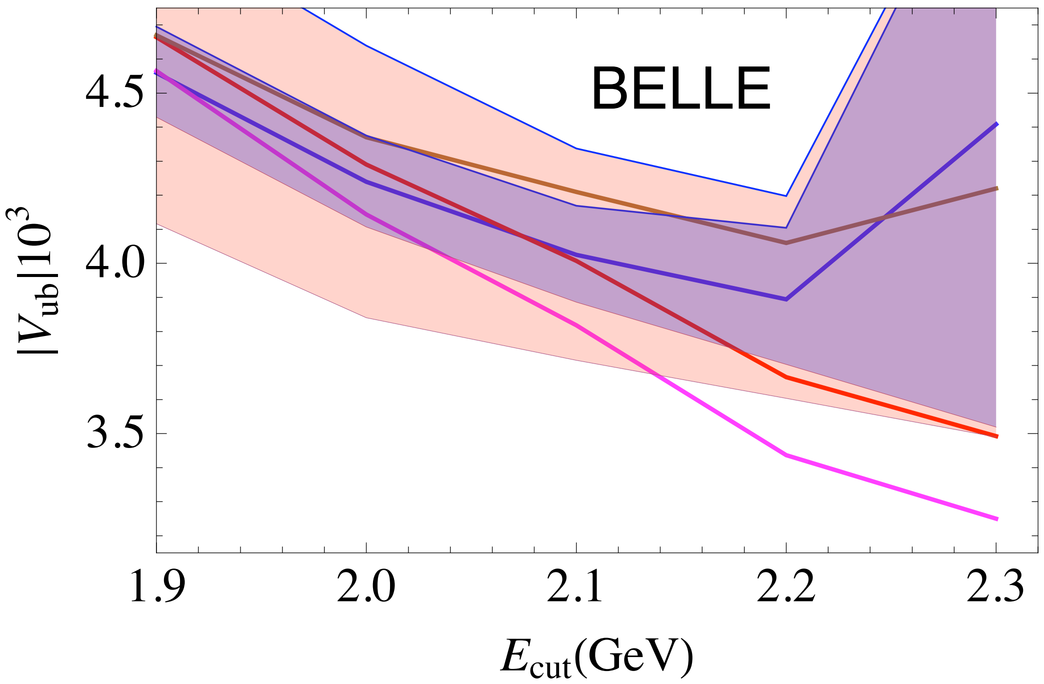


Belle *E_l determination*

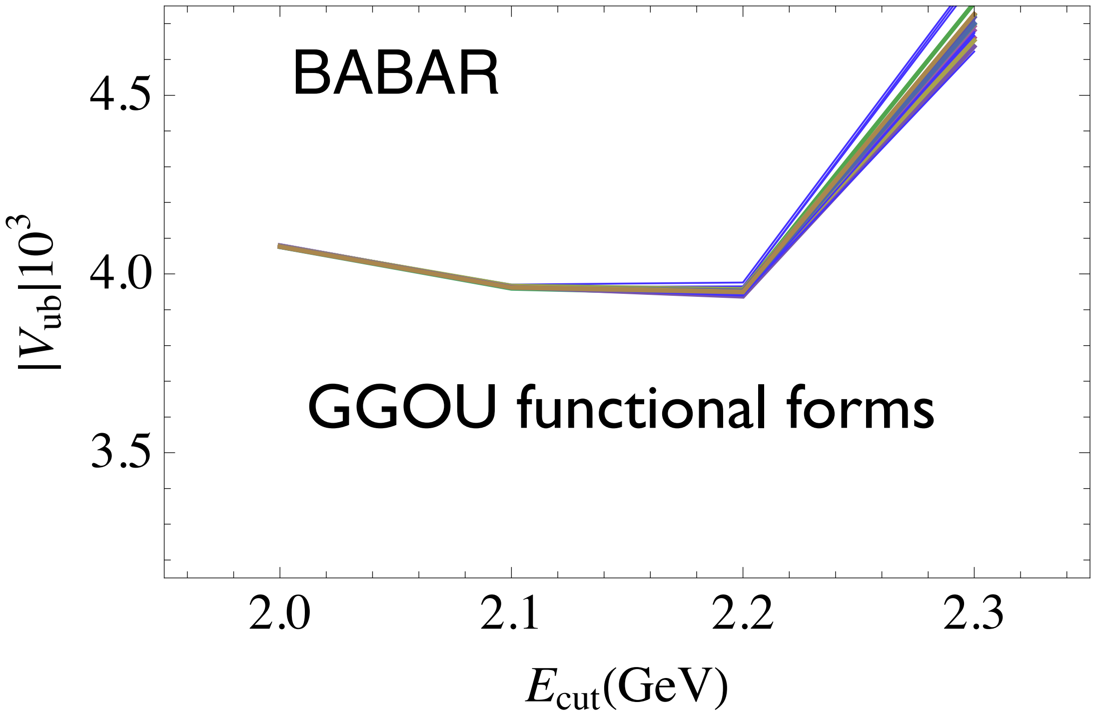
NB: error dominated by m_b and WA, strong correlation between different points.

The lepton spectrum



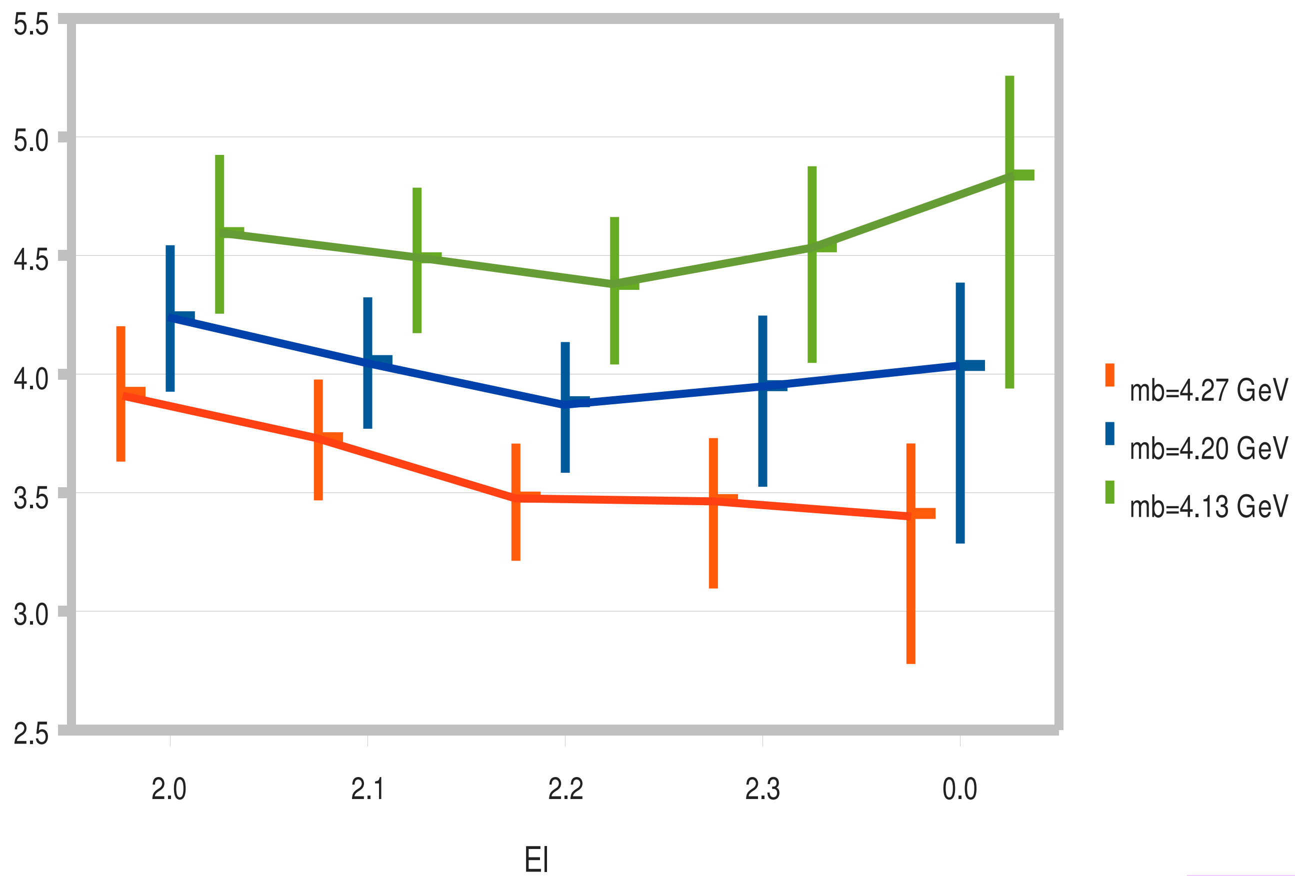


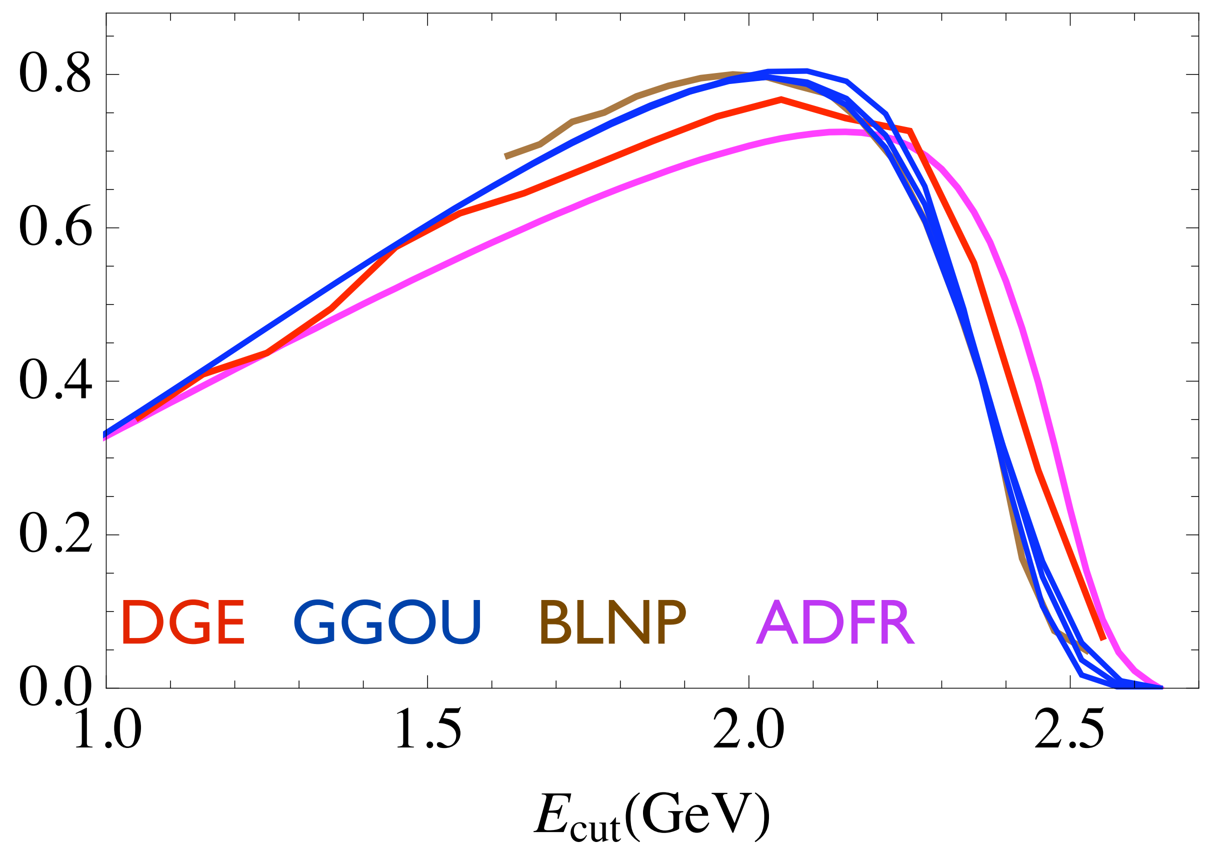
The spectrum does provide information: OPE based methods close to each other up to 2.2GeV, resummed methods show larger slope, *seem* to behave in same way



The lepton spectrum

DGE slope vs m_b





In DGE the slope depends on m_b, while ADFR have it fixed

The leptonic spectrum is not sensitive to the SF except quite close to the endpoint. At 1.5GeV all methods should agree (it's pQCD after all)

Conclusions

- ➡ New perturbative calculations important.
- ➡ Can we include additional constraints on m_b in the fit? which ones?
- ➡ *Not all observables are equivalent*, some are cleaner. For ex high q^2 tail is sensitive to WA: it decreases V_{ub}. Can we drop it? how much do the exp analyses depend on the high q^2 tail?
- ➡ Need spectra and/or analysis with varying cuts: only way to test current frameworks (see trial exercise on E_l spectrum). M_X cuts?
- ➡ More inclusive measurements would decrease the dependence of $|V_{ub}|$ on both SF *and* m_b
- ➡ Frameworks fairly compatible within non-par th errors. Convergence to OPE (normalization) to be checked. Use m_b uncertainty as exp error?
- ➡ The primary goal is the precise determination of $|V_{ub}|$. All frameworks are interesting, but they are ***not all equivalent***. After $|V_{ub}|$ is measured we can go back and study models of QCD dynamics.

back-up slides

Dressed Gluon Exponentiation

E.Gardi, J.Andersen

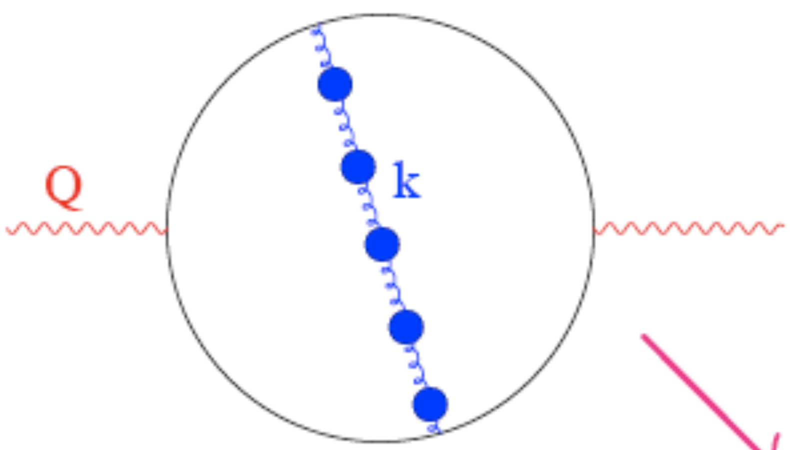
Renormalon resummation:

running–coupling corrections,

which dominate the large–order

asymptotics of the series, $n \to \infty$

$\sum_n n!\, {\alpha_s}^n \longrightarrow$ soft dynamics



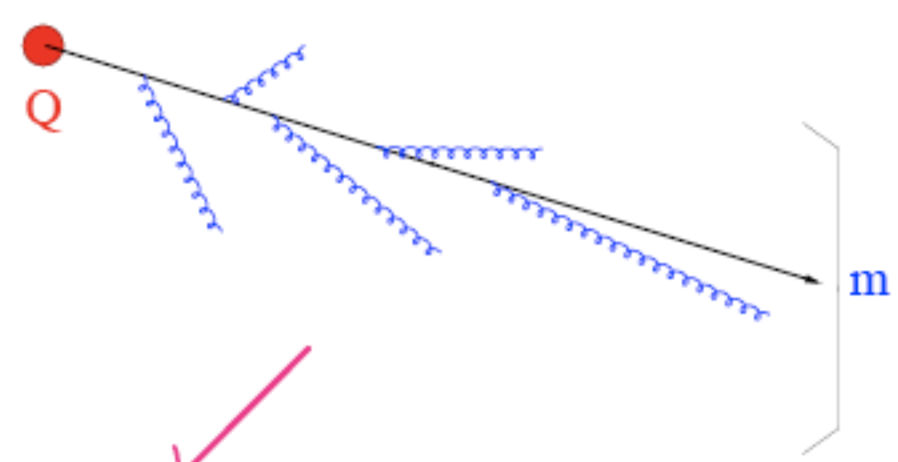
Sudakov resummation:

multiple soft and collinear radiation,

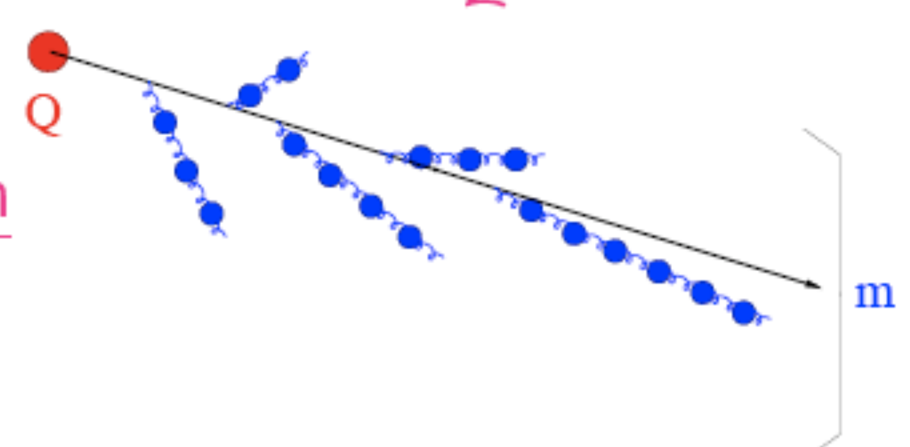
which dominate the dynamics

near threshold $\mathrm{m} \to 0$

$\sum_n {\alpha_s}^n \ln^{2n}(\mathrm{m}/\mathrm{Q})$



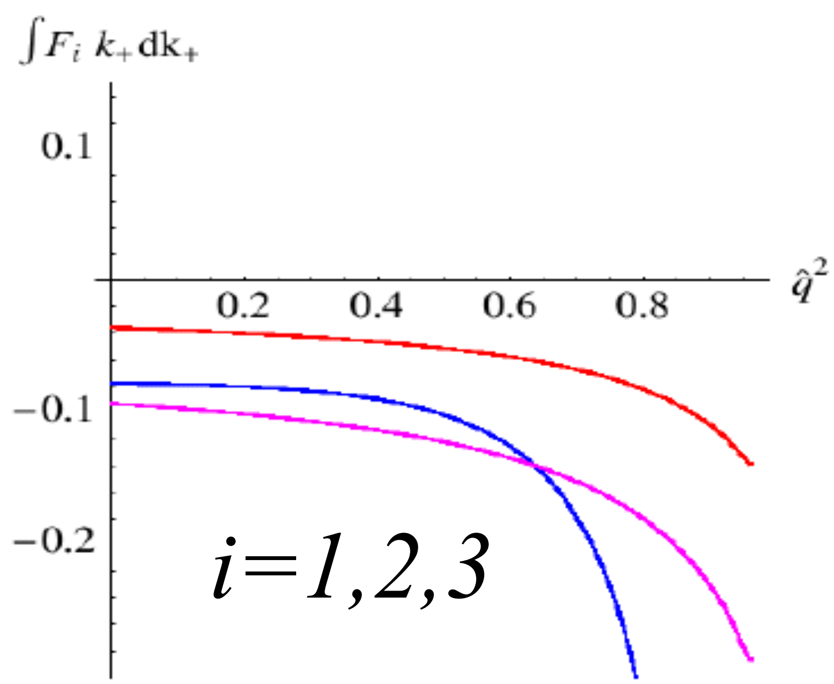
Dressed Gluon Exponentiation

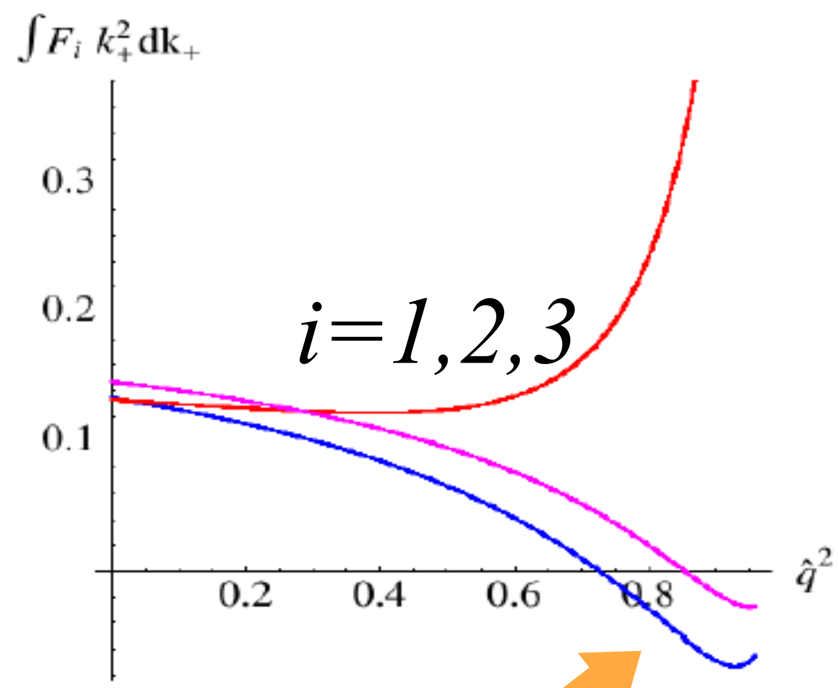


SF in GGOU (II)

crucial role of moments fit

$$\int dk_+ \, k_+^n \, F_i(k_+, q^2) = \text{local OPE prediction}$$



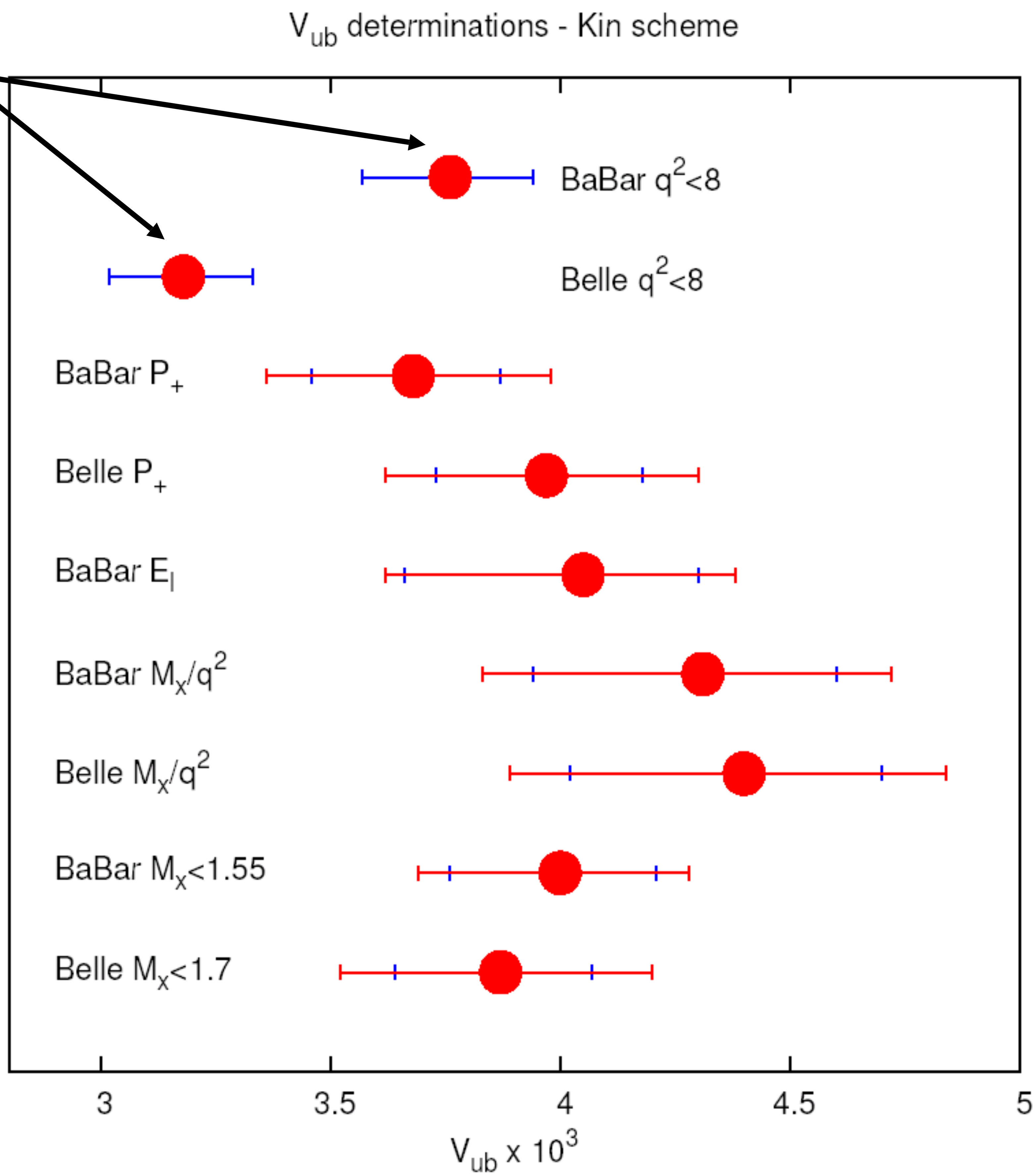


At each q^2 a new SF is computed, based on moments and a functional form (no q^2 parameterization)

The variance gets negative

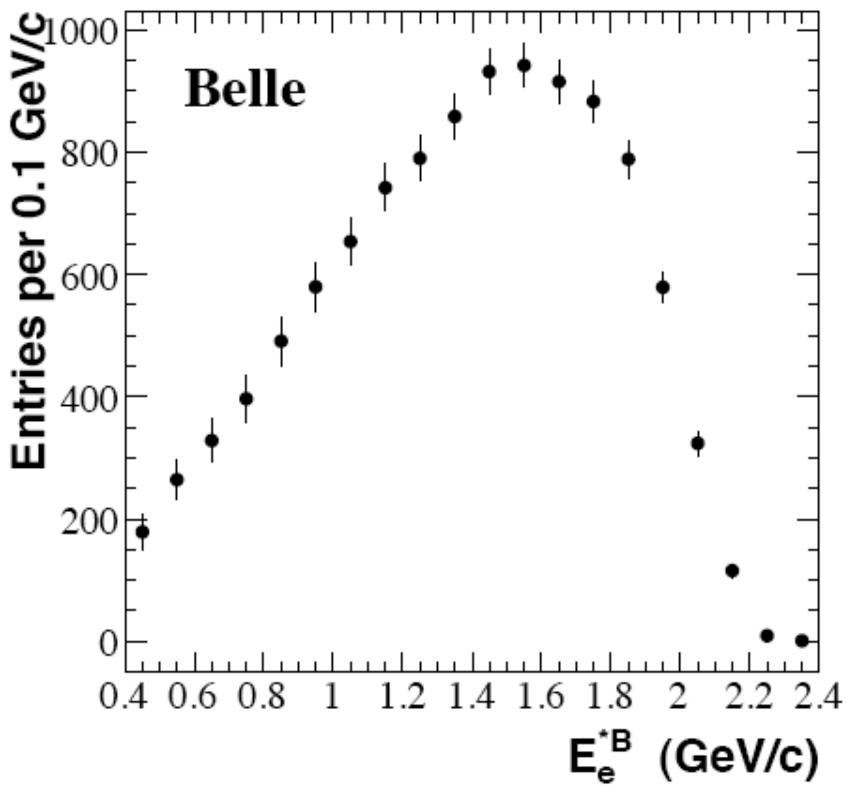
Importance of subleading effects

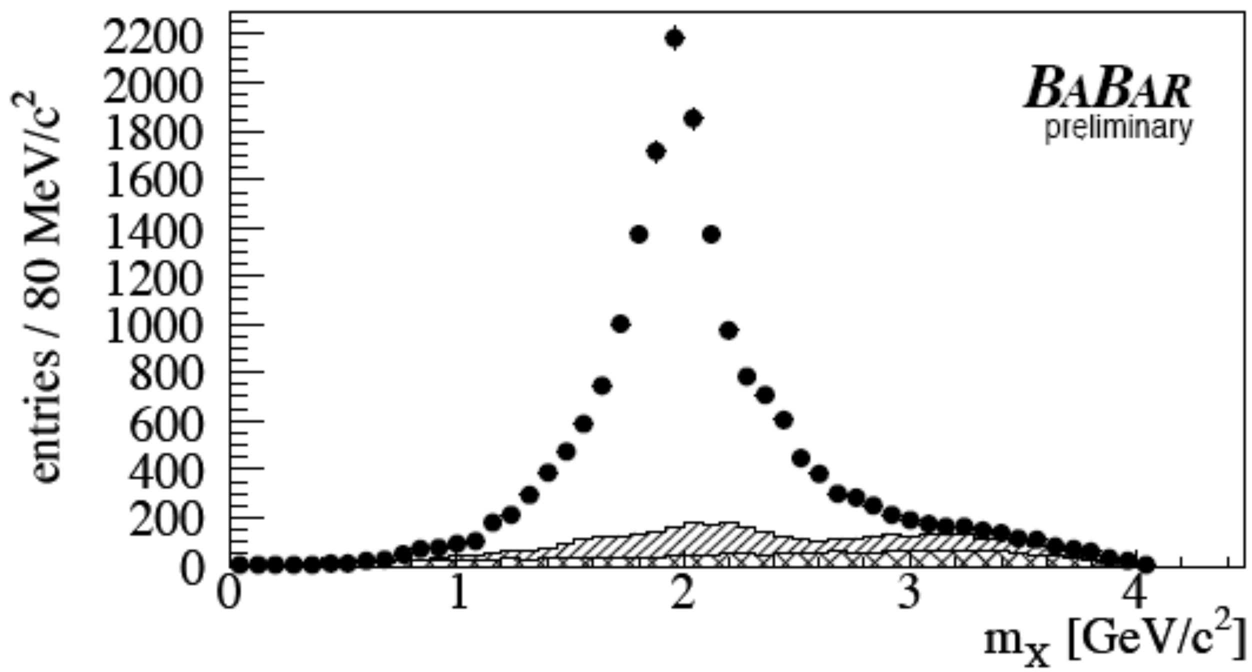
Theory errors only



This employs the HFAG average for m_b etc (bsγ included)

Fitting OPE parameters to the moments





*Total **rate** gives CKM elmnts; global **shape** parameters (moments of the distributions) tell us about B structure*

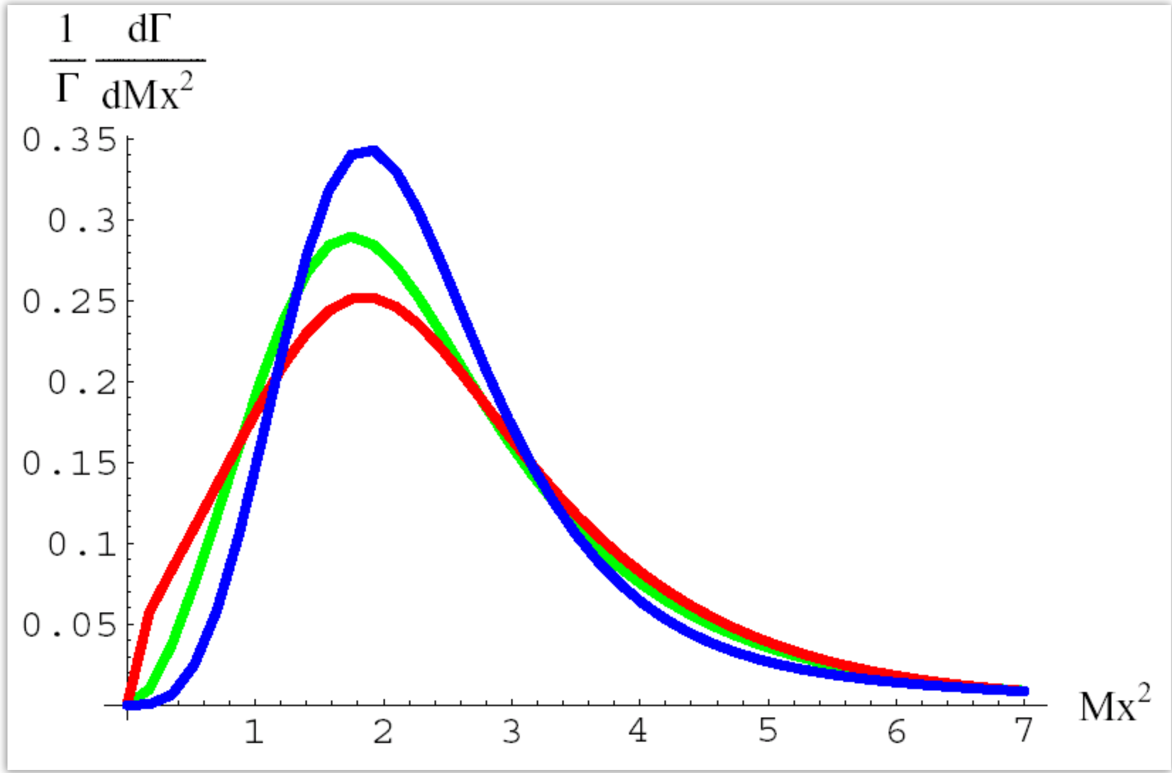
HQE parameters describe universal properties of the B meson and of the quarks

***Perturbative scheme**: $O(\alpha_s)$ Wilson coefficients depend on the exact definition of OPE parameters. They should be all short-distance parameters*

***In the kinetic scheme** the contributions of gluons with energy below $\mu \approx 1$GeV are absorbed in the OPE parameters*

Theoretical errors

- Parametric errors generally dominant, in particular m_b, 3-4%
- Perturbative corrections 2-3%
- Functional form 1-2%
- Modelling of the q^2 tail and WA depending on cut from 0 to 7%. WA tends to decrease V_{ub}



cuts	$\|V_{ub}\| \times 10^3$	f	exp	par	pert	tail model	q^2_*	X	ff	tot th
A [28]	3.87	0.71	6.7	3.5	1.7	1.6	2.0	$^{+0.0}_{-2.7}$	$^{+2.4}_{-1.1}$	$\pm 4.7^{+2.4}_{-3.8}$
B [28, 29]	4.44	0.38	7.3	3.5	2.6	3.0	4.0	$^{+0.0}_{-5.0}$	$^{+1.4}_{-0.5}$	$\pm 6.6^{+1.4}_{-5.5}$
C [30]	4.05	0.30	5.7	4.2	3.3	1.8	0.9	$^{+0.0}_{-6.2}$	$^{+1.2}_{-0.7}$	$\pm 5.7^{+1.2}_{-6.9}$

A= M_X cut Belle, B= (M_x,q^2) cut Belle+Babar, C= E_l cut Babar

Overall theory errors are 5-9%, depending on the cuts.