

# Birth and Raise of CKM physics

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# The Need for Flavor Mixing — 1

The inspiration for flavor mixing first arose from the 1958 paper by Feynman and Gell-Mann on the V-A theory of weak interactions,

PHYSICAL REVIEW

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## Theory of the Fermi Interaction

R. P. FEYNMAN AND M. GELL-MANN

*California Institute of Technology, Pasadena, California*

(Received September 16, 1957)

The representation of Fermi particles by two-component Pauli spinors satisfying a second order differential equation and the suggestion that in  $\beta$  decay these spinors act without gradient couplings leads to an essentially unique weak four-fermion coupling. It is equivalent to equal amounts of vector and axial vector coupling with two-component neutrinos and conservation of leptons. (The relative sign is not determined theoretically.) It is taken to be “universal”; the lifetime of the  $\mu$  agrees to within the experimental errors of 2%. The vector part of the coupling is, by analogy with electric charge, assumed to be not renormalized by virtual mesons. This requires, for example, that pions are also “charged” in the sense that there is a direct interaction in which, say, a  $\pi^0$  goes to  $\pi^-$  and an electron goes to a neutrino. The weak decays of strange particles will result qualitatively if the universality is extended to include a coupling involving a  $\Lambda$  or  $\Sigma$  fermion. Parity is then not conserved even for those decays like  $K \rightarrow 2\pi$  or  $3\pi$  which involve no neutrinos. The theory is at variance with the measured angular correlation of electron and neutrino in  $\text{He}^6$ , and with the fact that fewer than  $10^{-4}$  pion decay into electron and neutrino.

# The First Problem

## From the Feynman — Gell-Mann paper...

To account for all observed strange particle decays it is sufficient to add to the current a term like  $(\bar{p}\Lambda^0)$ ,  $(\bar{p}\Sigma^0)$ , or  $(\Sigma^-n)$ , in which strangeness is increased by one as charge is increased by one. For instance,  $(\bar{p}\Lambda^0)$  gives us the couplings  $(\bar{p}\Lambda^0)(\bar{e}\nu)$ ,  $(\bar{p}\Lambda^0)(\bar{\mu}\nu)$ , and  $(\bar{p}\Lambda^0)(\bar{n}p)$ . A direct consequence of the coupling  $(\bar{p}\Lambda^0)(\bar{e}\nu)$  would be the reaction



at a rate  $5.3 \times 10^7 \text{ sec}^{-1}$ , assuming no renormalization of the constants.<sup>18</sup> ..... we should observe process (14) in about 1.6% of the disintegrations. This is not excluded by experiments. If a term like  $(\Sigma^-n)$  appears, the decay  $\Sigma^- \rightarrow n + e^- + \bar{\nu}$  is possible at a predicted rate  $3.5 \times 10^8 \text{ sec}^{-1}$  and should occur .....

... in about 5.6% of the disintegrations of the  $\Sigma^-$ .

Around 1962 it became clear that these rates were  $\approx 20$  times smaller!

# The Need for Flavor Mixing — 2

The second hint is due to by Sam Berman, Feynman's student, and appeared at the end of 1958.

PHYSICAL REVIEW

VOLUME 112, NUMBER 1

OCTOBER 1, 1958

## Radiative Corrections to Muon and Neutron Decay

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(Received June 11, 1958)

The corrections to muon decay due to electromagnetic interactions have been recalculated. ....

.... With the radiative corrections to muon decay given here, the predicted value of the muon lifetime using the universal theory is  $(2.27 \pm 0.04) \times 10^{-6}$  sec. As a preliminary to studying the decay of particles with structure, the  $\beta$  decay of the neutron is examined. This leads to an increase in the Coulomb  $F$  factor independent of the nuclear charge and of amount approximately 2.6%. As a result the universal coupling constant obtained from the decay of  $O^{14}$  is decreased to  $G = (1.37 \pm 0.02) \times 10^{-49}$  erg cm<sup>3</sup> and increases the value of the muon lifetime to  $(2.33 \pm 0.05) \times 10^{-6}$  sec.

# The Second Problem

The radiative corrections tended to worsen the disagreement between the Fermi constant as measured in beta decay and in muon decay, making it serious.

The result decreases the universal coupling constant obtained from  $O^{14}$  to  $G = (1.37 \pm 0.02) \times 10^{-49} \text{ erg cm}^3$  and increases the value of the predicted value of the muon lifetime from the value given above to  $(2.33 \pm 0.05) \times 10^{-6} \text{ sec}$ , while the experimental value is  $(2.22 \pm 0.02) \times 10^{-6} \text{ sec}$ . The disagreement between experiment and theory appears to be outside of the limit of experimental error and might be regarded as an indication of the lack of universality even by the strangeness-conserving part of the vector interaction. However, it is very difficult to understand the mechanism for such a slight deviation from universality; that is, if universality is to be broken at all why should it be by such a small amount?

Taking muon decay as the standard we have beta decay a few % weaker and hyperon semileptonic decays about 20 times weaker.

# The Eightfold Way

In 1962 R. Gatto and I proposed that weak currents be classified in an SU(3) octet. This made the puzzle worse: the weakness of semileptonic  $\Delta S = 1$  could not be a renormalization effect. The missing clue, which I found the next year, was that one should not compare the strength of the two components of the hadronic weak current to the  $\mu - \nu_\mu$  or  $e - \nu_e$  current **separately** but **together**,

$$J^{\text{weak}} = J^{\mu-\nu_\mu} + J^{e-\nu_e} + (aJ^{\Delta S=0} + bJ^{\Delta S=1}) + \dots$$

This led to the condition

$$a^2 + b^2 = 1 \quad \text{or} \quad a = \cos \theta, \quad b = \sin \theta$$

and to a simultaneous solution of both problems: **the  $\Delta S = 1$  decays feed from a small decrease of the  $\Delta S = 0$  beta decay.**

# The “Angle” paper

## UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo  
CERN, Geneva, Switzerland  
(Received 29 April 1963)

We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as “eightfold way,”<sup>1</sup> and the  $V-A$  theory for weak interactions.<sup>2,3</sup> Our basic assumptions on  $J_\mu$ , the weak current of strong interacting particles, are as follows:

(1)  $J_\mu$  transforms according to the eightfold representation of  $SU_3$ . This means that we neglect currents with  $\Delta S = -\Delta Q$ , or  $\Delta I = 3/2$ , which should belong to other representations. This limits the scope of the analysis, and we are not

able to treat the complex of  $K^0$  leptonic decays, or  $\Sigma^+ \rightarrow n + e^+ + \nu$  in which  $\Delta S = -\Delta Q$  currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of  $J_\mu$  which is in the eightfold representation.

(2) The vector part of  $J_\mu$  is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For  $\Delta S = 0$ , this assumption is equivalent to vector-

The value of the angle  $\theta$  was here determined in two different ways:

From K13 decays  $\theta = 0.26$

From the  $\frac{K \rightarrow \mu\nu}{\pi \rightarrow \mu\nu}$  ratio  $\theta = 0.257$

Modern measurements of K13 decays lead to smaller values, and in 2008 the KLOE result is  $V_{us} = \sin(\theta) = 0.2237 \pm 0.0013$ . The different value from  $K \rightarrow \mu\nu$  is due to a violation of  $SU(3)$  symmetry, perfectly accounted by lattice QCD simulations.

# Lattice Gauge Theory

Since 1984 (N.C, G. Martinelli, R. Petronzio, Nuc, Phys. B244:381) lattice gauge theory has been an important tool in disentangling the QCD aspects of weak interaction processes. One of the nicer results was the computation of  $f_\pi, f_K$  by the MILC collaboration (hep-lat 0406324)

$$\begin{aligned}f_\pi &= 129.3 \pm 1.1 \pm 3.5 \text{ MeV} \\f_K &= 155.0 \pm 1.8 \pm 3.7 \text{ MeV} \\f_K/f_\pi &= 1.201(8)(15)\end{aligned}$$

From these results Marciano (hep-ph 0402299) obtained

$$\sin \theta = 0.2236(30)$$

More accurate values originate from recent LQCD simulations and the KLOE experimental data.

# Hyperon Semileptonic Decays

An important result of the “angle” paper was the prediction of the branching ratios and decay parameters for the possible  $\Delta S = 1$  hyperon decays.

Decay	Branching ratio		Type of interaction
	From reference 2	Present work	
$\Lambda \rightarrow p + e^- + \bar{\nu}$	1.4 %	$0.75 \times 10^{-3}$	$V - 0.72 A$
$\Sigma^- \rightarrow n + e^- + \bar{\nu}$	5.1 %	$1.9 \times 10^{-3}$	$V + 0.65 A$
$\Xi^- \rightarrow \Lambda + e^- + \bar{\nu}$	1.4 %	$0.35 \times 10^{-3}$	$V + 0.02 A$
$\Xi^- \rightarrow \Sigma^0 + e^- + \bar{\nu}$	0.14 %	$0.07 \times 10^{-3}$	$V - 1.25 A$
$\Xi^0 \rightarrow \Sigma^+ + e^- + \bar{\nu}$	0.28 %	$0.26 \times 10^{-3}$	$V - 1.25 A$

These were checked over many years, with correct results on the  $\Sigma^- \Rightarrow ne\bar{\nu}$  only appearing in the mid-eighties, and the first measurement of  $\Xi^0 \Rightarrow \Sigma^+ e\bar{\nu}$  by the KTeV group, presented in 2001.

# Board a Time Machine...

... and let events woooosh by

Quarks

CP Violation

Deep Inelastic,  $e^+e^-$  colliders

Charm,  $J/\psi$ , c-quark

Standard Model

The CKM matrix

$Y$ , b-quark, t-quark

Neutrino Oscillations — Neutrino Mixing

.....

# Quark Mixing - the CKM matrix

The quark mixing is described in terms of a matrix  $\mathbf{V}$  which can be expressed in terms of four parameters:

$$\mathbf{V} = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix} = \begin{vmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{vmatrix} + O(\lambda^4)$$

CP violation arises from the presence of phase factors in some of the  $V$ 's, i.e. from a non-vanishing value of  $\eta$ .

Unitarity of the CKM matrix implies relations such as

$$\begin{aligned} |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 1 && \dots \text{value of } V_{us} \\ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 && \dots \text{the Unitarity Triangle.} \end{aligned}$$

Each of these relations corresponds to areas that have seen substantial progress, and more is expected in the next few years

## $V_{us}$ and Unitarity.

There has been a long standing discrepancy between the requirement of unitarity and the experimental value of  $V_{us}$ . The situation (circa 2000) was:

$$\text{from CKM Unitarity and } |V_{ud}| \quad \rightarrow \quad |V_{us}| = 0.2265 \pm 0.0022$$

$$\text{PDG value, from } K_{\ell 3} \quad \rightarrow \quad |V_{us}| = 0.2196 \pm 0.0026$$

**Why not use hyperon data?** The  $f_+$  form factor for  $K_{\ell 3}$  decays is protected by the Ademollo-Gatto theorem from large corrections due to SU(3) symmetry breaking, **but the same is true of the vector parts of hyperon beta decays — the  $f_1$  form factor**. Both  $K_{\ell 3}$  decays and hyperon semileptonic decays are suitable for a precise determination of  $V_{us}$ .

In 2001 with R. Winston and E. Swallow we revisited hyperon decays and were pleasantly surprised: the bad reputation of Hyperon beta decays, of suffering large SU(3) breaking effects, turned out to be unfounded.

N.C., E. Swallow, R. Winston [PRL](#) **92**:251803 (2004) 

# SU(3) breaking in Hyperon decays

First order SU(3) symmetry breaking effects are expected to manifest themselves in  $g_1/f_1$ .

One can fit the data of the 5 semileptonic decays for the linear combinations  $F + D$  and  $F - D$  which have essentially uncorrelated errors. This fit yields

$$F + D = 1.2670 \pm 0.0035; \quad F - D = -0.341 \pm 0.016; \quad \chi^2 = 2.96/3d.f.$$

SU(3) symmetry breaking effects appear to be much smaller than expected!

The final word is coming from Lattice QCD:

recent results on  $\Sigma^- \Rightarrow ne\bar{\nu}$  indicate that SU(3) breaking effects are indeed small.

(D. Guadagnoli et al. — Nucl.Phys. B761 (2007) 63-91 )

# Determination of $V_{us}$ from hyperon decays

For each decay we have (apart from well known corrections)

$$\Gamma = (\text{Kin. Factors})[V_{us}f_1(0)]^2 \left(1 + 3\frac{g_1^2}{f_1^2}\right)$$

The Axial/Vector ratio  $g_1/f_1$  can be measured directly, so **each decay separately yields a determination of  $V_{us}f_1(0)$** . Neglecting flavor-SU(3) breaking for  $f_1(0)$ , we obtained a very consistent picture that agrees well with the unitarity requirement.

Decay	$g_1/f_1$	$V_{us}$
$\Lambda \rightarrow pe^-\bar{\nu}$	0.718(15)	$0.2224 \pm 0.0034$
$\Sigma^- \rightarrow ne^-\bar{\nu}$	-0.340(17)	$0.2282 \pm 0.0049$
$\Xi^- \rightarrow \Lambda e^-\bar{\nu}$	0.25(5)	$0.2367 \pm 0.0099$
$\Xi^0 \rightarrow \Sigma^+ e^-\bar{\nu}$	$1.32^{+0.22}_{-0.18}$	$0.209 \pm 0.027$
Combined	—	$0.2250 \pm 0.0027$

Could something have been wrong with  $K_{\ell 3}$ , theory or experiment?

## $V_{us}$ from $K_{\ell 3}$ decays.

Until 2002  $K_{\ell 3}$  decays seemed to point to a lower value for  $V_{us}$  than required by unitarity. The discrepancy started to clear in 2003 with new results from KTeV and then NA48. The most complete results come from the KLOE experiment in Frascati. Making use of both  $K_{\mu\nu}$  and  $K_{\ell 3}$ , and a new determination of the  $K^+$  lifetime, as well as the most recent Lattice computations of SU(3) breaking effects, KLOE obtains

$$|V_{us}| = 0.2249 \pm 0.0010$$

$$1 - |V_{us}|^2 - |V_{ud}|^2 = 0.0004 \pm 0.0007 (\sim 0.6 \sigma)$$

There is now no hint of a violation of unitarity at the 0.1% level!

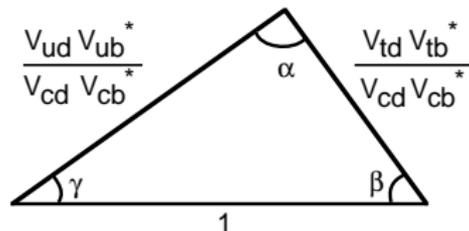
# The Unitarity Triangle

The unitarity relation,

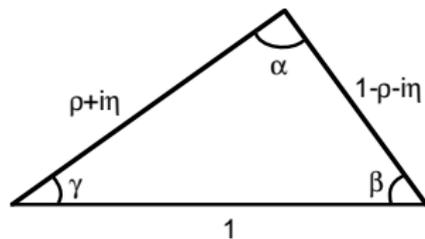
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

can be represented as a triangle in the complex plane:

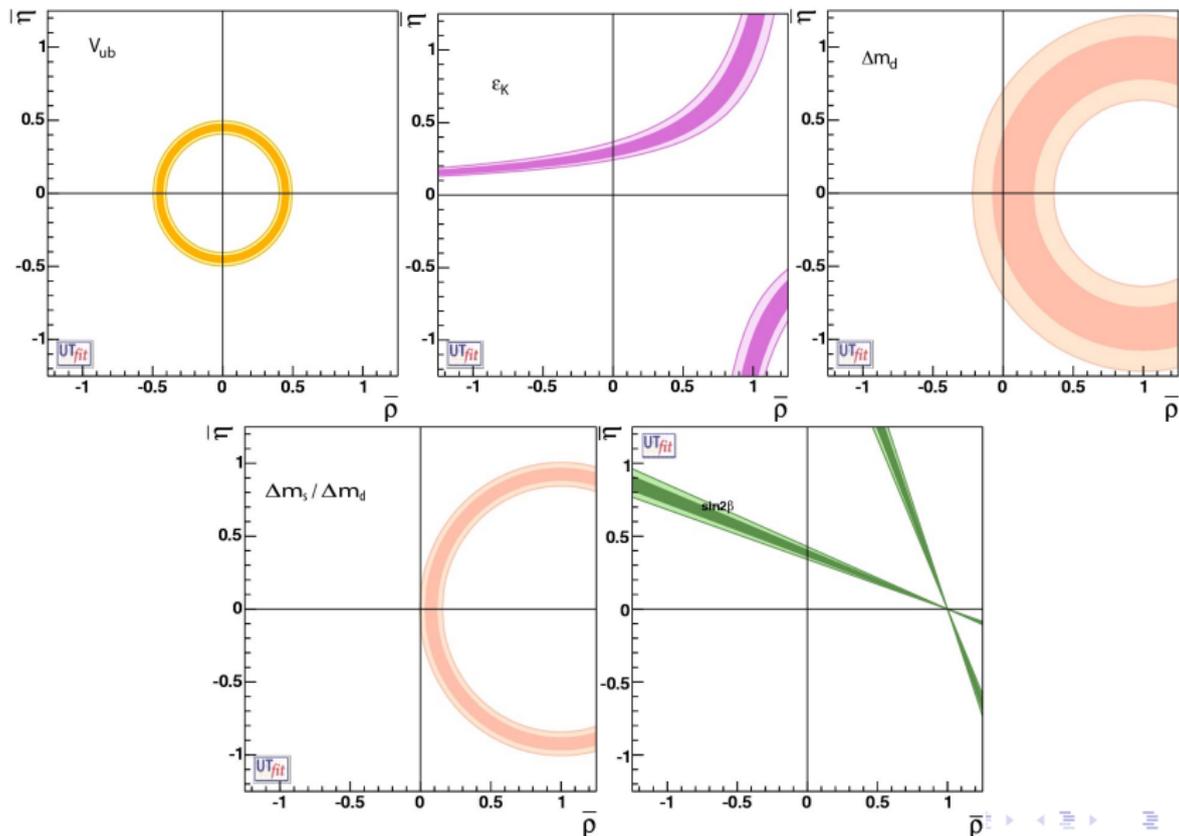
$\beta$  is the phase of  $V_{td}$ ,  
 $\gamma$  is the phase of  $V_{ub}^*$ .



This relation is trivially satisfied in the  $\rho/\eta$  parametrization:



# Determinations of the UT

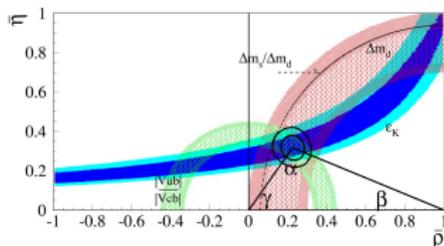


# The role of Lattice Gauge Theory

## Lattice QCD and the Unitarity Triangle

Three of the five determinations of the UT parameters depend in a critical way from Lattice QCD results.

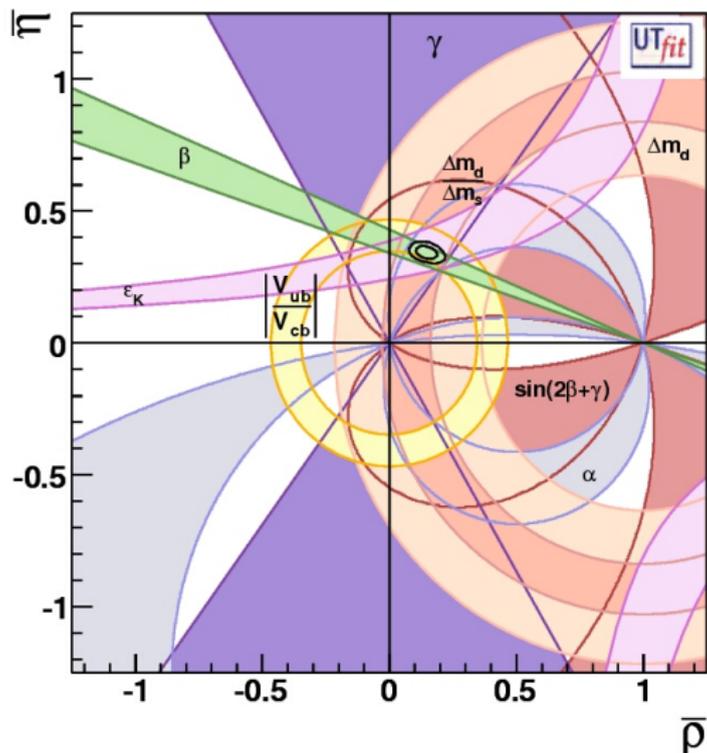
We would like measurements that are as far as possible independent from details of the hadron physics. The answer:  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ .



Measurement	$V_{CKM} \times \text{other}$	Constraint
$b \rightarrow u/b \rightarrow c$	$ V_{ub}/V_{cb} ^2$	$\bar{\rho}^2 + \bar{\eta}^2$
$\Delta m_d$	$ V_{td} ^2 f_{B_d}^2 B_{B_d} f(m_t)$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\frac{ V_{td} ^2 f_{B_d}^2 B_{B_d}}{ V_{ts} ^2 f_{B_s}^2 B_{B_s}}$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
$\epsilon_K$	$f(A, \bar{\eta}, \bar{\rho}, B_K)$	$\propto \bar{\eta}(1 - \bar{\rho})$

A. Stocchi, from analysis by M. Ciuchini et al.

# Putting all Together



# CP Violation is encoded in the mass matrix

Cecilia Jarslog's relation:

$$\det[M, M'] = i FF' J$$

Where:

$$F = (m_t - m_u)(m_t - m_c)(m_c - m_u)$$

$$F' = (m_b - m_s)(m_b - m_d)(m_s - m_d)$$

$$J \propto \text{Area of the Unitarity Triangle}$$

The mass matrices must contain complex numbers!.

...but in a gauge theory mass arises from the Higgs Mechanism...

# Quark Mixing in the Standard Model

The Higgs Boson and Symmetry Breaking (single Higgs):

$$\langle 0 | \phi | 0 \rangle = v$$

Higgs Boson Couplings:

$$\mathcal{L}_M = \frac{\phi}{v} [\bar{u}_R M u_L + \bar{d}_R M' d_L] + h.c.$$

In the Standard model we need complex Higgs coupling constants: the Higgs couplings directly break CP. More elegant alternatives to this simplest scheme — e.g. spontaneous breaking of CP symmetry would directly impact FCNC (Flavour Changing Neutral Currents), and the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  decays. [At a visible level?](#)

$$K^+ \rightarrow \pi^+ \nu \bar{\nu} \quad \& \quad K_L \rightarrow \pi^0 \nu \bar{\nu}$$

Given  $G_I^{(L,+)}$ , the branching ratios are directly related by isospin to that of the  $K_{e3}^+$  decay,

$$B(K^+ \rightarrow \pi^+ \bar{\nu} \nu) = 6 r_{K^+} B(K^+ \rightarrow \pi^0 e^+ \nu) \frac{|G_I^+|^2}{G_F^2 |V_{us}|^2} \quad (1)$$

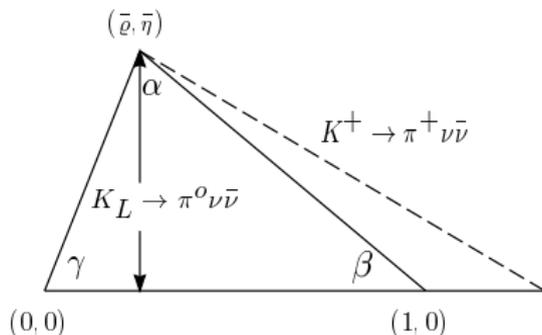
$$B(K_L \rightarrow \pi^0 \bar{\nu} \nu) = 6 \frac{\tau_{K_L}}{\tau_{K^+}} r_{K_L} B(K^+ \rightarrow \pi^0 e^+ \nu) \frac{(\text{Im } G_I^L)^2}{G_F^2 |V_{us}|^2} \quad (2)$$

$r_{K^+} = 0.901$  and  $r_{K_L} = 0.944$  are isospin breaking corrections (W.J. Marciano and Z. Parsa, – 1996) that include phase space and QED effects.

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are “Golden Channels”

These decays are sensitive to high energy ( $> M_t$ ) phenomena and to New Physics.

# $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , $K_L \rightarrow \pi^0 \nu \bar{\nu}$ , and the Unitarity Triangle



Theoretical errors in  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  are  $\sim 5 \div 7\%$ .

A combination of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and the  $\sin(2\beta)$  measurement in  $B^0 \rightarrow \Psi K_S$  would determine completely the unitarity triangle without any recourse to lattice gauge theory.

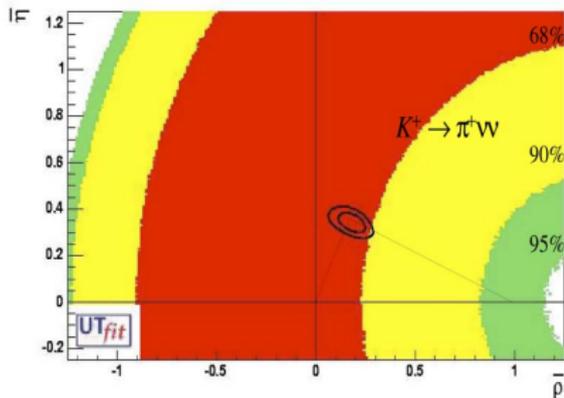
The uncertainties are even less for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ , whose measurement offers a direct determination of the area  $\eta/2$  of the unitarity triangle.

# The present situation in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The E787 and E949 experiments at Brookhaven have identified three events of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , and the branching ratio is somewhat high, although still compatible with the Standard model prediction.

$$\text{BR}(K^+)^{\text{exp}} = (1.47_{-0.9}^{+1.9}) \cdot 10^{-10}$$

E787+E949 [BNL]



A 100-event experiment as proposed at CERN (P-326) would reach the region where New Physics effects could emerge.

# Is New Physics already seen?

We will know...

## FIRST EVIDENCE OF NEW PHYSICS IN $b \leftrightarrow s$ TRANSITIONS (UTfit Collaboration)

M. Bona,<sup>1</sup> M. Ciuchini,<sup>2</sup> E. Franco,<sup>3</sup> V. Lubicz,<sup>2,4</sup> G. Martinelli,<sup>3,5</sup> F. Parodi,<sup>6</sup> M. Pierini,<sup>1</sup>  
P. Roudeau,<sup>7</sup> C. Schiavi,<sup>6</sup> L. Silvestrini,<sup>3</sup> V. Sordini,<sup>7</sup> A. Stocchi,<sup>7</sup> and V. Vagnoni<sup>8</sup>

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We combine all the available experimental information on  $B_s$  mixing, including the very recent tagged analyses of  $B_s \rightarrow J/\psi\phi$  by the CDF and DØ collaborations. We find that the phase of the  $B_s$  mixing amplitude deviates more than  $3\sigma$  from the Standard Model prediction. While no single measurement has a  $3\sigma$  significance yet, all the constraints show a remarkable agreement with the combined result. This is a first evidence of physics beyond the Standard Model. This result disfavors New Physics models with Minimal Flavour Violation with the same significance.

# Lepton mixing: the last frontier

Two different kinds of firmly established neutrino oscillations confirm Bruno Pontecorvo's hypothesis of a lepton mixing fully analogous to quark mixing. The "texture" of the mixing is very different from that of quark mixing, with two large mixing angles.

One would expect the lepton mixing matrix  $\mathbf{U}$  to have complex matrix elements which would then lead to CP and T breaking in neutrino oscillations, while one would not expect a violation of CPT,

$$P(\nu_a \Rightarrow \nu_b) \neq P(\bar{\nu}_a \Rightarrow \bar{\nu}_b) \quad \text{CP violation}$$

$$P(\nu_a \Rightarrow \nu_b) \neq P(\nu_b \Rightarrow \nu_a) \quad \text{T violation}$$

$$P(\nu_a \Rightarrow \nu_b) \neq P(\bar{\nu}_b \Rightarrow \bar{\nu}_a) \quad \text{CPT violation}$$

Can CP violation be detected in neutrino oscillations? It all depends on the size of the third mixing angle, known to be small, but not yet determined.

The End