Lifetimes and mixing parameters of heavy hadrons

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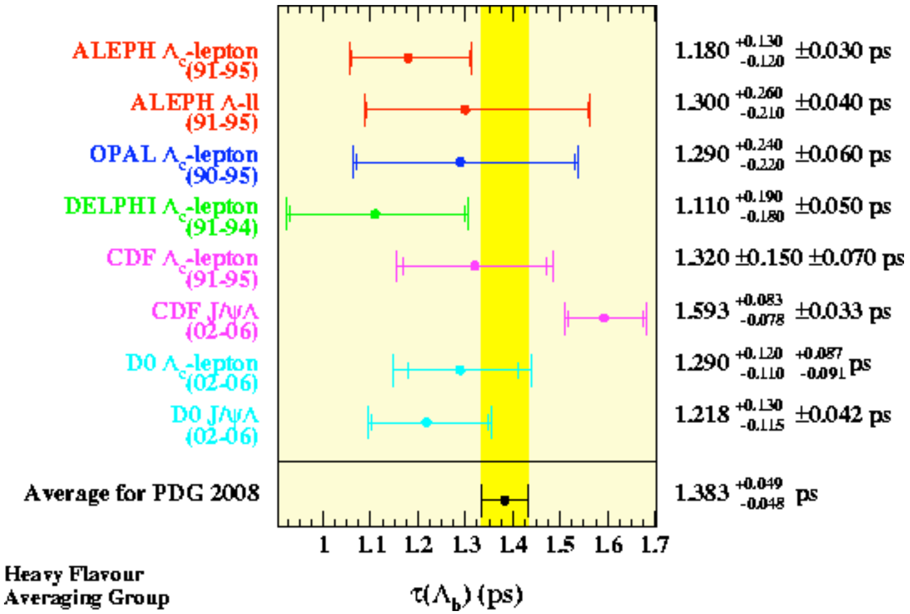
- Mixing parameters in heavy quark expansion
- New Physics in $\Delta\Gamma_{Bs}$
- Conclusions and outlook

1. Introduction: why do we care?

1. Nice test of our understanding of non-perturbative effects in QCD
2. One of the few unambiguous theoretical predictions that are "easy" to test experimentally
3. Theoretical uncertainty can be estimated: precision studies

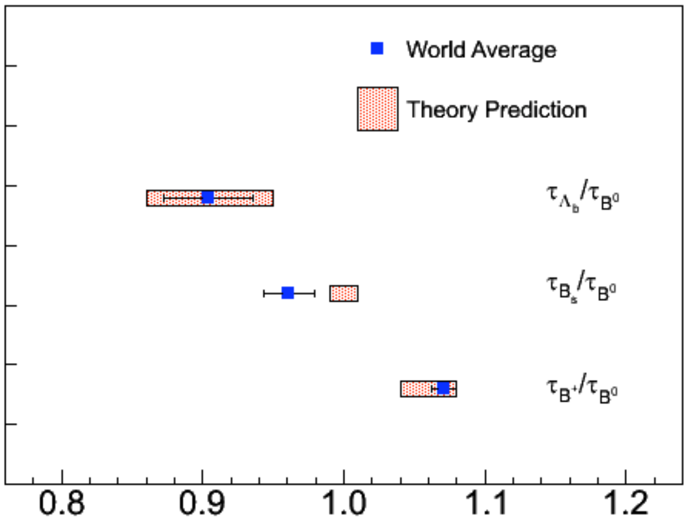


$$\Gamma(H_b) = \frac{G_F^2 m_Q^5}{192\pi^3}\left[A_0 + \frac{A_2}{m_Q^2} + \frac{A_2}{m_Q^3} + \ldots\right]$$

How good are the theoretical predictions?

2. Heavy hadron lifetimes

Not surprisingly, heavy hadron lifetimes were thoroughly measured…



C.Liu, FPCP 2008

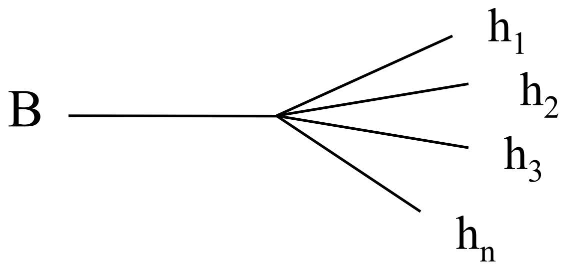
Theoretical predictions?

b hadron species	average lifetime, $\tau(H_b)$, ps	$\tau(H_b)/\tau(B^0)$
B^0	1.530 ± 0.009	1
B^+	1.638 ± 0.011	1.071 ± 0.009
B_s	$1.470^{+0.026}_{-0.027}$	0.961 ± 0.018
B_c	0.463 ± 0.071	
Λ_b	$1.383^{+0.049}_{-0.048}$	0.904 ± 0.032
Ξ_b^-, Ξ_b^0 mixture	$1.42^{+0.28}_{-0.24}$	
b-baryon mixture	$1.319^{+0.039}_{-0.38}$	0.862 ± 0.026
b-hadron mixture	1.568 ± 0.009	

PDG 2008/HFAG 2008

Theoretical framework

- Consider inclusive decay of a heavy hadron



$$\Gamma_{hadron}\left(H_b\right) = \sum_{\substack{all\ final\ state \\ hadrons}} \Gamma\left(H_b \to h_i\right)$$

This is NOT how we compute lifetimes!

- Use optical theorem to relate width to forward matrix element

M. Shifman, M. Voloshin,

$$\Gamma_{quark}\left(H_b\right) = \frac{1}{2M_b}\left\langle H_b \left| T \right| H_b \right\rangle = \frac{1}{2M_b}\left\langle H_b \left| \mathrm{Im}\, i \int d^4x\, T\left\{ H_{eff}^{\Delta B=1}\left(x\right) H_{eff}^{\Delta B=1}\left(0\right)\right\} \right| H_b \right\rangle$$

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum \left[c_1 Q_1^{u'd'} + c_2 Q_2^{u'd'}\right] + \mathrm{H.c.},$$

$$Q_1^{u'd'} = \bar{d}'_L \gamma_\mu u'_L \bar{c}_L \gamma^\mu b_L\,, \quad Q_2^{u'd'} = \bar{c}_L \gamma_\mu u'_L \bar{d}'_L \gamma^\mu b_L\,.$$

Theoretical framework

- What is the relation of Γ_{quark} to Γ_{hadron}?

E. Poggio, H. Quinn, S. Weinberg,
M. Shifman et al, B. Grinstein et al

$$\Gamma_{hadron}\left(H_b\right) = \sum_{\substack{all\ final\ state\\ hadrons}} \Gamma\left(H_b \to h_i\right)$$

$$\Updownarrow$$

$$\Gamma_{quark}\left(H_b\right) = \frac{1}{2M_b}\left\langle H_b \left| T \right| H_b \right\rangle$$

$$\Gamma_{quark}\left(H_b\right) = \Gamma_{hadron}\left(H_b\right)$$

Quark-hadron duality (local)

- Notes aside:
  1. Compute T in Eucledian space and analytically continue to Minkowski space [exact calculation in ES = exact result in MS]
  2. Expand T in α_S and "$1/Q \sim 1/m_Q$": series truncation
  3. Any deviation beyond "natural uncertainty" is treated as <u>violation of quark-hadron duality</u> [resonances, instantons,…]

This definition is due to M. Shifman

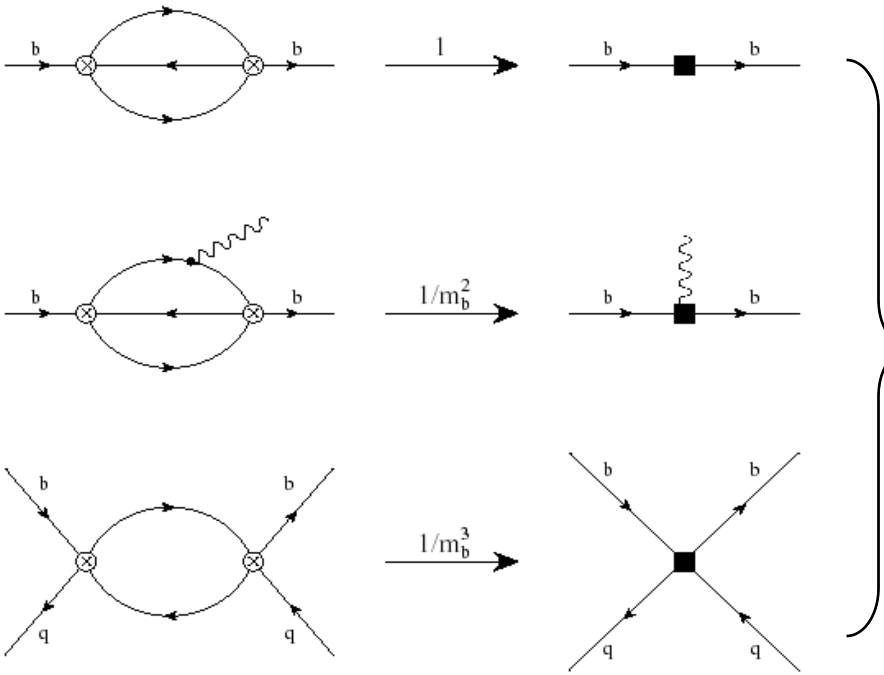
Theoretical expectations

- Assume quark-hadron duality: relate width to forward matrix element

$$\Gamma(H_b) = \frac{1}{2M_b}\langle H_b | T | H_b \rangle = \frac{1}{2M_b}\langle H_b | \mathrm{Im}\, i \int d^4x\, T\left\{ H_{eff}^{\Delta B=1}(x) H_{eff}^{\Delta B=1}(0) \right\} | H_b \rangle$$

- This correlator can be expanded using OPE

I. Bigi, M. Shifman, A. Vainshtein, M. Voloshin, N. Uraltsev, A. Falk, A. Manohar, M. Wise, M. Neubert, C. Sachrajda, P. Colangelo, F. de Fazio, …

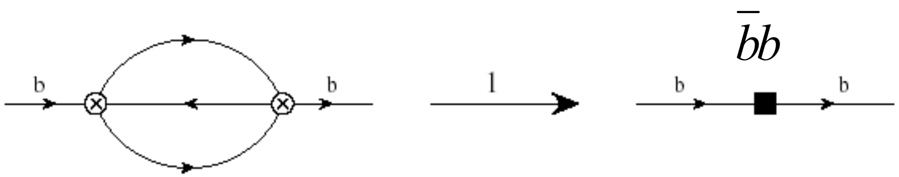


$$\Gamma(H_b) = \sum_k \frac{C_k(\mu)}{m_b^k}\langle H_b | O_k^{\Delta B=0}(\mu) | H_b \rangle$$

What are the results?

Leading order

- Leading order (in $1/m_b$ expansion)



$$\tau\left(B^+\right)=\tau\left(B^0\right)=\tau\left(\Lambda_b\right)=1/\Gamma\left(H_b\right),$$

$$\Gamma\left(H_b\right)=\eta c_3 \frac{G_F^2 m_b^5}{192\pi^3}$$

Must include explicit spectator interaction to see the differences in lifetimes of different hadrons…

$$\eta = \langle H_b | \bar{b}b | H_b \rangle \equiv \langle \bar{b}b \rangle_{H_b} \xrightarrow{m_b \to \infty} 1$$

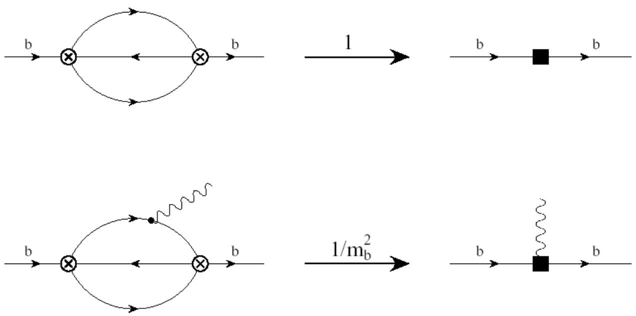
- Subleading $1/m_b$ corrections? No!

$$\langle H_b | \bar{b} i \not{D} b | H_b \rangle \xrightarrow{Eq.of\ Motion} \langle H_b | m_b \bar{b}b | H_b \rangle$$

Dimension 4 operators are eliminated through equations of motion

Subleading orders – $1/m_b^2$ corrections

- Subleading $1/m_b^2$ corrections



$$\Gamma(H_b) = \frac{G_F^2 m_b^5}{192\pi^3}\left[c_3 \langle \bar{b}b \rangle_{H_b} + c_5 \frac{\langle g_s \bar{b}\sigma_{\mu\nu} G^{\mu\nu} b \rangle_{H_b}}{m_b^2} \right],$$

These matrix elements can be systematically expanded in $1/m_b$

$$\langle \bar{b}b \rangle_{H_b} = 1 - \frac{1}{2m_b^2}\left[\mu_\pi^2(H_b) - \mu_G^2(H_b) \right] + O\left(1/m_b^2\right),$$

$$\langle g_s \bar{b}\sigma_{\mu\nu} G^{\mu\nu} b \rangle_{H_b} = 2\mu_G^2(H_b) + O\left(1/m_b\right)$$

$$\mu_\pi^2(H_b) = \frac{1}{2m_b^2}\langle H_b(v) | \bar{b}_v \left(i\vec{D}\right)^2 b_v | H_b(v) \rangle,$$

$$\mu_G^2(H_b) = \frac{1}{2m_b^2}\langle H_b(v) | \bar{b}_v \frac{g_s}{2}\sigma_{\mu\nu} G^{\mu\nu} b_v | H_b(v) \rangle$$

HQET matrix elements

$$\mu_\pi^2(B) = (0.4 \pm 0.2)\, GeV^2,$$

$$\mu_\pi^2(B) - \mu_\pi^2(\Lambda_b) = (0.01 \pm 0.03)\, GeV^2,$$

$$\mu_G^2(B) = \frac{3}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.36\, GeV^2,$$

$$\mu_G^2(\Lambda_b) = 0.$$

$$\tau(B^+) = \tau(B^0)$$

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 1 + \frac{1}{2m_b^2}\left[\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B^0)\right] + \frac{C_G}{m_b^2}\left[\mu_G^2(\Lambda_b) - \mu_G^2(B^0)\right]$$

About 1-2% effect…

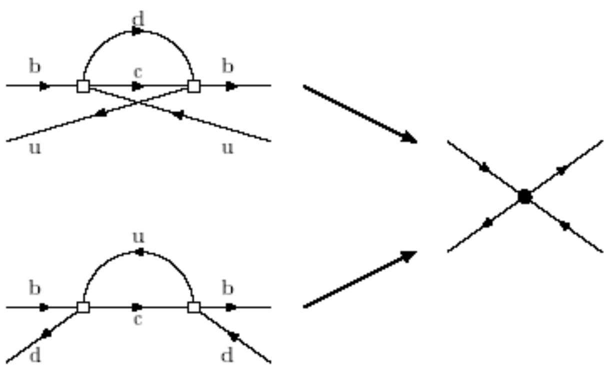
Is there anything else???

"Natural" uncertainty: $O\left(\Lambda^3/m_b^3\right) \sim 0.9\,\%$

Subleading orders – main effect?

- Subset of subleading $1/m_b^3$ corrections

$$\Gamma(H_b) = \sum_k \frac{C_k(\mu)}{m_b^3} \langle H_b | O_k^{\Delta B=0}(\mu) | H_b \rangle$$



$$O^q = \bar{b}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu b_L, \qquad O_S^q = \bar{b}_R q_L \bar{q}_R b_L,$$
$$T^q = \bar{b}_L \gamma_\mu t^a q_L \bar{q}_L \gamma^\mu t^a b_L, \quad T_S^q = \bar{b}_R t^a q_L \bar{q}_R t^a b_L$$

For the mesons…

$$\frac{1}{2m_{B_q}} \langle B_q | Q^q | B_q \rangle = \frac{f_{B_q}^2 m_{B_q}}{8} B_1, \quad \frac{1}{2m_{B_q}} \langle B_q | Q_S^q | B_q \rangle = \frac{f_{B_q}^2 m_{B_q}}{8} B_2$$

$$\frac{1}{2m_{B_q}} \langle B_q | T^q | B_q \rangle = \frac{f_{B_q}^2 m_{B_q}}{8} \varepsilon_1, \quad \frac{1}{2m_{B_q}} \langle B_q | T_S^q | B_q \rangle = \frac{f_{B_q}^2 m_{B_q}}{8} \varepsilon_2$$

Two intermediate quarks: $16\pi^2$ enhanced

… and for the baryons

$$\langle \Lambda_b | O_1^q | \Lambda_b \rangle = -\tilde{B} \langle \Lambda_b | \tilde{O}_1^q | \Lambda_b \rangle = \frac{\tilde{B}}{6} f_{B_q}^2 m_{B_q} m_{\Lambda_b} r,$$

As a result: $$\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.98 - \left(d_1 + d_2 \bar{B}\right) r - \left(d_3 \varepsilon_1 + d_4 \varepsilon_2\right) - \left(d_5 B_1 + d_6 B_2\right)$$

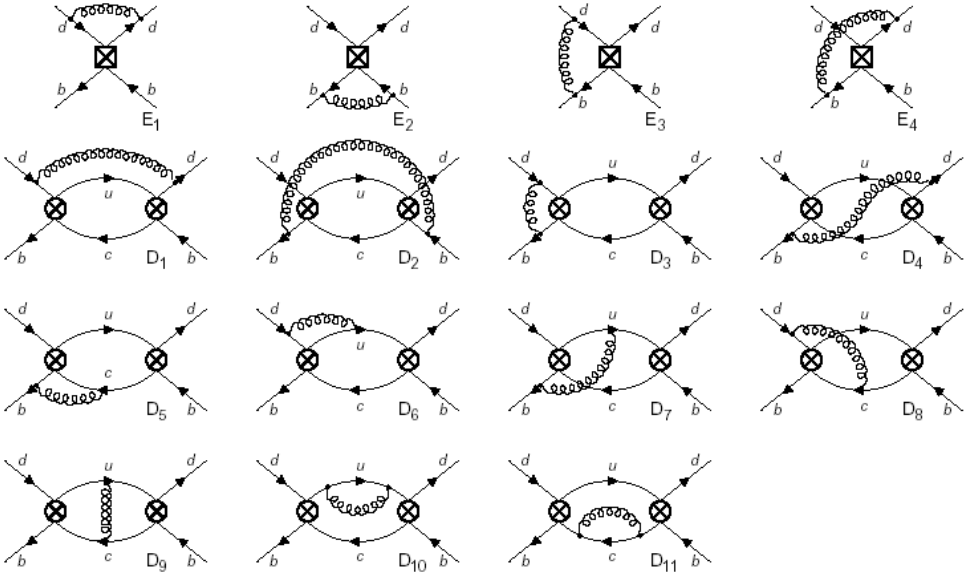
About 5-8% effect?

Effects of radiative corrections

- Numerical studies reveal "accidental" cancellations:

$$\frac{\tau(B^+)}{\tau(B_d^0)} - 1 = \tau(B^+)\left[\Gamma(B_d^0) - \Gamma(B^+)\right]$$
$$= 0.0325 \left(\frac{|V_{cb}|}{0.04}\right)^2 \left(\frac{m_b}{4.8\,\mathrm{GeV}}\right)^2 \left(\frac{f_B}{200\,\mathrm{MeV}}\right)^2 \times$$
$$\left[(1.0 \pm 0.2)\, B_1 + (0.1 \pm 0.1)\, B_2 - (18.4 \pm 0.9)\, \epsilon_1 + (4.0 \pm 0.2)\, \epsilon_2\right].$$

- Radiative corrections can be quite large:



M. Beneke et al
C. Franco et al

Radiative corrections enhance coefficients in front of $B_{1,2}$ ~$O(1)$!

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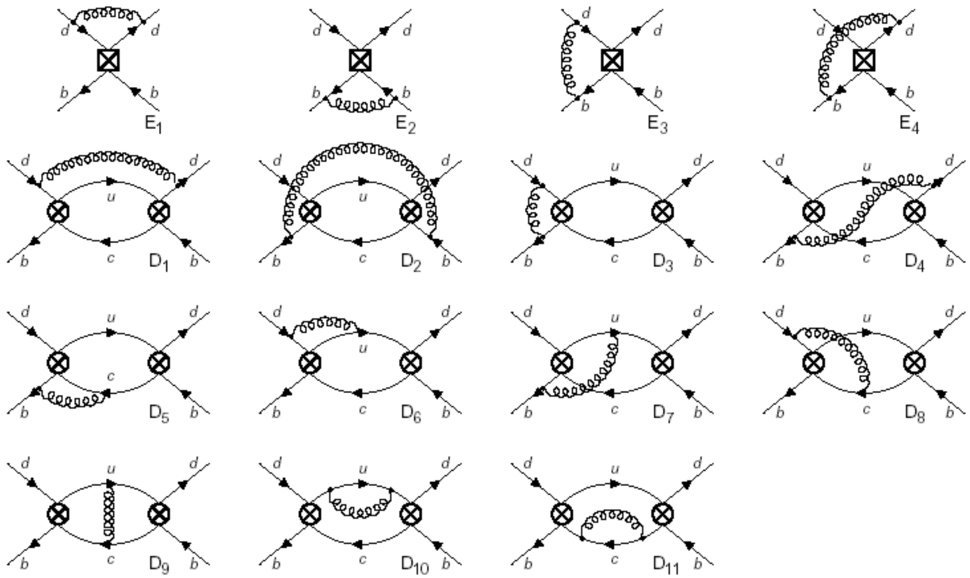

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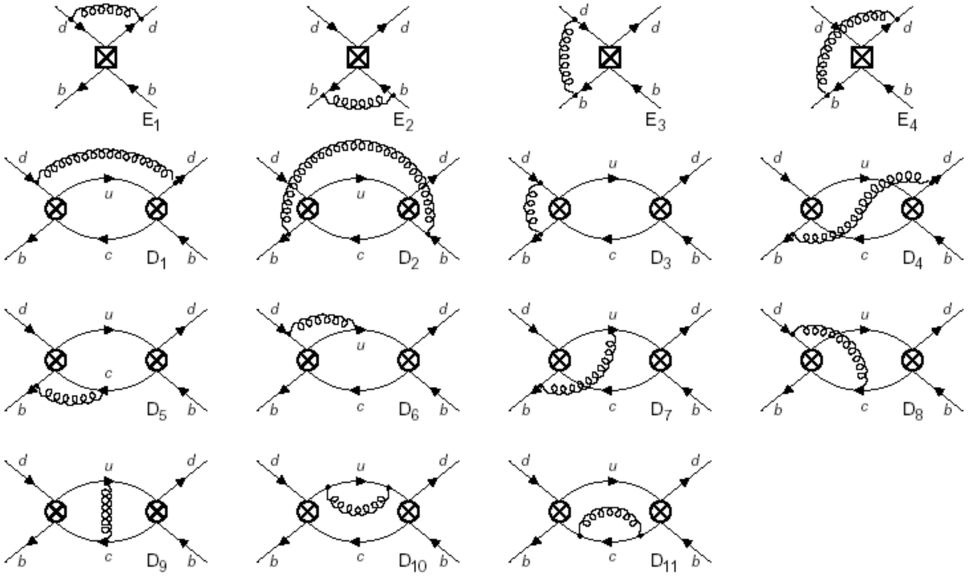

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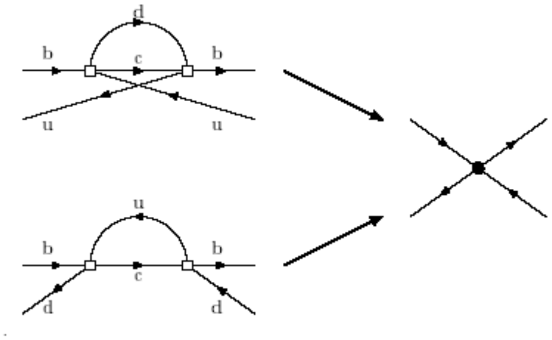
Spectator effects: punch line

- Look again at $1/m^3$ corrections



weak annihilation/scattering occurs for the bound quarks!

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.98 - m_b^2 \left(d_1^* + d_2^* \overline{B}\right) r - m_b^2 \left[\left(d_3^* \varepsilon_1 + d_4^* \varepsilon_2\right) - \left(d_5^* B_1 + d_6^* B_2\right)\right]$$

$$\Rightarrow 0.98 - m_{\Lambda_b}^2 \left(d_1^* + d_2^* \overline{B}\right) r - m_{B^0}^2 \left[\left(d_3^* \varepsilon_1 + d_4^* \varepsilon_2\right) - \left(d_5^* B_1 + d_6^* B_2\right)\right]$$

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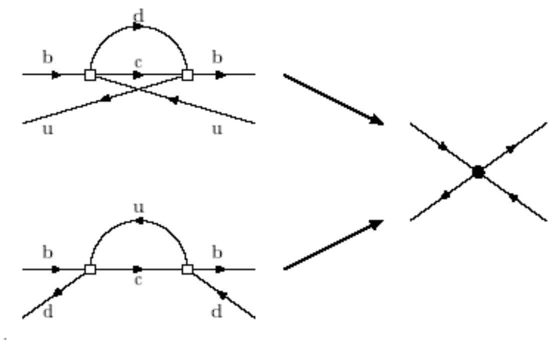

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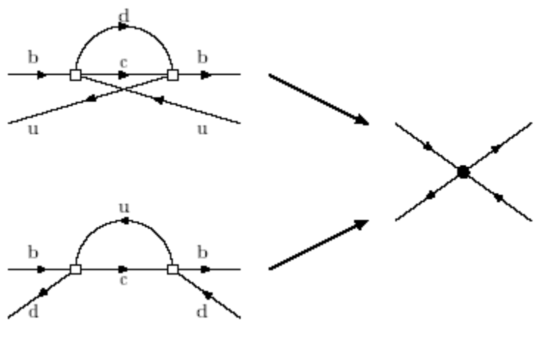
0.91 ↓ 0.89 **?**

Hint: go to higher order in $1/m_b$!

Spectator effects: punch line

- Look again at $1/m^3$ corrections



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$$\downarrow \quad ?$$

$$\Rightarrow 0.98 - m_{\Lambda_b}^2\left(d_1^* + d_2^*\overline{B}\right)r - m_{B^0}^2\left[\left(d_3^*\varepsilon_1 + d_4^*\varepsilon_2\right) - \left(d_5^* B_1 + d_6^* B_2\right)\right] \quad 0.89$$

Hint: go to higher order in $1/m_b$!

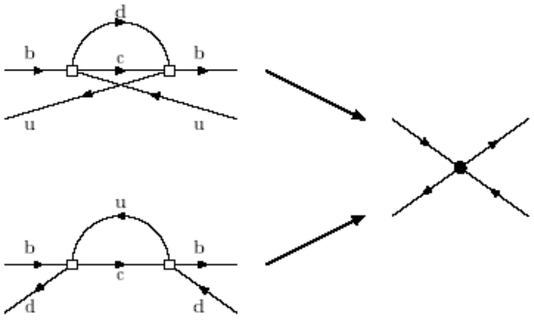
- Can this set of corrections change anything? Look at it once more…

$$\Gamma(\Lambda_b)_{1/m^3} \sim \frac{G_F^2}{2\pi}\left|\Psi(0)\right|_{bu}^2 \left|V_{ud}\right|^2 \left|V_{cb}\right|^2 m_b^2\left(1-z^2\right)\left[c_-^2 - (1-z)c_+\left(c_- - \frac{c_+}{2}\right)\right]$$

Weak annihilation **Pauli Interference**

Spectator effects: punch line

- Look again at $1/m^3$ corrections



weak annihilation/scattering
occurs for the bound quarks!

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.98 - m_b^2\left(d_1^* + d_2^* \overline{B}\right)r - m_b^2\left[\left(d_3^*\varepsilon_1 + d_4^*\varepsilon_2\right) - \left(d_5^* B_1 + d_6^* B_2\right)\right] \quad 0.91$$

$$\downarrow \quad ?$$

$$\Rightarrow 0.98 - m_{\Lambda_b}^2\left(d_1^* + d_2^* \overline{B}\right)r - m_{B^0}^2\left[\left(d_3^*\varepsilon_1 + d_4^*\varepsilon_2\right) - \left(d_5^* B_1 + d_6^* B_2\right)\right] \quad 0.89$$

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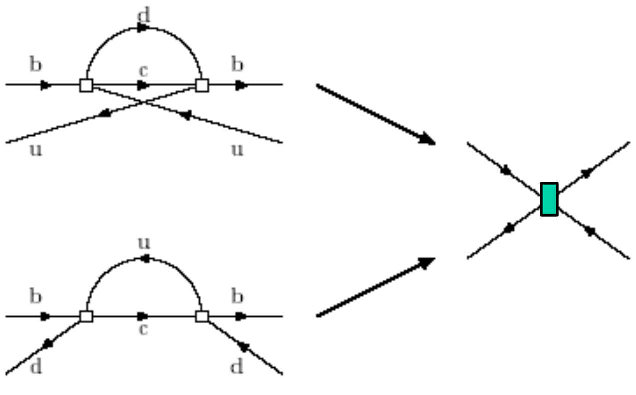
Weak annihilation ↑ **Pauli Interference**

Subleading spectator effects

- Compute subleading corrections to spectator effects

$$\Gamma_{spec,1/m}(H_b) = \frac{1}{2M_b}\langle H_b | T_{spec,1/m} | H_b \rangle$$



- Expand $T_{spec,\,1/m}$ in the light-quark momentum and match onto operators with derivatives…

$$T^{u}_{spec,1/m} = -2\left(c_1^2 + c_2^2\right)\frac{1+z}{1-z} R_1^u - c_1 c_2 \frac{1+z}{1-z}\overline{R}_1^u ,$$

$$T^{d'}_{spec,1/m} = c_1^2 \left[\frac{8z^2}{1-z} R_0^{d'} + \frac{2}{3}\frac{1+z+10z^2}{1-z} R_1^{d'} + \frac{2}{3}(1+2z)\left(R_2^{d'} - R_3^{d'}\right)\right]$$
$$+ \left(N_c c_1^2 + 2c_1c_2\right)\left[\frac{8z^2}{1-z}\overline{R}_0^{d'} + \frac{2}{3}\frac{1+z+10z^2}{1-z}\overline{R}_1^{d'} + \frac{2}{3}(1+2z)\left(\overline{R}_2^{d'} - \overline{R}_3^{d'}\right)\right],$$

$$T^{s'}_{spec,1/m} = c_1^2 \left[\frac{16z^2}{1-4z} R_0^{s'} + \frac{2}{3}\frac{1-2z+16z^2}{1-4z} R_1^{s'} + \frac{2}{3}(1+2z)\left(R_2^{s'} - R_3^{s'}\right)\right]$$
$$+ \left(N_c c_1^2 + 2c_1c_2\right)\left[\frac{16z^2}{1-4z}\overline{R}_0^{s'} + \frac{2}{3}\frac{1-2z+16z^2}{1-4z}\overline{R}_1^{s'} + \frac{2}{3}(1+2z)\left(\overline{R}_2^{s'} - \overline{R}_3^{s'}\right)\right].$$

Subleading spectator effects, cont.

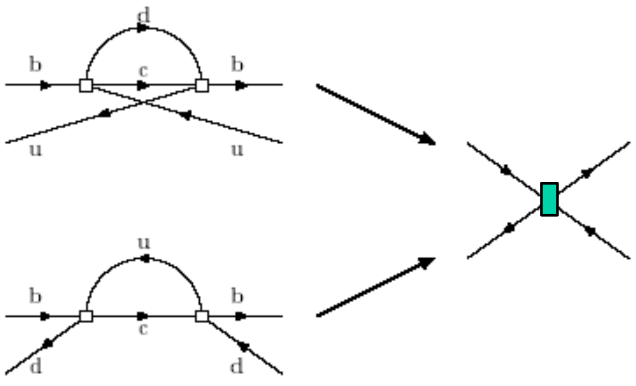
- … with the following set of operators…

$$R_0^q = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu \gamma_5 D^\alpha b_i \, \bar{q}_j \gamma_\mu (1-\gamma_5) D_\alpha q_j,$$

$$R_1^q = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1-\gamma_5) D^\alpha b_i \, \bar{q}_j \gamma_\mu (1-\gamma_5) D_\alpha q_j,$$

$$R_2^q = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1-\gamma_5) D^\alpha b_i \, \bar{q}_j \gamma_\alpha (1-\gamma_5) D_\mu q_j,$$

$$R_3^q = \frac{m_q}{m_b} \bar{b}_i (1-\gamma_5) b_i \, \bar{q}_j (1-\gamma_5) q_j .$$



- … with explicit power counting after taking matrix elements

$$\langle B_q | R_1^q | B_q \rangle = \langle B_q | \tilde{R}_1^q | B_q \rangle / N = \frac{\beta_1}{2N_c} f_{B_q}^2 m_{B_q}^2 \left[\frac{m_{B_q}^2}{m_b^2} - 1 \right],$$

$$\langle B_q | R_{2,3}^q | B_q \rangle = -\frac{\beta_{2,3}}{4N_c} f_{B_q}^2 (m_{B_q}^2 - m_b^2),$$

$$\langle B_q | \tilde{R}_{2,3}^q | B_q \rangle = -\frac{\beta_{2,3}}{4} f_{B_q}^2 (m_{B_q}^2 - m_b^2),$$

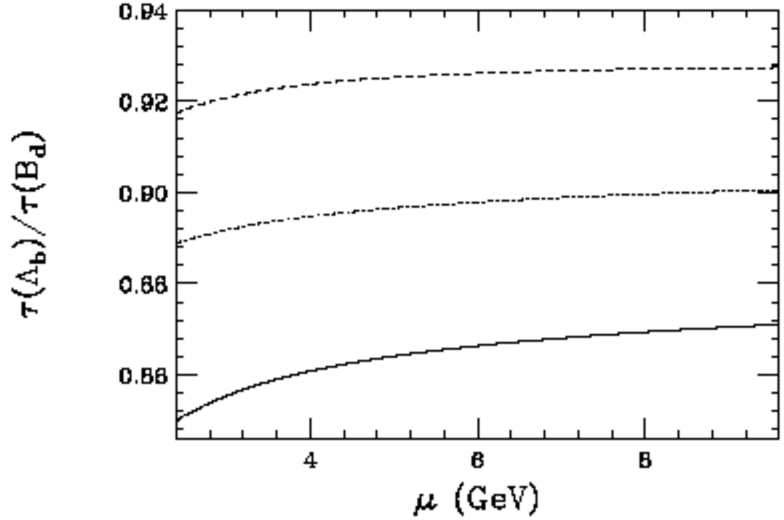
$$\langle \Lambda_b | R_1^q | \Lambda_b \rangle = -\langle \Lambda_b | \tilde{R}_1^q | \Lambda_b \rangle = -\frac{\tilde{\beta}_1}{24} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \left[\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right],$$

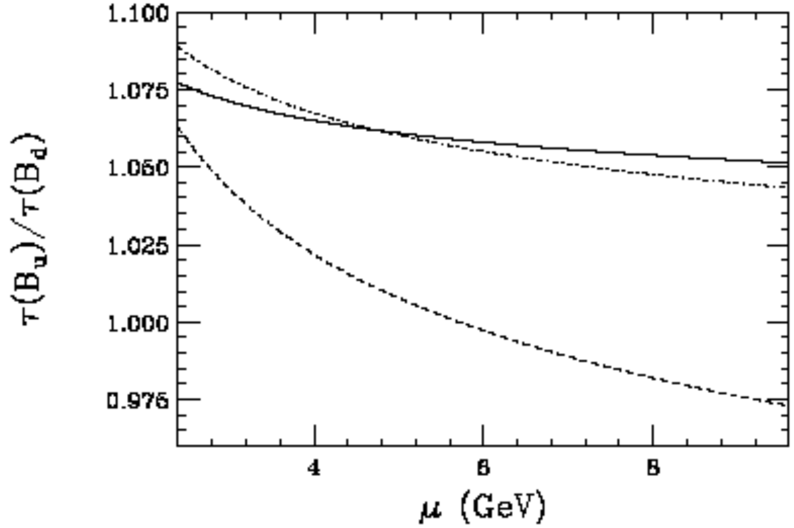
$$\langle \Lambda_b | R_2^q | \Lambda_b \rangle = -\langle \Lambda_b | \tilde{R}_2^q | \Lambda_b \rangle = -\frac{\tilde{\beta}_2}{48 m_b^2} f_{B_q}^2 \frac{m_{B_q}}{m_{\Lambda_b}} (m_{\Lambda_b}^4 - m_b^4),$$

$$\langle \Lambda_b | R_3^q | \Lambda_b \rangle = -\langle \Lambda_b | \tilde{R}_3^q | \Lambda_b \rangle = -\frac{\tilde{\beta}_3}{\tilde{\beta}_2} \langle \Lambda_b | R_2^q | \Lambda_b \rangle,$$

Subleading spectator effects

- As a result, the lifetime ratios become…



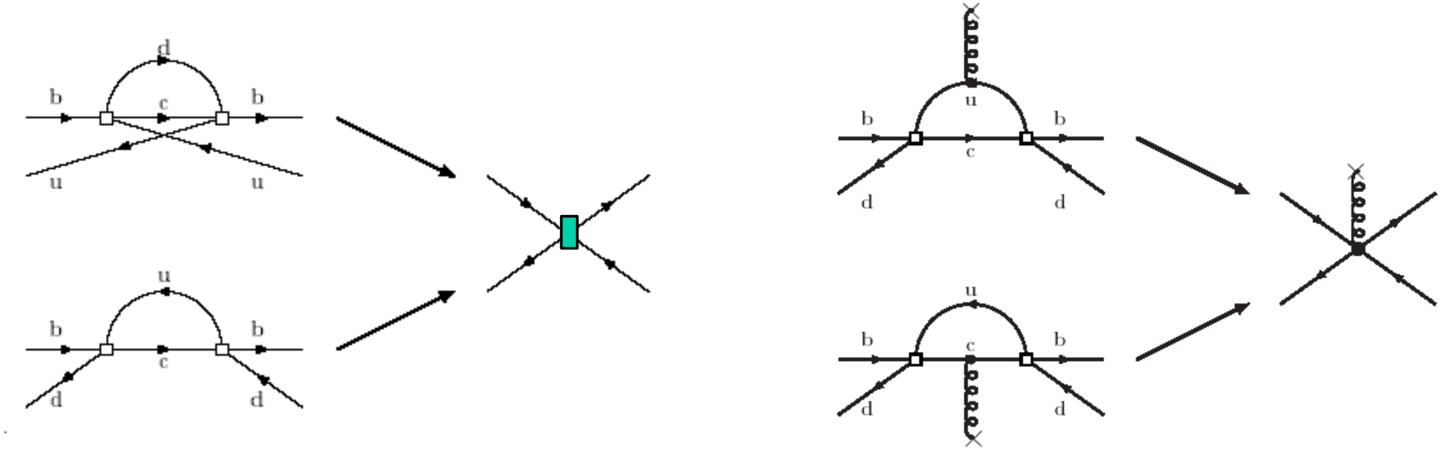


Dashed: LL, dash-dotted : pQCD NLL, solid: pQCD NLL + subleading spectator corrections

- The effect is <u>negligible</u> in meson ratio and is about –(2-4)% in baryon-meson ratio
  - ✓ no cancellation b/w WS and PI (enter with the same sign)
  - ✓ reduces baryon/meson ratio
- How "good" is this expansion? Let's estimate next term in 1/m…

Estimate of higher order effects

- Let's estimate convergence of expansion



- Expand one order further and add background gluon operator contributions entering at this order…

F. Gabbiani, A. Onishchenko, A.A.P.
Phys. Rev. D70, 094031 (2004)

Estimate of higher order effects

- … as a result, we get a collection of operators…

$$\delta^u_{1/m^2} = \left(c_1^2 + c_2^2\right)\left[\frac{m_u^2}{m_b^2}\frac{1+z}{1-z}O_1^u + \frac{8z^2}{(1-z)^2}W_1^u\right] + 2c_1c_2\left[\frac{m_u^2}{m_b^2}\frac{1+z}{1-z}\bar{O}_1^u + \frac{8z^2}{(1-z)^2}\bar{W}_1^u\right],$$

$$\delta^{d'}_{1/m^2} = c_1^2\left[\frac{m_{d'}^2}{m_b^2}\frac{1+z+2z^2}{1-z}O_1^{d'} + \frac{m_{d'}^2}{m_b^2}\frac{4z^2}{1-z}O_2^{d'} + \frac{m_{d'}^2}{m_b^2}\frac{2(1+2z)}{3}O_3^{d'}\right.$$
$$\left. + \frac{4z^2(7z-5)}{(1-z)^2}W_1^{d'} + \frac{8z^2(4z-3)}{(1-z)^2}W_2^{d'} + \frac{8z^2}{1-z}\left(W_3^{d'} - W_4^{d'}\right)\right]$$
$$+ \left(N_c c_2^2 + c_1c_2\right)\left[\frac{m_{d'}^2}{m_b^2}\frac{1+z+2z^2}{1-z}\bar{O}_1^{d'} + \frac{m_{d'}^2}{m_b^2}\frac{4z^2}{1-z}\bar{O}_2^{d'} + \frac{m_{d'}^2}{m_b^2}\frac{2(1+2z)}{3}\bar{O}_3^{d'}\right.$$
$$\left. + \frac{4z^2(7z-5)}{(1-z)^2}\bar{W}_1^{d'} + \frac{8z^2(4z-3)}{(1-z)^2}\bar{W}_2^{d'} + \frac{8z^2}{1-z}\left(\bar{W}_3^{d'} - \bar{W}_4^{d'}\right)\right],$$

$$\delta^{s'}_{1/m^2} = c_1\left[\frac{m_{s'}^2}{m_b^2}\frac{1-2z}{1-4z}O_1^{s'} + \frac{m_{s'}^2}{m_b^2}\frac{8z^2}{1-4z}O_2^{s'} + \frac{m_{s'}^2}{m_b^2}\frac{2(1+2z)}{3}O_3^{s'}\right.$$
$$\left. + \frac{8z^2(16z-5)}{(1-4z)^2}W_1^{s'} + \frac{16z^2(10z-3)}{(1-4z)^2}W_2^{s'} + \frac{16z^2}{1-4z}\left(W_3^{s'} - W_4^{s'}\right)\right]$$
$$+ \left(N_c c_2^2 + 2c_1c_2\right)\left[\frac{m_{s'}^2}{m_b^2}\frac{1-2z}{1-4z}\bar{O}_1^{s'} + \frac{m_{s'}^2}{m_b^2}\frac{8z^2}{1-4z}\bar{O}_2^{s'} + \frac{m_{s'}^2}{m_b^2}\frac{2(1+2z)}{3}\bar{O}_3^{s'}\right.$$
$$\left. + \frac{8z^2(16z-5)}{(1-4z)^2}\bar{W}_1^{s'} + \frac{16z^2(10z-3)}{(1-4z)^2}\bar{W}_2^{s'} + \frac{16z^2}{1-4z}\left(\bar{W}_3^{s'} - \bar{W}_4^{s'}\right)\right].$$

$$\mathcal{T}^u_{\mathrm{spec},G} = 0,$$

$$\mathcal{T}^{d'}_{\mathrm{spec},G} = -\frac{G_F^2|V_{bc}|^2}{4\pi}\left\{c_1^2\left[(1-z^2)P_1^{d'} - (1-z^2)P_2^{d'} + 2z(1-z)P_3^{d'} + 4z^2P_4^{d'}\right]\right.$$
$$\left. + 2c_1c_2z\left[(1-z)P_5^{d'} + (1-z)P_6^{d'} + 2zP_7^{d'} + 2zP_8^{d'}\right]\right\},$$

$$\mathcal{T}^{s'}_{\mathrm{spec},G} = -\frac{G_F^2|V_{bc}|^2}{4\pi\sqrt{1-4z}}\left\{c_1^2\left[(1-2z)P_1^{s'} - (1-4z)P_2^{s'} + 2zP_3^{s'} + 4zP_4^{s'}\right]\right.$$
$$\left. + 4c_1c_2z\left[P_7^{s'} + P_8^{s'} + P_9^{s'} + P_{10}^{s'}\right]\right\}.$$

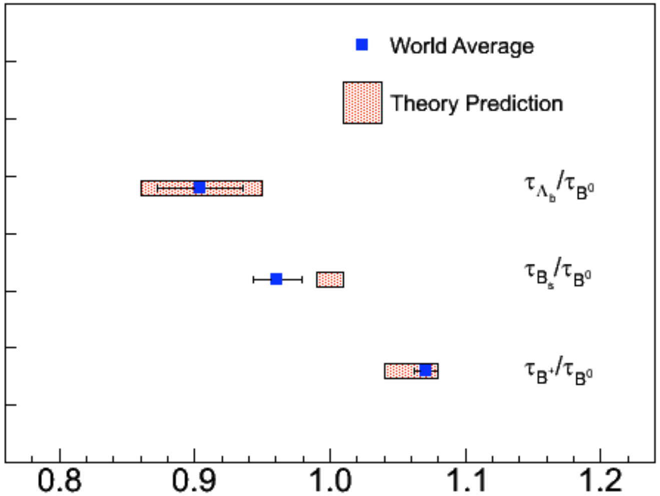
F. Gabbiani, A. Onishchenko, A.A.P.
Phys. Rev. D68, 114006 (2003)
Phys. Rev. D70, 094031 (2004)

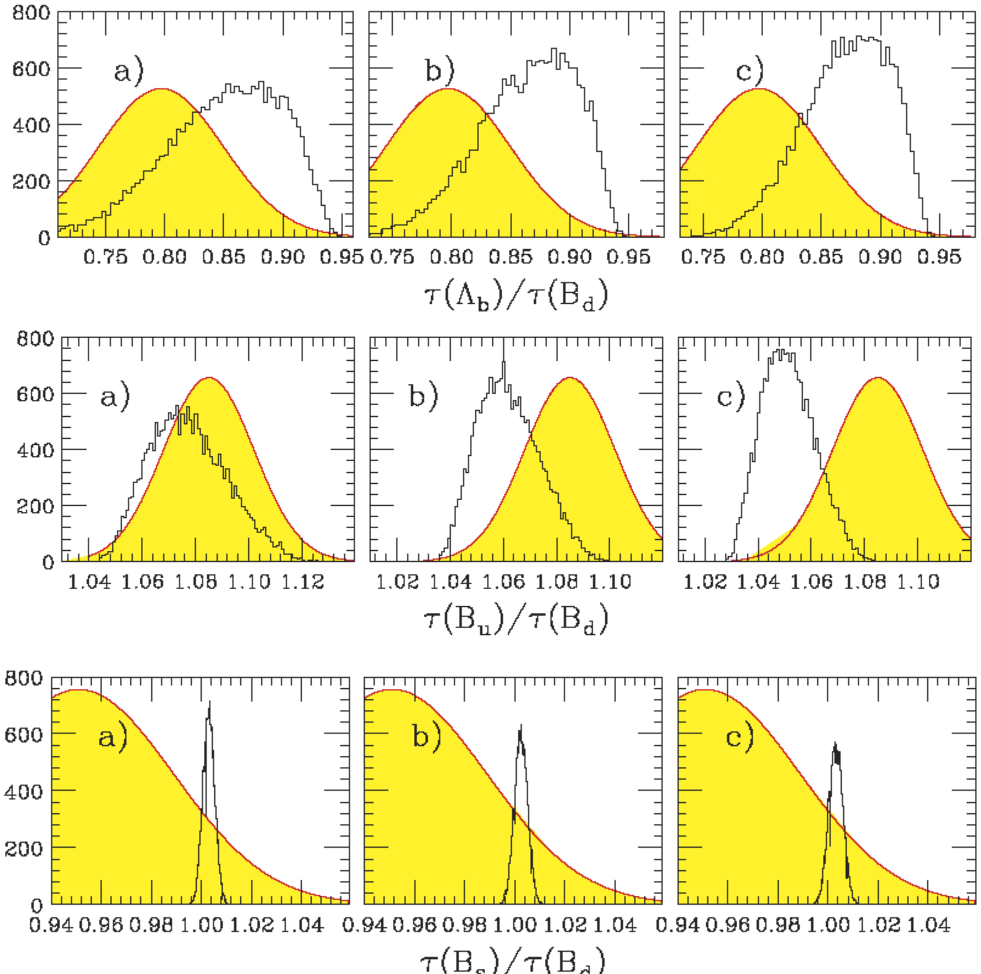
- … so estimating their matrix elements we obtain the answer!
  - ✓ Problem: too many matrix elements for meaningful answer
  - ✓ Solution: generate random values for parameters/matrix elements (±30% of "factorized" value)

Lifetimes: results

★ The expansion appears well convergent for b-quark

Lifetime ratio	Measured value	Predicted range
$\tau(B^+)/\tau(B^0)$	1.076 ± 0.008	1.04 – 1.08
$\overline{\tau}(B_s^0)/\tau(B^0)^a$	0.950 ± 0.019	0.99 – 1.01
$\tau(\Lambda_b^0)/\tau(B^0)$	0.912 ± 0.032	0.86 – 0.95





★ "Λ_b problem" is no longer a problem... ☺

★ ... but what's with the B_s lifetime?

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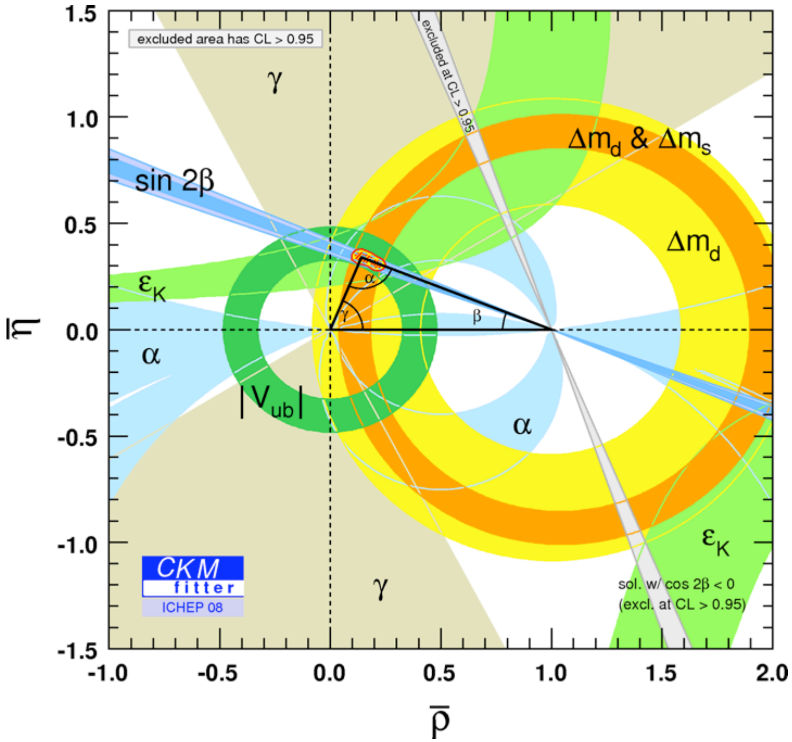
3. Mixing in heavy hadrons

Mixing parameters are sensitive probes of new physics



Theoretical predictions?

★ Time development of B_s system

$$i\frac{d}{dt}\begin{pmatrix} B_q(t) \\ \overline{B}_q(t) \end{pmatrix} = \left[M - \frac{i}{2}\Gamma\right]_{ij} \begin{pmatrix} B_q(t) \\ \overline{B}_q(t) \end{pmatrix}$$

★ Mixing parameters (concentrate on B_s)

$$\Delta M_{B_s} = 2\left|M_{12}\right|, \quad \Delta\Gamma_{B_s} = \frac{4Re\left(M_{12}\Gamma_{12}^*\right)}{\Delta M_{B_s}}$$

✦ NP in phase of ΔM_{Bs}:

$$\Delta\Gamma_{B_s} = 2\left|\Gamma_{12}\right| \cos 2\phi_s$$

$\arg(M_{12})$

✦ "direct" NP in $\Delta\Gamma_{Bs}$:

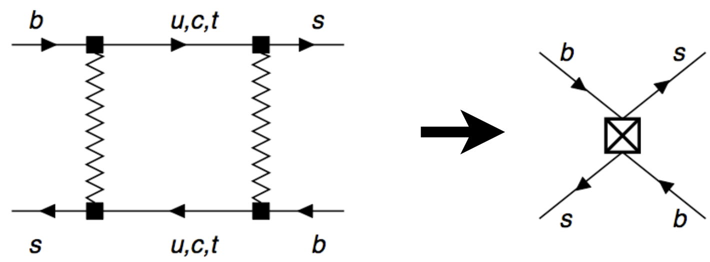
$$\Delta\Gamma_{B_s} = \Delta\Gamma_{B_s}^{SM} + \Delta\Gamma_{B_s}^{NP} \cos 2\phi_s'$$

$\arg(\Gamma_{12})$

Standard Model contributions

Both ΔM_{Bs} and $\Delta\Gamma_{Bs}$ can be computed in the limit $m_b \to \infty$:

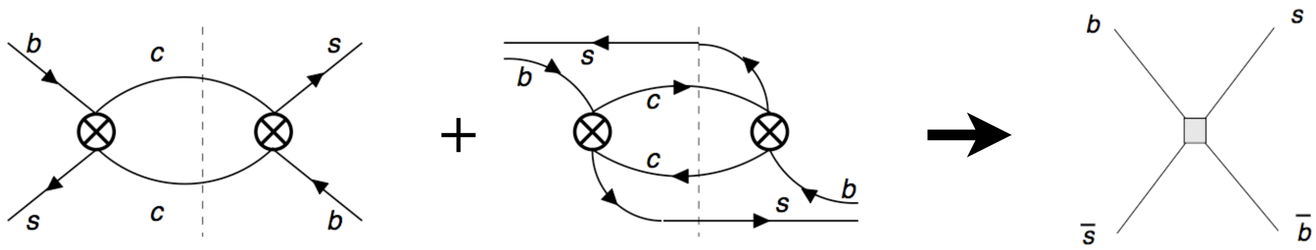
ΔM_{Bs}:



A.Buras, M.Jamin, P.Weisz

$$M_{12}(B_s) = \frac{G_F^2 M_{B_s}}{12\pi^2} M_W^2 \left(V_{tb}V_{ts}^*\right)^2 \hat{\eta}_B S_0(x_t) f_{B_s}^2 B$$

$\Delta\Gamma_{Bs}$:



$$\Gamma_{21}(B_s) = \sum_k \frac{C_k(\mu)}{m_b^k} \langle \overline{B}_s | \mathcal{O}_k^{\Delta B=2}(\mu) | B_s \rangle .$$

SM contributions to $\Delta\Gamma_{Bs}$

$\Delta\Gamma_{Bs}$: similar (to lifetimes) calculation yields:

$$\Gamma_{21}(B_s) = - \frac{G_F^2 m_b^2}{12\pi(2M_{B_s})}(V_{cb}^* V_{cs})^2 \left[[F(z)+P(z)] \langle Q \rangle + [F_S(z)+P_S(z)] \langle Q_S \rangle + \delta_{1/m} + \delta_{1/m^2} \right]$$

WC (incl. pQCD corr): Beneke et al, Ciuchini et al

★ ... with operators

$$Q = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \quad Q_S = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}$$

$$\tilde{Q} = (\bar{b}_i s_j)_{V-A}(\bar{b}_j s_i)_{V-A}, \quad \tilde{Q}_S = (\bar{b}_i s_j)_{S-P}(\bar{b}_j s_i)_{S-P}$$

$$\langle Q \rangle = 2\frac{1+N_c}{N_c} f_{B_s}^2 M_{B_s}^2 B$$

$$\langle Q_S \rangle = \frac{1-2N_c}{N_c}\frac{M_{B_s}^4}{(m_b+m_s)^2} f_{B_s}^2 B_S$$

★ ... so the result (up to $1/m_b^2$) is:

$$\begin{aligned} \Delta\Gamma_{B_s} = & \Big[0.0005B + 0.1732B_s + 0.0024B_1 - 0.0237B_2 - 0.0024B_3 - 0.0436B_4 \\ + & 2\times10^{-5}\alpha_1 + 4\times10^{-5}\alpha_2 + 4\times10^{-5}\alpha_3 + 0.0009\alpha_4 - 0.0007\alpha_5 \\ + & 0.0002\beta_1 - 0.0002\beta_2 + 6\times10^{-5}\beta_3 - 6\times10^{-5}\beta_4 - 1\times10^{-5}\beta_5 \\ - & 1\times10^{-5}\beta_6 + 1\times10^{-5}\beta_7 + 1\times10^{-5}\beta_8\Big] \quad (\text{ps}^{-1}). \end{aligned}$$

A.Badin, F. Gabbiani, A.A.P.
Phys. Lett. B653, 230 (2007)

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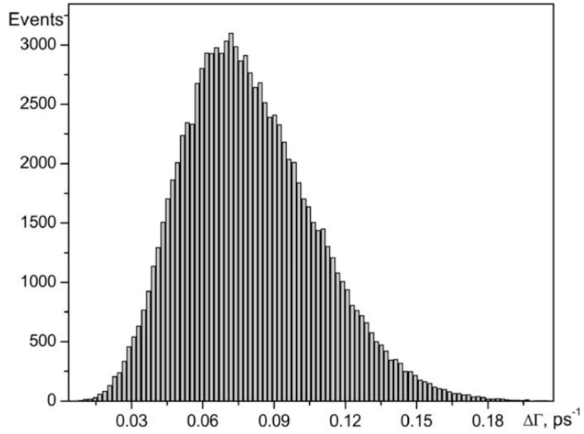
SM contributions to $\Delta\Gamma_{Bs}$

★ Varying unknown parameters:



★ Old 1998 result:

$$\left(\frac{\Delta\Gamma_s}{\Gamma_s}\right) = \left(\frac{f_{B_s}}{210\ \text{MeV}}\right)^2 [0.006\, B + 0.150\, B_S - 0.063]$$

- Small impact of $1/m_b^2$ corrections
- Large dependence on B_S !!!

★ Change LO operator basis:

$$\begin{aligned} Q &= (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \\ Q_S &= (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}. \end{aligned}$$

→

$$\begin{aligned} Q &= (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \\ \tilde{Q}_S &= (\bar{b}_i s_j)_{S-P}(\bar{b}_j s_i)_{S-P}, \end{aligned}$$

$$R_0 = Q_S + \alpha_1 \widetilde{Q}_S + \frac{\alpha_2}{2} Q$$

$\mathcal{O}(1/m_b)$

Idea: try to avoid accidental cancellations in perturbative coefficient of B

A. Lenz, U. Nierste, JHEP 06, 072 (2007)

SM contributions to $\Delta\Gamma_{Bs}$

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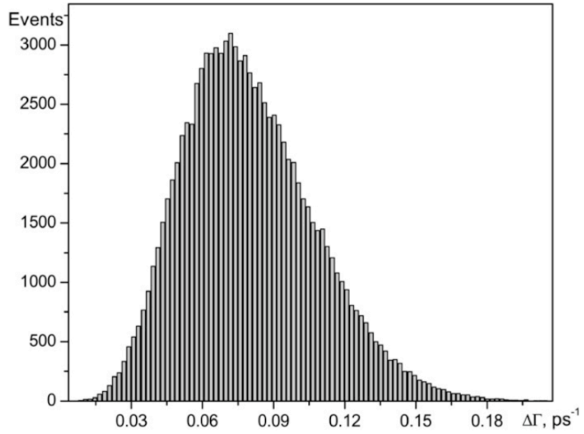

★ Old 1998 result:

M.Beneke, G.Buchalla, C.Greub, U.Nierste

$$\left(\frac{\Delta\Gamma_s}{\Gamma_s}\right) = \left(\frac{f_{B_s}}{210\text{ MeV}}\right)^2 [0.006\, B + 0.150\, B_S - 0.063]$$

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$$R_0 = Q_S + \alpha_1 \widetilde{Q}_S + \frac{\alpha_2}{2} Q$$

$\mathcal{O}(1/m_b)$

Idea: try to avoid accidental cancellations in perturbative coefficient of B

A. Lenz, U. Nierste, JHEP 06, 072 (2007)

SM contributions to $\Delta\Gamma_{Bs}$

★ Result in the new basis:

A. Lenz, U. Nierste, JHEP 06, 072 (2007)

$$\Delta\Gamma_s = \left(\frac{f_{B_s}}{240\,\mathbf{MeV}}\right)^2 \left[(0.105\pm0.016)B + (0.024\pm0.004)\tilde{B}'_S - \left((0.030\pm0.004)B_{\tilde{R}_2} - (0.006\pm0.001)B_{R_0} + 0.003B_R\right)\right]\ \mathbf{ps}^{-1}$$

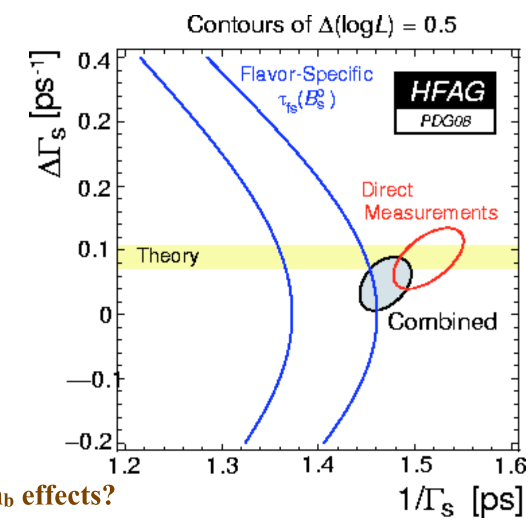
$$a^s_{\rm fs} = \left[(9.7\pm1.6) + 0.3\frac{\tilde{B}'_S}{B} + 0.3\frac{B_R}{B}\right]\mathbf{Im}\left(\frac{\lambda_u}{\lambda_t}\right)\cdot 10^{-4} + \left[(0.08\pm0.01) + 0.02\frac{\tilde{B}'_S}{B} + (0.05\pm0.01)\frac{B_R}{B}\right]\mathbf{Im}\left(\frac{\lambda_u}{\lambda_t}\right)^2\cdot 10^{-4}$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = \left[(46.2\pm4.4) + (10.6\pm1.0)\frac{\tilde{B}'_S}{B} - \left((13.2\pm1.3)\frac{B_{\tilde{R}_2}}{B} - (2.5\pm0.2)\frac{B_{R_0}}{B} + (1.2\pm0.1)\frac{B_R}{B}\right)\right]\cdot 10^{-4}$$

★ Assuming no New Physics contributions to ΔM_{Bs}:

$$\Delta\Gamma_s = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm Theory}\cdot\Delta M_s^{\rm Exp.} = 0.088\pm0.017\,\mathbf{ps}^{-1}$$

$$\Rightarrow \frac{\Delta\Gamma_s}{\Gamma_s} = \Delta\Gamma_s\cdot\tau_{B_d} = 0.127\pm0.024\,.$$



Notice shift from the old result: α_s/m_b effects?

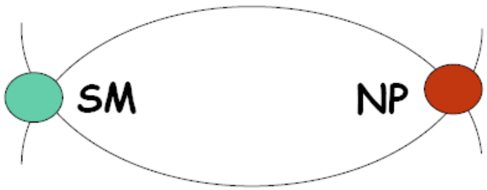
"Direct" NP contributions to $\Delta\Gamma_{Bs}$

★ Since $\Delta\Gamma_{Bs}$ is known relatively well in SM: constrain NP contributions!

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left(\overline{A}_n^{SM} + \overline{A}_n^{NP}\right)\left(A_n^{SM} + A_n^{NP}\right) \approx \frac{1}{2\Gamma}\sum_n \rho_n \overline{A}_n^{SM} A_n^{SM} + \frac{1}{2\Gamma}\sum_n \rho_n \left(\overline{A}_n^{SM} A_n^{NP} + \overline{A}_n^{NP} A_n^{SM}\right)$$

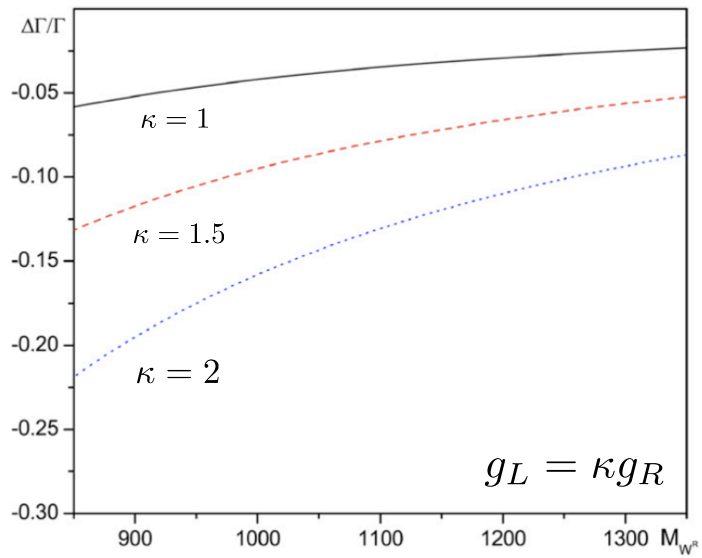
$$\left.\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}\right|_{NP} = \frac{1}{M_{B_s}\Gamma_{B_s}} \langle \overline{B}_s | \mathrm{Im}\, \mathcal{T} | B_s \rangle \quad , \quad \text{where}$$

$$\mathcal{T} = i \int d^4x\, T\left(\mathcal{H}_{SM}^{\Delta B=1}(x)\, \mathcal{H}_{NP}^{\Delta B=1}(0)\right) \quad .$$



★ For example, for the Left-Right Models:

$$\left.\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}\right|_{LR} = -V_{cb}^* V_{cs} V_{cb}^{*(R)} V_{cs}^{(R)} \frac{2\kappa^2 G_F^2 m_b^2 z_c \sqrt{1-4z_c}}{\pi M_B \Gamma_{B_s}} \times \left(\frac{M_W}{M_W^{(R)}}\right)^2 \left[C_1 \langle Q_2 \rangle - 2C_2 \langle Q_1 \rangle\right].$$



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Phys. Lett. B653, 230 (2007)

Conclusions

- Calculations of lifetimes and mixing parameters of heavy mesons and baryons are quite mature
  - $1/m$ and $1/m^2$ corrections to spectator effects in lifetime ratios and mixing parameters of heavy mesons and baryons are calculated
  - perturbative QCD corrections to leading effects are done (including "hybrid logs")
- Reasonably good agreement between theory and experiment
  - "short lifetime problem" of Λ_b no longer exists
  - some disagreement on B_s lifetime (experiment?)
  - it appears no duality violations are needed
  - it appears that $1/m$-expansion is well convergent here
  - B_s mixing: is there New Physics in $\arg(M_{12})$?
  - additional input from LHC experiments/Super-B AND lattice?
- Can constrain some NP models from measurement of lifetime differences in heavy mesons (both B_s and B_d)

WAYNE STATE UNIVERSITY

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July 26-31, 2009, Detroit, Michigan

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http://www.dpf2009.wayne.edu/

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