Δm_K ϵ_K ϵ'/ϵ B_K

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neutral kaon mass difference:

$$\Delta m_K = 3.483(6) \cdot 10^{-12}\text{MeV}$$

indirect CP-violation $K \to \pi\pi$:

$$|\epsilon| = 2.229(10) \cdot 10^{-3}$$

direct/indirect CP-violation $K \to \pi\pi$:

$$\text{Re}(\epsilon'/\epsilon) = 1.65(26) \cdot 10^{-3}$$

$\Delta I = 1/2$-rule:

$$|A_2/A_0| \sim 1/20$$

(PDG '08)

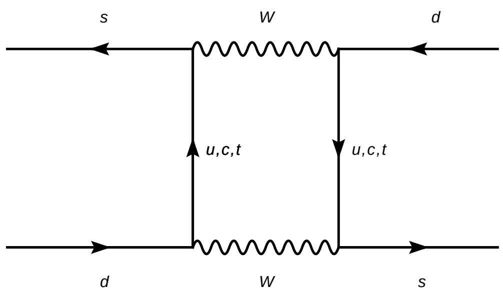
Operator Product Expansion

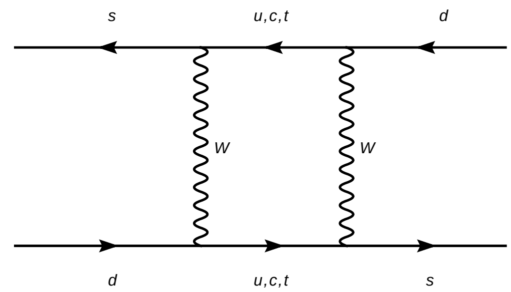
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i V_i^{\text{CKM}} C_i(\mu)\, Q_i(\mu)$$

- CKM-matrix elements: weak- $\leftrightarrow$ mass-eigenstates
- separation of scales
  - ∗ electroweak scale
  - ∗ perturbative QCD — short-distance
  - ∗ non-perturbative QCD — long-distance
- hadronic matrix elements $\langle A|Q_i(\mu)|B\rangle$
- Wilson coefficients short-distance effects

  usually known to NLO (Munich, Rome groups)

neutral kaon mixing





- flavor eigenstates $\qquad K^0 \;=\; (\bar{s}d) \qquad \bar{K}^0 \;=\; (s\bar{d})$
  - $*$ flavor-mixing $\hat{M} - \frac{i}{2}\hat{\Gamma} \qquad 2m_K\, M^*_{12} \;=\; \langle \bar{K}^0 | \mathcal{H}^{\Delta S=2} | K^0 \rangle$
  - mixing parameter: $\bar{\boldsymbol{\epsilon}} \simeq \mathcal{O}(10^{-3})$
  - $*$ $\quad K_S = [(1+\bar{\epsilon})K^0 - (1-\bar{\epsilon})\bar{K}^0]/N' \qquad K_L = [(1+\bar{\epsilon})K^0 + (1-\bar{\epsilon})\bar{K}^0]/N'$
- CP eigenstates $\qquad K_1 \;=\; (K^0 - \bar{K}^0)/\sqrt{2} \qquad K_2 \;=\; (K^0 + \bar{K}^0)/\sqrt{2}$

$$K_S = (K_1 + \bar{\epsilon}K_2)/N \qquad K_L = (K_2 + \bar{\epsilon}K_1)/N$$

- $\bar{\epsilon} \simeq \mathcal{O}(10^{-3})$

$$\Delta m_K \;=\; 2\mathrm{Re}M_{12} \quad \Delta\Gamma_K \;=\; 2\mathrm{Re}\Gamma_{12}$$

indirect and direct CP-violation

- indirect CP-violation
  - $*$ CP-conserving dominant: $K_L \stackrel{K_2}{\longrightarrow} 3\pi$, $K_S \stackrel{K_1}{\longrightarrow} 2\pi$
  - $*$ from mixing: $K_L \stackrel{K_1}{\longrightarrow} 2\pi$, $K_S \stackrel{K_2}{\longrightarrow} 3\pi$
- $\boldsymbol{\epsilon_K = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = \bar{\epsilon} + \mathrm{i}\xi \approx \mathrm{Im} M_{12} \exp(\mathrm{i}\pi/4) \,/\, (\sqrt{2}\Delta m_K)}$
- direct CP-violation $K_L \longrightarrow 2\pi$

$$\epsilon' = \frac{1}{\sqrt{2}}\left[\frac{A(K_L \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})} - \frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})}\frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}\right]$$

$$= \frac{1}{\sqrt{2}}\mathrm{Im}\left(\frac{A_2}{A_0}\right)\exp(\mathrm{i}\phi_{\epsilon'})$$

amplitudes $A_{0,2}$: $K \longrightarrow 2\pi$ with $I = 0,\ 2$ $\Delta I = 1/2,\ 3/2$

$$\langle(\pi\pi)_I|\mathcal{H}^{\Delta S=1}|K\rangle$$

Outline

neutral kaon mass difference

ϵ_K and B_K

ϵ'/ϵ

neutral kaon mass difference Δm_K

- PDG '08: $\Delta m_K \;=\; m_{K_L} - m_{K_S} \;=\; 3.483(6) \cdot 10^{-12}$ MeV
- $\Delta m_K \;=\; 2\mathrm{Re}M_{12}$

$$2m_K\, M_{12}^* \;=\; \langle \bar{K}^0 | \mathcal{H}^{\Delta S=2} | K^0 \rangle$$

  * main contribution: **charm**- and top-exchange box diagrams
  * simultaneous charm/top exchange

- Herrlich, Nierste (1994):
  * charm-contribution $\approx 64\%$
  * top-contribution $\approx 6\%$
    of measured Δm_K

  * scales linearly with $\hat{B}_K$ (used 0.7)

  * vacuum saturation ($\hat{B}_k = 1$) would completely explain Δm_K

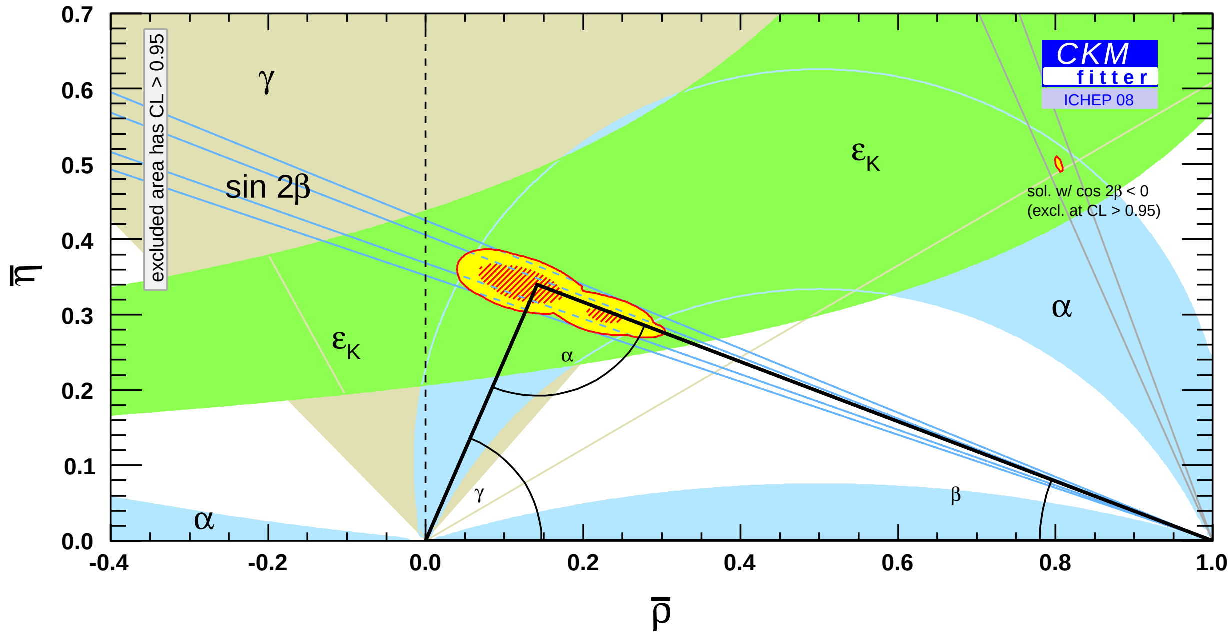
- $\hat{B}_K \neq 1 \Rightarrow\;\approx\; 30\%$ long distance contribution

indirect CP-violation: ϵ_K and B_K

$$B_K(\mu) \;=\; \langle \bar{K}^0 | Q^{\Delta S=2} | K^0 \rangle \;/\; (\tfrac{8}{3} f_K^2 m_K^2)$$

$$|\epsilon_K| \;=\; C_\epsilon \,\hat{B}_K \lambda^2 \bar{\eta}^2 |V_{cb}|^2 \Big[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} S_0(x_c) \Big]$$

PDG '08: $|V_{cb}| \;=\; 0.0412(11)$ 2.7% $\rightarrow \quad \delta |V_{cb}|^4 \;\simeq\; \delta B_K^{\text{lat}}$



(CKM fitter 2008, prelim.)

B_K on the lattice

- measure

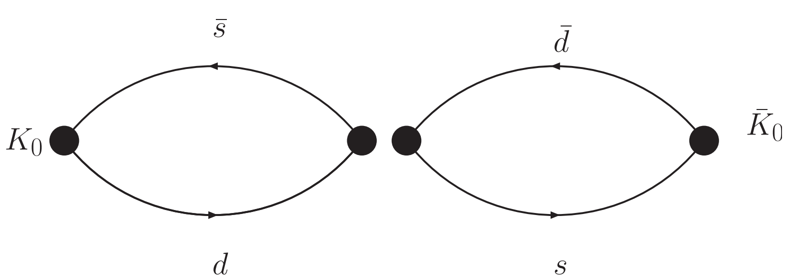
$$B_K = \frac{\langle \bar{K}^0 | Q_{\Delta S=2} | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

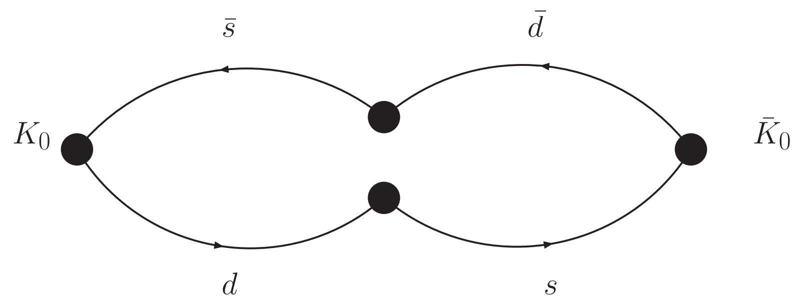
via ratio of 4-point to 2-point functions

$$B_K(t) = \frac{3}{8} \frac{\mathcal{C}_{PQP}(t_{\mathsf{src}}, t, t_{\mathsf{snk}})}{\mathcal{C}_{PA}(t_{\mathsf{src}}, t)\mathcal{C}_{AP}(t, t_{\mathsf{snk}})}$$

$$\mathcal{C}_{PQP} = \langle q(t_{\mathsf{src}}) P \bar{q}(t_{\mathsf{src}}) Q(t) q(t_{\mathsf{snk}}) P \bar{q}(t_{\mathsf{snk}}) \rangle$$

- construction of Q depends on lattice fermion formulation
- two color-traces for 4-pt correlator





- lattice fermion action
  - chiral properties (affects renormalization)
  - dynamical fermions? quenched ($N_f = 0$), unquenched ($N_f = 2$, $N_f = 2 + 1$)
  - improved continuum limit
  - cost

- renormalization (perturbative — non-perturbative)
  "wrong chirality" operator-mixing

- non-degenerate (m_{ud}, m_s) or degenerate kaons ($m_s/2$, $m_s/2$)

- extrapolation to light physical quark masses ($m_{ud} = (m_u + m_d)/2$), strange quark mass
  - (Partially Quenched) Chiral Perturbation Theory
  - SU(2) — SU(3)

operator renormalization

parity conserving part of $(V-A)\otimes(V-A) \;=\; VV+AA$

- exact chiral symmetry: $\mathcal{O}_{VV+AA}$
- broken chiral symmetry: operator mixing

$$\mathcal{O}_{VV-AA} \quad \mathcal{O}_{SS+PP} \quad \mathcal{O}_{SS-PP} \quad \mathcal{O}_{TT}$$

$$\langle \bar{K}^0 | \mathcal{O}_{VV+AA} | K^0 \rangle \;\propto\; m_K^2$$

$$\langle \bar{K}^0 | \mathcal{O}_{\text{other}} | K^0 \rangle \;\propto\; \text{const}$$

multiplicative renormalization: non-perturbative RI-MOM, Schrödinger-functional

- broken flavor symmetry
  * staggered fermions: perturbative 1-loop matching systematic error!
  * twisted mass fermions: mixed action (TM/OS): NPR/RI-MOM

Chiral Perturbation Theory

- most lattice simulations:
  - $*$ not at physical light quark masses currently $\gtrsim 300$ MeV pion mass
  - $*$ close to physical strange quark mass (if $N_f = 2+1$)
- Partially Quenched ChPT (Sharpe, Van de Water)
  - $*$ evaluate lattice correlators at quark masses $m_{\text{valence}} \neq m_{\text{dynamical}}$
  - $*$ still dynamical(sea) content from $m_{\text{dynamical}}$ (fermion loops)
  - $*$ better handle to perform PQChPT-extrapolation: different dependence on m_{valence}, $m_{\text{dynamical}}$
- SU(3) ChPT: expand around $m_{ud}, m_s = 0$
  - $*$ up to NLO bad convergence at strange quark mass
  - $*$ complete NNLO difficult (number of parameters)
  - $*$ adding analytical NNLO-terms (change of NLO-behavior?)
- SU(2) ChPT: expand around $m_{ud} = 0$, m_s fixed (RBC/UKQCD '08)
  - $*$ NLO seems to be sufficient for $\lesssim 420$ MeV pion masses
  - $*$ no ad-hoc terms need to be added — reduced syst. error
  - $*$ interpolation in m_s possible after $m_l \longrightarrow m_{ud}$

unquenched results for B_K

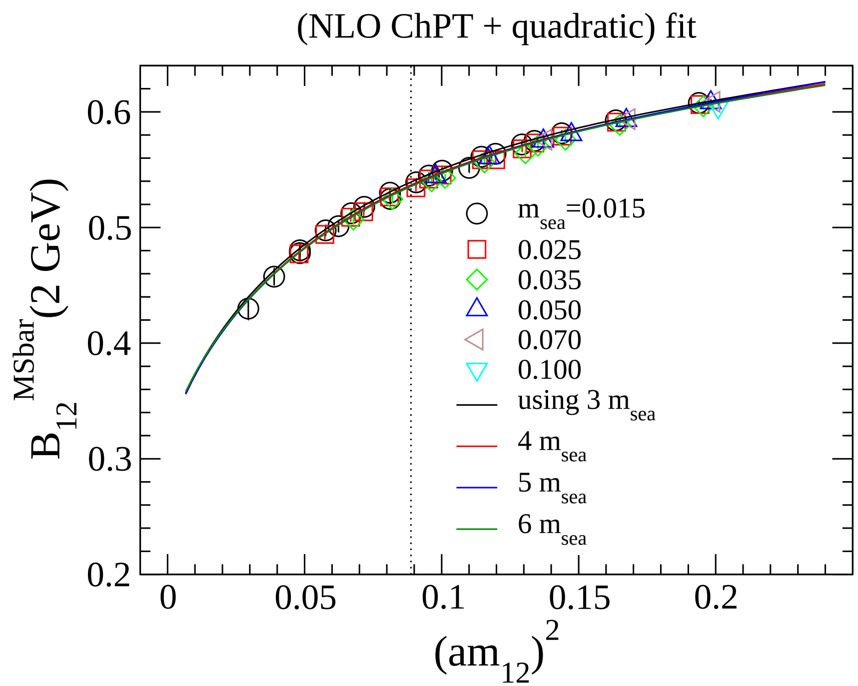
- $N_f = 2$
  - ∗ ETMC '08 twisted mass fermions (prelim.)
  - ∗ JLQCD '08 overlap fermions

- $N_f = 2 + 1$
  - ∗ HPQCD '06 staggered fermions
  - ∗ RBC/UKQCD '08 Domain Wall Fermions
  - work in progress:
    - ∗ mixed action (DWF on stagg. sea) Aubin, Laiho, van de Water
    - ∗ staggered fermions: Lee et al.

$N_f = 2$ overlap fermions by JLQCD

S. Aoki et al., Phys. Rev. **D 77** (2008) 094503

- dynamical(!) overlap fermions (Ginsparg-Wilson)
  - ∗ exact chiral symmetry
  - ∗ fixed topology ($Q = 0, -2, -4$), corrected in ChPT (1.4% syst. err.)
- $a \approx 0.12$ fm, $1/a \approx 1.67$ GeV (no cont. extr., but 5% scale setting syst. err.)
- $V \approx (1.9\,\text{fm})^3$ ($16^3 \times 32$) (5% syst. err.)
- pion mass: 290–750 MeV
  - ∗ NLO-PQChPT works up to $m_s/2$
  - ∗ added analytic terms to extend range $\longrightarrow m_s$
- renormalization: NPR (2% syst. err.)



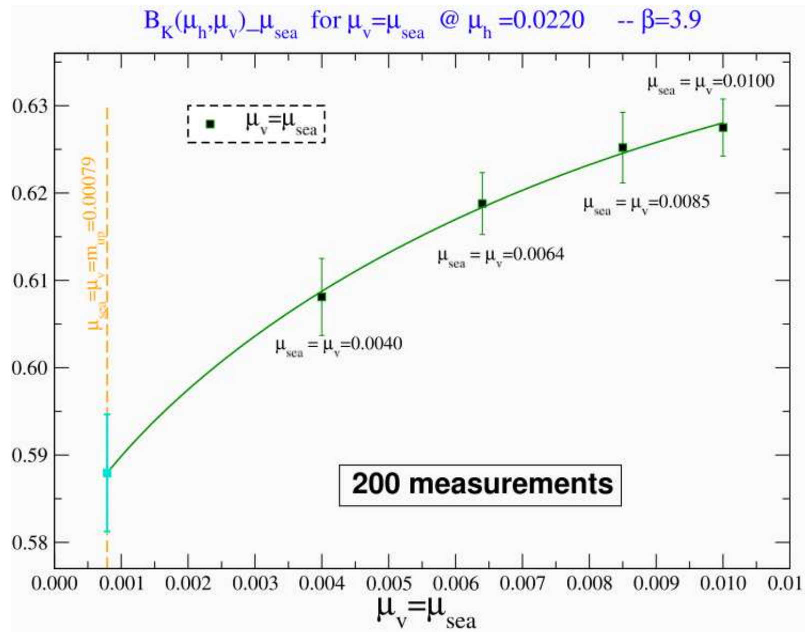
$$B_K^{\overline{\text{MS}}}(2\text{GeV}) = 0.537(4)_{\text{stat}}(40)_{\text{syst}} \quad (\hat{B}_K = 0.758(6)_{\text{stat}}(56)_{\text{syst}})$$

(no systematic error for missing dynamical strange quark included)

$N_f = 2$ twisted mass fermions by ETMC

A. Vladikas, LATTICE 2008

- dynamical twisted mass fermions
  - ∗ formulation explicitly breaks flavor-, chiral-symmetry
  - ∗ using Osterwalder-Seiler valence quarks for B_K automatic $\mathcal{O}(a)$-impr. + mult. renormalization
- $a \approx 0.09$ fm, $1/a \approx 2.3$ GeV (scaling study with $a \approx 0.07$ fm)
- $V \approx (2.2\,\text{fm})^3$ $(24^3 \times 48)$ (finite volume study with $V \approx (2.9\,\text{fm})^3$)
- pion masses 300 – 550 MeV
  - ∗ SU(2) PQChPT for m_{ud}
  - ∗ linear interpolation in m_s
  - ∗ polynomial fits?
- non-perturbative renormalization (RI/MOM)



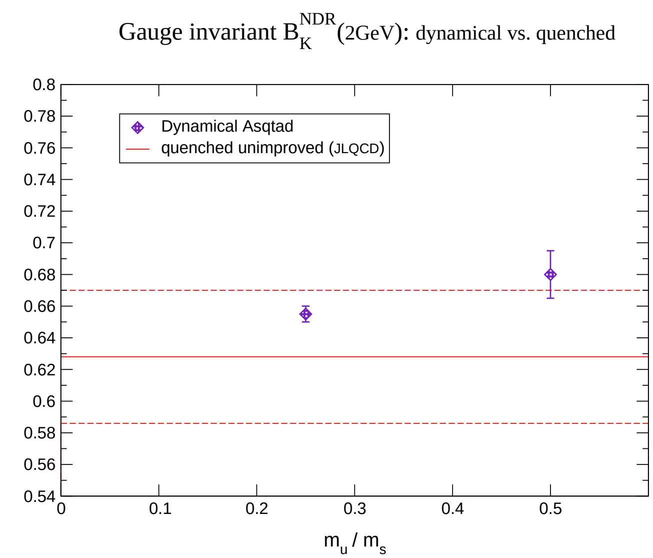
PRELIMINARY $B_K^{\overline{\text{MS}}}(2\text{GeV}) = 0.56(2)$ $\hat{B}_K = 0.77(3)$ PRELIMINARY

complete systematic error analysis missing

$N_f = 2 + 1$ staggered fermions by HPQCD

E. Gámiz et al., Phys. Rev. **D 73** (2006) 114502

- improved staggered fermions
  - ∗ 4 fermion tastes
  - ∗ taste-mixing
  - ∗ remaining U(1) chiral symmetry
  - ∗ rooting procedure, see LATTICE 07 Creutz ↔ Kronfeld
- $a \approx 0.125$ fm, $1/a \approx 1.6$ GeV
- $V \approx (2.5\text{ fm})^3$
- pion masses 360, 500 MeV
- "degenerate valence kaon" $(m_s/2)$ no extrapolation
- linear interpolation in m_{sea} (NLO SU(3) ChPT)
- perturbative renormalization
  - ∗ pert. 1-loop matching for mixing operators
  - ∗ $\mathcal{O}(\alpha_S^2)$ uncertainty main syst. error source

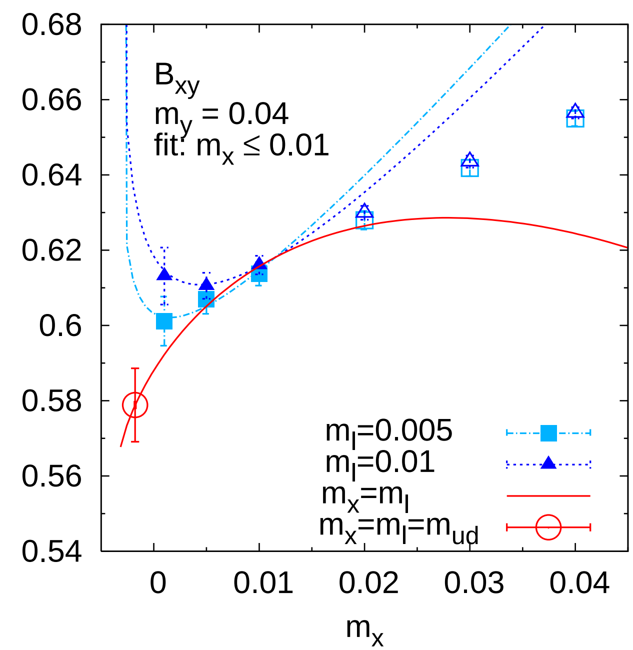


$$B_K^{\overline{\text{MS}}}(2\text{GeV}) = 0.618(18)_{\text{stat}}(135)_{\text{comb}} \quad \hat{B}_K = 0.83(18)_{\text{total}}$$

$N_f = 2+1$ Domain Wall fermions by RBC/UKQCD

D.J. Antonio et al., Phys. Rev. Lett. **100** (2008) 032001
C. Allton et al., arXiv:0804.0473 [hep-lat]

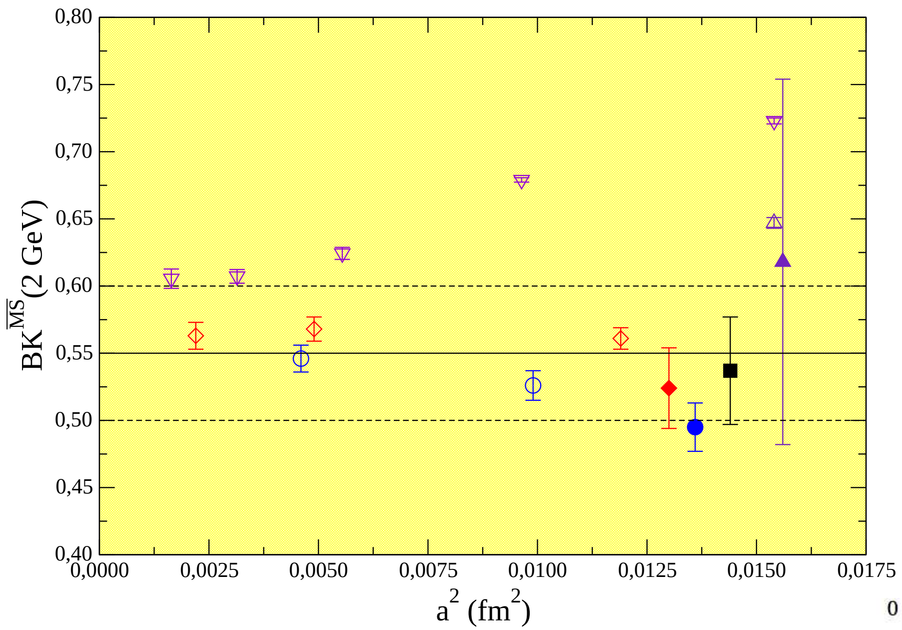
- Domain Wall Fermions
  * left-, right-handed separated in 5th dim.
  * only small residual chiral symmetry breaking
  * wrong chiral op. mixing sufficiently suppressed
- $a \approx 0.11$ fm, $1/a \approx 1.73$ GeV (incl. 4% syst. err.)
  (PRELIM.: $a \approx 0.08$ fm, $1/a \approx 2.42$ GeV)
- $V \approx (2.74\,\text{fm})^3$ $(24^3 \times 64 \times 16)$ (1% syst. err.)
- pion masses 330, 420 MeV
  * SU(2) PQChPT (2% syst. err.+1% m_s)
  * lightest valence pion mass 240 MeV
  * new data set: $m_\pi \approx 300$ MeV
- non-perturbative renormalization (2% uncertainty)

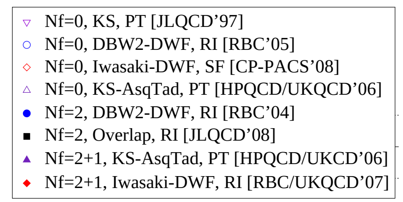


$$B_K^{\overline{\text{MS}}}(2\text{GeV}) = 0.524(10)_{\text{stat}}(28)_{\text{comb}} \quad \hat{B}_K = 0.720(13)_{\text{stat}}(37)_{\text{comb}}$$

V. Lubicz, C. Tarantino, IFAE 2008

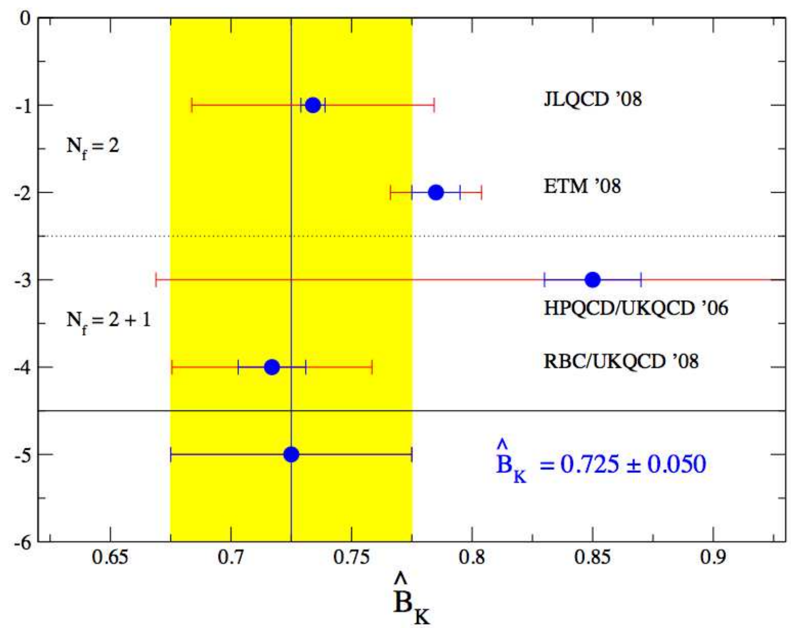
- scale dependence from quenched sim.
- $B_K^{\overline{\text{MS}}}(2\text{ GeV}) = 0.55(.05)$
  $\hat{B}_K = 0.75(.07)$





L. Lellouch, LATTICE 2008

- only dynamical $N_f = 2+1$ data used
- $\delta B_K \approx 7\%$
- $\hat{B}_K = 0.725(.050)$



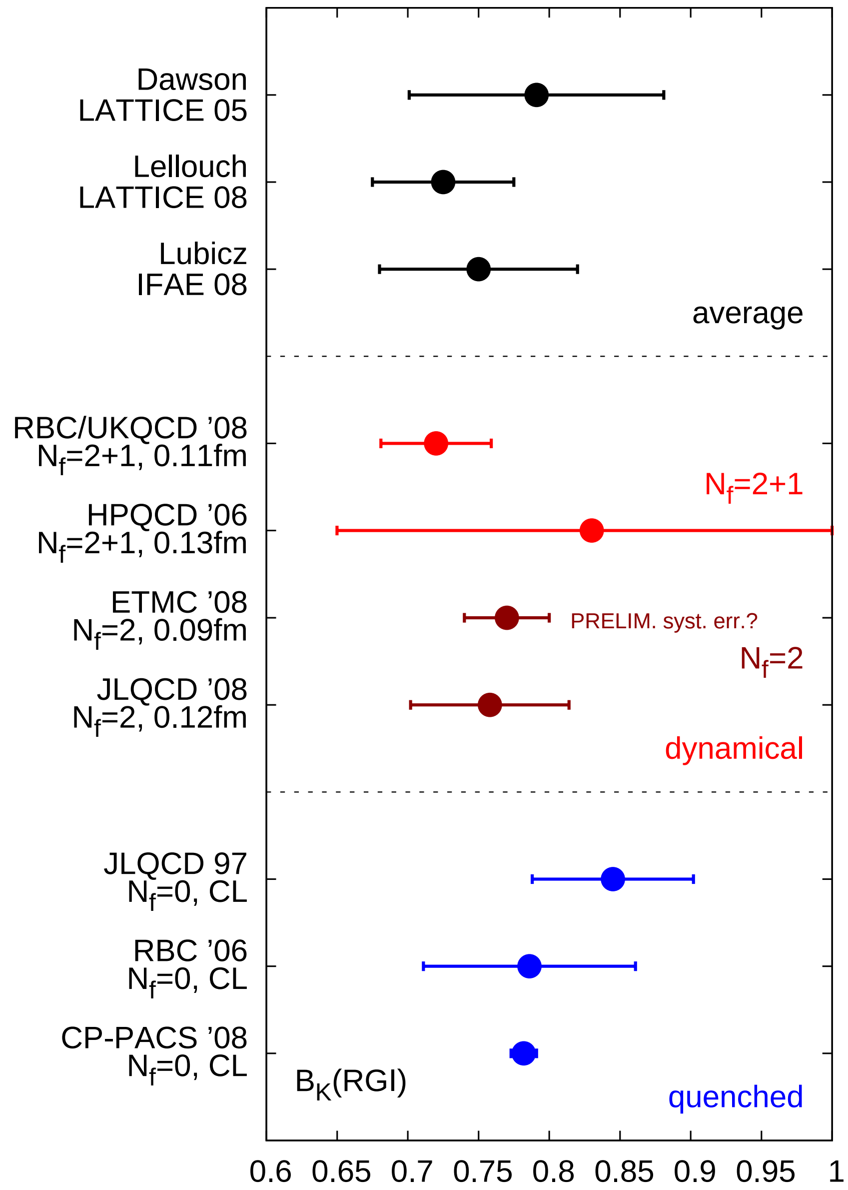
- $N_f = 2$ dynamical strange quark dependence?
  - current accuracy: $N_f = 2 \;\simeq\; N_f = 2 + 1$
  - error from quenched strange quark?

- continuum limit studies with $N_f = 2{+}1$ needed
  - RBC/UKQCD: $a \approx 0.08$ ongoing
  - mixed action results from Aubin et al.
  - staggered results from Lee et al.

- check for finite volume corrections

(Bećirević, Villadoro, Phys. Rev. **D69** (2004) 054010)

currently max. $V \approx (2.7\,\mathrm{fm})^3$



B_K in the chiral limit

- lattice result from RBC/UKQCD '08

$$\hat{B}_K^{\chi} = 0.339(33)(47)$$

  - $*$ obtained from NLO-SU(3) fit ($m_{PS} \leq 420\,\text{MeV}$)
  - caveat: convergence, NNLO?

- large N_C (Bijnens, Gámiz, Prades '06)

$$\hat{B}_K^{\chi} \;=\; 0.38(.15)$$

(expect small corrections for large N_C value $\hat{B}_K = 3/4$)

- QCD-hadronic duality (Prades et al. '91)

$$\hat{B}_K^{\chi} \;=\; 0.39(.10)$$

ϵ'/ϵ

- indirect CP-violation $\epsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}$
- direct CP-violation in $K_L \to \pi\pi$

$$\epsilon' = \frac{1}{\sqrt{2}}\left[\frac{A(K_L \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})} - \frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})}\frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}\right]$$

$$= \frac{1}{\sqrt{2}}\text{Im}\left(\frac{A_2}{A_0}\right)\exp(i\phi_{\epsilon'})$$

- $A_0 \leftrightarrow \Delta I = 1/2$, $A_2 \leftrightarrow \Delta I = 3/2$ in $K \to \pi\pi$
- $\Delta I = 1/2$-rule: $1/\omega = \text{Re}A_0/\text{Re}A_2 \approx 22.2$
- relevant hadronic matrix elements: Q_6, Q_8
- isospin-breaking Ω_{IB}, $V_{ts}^* V_{td}$, α_s, m_s, m_t
- NLO-analysis
  * main uncertainty: Q_6 (Q_8)

(Ciuchini et al., Z. Phys. **C68** (1995) 239
Buras, Jamin, JHEP **01** (2004) 048)

lattice calculations for ϵ'/ϵ

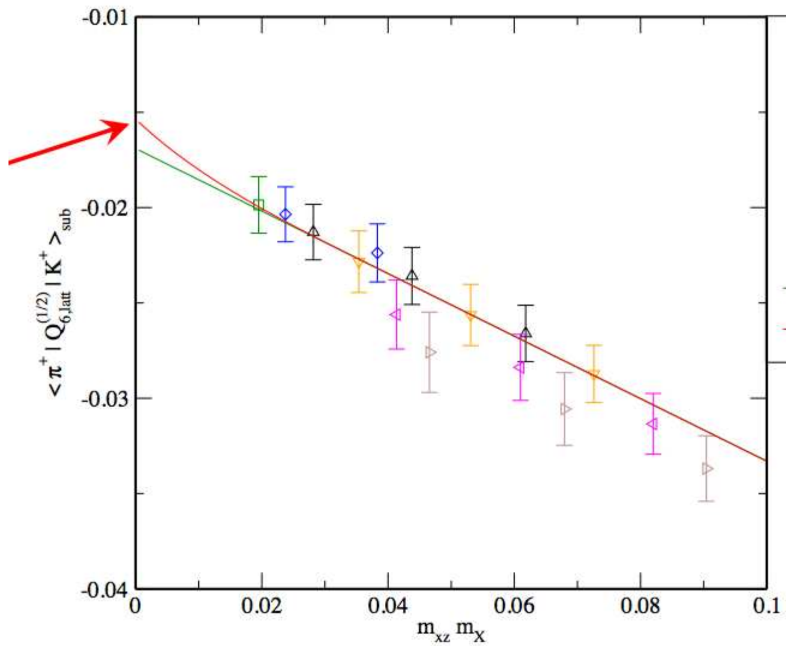
- direct calculation difficult: 2π state
  - Maiani-Testa: pions at rest $\pi(0)\pi(0)$ vs. $\pi(p)\pi(-p)$
  - $\Delta E = 2m_\pi - m_K$
- indirect method: Chiral Perturbation Theory (Bernard et al.)
  - calculate $K \longrightarrow \pi$ and $K \longrightarrow$ vacuum
  - $\langle \pi^+\pi^- | Q_i^{(2)} | K^0 \rangle = \frac{(m_K^2 - m_\pi^2)}{\sqrt{2} f (p_K \cdot p_\pi)} \langle \pi^+ | Q_i^{(2)} | K^+ \rangle + \mathcal{O}(p^2)$
  - . . .
  - $K \longrightarrow$ vacuum: subtraction of unphysical contributions
- quenched calculations
  - CP-PACS 2003 and RBC 2003 ($\Delta I = 1/2$)
  - both used Domain Wall Fermions
  - quenching artefacts lead to large systematic errors
    - Golterman, Pallante; RBC: Quenched ChPT dominated by unphysical operator
- quenched, ϵ-regime (small box) (Hernández et al., Phys. Rev. Lett. **98** (2007) 082003)
  - 4 flavor theory, GIM-limit: light charm (Ginsparg-Wilson fermions)
  - already $\Delta I = 1/2$ enhancement
  - further steps: $\Lambda_{\text{ChPT}} \gg m_c \gg m_{u,d,s}$, $m_c \geq \Lambda_{\text{ChPT}}$

ϵ'/ϵ from dynamical lattice simulations

- recent improvement: Aubin, Laiho, Li, Lin (2008)
  2+1 flavor NLO PQChPT for $K \to \pi$, $K \to$ vacuum
- RBC/UKQCD: $N_f = 2 + 1$ Domain Wall Fermions PRELIMINARY

Christ, Li, LATTICE 08

- ∗ convergence of SU(3) ChPT
- ∗ some LEC's have 100% error
- ∗ complete NLO $K \to \pi\pi$ missing
- ∗ Re$(\epsilon'/\epsilon) = 7.6(6.8)(25.6) \cdot 10^{-4}$
  $1/\omega = 50(13)(62)$ PRELIMINARY
- serious problems
  currently studying 2π alternative
  (Lellouch-Lüscher method)



- improvement on $\hat{B}_K$ from lattice
  - ∗ dynamical 2+1 simulations with good chiral properties
  - ∗ scaling studies underway
  - ∗ use of SU(2)-ChPT leads to more reliable results (at NLO)

$$\delta B_K \approx 7\%$$

comparable to $\delta|V_{cb}|^4$

- neutral kaon mass difference: still 20–30% long distance contr. missing

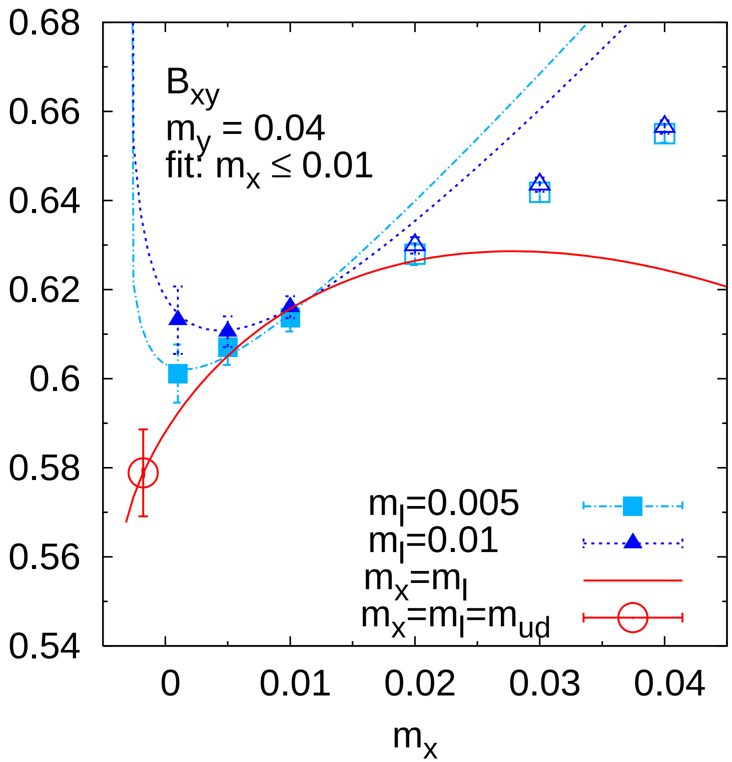
- ϵ'/ϵ
  - ∗ main uncertainty still Q_6, Q_8
  - ∗ current lattice results show "some trouble":

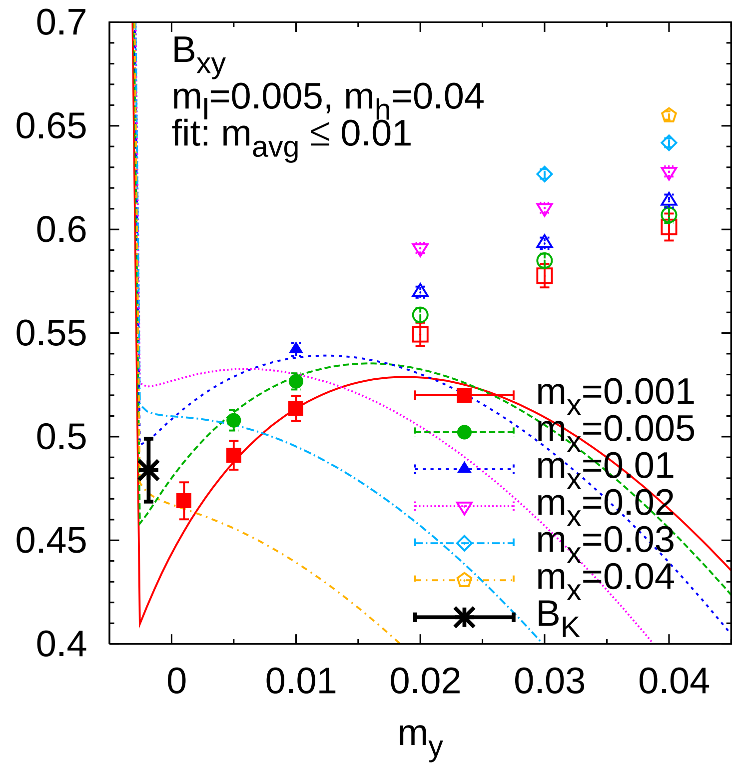
NLO-SU(3) has bad convergence at m_K

BACKUP

SU(2) — SU(3) (NLO)

RBC/UKQCD '08





SU(3) — extended fit range (NLO)

RBC/UKQCD '08