future prospects for LQDC form factors calculations

nazario tantalo

INFN sez. "Tor Vergata" & Centro Ricerche e Studi "E. Fermi"

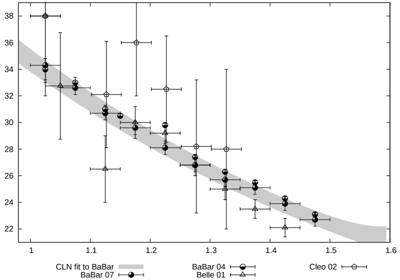
10-09-2008

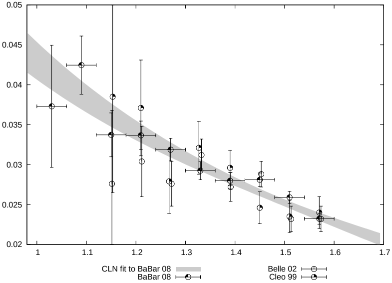
why we need LQCD calculations of ffs?

$$\frac{d\Gamma^{B\to D^\star \ell\nu}}{dw} = (\text{kin. fact.}) \times$$

$$|\mathbf{V_{cb}}|^2 \sqrt{w^2 - 1}(1+w)^2 \lambda(w) \left[\mathbf{F^{B\to D^\star}(w)}\right]^2$$

$$F^{B\to D^\star}(w) = h_{A_1}(w)\sqrt{\frac{H_0^2(w) + H_+^2(w) + H_-^2(w)}{\lambda(w)}}$$





$$\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = (\text{kin. fact.}) \times$$

$$\times |\mathbf{V_{cb}}|^{\mathbf{2}} (\omega^2 - 1)^{\frac{3}{2}} \left[\mathbf{G^{B\to D}}(\omega)\right]^2$$

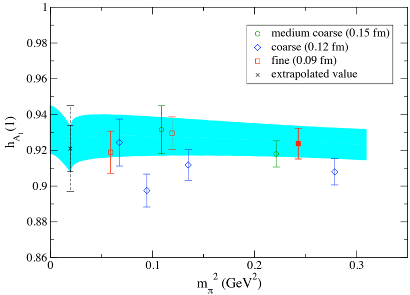
$$G^{B\to D}(\omega) = h_+^{B\to D}(w) - \frac{M_D - M_B}{M_D + M_B} h_-^{B\to D}(w)$$

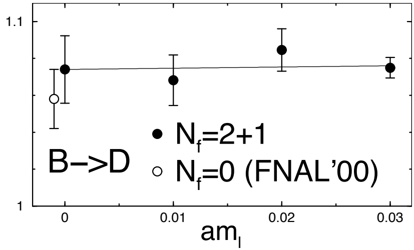
what do we have now?

- we already have $N_f = 2+1$ calculations with percent accuracy for $B \to D^\star \ell\nu$ at zero recoil. . .

  [see J.Laiho talk]

  [C.Bernard et al. arXiv:0808.2519]





- we already have $N_f = 2+1$ calculations with percent accuracy for $B \to D\ell\nu$ at zero recoil. . .

  [M.Okamoto et al. Nucl.Phys.Proc.Suppl.140(2005)]

can we do better? ... differently?

- limited computer resources forced/forces us to cope with **chiral extrapolations**,**continuum extrapolations**, **finite volume effects**,...
  - these are sources of systematics that cannot be eliminated
  - although the associated errors can be reduced

- different lattice formulations could be useful in assessing possible **rooting** effects

$$Z^{\rm root}_{N_f=3} = \int DU e^{-S_g} \left\{ \det[D_{\rm stag}(m_u)] \det[D_{\rm stag}(m_d)] \det[D_{\rm stag}(m_s)] \right\}^{1/4}$$

[S.Sharpe PoS LAT2006:022,2006]
[A.Kronfeld PoS LAT2007:016,2007]
[M.Creutz PoS LAT2007:007,2007]
[...]

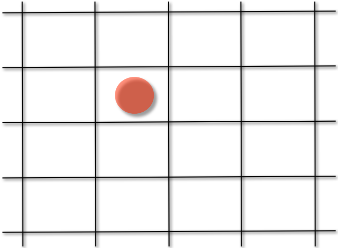
- form factors can be calculated where experimental data are directly available, thus eliminating systematics associated with w-extrapolations

- different **heavy quark technologies** ...

B-mesons matrix elements are challenging

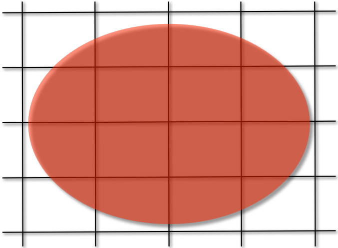
A *B*-meson is a system characterized by two largely separated energy scales: the *b*-quark mass (cutoff effects) and the confinement scale (finite volume effects)

large volume approach: L $\simeq$ 2 fm



- HQET
- FNAL
- simulate charm region + HQET

small volume approach: L $\simeq$ 0.5 fm



- step scaling method (SSM)
- match/renormalize HQET
- match/renormalize FNAL

large volume approaches require small volume matching: HQET

at order $O(\Lambda_{QCD}/m_h)$ the action is: **[Eichten,Hill Phys.Lett.B234:511,1990]**

$$\psi_h = \frac{1+\gamma_0}{2}\,\psi$$

$$\mathcal{L}_{HQET} = \bar{\psi}_h \left[(D_0 + \delta m) - \underbrace{\left(\omega_{spin}\ \vec{\sigma}\cdot\vec{B} + \frac{\omega_{kin}}{2}\vec{D}^2 \right)}_{insertions} \right] \psi_h$$

this action needs to be renormalized and matched with QCD at $\mu \simeq m_b$:

- perturbative renormalization:
  - perturbative errors $\propto \alpha(m_h)^n \propto [\log(m_h/\Lambda_{QCD})]^{-n}$
  - on the lattice, power divergences $1/a^n$
- non-perturbative renormalization: **[Heitger,Sommer JHEP 0402:022,2004]**
  - matching on a **small volume**

large volume approaches require small volume matching: FNAL

the Fermilab approach consists in simulating the following action with $\mathbf{am_0} \simeq \mathbf{1}$

[El-Khadra et al Phys.Rev.D55:3933, 1997]
[Aoki et al Prog.Theor.Phys.109:383, 2003]
[Oktay,Kronfeld arXiv:0803.0523]

$$S = \sum_n \bar{\psi}_n \left[m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - r_t \frac{a D_0^2}{2} - r_s \frac{a \vec{D}^2}{2} + c_B \frac{i \sigma_{ij} F_{ij}}{4} + c_E \frac{i \sigma_{0i} F_{0i}}{2} \right] \psi_n$$

i.e. the Symanzik effective action for quarks with $|a\vec{p}| \ll 1$ with **mass dependent** coefficients usually computed perturbatively

- the number of parameters in the action can be reduced to **3**

  [Christ,Li,Lin Phys.Rev.D76:074505,2007]

- and determined non-perturbatively by matching QCD on a **small volume**

  [Lin,Christ Phys.Rev.D76:074506,2007]

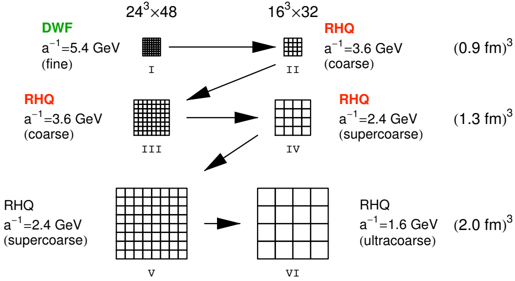
go on a small volume . . .

given a set of matrix elements involving B_q mesons in the initial and/or final states, we have

$$\mathcal{O}_i(m_b, m_l) \quad = \quad \mathcal{O}_i(m_b, m_l; \mathbf{L_0}) \quad \times \quad [\textbf{finite volume effects}]_i(m_b, m_l; \mathbf{L_0})$$

- **effective theory strategy**
  - calculate small volume observables in QCD, $\mathcal{O}_i^{QCD}(m_b, m_l; \mathbf{L_0})$, and in the effective theory (HQET,FNAL), $\mathcal{O}_i^{eff}(\mu, m_b, m_l; \mathbf{L_0})$, and extract action parameters, $c_i(\mu = \mathbf{m_b})$
  - calculate finite volume effects and/or the running of the parameters within the effective theory
- **step scaling method (SSM) strategy**
  - calculate small volume observables in QCD at the physical value of the beauty mass
  - calculate finite volume effects within QCD: FVE should be almost insensitive to the high energy scale

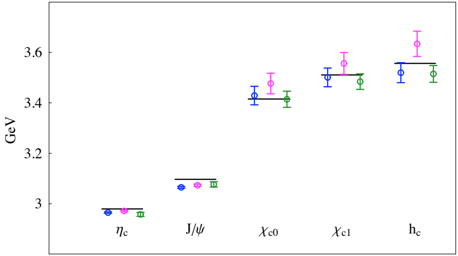
non-perturbative FNAL



such a strategy should be considered in order to remove/reduce **perturbative uncertainties** from FNAL calculation of form factors

[Lin PoS LAT:184,2006]
[Li,Lin PoS LAT2007:117,2007]

- small volume QCD simulations with domain wall fermions
- small volume Fermilab simulations
- matching to reproduce physical spectrum + FVE
- evolution toward large volumes with Fermilab simulations



step scaling method

[Guagnelli,Palombi,Petronzio,N.T. Phys.Lett.B546:237,2002]

$$\mathcal{O}(m_b, m_l) \quad = \quad \mathcal{O}(m_b, m_l; \mathbf{L_0}) \quad \underbrace{\frac{\mathcal{O}(m_b, m_l; \mathbf{2L_0})}{\mathcal{O}(m_b, m_l; \mathbf{L_0})}}_{\sigma(\mathbf{m_b}, \mathbf{m_l}; \mathbf{L_0})} \quad \frac{\mathcal{O}(m_b, m_l; \mathbf{4L_0})}{\mathcal{O}(m_b, m_l; \mathbf{2L_0})} \quad \dots$$

- step scaling functions, the σ's, have to be calculated at lower values of the high energy scale

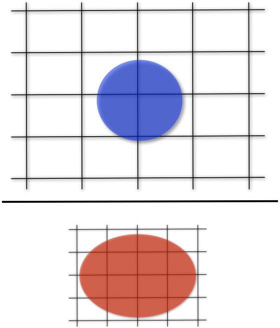
$$\mathcal{O}(m_b, m_l; \mathbf{L_0}) \leftarrow \mathbf{m_b} = \mathbf{m_b^{phys}}$$

$$\sigma(m_b, m_l; \mathbf{n}L_0) \leftarrow \mathbf{m_b} \leq \frac{\mathbf{m_b^{phys}}}{\mathbf{n}}$$

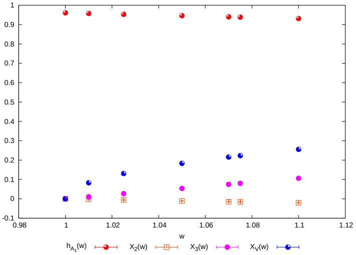
- but extrapolating the step scaling functions is much easier than extrapolating the observable itself

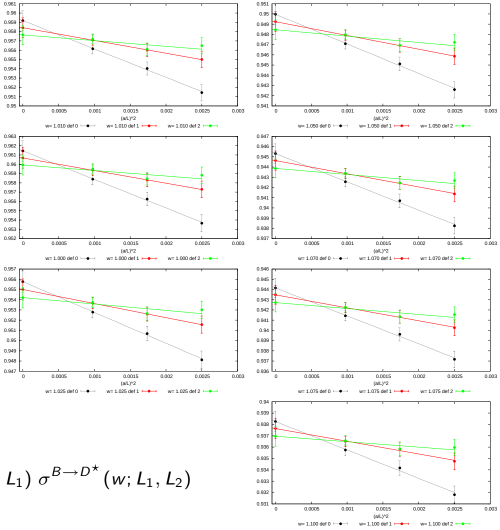
$$\mathcal{O}(m_b, m_l; L) = \mathcal{O}^0(m_l; L) \left[1 + \frac{\mathcal{O}^1(m_l; L)}{m_b} \right]$$

$$\sigma(m_b, m_l; L) = \frac{\mathcal{O}^0(m_l; 2L)}{\mathcal{O}^0(m_l; L)} \left[1 + \frac{\mathcal{O}^1(m_l; 2L) - \mathcal{O}^1(m_l; L)}{m_b} \right]$$



$B \to D^\star \ell\nu$: the small volume





$$F^{B\to D^\star}(w) =$$

$$h_{A_1}(w)\sqrt{\frac{H_0^2(w)+H_+^2(w)+H_-^2(w)}{\lambda(w)}}$$

$$F^{B\to D^\star}(w;L_2) = F^{B\to D^\star}(w;L_0)\;\sigma^{B\to D^\star}(w;L_0,L_1)\;\sigma^{B\to D^\star}(w;L_1,L_2)$$

$B \to D^\star \ell\nu$: continuous momenta & 3-point functions

[de Divitiis,Petronzio,N.T. arXiv:0807.2944]

[Martinelli,Parisi,Petronzio,Rapuano Phys.Lett.B122:283,1983]
[de Divitiis,Petronzio,N.T. Phys.Lett.B595:408,2004]
[Bedaque Phys.Lett.B593:82,2004]
[Sachrajda, Villadoro Phys.Lett.B609:73,2005]
[...]

$$\psi(x + \mathbf{e}_i L) = e^{i\theta_i}\psi(x) \qquad \theta_0 = 0$$

$$\int d\mathbf{p}\ e^{i\mathbf{p}\cdot(\mathbf{x}+\mathbf{e}_i L)}\psi(t;\mathbf{p}) = \int d\mathbf{p}\ e^{i(\mathbf{p}\cdot\mathbf{x}+\theta_i)}\psi(t;\mathbf{p})$$

$$p_i = \frac{\theta_i}{L} + \frac{2\pi n}{L}, \quad n \in Z^3$$

$$\langle PVP\rangle^{\mu}_{if} = \hat{Z}_V \sum_{\vec{x}} \langle \mathbf{P}_{li}\ \mathcal{V}^{\mu}_{if}(x)\ \mathbf{P'}_{fl}\rangle$$

$$\langle VVV\rangle^{l\mu l}_{if} = \hat{Z}_V \sum_{\vec{x}} \langle \mathbf{V}^{l}_{li}\ \mathcal{V}^{\mu}_{if}(x)\ \mathbf{V'}^{l}_{fl}\rangle$$

$$\langle PVV\rangle^{\mu l}_{if} = \hat{Z}_V \sum_{\vec{x}} \langle \mathbf{P}_{li}\ \mathcal{V}^{\mu}_{if}(x)\ \mathbf{V'}^{l}_{fl}\rangle$$

$$\langle PAV\rangle^{\mu l}_{if} = \hat{Z}_A \sum_{\vec{x}} \langle \mathbf{P}_{li}\ \mathcal{A}^{\mu}_{if}(x)\ \mathbf{V'}^{l}_{fl}\rangle$$

- "small" w values can be simulated by using flavour-twisted boundary conditions
- form factors can be expressed entirely in terms of 3-point correlation functions

$$X_V(w) = \sqrt{\frac{w-1}{w+1}}\,\frac{h_V(w)}{h_{A_1}(w)}$$

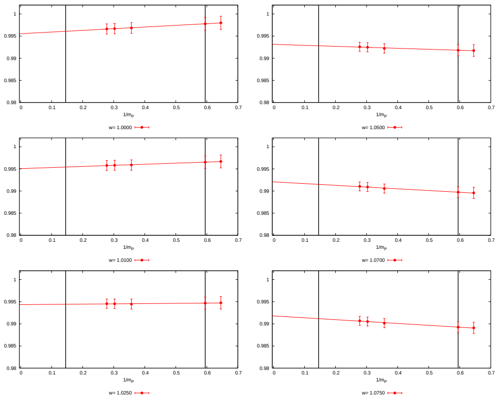
$$X_2(w) = (w-1)\frac{h_{A_2}(w)}{h_{A_1}(w)}$$

$$X_3(w) = (w-1)\frac{h_{A_3}(w)}{h_{A_1}(w)}$$

$$H_0(w) = \frac{w - r - X_3(w) - rX_2(w)}{1-r}$$

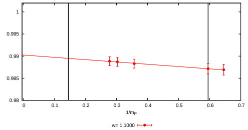
$$H_\pm(w) = t(w)\,[1 \mp X_V(w)]$$

$B \longrightarrow D^{\star}\ell\nu$: first volume step



step scaling functions are very flat:
extrapolated values differ by simulated ones by a few per mille

$$\sigma^{P\longrightarrow D^{\star}}(w; L_0, L_1) = \frac{F^{P\longrightarrow D^{\star}}(w; L_1^3 \times L_1)}{F^{P\longrightarrow D^{\star}}(w; L_0^3 \times L_0)}$$

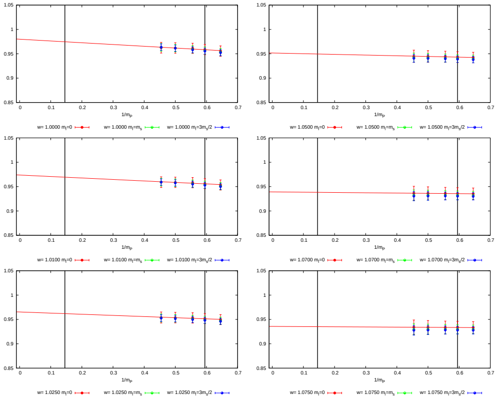


$$F^{B\longrightarrow D^{\star}}(w; L_2) = F^{B\longrightarrow D^{\star}}(w; L_0)\, \sigma^{B\longrightarrow D^{\star}}(w; L_0, L_1)\, \sigma^{B\longrightarrow D^{\star}}(w; L_1, L_2)$$

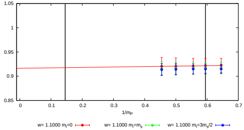
$B \to D^\star \ell \nu$: second volume step

step scaling functions are very flat: extrapolated values differ by simulated ones by a few per mille

$$\sigma^{P\to D^\star}(w; L_0, L_1) = \frac{F^{P\to D^\star}(w; L_1^3 \times L_1)}{F^{P\to D^\star}(w; L_0^3 \times L_0)}$$

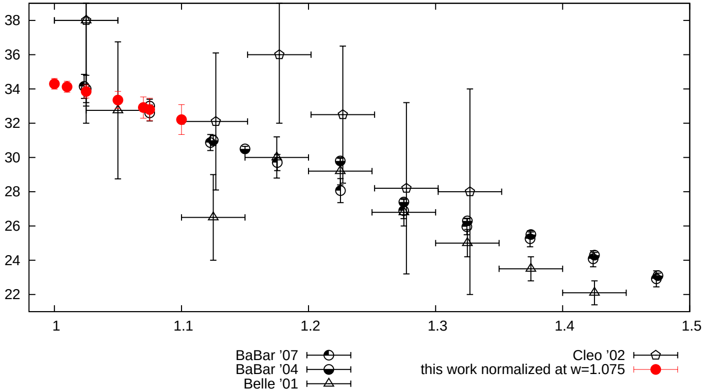


$$F^{B\to D^\star}(w; L_2) = F^{B\to D^\star}(w; L_0)\ \sigma^{B\to D^\star}(w; L_0, L_1)\ \sigma^{B\to D^\star}(w; L_1, L_2)$$



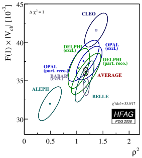
$B \to D^\star \ell \nu$: theory vs. experiment

[de Divitiis,Petronzio,N.T. arXiv:0807.2944]



we get: $V_{cb}(@w = 1.075) \; = \; 3.74(8)(5) \times 10^{-2}$

$F^{B \to D^\star}(w = 1.075) = 0.877(18)(04)$



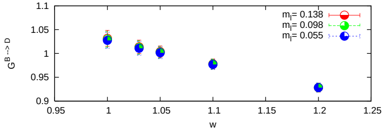
$B \to D\ell\nu$

the same game has been played in the pseudoscalar-pseudoscalar case:

$$\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = (\text{kin. fact.}) \times$$

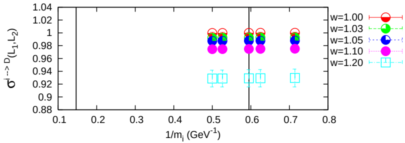
$$\times |\mathbf{V_{cb}}|^2 (\omega^2 - 1)^{\frac{3}{2}} \left[\mathbf{G^{B\to D}}(\omega)\right]^2$$

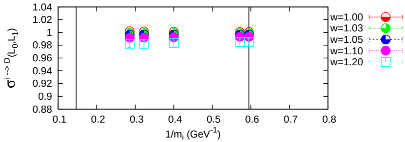
$$G^{B\to D}(\omega) = h_+^{B\to D}(w) - \frac{M_D - M_B}{M_D + M_B} h_-^{B\to D}(w)$$



also in this case the step scaling functions are very flat!

$$\sigma^{i\to D}(w; L_0, L_1) = \frac{G^{i\to D}(w; L_1)}{G^{i\to D}(w; L_0)}$$

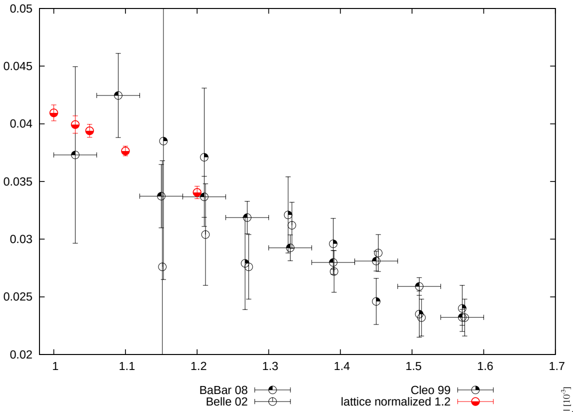




$$G^{B\to D}(w; L_2) = G^{B\to D}(w; L_0)\ \sigma^{B\to D}(w; L_0, L_1)\ \sigma^{B\to D}(w; L_1, L_2)$$

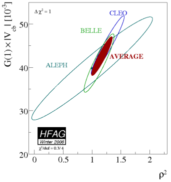
$B \to D\ell\nu$: theory vs. experiment

[de Divitiis,Molinaro,Petronzio,N.T. Phys.Lett.B655:45,2007]



we get: $V_{cb}(@w = 1.2) \; = \; 3.84(9)(42) \times 10^{-2}$

$G^{B \to D}(w = 1.2) = 0.853(21)$



outlooks

- FNAL calculations already provide percent relative accuracy for $B \to D^{(\star)}\ell\nu$ at zero recoil
  - alternative lattice formulations for sea quarks would asses possible rooting effects
  - non-perturbative matching could reduce or provide a better estimate of perturbative uncertainties

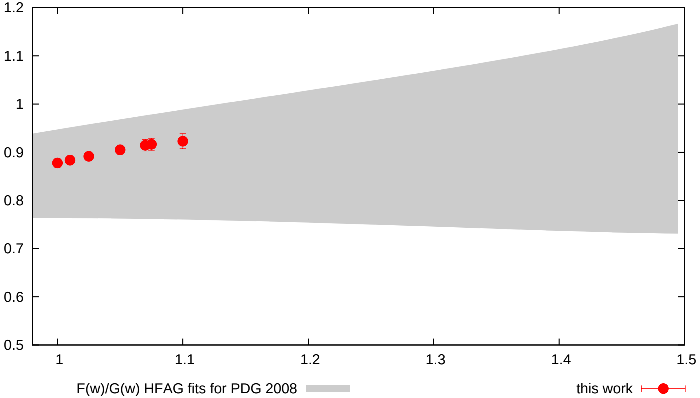
- the step scaling method is a viable alternative
  - it works very well in the quenched approximation

- form factors at $w > 1$ can be calculated with good accuracy
  - strategy devised and tested in the quenched approximation
  - avoid extrapolations on the experimental side: experimentalists should quote differential decay rates at $w > 1$
  - if available, experimentalists should quote the ratio of form factors at $w \geq 1$ in order to benchmark lattice calculations
  - not tested but in principle should work also for $B \to$ *light* semileptonic transitions

- **presently**: do the quenched numbers have any phenomenological relevance?

presently: do the quenched numbers have any relevance?

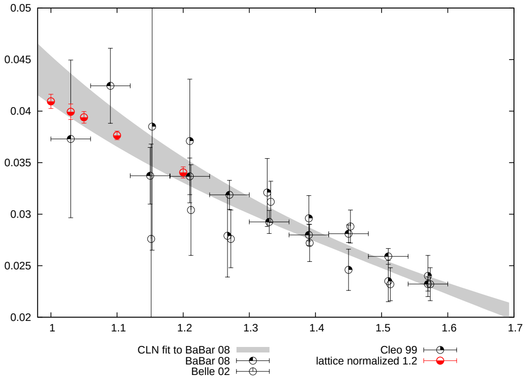
[de Divitiis,Petronzio,N.T. arXiv:0807.2944]



- quenched form factors are in very good agreement with $N_f = 2 + 1$ at zero recoil
- the ratio $F^{B\to D^\star}(w)/G^{B\to D}(w)$ is in good agreement with experimental data

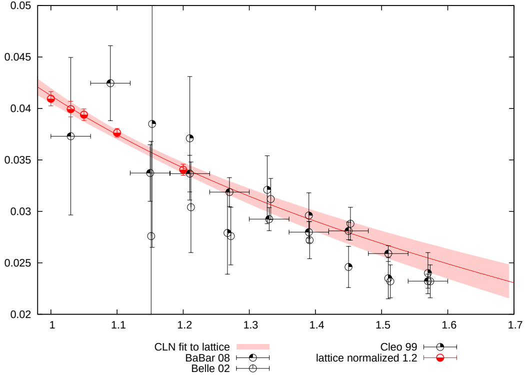
presently: do the quenched numbers have any relevance?

[de Divitiis,Petronzio,N.T. arXiv:0807.2944]
[Caprini,Lellouch,Neubert Nucl.Phys.B530:153,1998]



- in the $B \longrightarrow D\ell\nu$ channel experimental extrapolations have a big impact on V_{cb}...

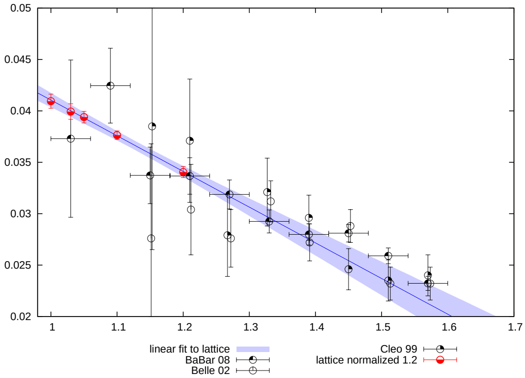
[de Divitiis,Petronzio,N.T. arXiv:0807.2944]
[Caprini,Lellouch,Neubert Nucl.Phys.B530:153,1998]



- in the $B \to D\ell\nu$ channel experimental extrapolations have a big impact on V_{cb}...

presently: do the quenched numbers have any relevance?

[de Divitiis,Petronzio,N.T. arXiv:0807.2944]
[Caprini,Lellouch,Neubert Nucl.Phys.B530:153,1998]



- in the $B \to D\ell\nu$ channel experimental extrapolations have a big impact on V_{cb}...
- here quenched data at $w > 1$ can be used to estimate the uncertainties on V_{cb} due to extrapolations at zero recoil
- and/or to calculate experimental efficiency

[D.Lopes-Pegna]