Determination of $\mathcal{F}(1)$ and $\mathcal{G}(1)$

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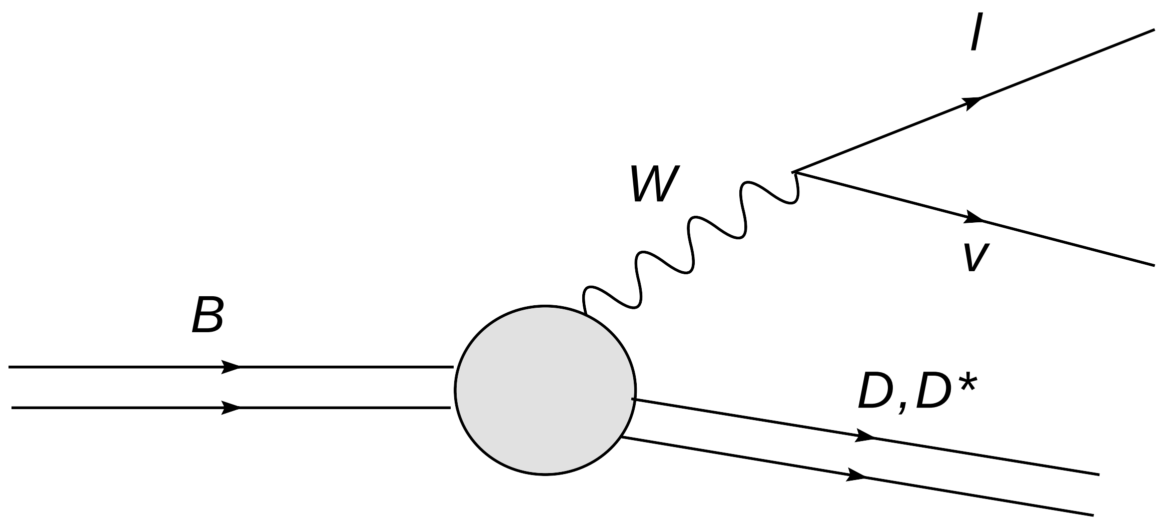
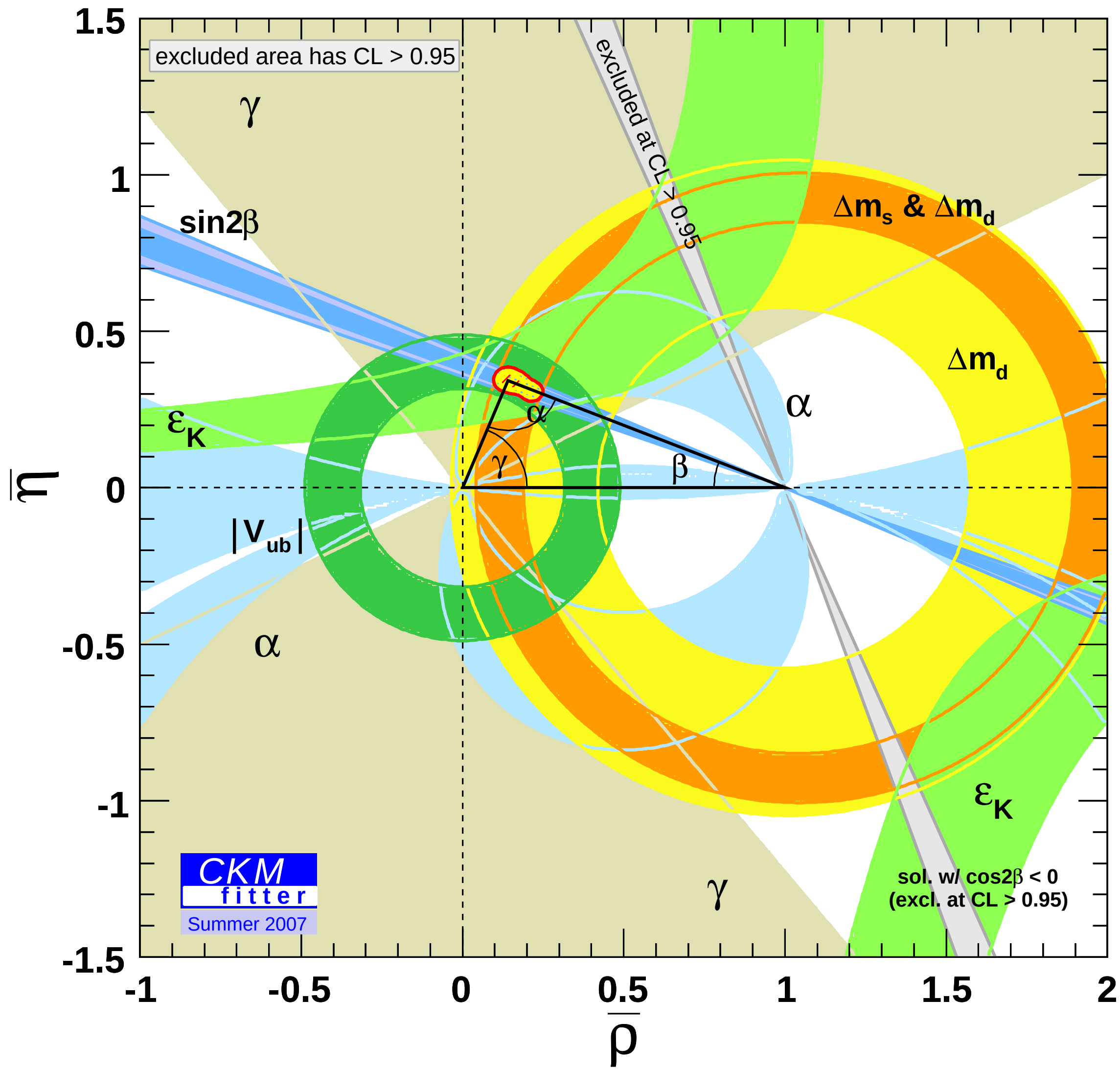
Charmed B semileptonic decays



Vertex proportional to $|V_{cb}|$. In order to extract it, nonperturbative input is needed.

Constraining the Unitarity Triangle



Importance of $|V_{cb}|$

$|V_{cb}|$ is needed to constrain the apex of the unitarity triangle from kaon mixing. Given that

$$A = \frac{|V_{cb}|}{\lambda^2} \tag{1}$$

has $\approx 2\%$ error, we see that this contributes a 9% error to ϵ_K because it appears in the formula below to the fourth power.

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

Given expected progress in B_K, we must lower the errors on $|V_{cb}|$. This puts pressure on the continuum perturbation theory community since the two-loop calculation of the Wilson coefficients has $\sim 7\%$ errors.

Methods for extracting $|V_{cb}|$

- Inclusive $b \to c\ell\nu$ can be calculated using the OPE and perturbation theory. Requires non-perturbative input from experiment: moments of inclusive form factor $\overline{B} \to X_c \ell \overline{\nu}_\ell$ as a function of minimum electron momentum. Theoretical uncertainties from truncating the OPE and PT, and also perhaps from duality violations.

- Exclusive $B \to D\ell\nu$ has seen much experimental progress in the last month. No problem in principle of going to small recoil on the lattice.

- Exclusive $B \to D^*\ell\nu$ is slightly cleaner ($\sim 1.7\%$ experimental error at zero-recoil).

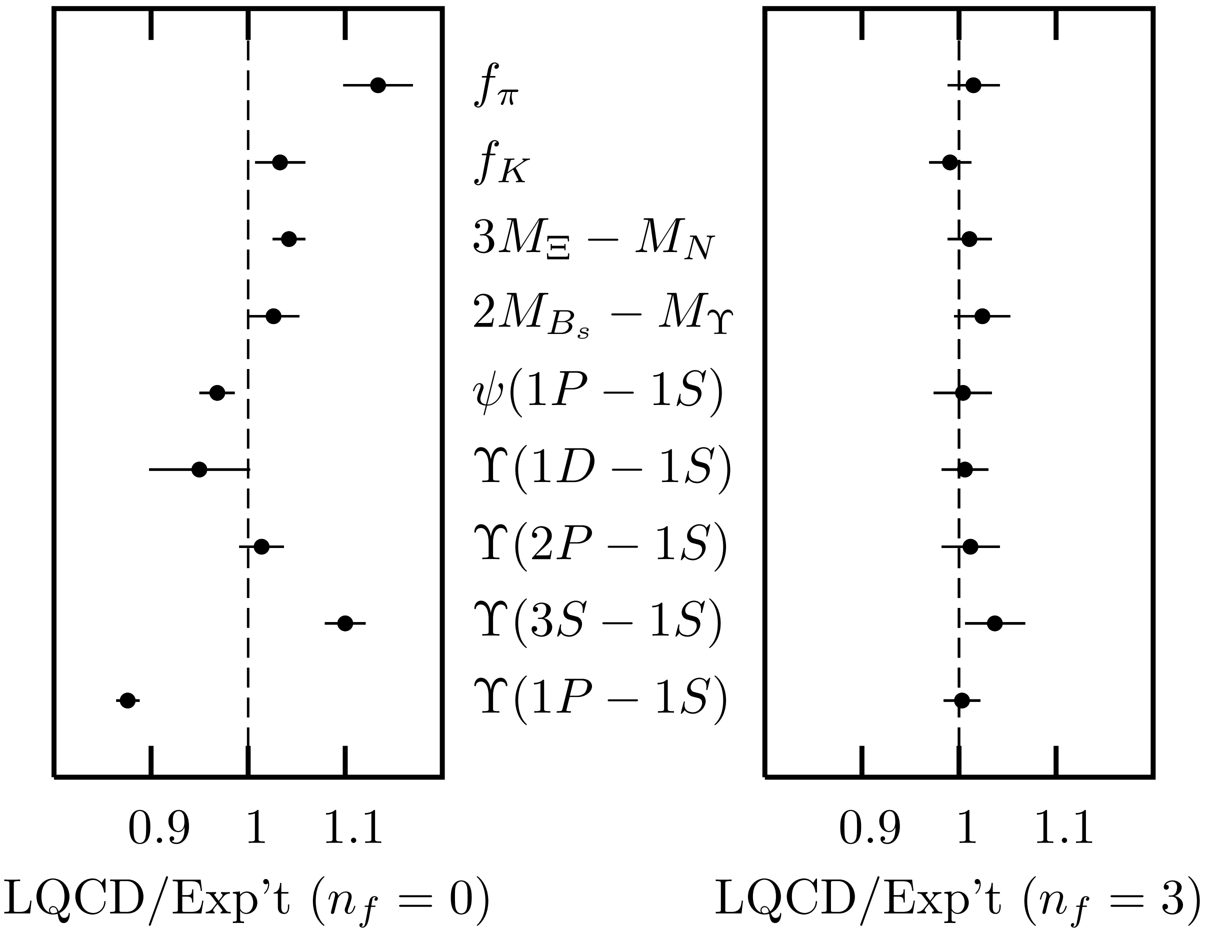
Staggered fermions

- Staggered fermions are the cheapest fermions on the market at the present time.
- The staggered action has extra unphysical species of fermions (called "tastes") due to lattice artifacts which vanish in the continuum limit.
- This complicates the analysis with staggered fermions, as compared to "chiral" fermions such as domain-wall or overlap, which are many times more expensive.
- Staggered chiral perturbation theory gives good control over staggered discretization effects (MILC, arXiv:hep-lat/0407028).

Staggered quarks and rooting

- In the continuum limit, the four staggered tastes become degenerate
- In principle, taste breaking can be removed by taking the continuum limit, but in practice one must take the fourth root at finite lattice spacing.
- There is no rigorous proof that this procedure recovers QCD in the continuum limit, though there has been much recent progress on this issue, which is reviewed in hep-lat/0610094 by Steve Sharpe. Recent criticism has been refuted.
- It appears plausible that this procedure recovers QCD in the continuum limit, and we work under this assumption.
- There is no reason why these calculations could not be repeated with other types of lattice fermions.

Unquenching with staggered quarks



- Hadron spectroscopy – masses and decay constants
- *Good agreement for simple quantities!*

$|V_{cb}|$ from $B \to D\ell\nu$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} \times |V_{cb}|^2 |\mathcal{G}_{B\to D}(w)|^2 \quad (2)$$

where $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v'), and where

$$\mathcal{G}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w). \quad (3)$$

Quenched Fermilab calculation

Hashimoto et al, hep-ph/990637 computed $h_+(1)$ and $h_-(1)$ in order to construct $\mathcal{G}_{B\to D}(1)$ and extract $|V_{cb}|$. This was done using the Fermilab action for heavy quarks. Double ratios were constructed.

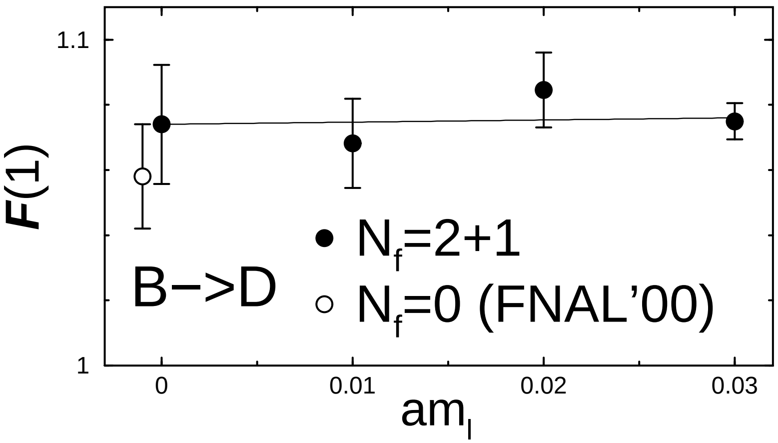
Advantages of the double ratios:

- Statistical errors cancel in the ratios
- Most of the current renormalization cancels. The remainder can be computed perturbatively.
- As shown by Kronfeld (hep-lat/0002008), heavy quark symmetry constrains the discretization errors in the double ratio for $h_+(1)$, so that for this quantity the leading corrections are of the order $\alpha_s(\overline{\Lambda}/m_Q)^2$ and $\overline{\Lambda}/m_Q^3$.
- All errors in double ratios $\mathcal{R}$ scaled as $\mathcal{R}-1$ rather than as $\mathcal{R}$, since when $m_c = m_b$ the ratio for $h_+(1)$ was one by construction. This was especially important since Hashimoto et al were working in the quenched approximation.

$\mathcal{G}_{B\to D}(1) = 1.058(^{+21}_{-17})$, $\sim 2\%$ error

Preliminary unquenched calculation

Okamoto, et al, hep-lat/0409116, for Fermilab/MILC Collaborations

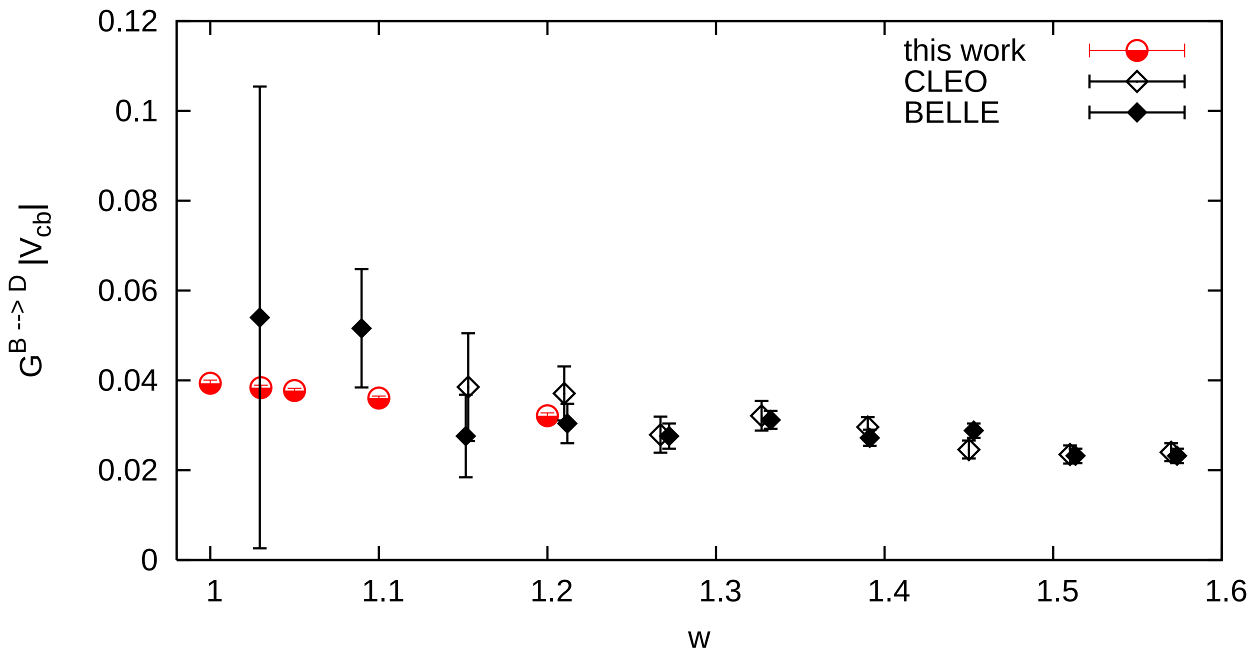


The preliminary result $\mathcal{G}(1) = 1.074(18)(16)$ was quoted, where the first error was statistical and the second was the sum of all systematic errors in quadrature.

uncertainty	$\mathcal{G}(1)$
statistical	1.7%
chiral extrapolation	$\sim 1\%$
discretization errors	$\sim 1\%$
perturbation theory	$\sim 1\%$
Total	$2-3\%$

New (quenched) result for $w \geq 1$

de Divitiis, et al, arXiv:0707.0582



New result using a step-scaling method.

A result is quoted of $\mathcal{G}_{B\to D}(1) = 1.026(17)$, with results also for $w \geq 1$. This is consistent with the quenched Hashimoto et al result of $\mathcal{G}_{B\to D}(1) = 1.058(^{+21}_{-17})$.

New (quenched) result for $w \geq 1$

A few caveats:

- Theoretical analysis of mass dependence is not fully understood. However, this appears to be unimportant because the mass dependence is so mild.
- Papers do not contain a table of the full error budget, so it is not clear if the error bar encompasses all sources of uncertainty.

Even so, the w dependence looks very promising!

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \times |V_{cb}|^2 \mathcal{G}(w) |\mathcal{F}_{B \to D^*}(w)|^2 \tag{4}$$

where $\mathcal{G}(w)|\mathcal{F}_{B \to D^*}|^2$ contains a combination of form-factors which must be computed non-perturbatively.

$w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v').

Calculating $B \to D^*$ form factor

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1), \tag{5}$$

$$\langle D^*(v)|\mathcal{A}^\mu|\overline{B}(v)\rangle = i\sqrt{2m_B 2m_{D^*}}\,\overline{\epsilon'}^\mu h_{A_1}(1). \tag{6}$$

$h_{A_1}(1)$ is constrained by heavy quark symmetry:

$$h_{A_1}(1) = \eta_A \left[1 - \frac{l_V}{(2m_c)^2} + \frac{2l_A}{2m_c 2m_b} - \frac{l_P}{(2m_b)^2}\right] \tag{7}$$

Quenched Fermilab calculation

Hashimoto et al, hep-ph/0110253 proposed three double ratios, one for each of the $1/m_Q^2$ coefficients on the previous slide. Fits to the three ratios using the HQET dependence on heavy quark masses yielded the $1/m_Q^2$ (and most of the $1/m_Q^3$) coefficients.
Again, the advantages of the double ratios are:

- Statistical errors cancel in the ratios
- Most of the axial current renormalization cancels with the vector current renormalization. The remainder can be computed perturbatively.
- As shown by Kronfeld (hep-lat/0002008), heavy quark symmetry constrains the discretization errors in the double ratios, so that for this quantity the leading corrections are of the order $\alpha_s(\overline{\Lambda}/m_Q)^2$ and $\overline{\Lambda}/m_Q^3$.
- All errors in double ratios $\mathcal{R}$ scaled as $\mathcal{R}-1$ rather than as $\mathcal{R}$, since when $m_c = m_b$ the ratios were one by construction. This was especially important since Hashimoto et al were working in the quenched approximation.

$\sim 4\%$ error was quoted for $\mathcal{F}_{B\to D^*}(1)$

New calculation (Fermilab/MILC)

- We still use the Fermilab method to treat heavy quarks, as in the original quenched calculation of Hashimoto et al, hep-ph/0110253.
- Now using the MILC 2+1 flavor lattices, so the calculation is unquenched, with improved staggered (asqtad) light fermions in valence and sea
- Staggered quarks allow us to go to much lighter quark masses. Staggered chiral perturbation theory (SχPT) allows us to control systematic errors from staggered quarks in heavy-light quantities. (Aubin and Bernard, arXiv:hep-lat/0510088)
- Many MILC lattice ensembles were used. This work uses three lattice spacings ($a \approx 0.15$ fm, $a \approx 0.12$ fm, $a \approx 0.09$ fm).
- New double ratio is constructed which gives the answer more directly, allowing a cleaner determination and a huge savings in computing cost ($\sim$ factor of 10)

New Method

$$\frac{\langle D^* | \bar{c}\gamma_j\gamma_5 b | \overline{B} \rangle \langle \overline{B} | \bar{b}\gamma_j\gamma_5 c | D^* \rangle}{\langle D^* | \bar{c}\gamma_4 c | D^* \rangle \langle \overline{B} | \bar{b}\gamma_4 b | \overline{B} \rangle} = |h_{A_1}(1)|^2 \,. \tag{8}$$

- Statistical errors cancel in the ratio
- Most of the axial current renormalization cancels with the vector current renormalization. The remainder can be computed perturbatively.
- This ratio gives (the lattice approximation of) h_{A_1} directly to all orders in HQET
- The ratio can then be calculated at the tuned $m_{b,c}$, so that many heavy quark mass values are not needed.
- Fewer masses and fewer ratios means a factor of ~ 10 less computer time
- Not all errors scale as $\mathcal{R} - 1$, but in a full-QCD setting, it is no longer essential. One must simply compare (total error)/(computer time).

JL for the Fermilab and MILC Collaborations, arXiv:0710.1111

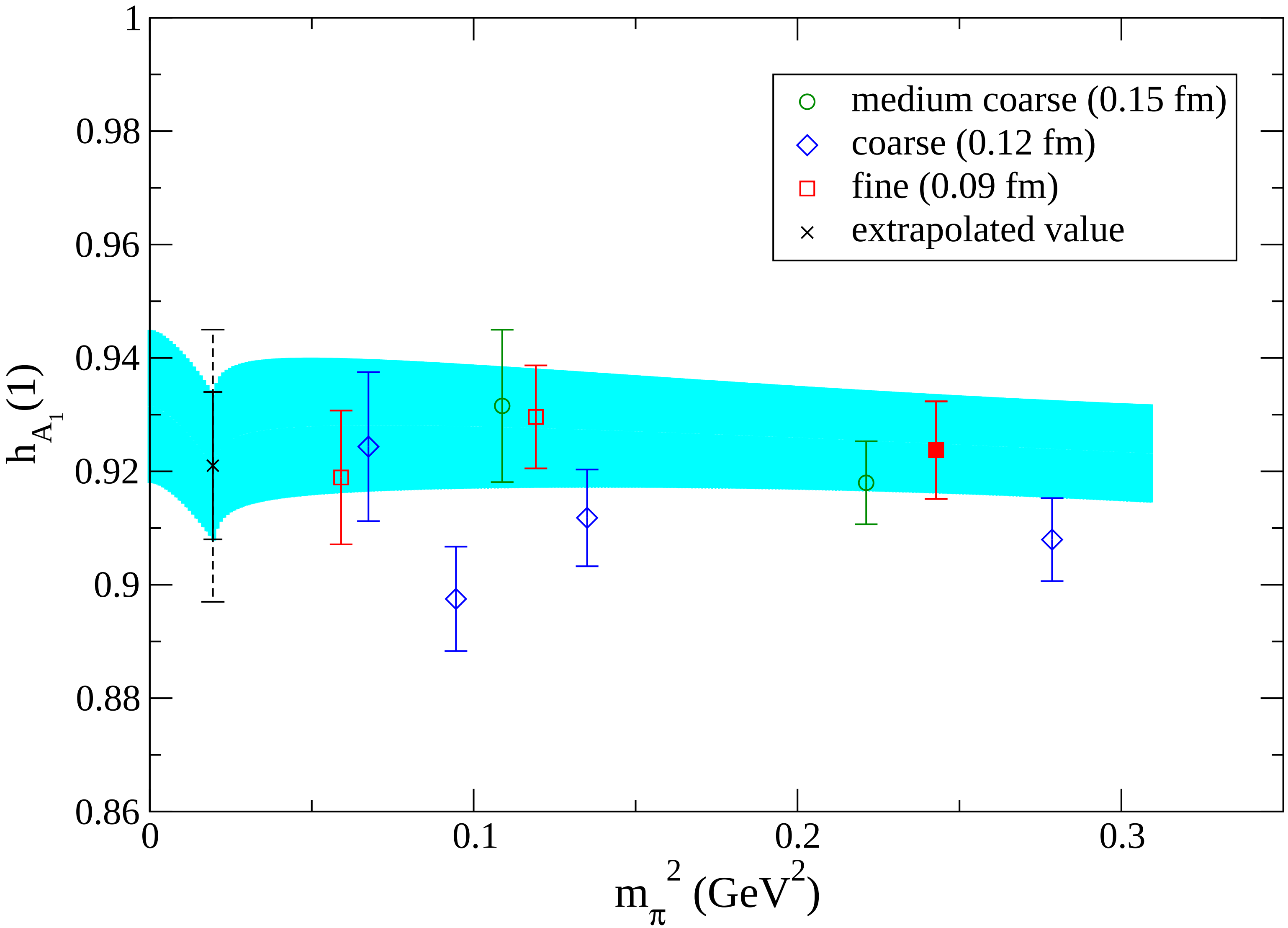
Staggered ChPT formula

[from J.L. and Van de Water, PRD74 (2006) 034510]

$$
\begin{aligned}
h_{A_1}^{2+1}(1) \quad = \quad & 1 + X_A + \frac{g_\pi^2}{48\pi^2 f^2}\left[\frac{1}{16}\sum_B (2\overline{F}_{\pi_B} + \overline{F}_{K_B}) - \frac{1}{2}\overline{F}_{\pi_I} + \frac{1}{6}\overline{F}_{\eta_I}\right. \\
& + a^2\delta'_V \left(\frac{m_{S_V}^2 - m_{\pi_V}^2}{(m_{\eta_V}^2 - m_{\pi_V}^2)(m_{\pi_V}^2 - m_{\eta'_V}^2)}\overline{F}_{\pi_V} \right. \\
& + \frac{m_{\eta_V}^2 - m_{S_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta_V} \\
& \left.\left. + \frac{m_{S_V}^2 - m_{\eta'_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta'_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta'_V} \right) + (V \to A) \right],
\end{aligned}
\tag{9}
$$

where a is the lattice spacing, δ'_V, g_π and X_A are constants, and $\overline{F}$ is a complicated function involving logs.

Chiral Extrapolation

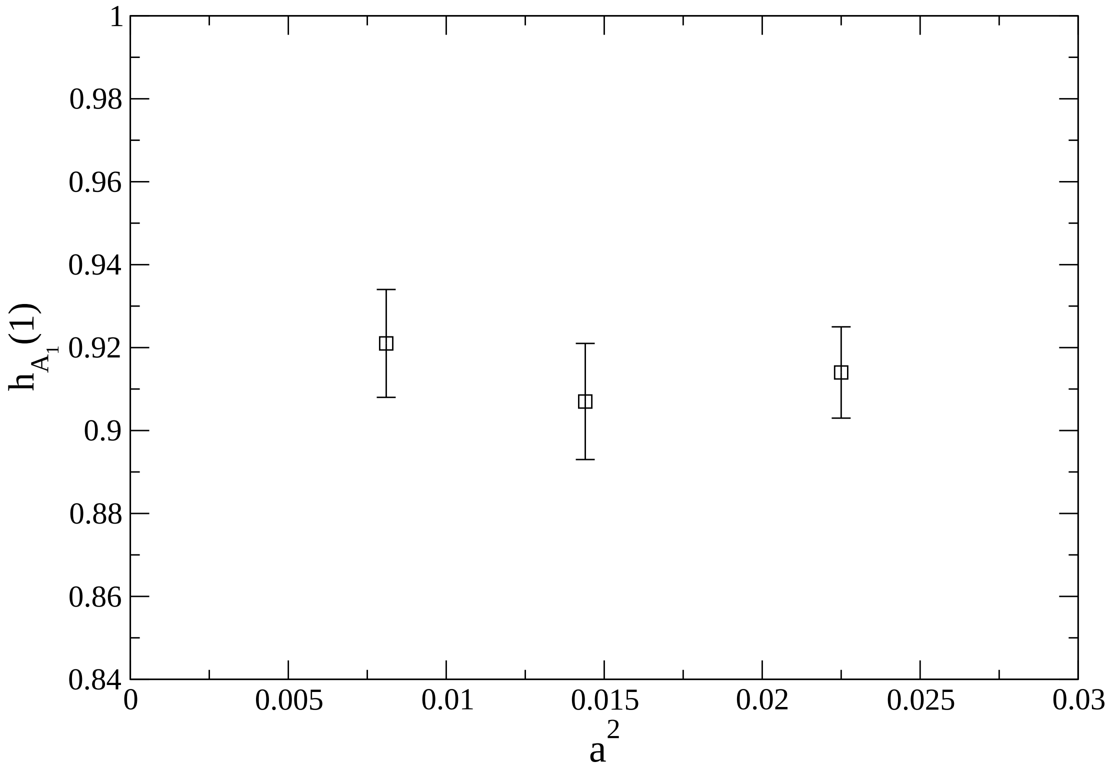


Total error budget

uncertainty	$h_{A_1}(1)$
statistical	1.4%
g_π	0.9%
NLO vs partial NNLO ChPT fits	0.9%
discretization errors	1.5%
kappa tuning	0.7%
perturbation theory	0.3%
u_0 tuning	0.4%

Total	2.6%

Discretization errors



New result for F(1)

$$h_{A_1}(1) = 0.921(13)(20)$$

where the first error is statistical and the second systematic (Fermilab/MILC, arXiv:0808.2519).

This is consistent with the earlier quenched result of [$0.913^{+0.024+0.017}_{-0.017-0.030}$].

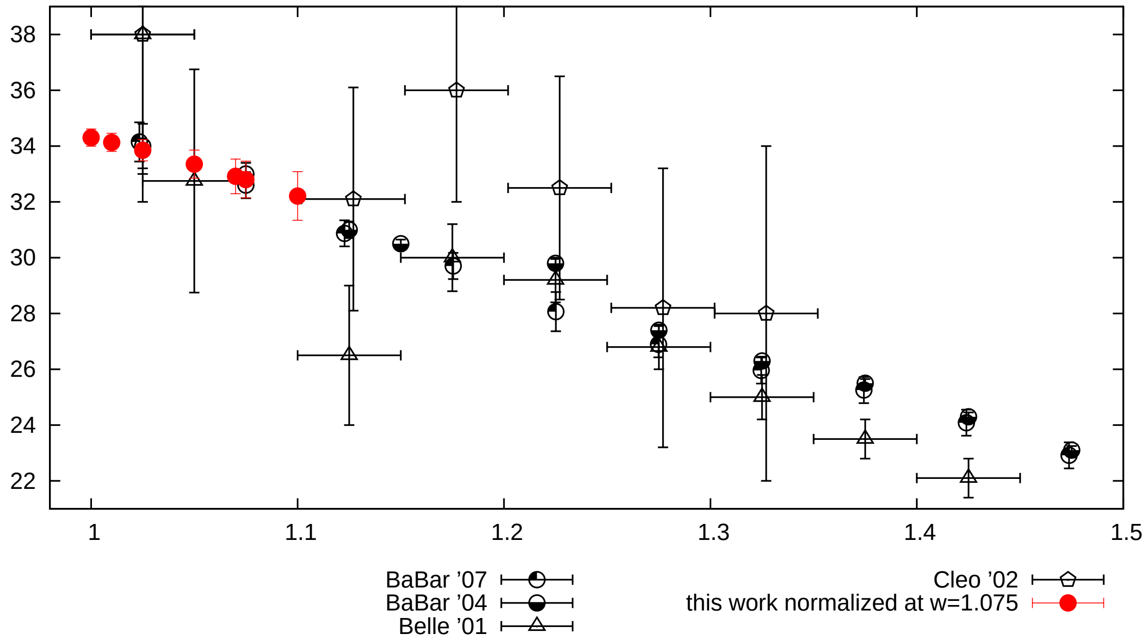
Applying a QED correction of 0.7%, and taking the PDG 08 value, $F(1)|V_{cb}| = (35.9 \pm 0.8) \times 10^{-3}$, we get

$$|V_{cb}| = (38.7 \pm 0.9_{exp} \pm 1.0_{theo}) \times 10^{-3}.$$

For comparison, the inclusive number is (PDG 2008)

$$|V_{cb}| = (41.6 \pm 0.6_{tot}) \times 10^{-3}.$$

New method at non-zero recoil



de Divitiis, Petronzio, and Tantalo, arXiv:0807.2944

Comparison of error budgets

uncertainty	Fermi/MILC $h_{A_1}(1)$	de Divitiis et. al. $h_{A_1}(1)$
statistical	1.4%	0.9%
g_π	0.9%	?
NLO vs partial NNLO ChPT fits	0.9%	?
discretization errors	1.5%	?
kappa tuning	0.5%	?
renormalization factor	0.3%	0.6%
u_0 tuning	0.4%	NA
Total	2.5%	1.0%?

$|V_{cb}| = 37.4(8)(5) \times 10^{-3}$ from de Divitiis et. al. Were the error analysis complete, this is 3.7σ from inclusive result.

More work is necessary...

Given the ~ 2 sigma disagreement between inclusive and exclusive $|V_{cb}|$, it is important to revisit all of the systematic errors.

For the lattice:

- Go to finer lattice spacings
- Better statistics
- Cross-check with $B \to D\ell\nu$ for $w \geq 1$
- Improve the quark actions
- Go to $B \to D^*\ell\nu$ with $w > 1$

Inclusive decay errors?

Kappa tuning error