Measurements of ϕ_1^{eff} from Time-dependent Dalitz Plot Analyses

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Time-dependent Dalitz Amplitude

Quasi-2-body approach to 3-body decays is limited

Other resonances may interfere with signal

Precise study can be performed with time-dependent Dalitz plot fit

The time-dependent decay rate is given by

$$|A(\Delta t, q)|^2 = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \Big[(|A|^2 + |\bar{A}|^2) - q(|A|^2 - |\bar{A}|^2) \cos \Delta m_d \Delta t \\ + 2q \Im(\bar{A} A^*) \sin \Delta m_d \Delta t \Big]$$

where $A(\bar{A})$ is the Lorentz-invariant amplitude of B^0($\bar{B}^0$)

Time-dependent Dalitz Amplitude

In the isobar approximation, the total amplitude is written as sum of decay channels,

$$A(s_+, s_-) = \sum_i a_i F_i(s_+, s_-), \quad \bar{A}(s_-, s_+) = \sum_i \bar{a}_i \bar{F}_i(s_-, s_+)$$

The Dalitz plot variables are defined as the invariant squared masses

$$s_\pm = (p_\pm + p_0)^2$$

where $p_{+,-,0}$ is the daughter 4-momentum

a_i are complex coefficients describing the relative magnitude and phase between decay channels

$a_i \equiv a_i(1 + c_i)e^{i(b_i + d_i)}$ for A, $\quad$ $a_i \equiv a_i(1 - c_i)e^{i(b_i - d_i)}$ for $\bar{A}$

For CP eigenstates and some resonance, i,

$$\mathcal{A}_{CP}(i) = \frac{|\bar{a}_i|^2 - |a_i|^2}{|\bar{a}_i|^2 + |a_i|^2} = \frac{-2c_i}{1 + c_i^2}, \quad \phi_1^{\text{eff}}(i) = d_i$$

Note that $\mathcal{A}_{CP}(i)$ must lie in physical region

Time-dependent Dalitz Amplitude

$F_i(s_+, s_-)$ are the Dalitz-dependent amplitudes for each decay channel

Expand in terms of invariant mass and angular distribution probabilities

$$F_i^L(s_+, s_-) \equiv X_i^L(\vec{p}^*) \times X_i^L(\vec{q}) \times Z_i^L(\vec{p}, \vec{q}) \times R_i(s_+, s_-)$$

$\vec{p}$ and $\vec{q}$ are momenta of bachelor and one resonance daughter in the resonance frame

L is the orbital angular momentum between the resonance and the bachelor

X_i^L are the Blatt-Weisskopf barrier factors

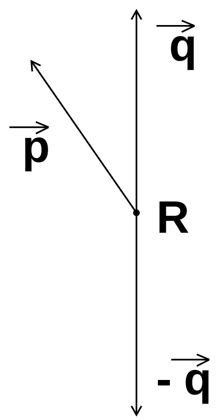
$Z_i^L(\vec{p}, \vec{q})$ is the angular distribution where

$$Z_i^0(\vec{p}, \vec{q}) = 1$$

$$Z_i^1(\vec{p}, \vec{q}) = -4\vec{p} \cdot \vec{q}$$

$$Z_i^2(\vec{p}, \vec{q}) = \frac{8}{3}[3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}||\vec{q}|)^2]$$

$R_i(s_+, s_-)$ are the Dalitz lineshapes



$B^0 \to K_S^0\pi^+\pi^-$ Decay Amplitude

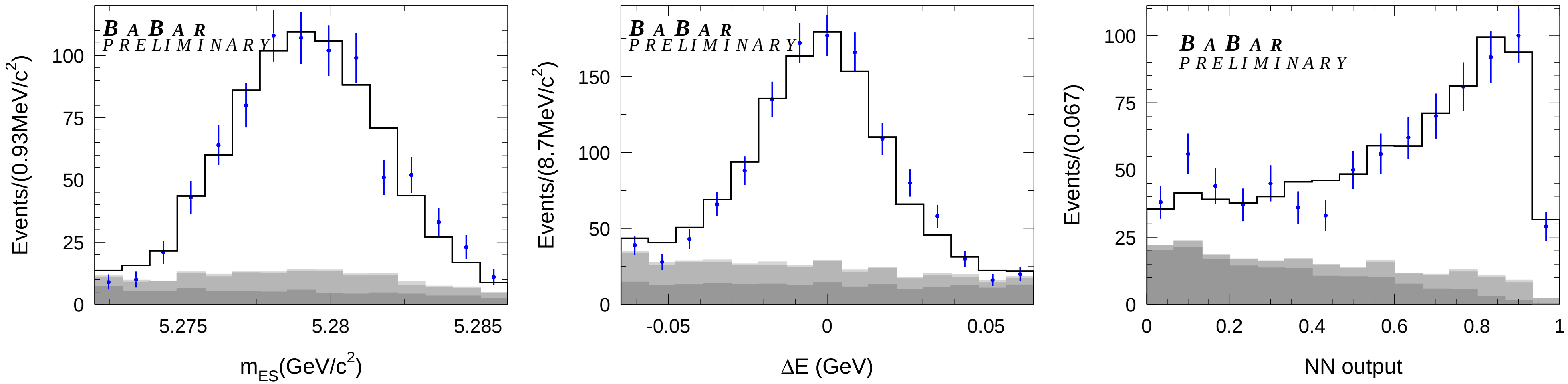
Resonance	BaBar Form Factor	Belle Form Factor	CP Eigenvalue
$K^{*\pm}(892)$	RBW	RBW	-
$K_0^{*\pm}(1430)$	LASS	RBW	-
$\rho^0(770)$	GS	GS	-1
$f_0(980)$	Flatté	Flatté	1
$f_2(1270)$	RBW	RBW	1
$f_X(1300)$	RBW	RBW	1
$(K_S^0\pi^+)_{\rm NR}\pi^-$	Flat	$e^{-\alpha s^+}$	1
$(K_S^0\pi^-)_{\rm NR}\pi^+$	Flat	$e^{-\alpha s^-}$	1
$(\pi^+\pi^-)_{\rm NR}K_S^0$	Flat	$e^{-\alpha s^0}$	1

Belle: $f_2(1270)$, $f_X(1300)$ and the non-resonant component share common CP parameters

$B^0 \to K^0_S \pi^+ \pi^-$ Signal Yield

BaBar

- Based on 383×10^6 $B\bar{B}$ pairs
- Peaking charm background modelled
- Described with m_{ES}, ΔE and NN
- Signal yield: 2172 ± 70 events

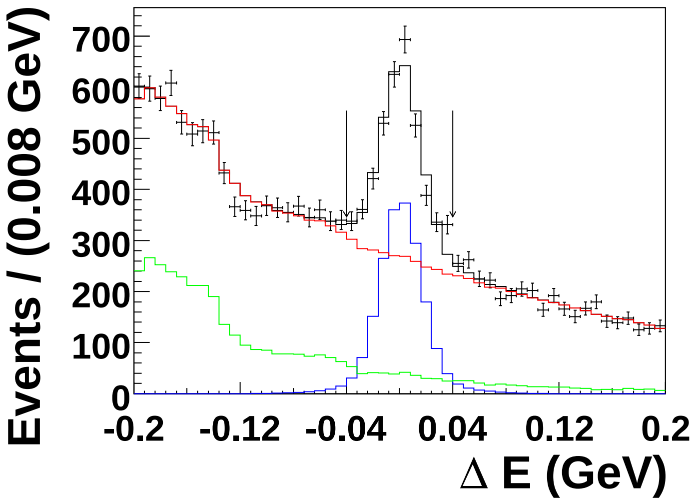


Dark grey: $q\bar{q}$, Light grey: $q\bar{q}$ + $B\bar{B}$, Removed $D^-\pi^+$, $J/\psi K^0_S$ from plot

$B^0 \to K^0_S \pi^+ \pi^-$ Signal Yield

Belle

- Based on 657×10^6 $B\bar{B}$ pairs
- Peaking charm background vetoed
- Described with ΔE
- Signal yield: 1944 ± 98 events

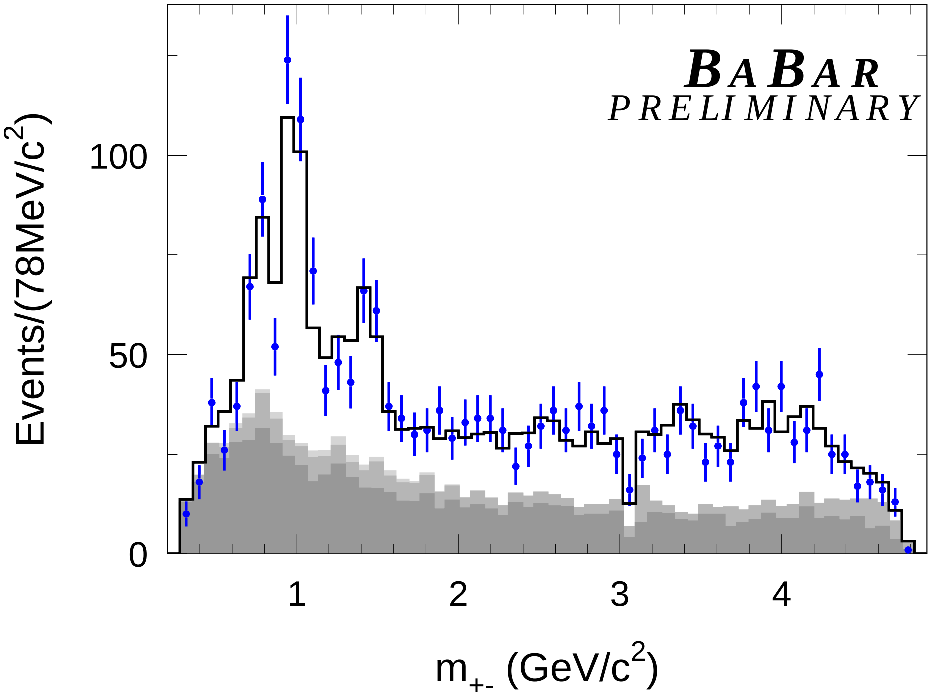


Green: $B\bar{B}$, Red: $B\bar{B}$ + $q\bar{q}$

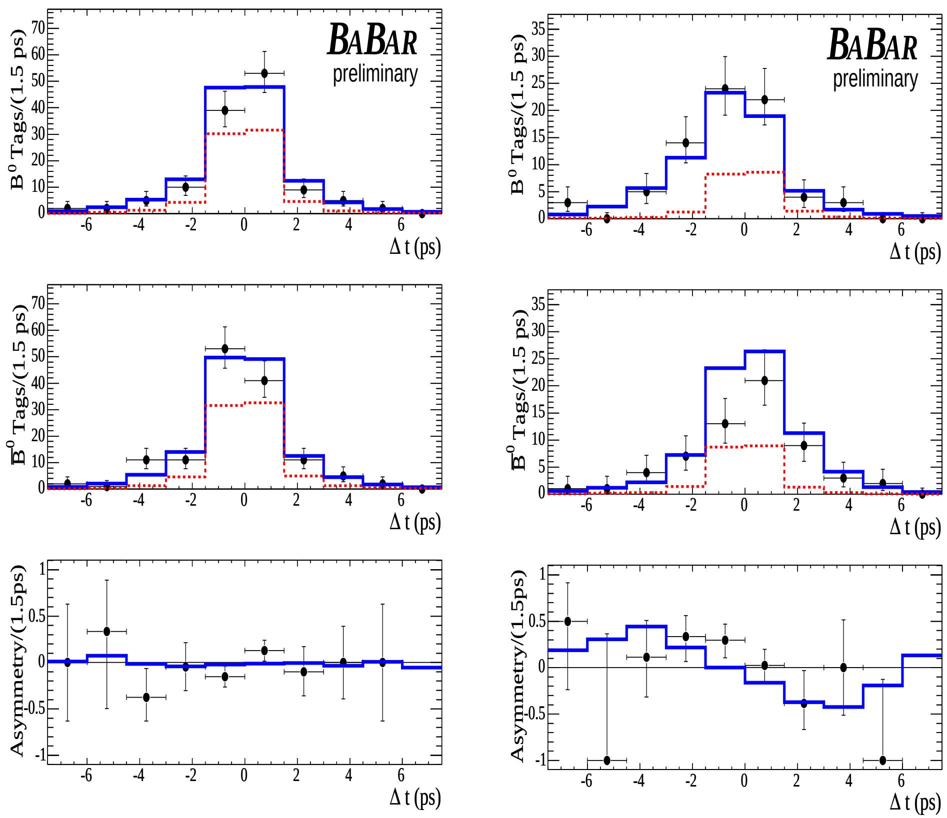
$B^0 \to K^0_S \pi^+ \pi^-$ Fit Results

BaBar

Time-dependent DP fit and signal yield extraction performed simultaneously



$\rho^0(770) K^0_S$ $f_0(980) K^0_S$



Dark grey: $q\bar{q}$, Light grey: $q\bar{q}$ + $B\bar{B}$

Removed $D^-\pi^+$, $J/\psi K^0_S$ from plot

Top: B^0 tags

Middle: $\bar{B}^0$ tags

Red: Background

$B^0 \to K^0_S \pi^+ \pi^-$ Fit Results

BaBar

$$
\begin{aligned}
\mathcal{A}_{CP}(\rho^0(770)K^0_S) &= -0.02 \pm 0.27 \pm 0.08 \pm 0.06 \\
\phi_1^{\rm eff}(\rho^0(770)K^0_S) &= (18.5^{+9.5}_{-8.5} \pm 2.5 \pm 3.0)^\circ \\
\mathcal{A}_{CP}(f_0(980)K^0_S) &= -0.35 \pm 0.27 \pm 0.07 \pm 0.04 \\
\phi_1^{\rm eff}(f_0(980)K^0_S) &= (44.5^{+11.0}_{-10.0} \pm 2.5 \pm 4.0)^\circ
\end{aligned}
$$

The first error is statistical, the second systematic, the third the model uncertainty

$B^0 \to K^0_S \pi^+ \pi^-$ Fit Results

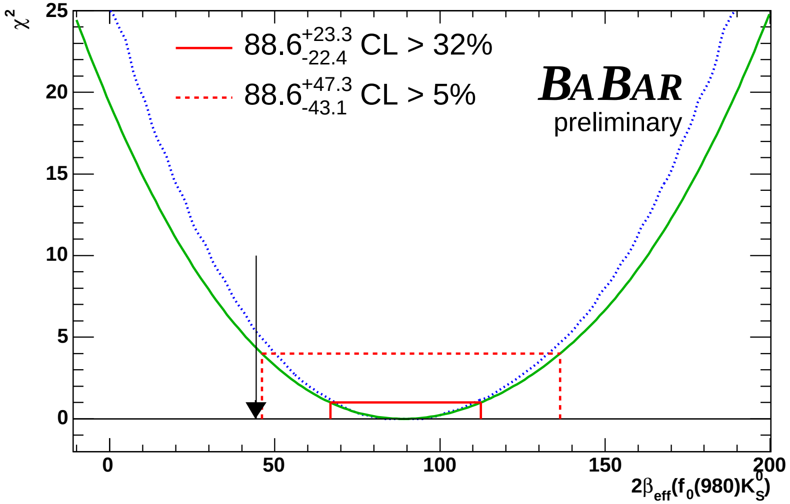
BaBar

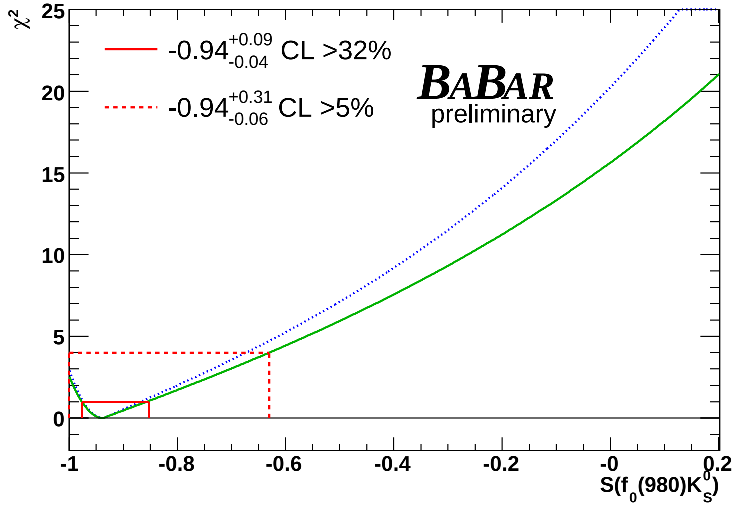
Calculate $\mathcal{S}_{CP}(i)$ for some resonance, i, restricted to the physical region

$$\mathcal{S}_{CP}(i) = \frac{2\eta_{CP}\Im(\bar{a}_i a_i^*)}{|a_i|^2 + |\bar{a}_i|^2}$$

$\phi_1^{\mathrm{eff}}(f_0(980)K^0_S)$ close to $45°$ $\rightsquigarrow$ non-Gaussian errors on $\mathcal{S}_{CP}$

$\phi_1^{\mathrm{eff}}(f_0(980)K^0_S)$	$\mathcal{S}_{CP}(f_0(980)K^0_S)$



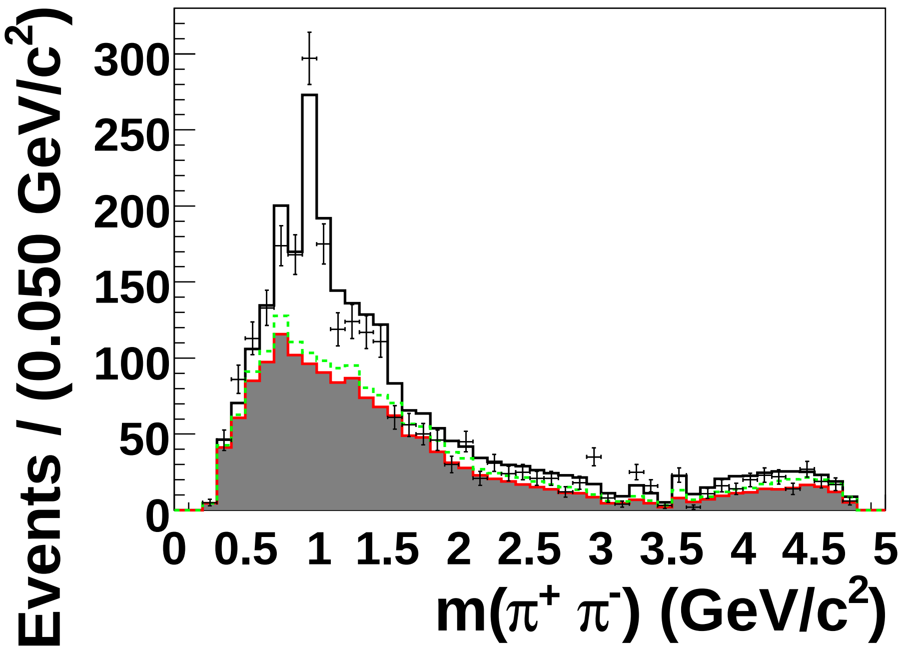


Blue: Statistical errors, Green: Statistical + Systematic errors, Red: $\pm 1\sigma$

$B^0 \to K^0_S \pi^+ \pi^-$ Fit Results

Belle

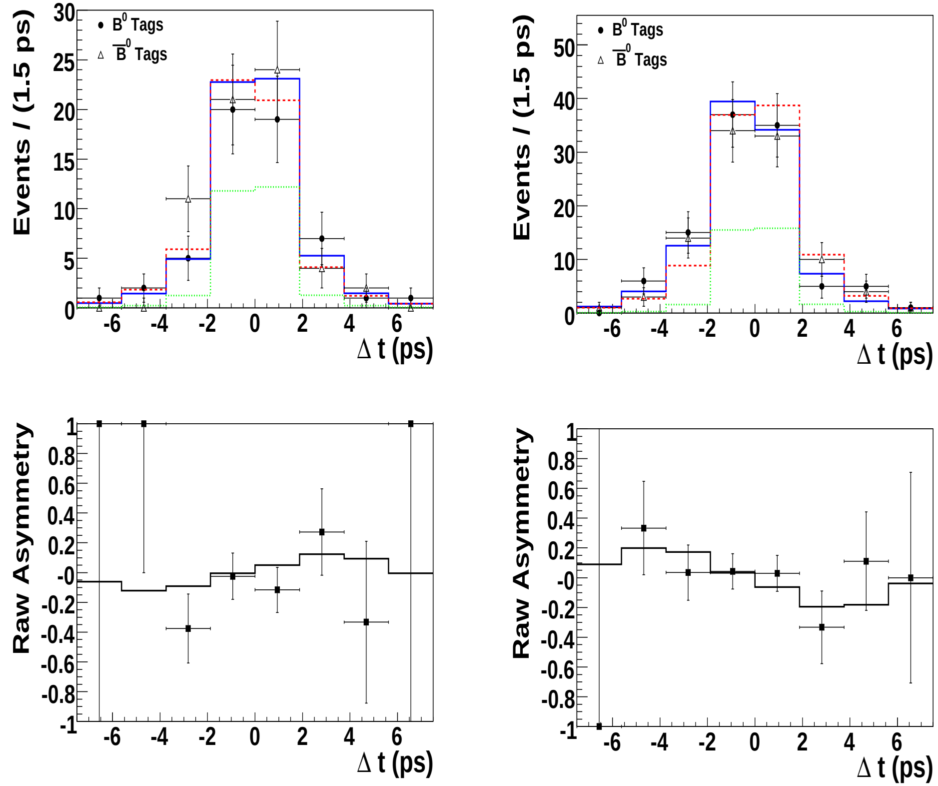
Signal yield extraction gives signal probability for time-dependent DP fit



Dark grey: $q\bar{q}$, Light grey: $q\bar{q}$ + $B\bar{B}$

Signal enhanced

$\rho^0(770)K^0_S$ $f_0(980)K^0_S$



Blue curve: B^0 tags

Red curve: $\bar{B}^0$ tags

Green: Background

$B^0 \to K^0_S \pi^+ \pi^-$ Fit Results

Belle

Solution 1: $f(K_0^{*+}(1430)\pi^-) = (61.7 \pm 10.4)\%$

$$\begin{aligned}
\mathcal{A}_{CP}(\rho^0(770)K^0_S) &= +0.03^{+0.23}_{-0.24} \pm 0.11 \pm 0.10 \\
\phi_1^{\rm eff}(\rho^0(770)K^0_S) &= (20.0^{+8.6}_{-8.5} \pm 3.2 \pm 3.5)^\circ \\
\mathcal{A}_{CP}(f_0(980)K^0_S) &= -0.06^{+0.17}_{-0.17} \pm 0.07 \pm 0.09 \\
\phi_1^{\rm eff}(f_0(980)K^0_S) &= (12.7^{+6.9}_{-6.5} \pm 2.8 \pm 3.3)^\circ
\end{aligned}$$

Solution 2: $f(K_0^{*+}(1430)\pi^-) = (17.4 \pm 5.0)\%$

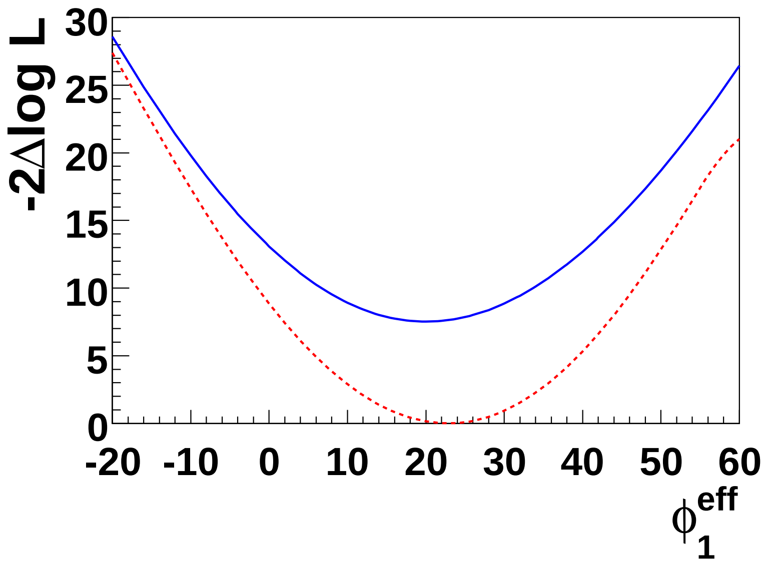
$$\begin{aligned}
\mathcal{A}_{CP}(\rho^0(770)K^0_S) &= -0.16^{+0.24}_{-0.24} \pm 0.12 \pm 0.10 \\
\phi_1^{\rm eff}(\rho^0(770)K^0_S) &= (22.8^{+7.5}_{-7.5} \pm 3.3 \pm 3.5)^\circ \\
\mathcal{A}_{CP}(f_0(980)K^0_S) &= +0.00^{+0.17}_{-0.17} \pm 0.06 \pm 0.09 \\
\phi_1^{\rm eff}(f_0(980)K^0_S) &= (14.8^{+7.3}_{-6.7} \pm 2.7 \pm 3.3)^\circ
\end{aligned}$$

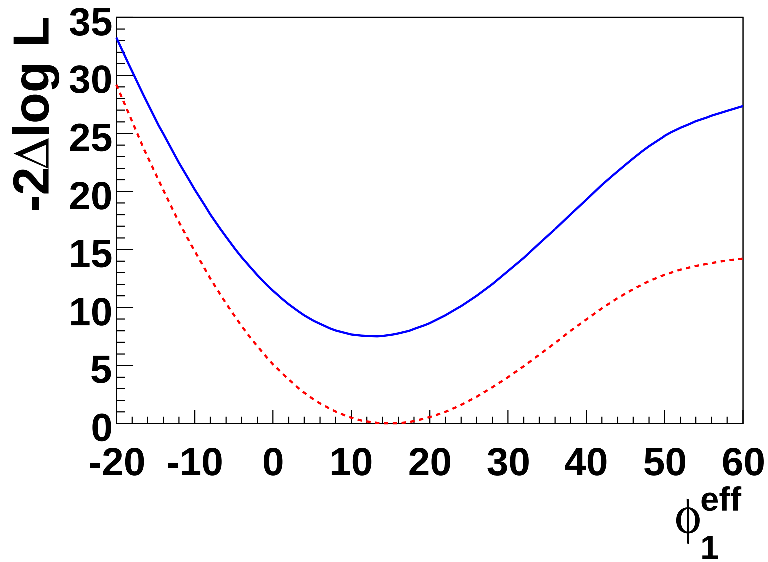
The first error is statistical, the second systematic, the third the model uncertainty

$B^0 \to K_S^0 \pi^+ \pi^-$ Fit Results

Likelihood for Solution 2 is slightly better but not significantly

$\rho^0(770)K_S^0$ $f_0(980)K_S^0$





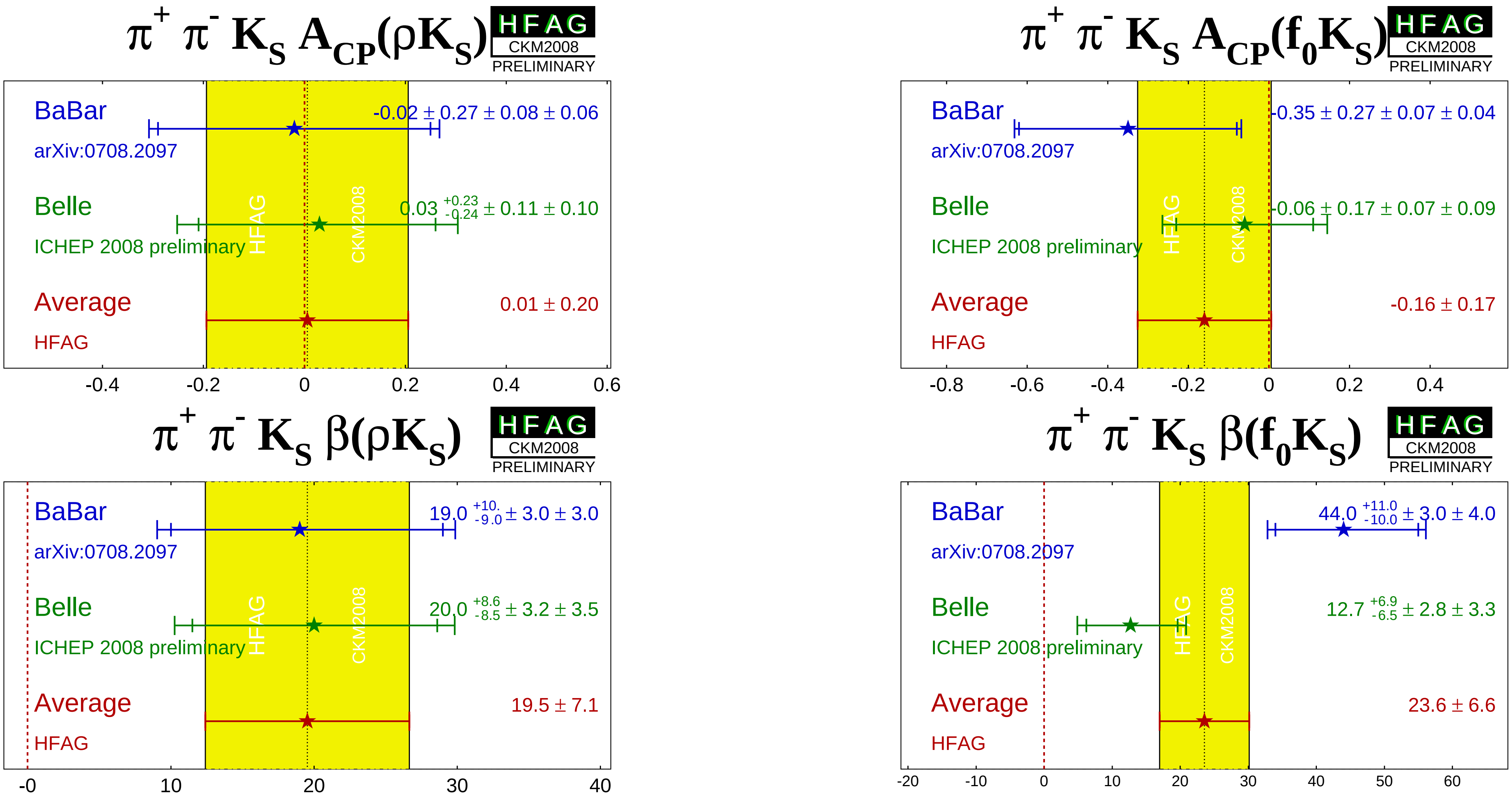
Blue: Solution 1, Red: Solution 2

Phase shift of total $K - \pi$ S-wave amplitude compared with LASS may favour Solution 1

LASS Collaboration, D. Aston *et al.*, Nucl. Phys. B **296**, 493 (1988)

We do not rule out any solution

$B^0 \to K_S^0 \pi^+ \pi^-$ Summary



$B^0 \to K^0_S K^+ K^-$ Decay Amplitude

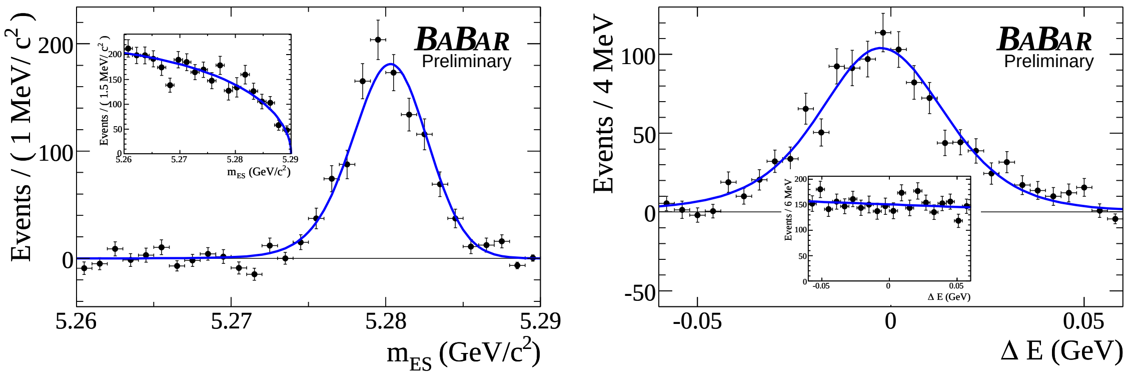
Resonance	BaBar Form Factor	Belle Form Factor	CP Eigenvalue
$f_0(980)$	Flatté	Flatté	1
$\phi(1020)$	RBW	RBW	-1
$f_X(1500)$	RBW	RBW	1
χ_{c0}	RBW	RBW	1
$(K^0_S K^+)_{\rm NR} K^-$	$e^{-\alpha s^+}$	$e^{-\alpha s^+}$	1
$(K^0_S K^-)_{\rm NR} K^+$	$e^{-\alpha s^-}$	0	1
$(K^+ K^-)_{\rm NR} K^0_S$	$e^{-\alpha s^0}$	$e^{-\alpha s^0}$	1

Belle: $f_X(1500)$, χ_{c0} and the non-resonant component share common CP parameters

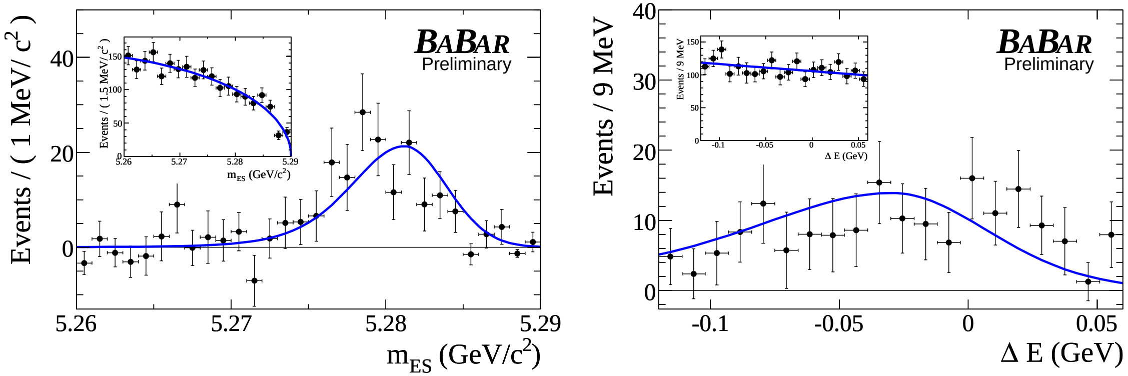
$B^0 \to K_S^0 K^+ K^-$ Signal Yield

- BaBar
  - Based on 465×10^6 $B\bar{B}$ pairs
  - Peaking charm background modelled
  - $K_S^0 \to \pi^0\pi^0$ included
  - Described with $m_{\rm ES}$, ΔE
- Inset: Background

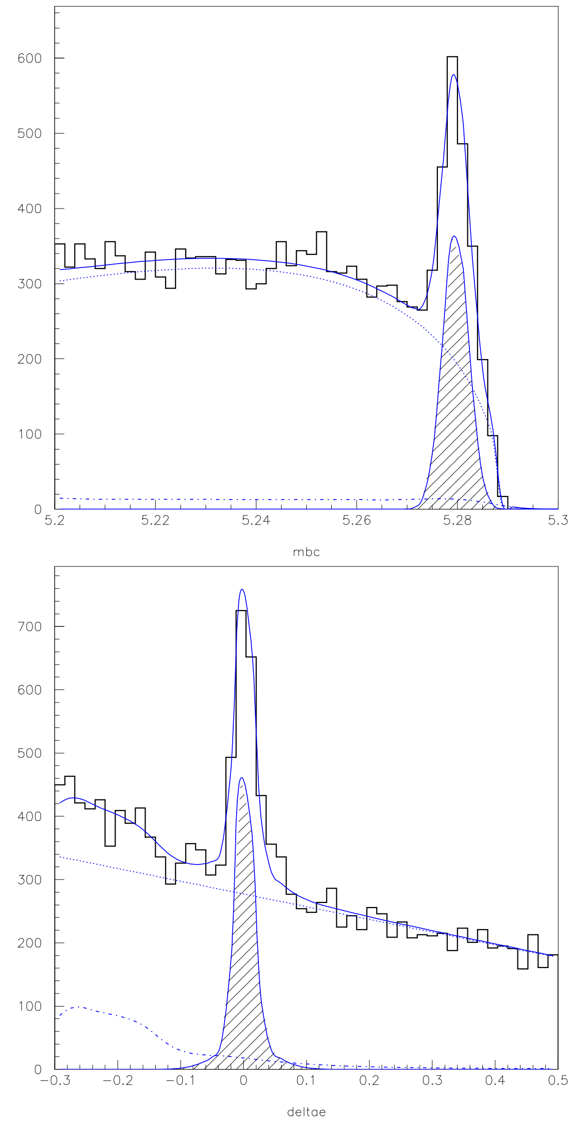
$K_S^0 \to \pi^+\pi^-$



$K_S^0 \to \pi^0\pi^0$



$B^0 \to K^0_S K^+ K^-$ Signal Yield



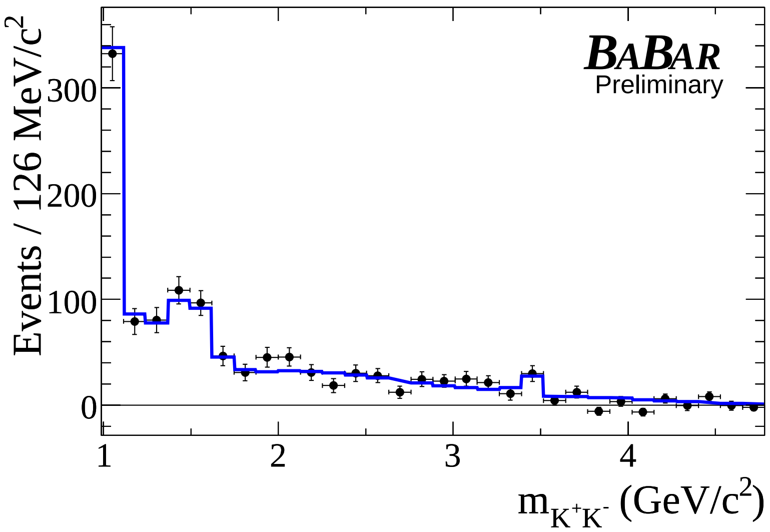
Belle

- Based on 657×10^6 $B\bar{B}$ pairs
- Peaking charm background vetoed
- Described with $M_{\rm bc}$, ΔE
- Signal yield: 1269 ± 51 events

$B^0 \to K_S^0 K^+ K^-$ Fit Results

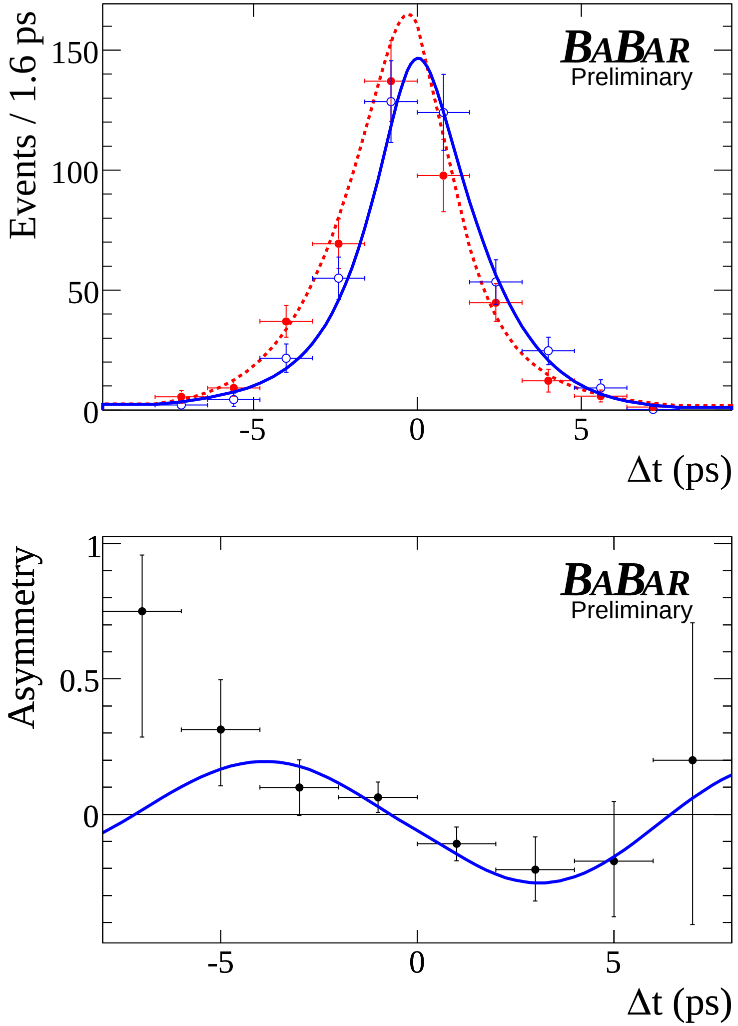
BaBar

- Time-dependent DP fit and signal yield extraction performed simultaneously
- Common CP parameters for whole DP



Signal yield: 1269 ± 51 events

$$\mathcal{A}_{CP} = +0.03 \pm 0.07 \pm 0.02$$
$$\phi_1^{\rm eff} = (25.2 \pm 4.0 \pm 1.1)^\circ$$

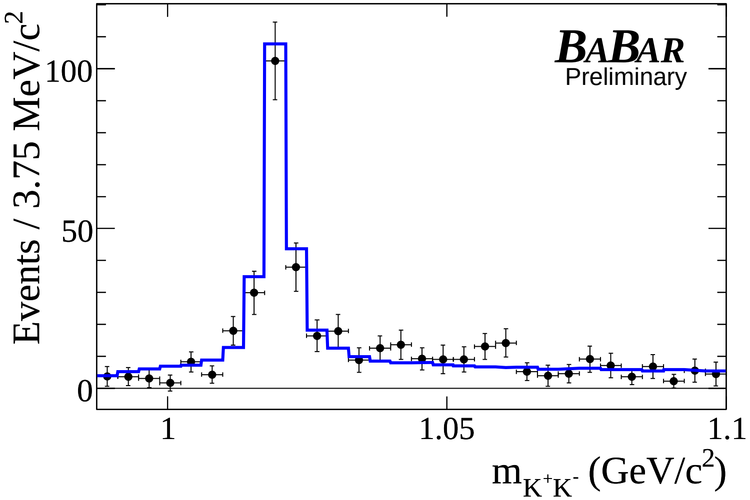


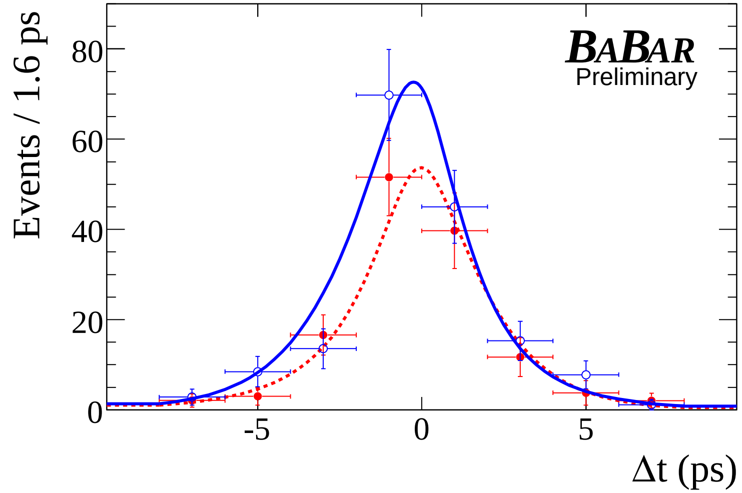
Red: B^0 tags, Blue $\bar{B}^0$ tags

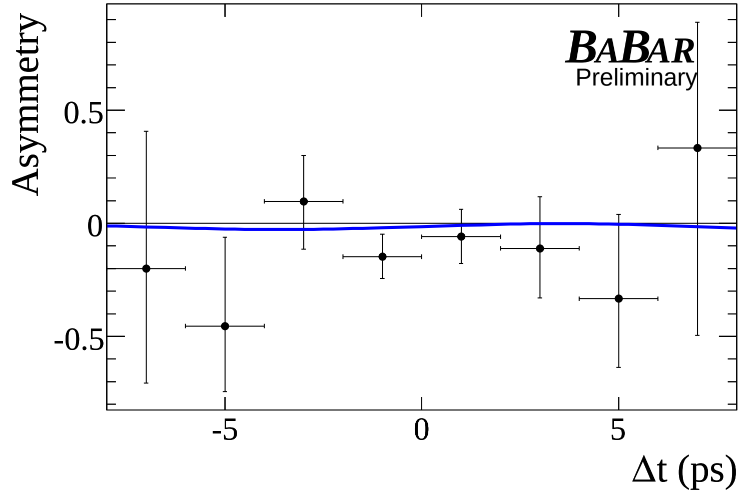
$B^0 \to K_S^0 K^+ K^-$ Fit Results

BaBar

- Time-dependent DP fit to low mass, $m_{K^+K^-} < 1.1 \text{ GeV}$
- Amplitudes and phases fixed from full DP fit, except $\phi(1020)$
- Separate $f_0(980)$ and $\phi(1020)$ CP parameters
- Other CP parameters set to zero
- Signal yield: 421 ± 25 events







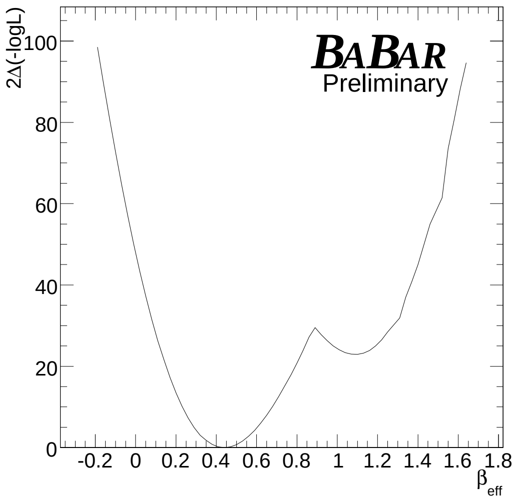
Red: B^0 tags, Blue $\bar{B}^0$ tags

$B^0 \to K^0_S K^+ K^-$ Fit Results

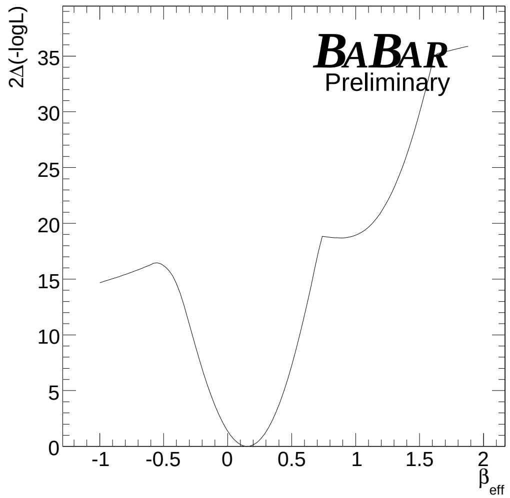
BaBar

$$\mathcal{A}_{CP}(f_0(980)K^0_S) = +0.01 \pm 0.26 \pm 0.07$$
$$\phi_1^{\rm eff}(f_0(980)K^0_S) = (8.6 \pm 7.4 \pm 1.7)^\circ$$
$$\mathcal{A}_{CP}(\phi(1020)K^0_S) = +0.14 \pm 0.19 \pm 0.02$$
$$\phi_1^{\rm eff}(\phi(1020)K^0_S) = (7.4 \pm 7.4 \pm 1.1)^\circ$$

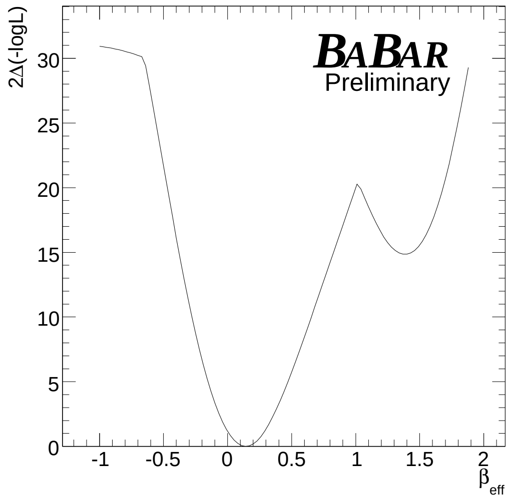
DP



$f_0(980)K^0_S$



$\phi(1020)K^0_S$



$B^0 \to K^0_S K^+ K^-$ Fit Results

Belle

Decay Mode Fractions (Sum may be $>$ 100%)

Decay Mode	S1(%)	S2(%)	S3(%)	S4(%)
$f_0(980) K^0_S$	18.4	15.3	31.0	31.2
$\phi(1020) K^0_S$	15.9	16.3	16.3	16.5
$f_X(1500) K^0_S$	5.2	50.2	64.3	6.1
$\chi_{c0}\, K^0_S$	4.1	3.8	4.1	4.4
NR	83.5	67.1	58.9	58.4
$-\log \mathcal{L}$	8079.9	8063.9	8066.3	8067.1

Source is interference between two S-wave amplitudes

- 2 solutions from interference between $f_0(980)$ and NR
- 2 solutions from interference between $f_X(1500)$ and NR

$B^0 \to K_S^0 K^+ K^-$ Fit Results

Belle

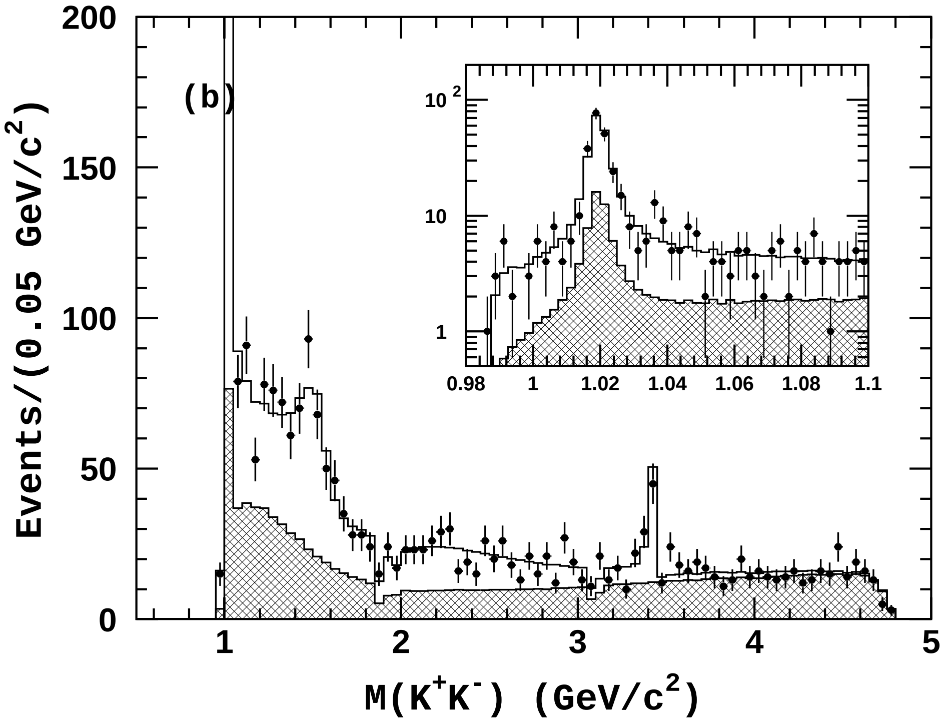
- Cannot distinguish from $\log \mathcal{L}$
- External information from $B^0 \to K_S^0 \pi^+ \pi^-$ might help
  - If $f_X(1500) = f_0(1500)$ for $B^0 \to K_S^0 K^+ K^-$ and $B^0 \to K_S^0 \pi^+ \pi^-$
  - Calculate $\frac{\mathcal{B}(f_0(1500) \to \pi^+\pi^-)}{\mathcal{B}(f_0(1500) \to K^+K^-)}$ and compare with PDG
  - Solution with low $f_X(1500) K_S^0$ fraction preferred

  - Observation of $f_0(980)$ signal in $B^0 \to K_S^0 \pi^+ \pi^-$
  - Calculate $\frac{\mathcal{B}(f_0(980) \to \pi^+\pi^-)}{\mathcal{B}(f_0(980) \to \pi^+\pi^-) + \mathcal{B}(f_0(980) \to K^+K^-)}$ and compare with PDG
  - Solution with low $f_0(980) K_S^0$ fraction preferred
- Solution 1 is preferred

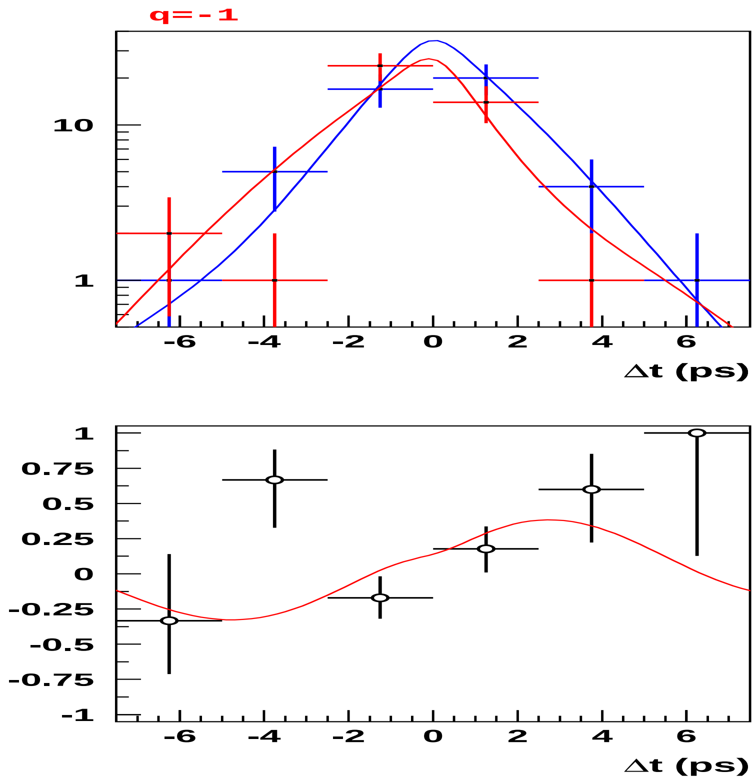
$B^0 \to K_S^0 K^+ K^-$ Fit Results

Belle

m(K^+K^-) mass projection



$\phi(1020)K_S^0$ and good tags



Black histogram: Total PDF

Shaded histogram: Background

Blue curve: B^0 tags

Red curve: $\bar{B}^0$ tags

$B^0 \to K^0_S K^+ K^-$ Fit Results

Belle

Solution	CP Parameter	Fit Result
1	$\phi_1^{\rm eff}(f_0(980)K^0_S)$	$(28.2^{+9.8}_{-9.9})^\circ$
	$\phi_1^{\rm eff}(\phi(1020)K^0_S)$	$(21.2^{+9.8}_{-10.4})^\circ$
2	$\phi_1^{\rm eff}(f_0(980)K^0_S)$	$(64.1^{+7.6}_{-8.0})^\circ$
	$\phi_1^{\rm eff}(\phi(1020)K^0_S)$	$(62.1^{+8.3}_{-8.8})^\circ$
3	$\phi_1^{\rm eff}(f_0(980)K^0_S)$	$(61.5^{+6.5}_{-6.5})^\circ$
	$\phi_1^{\rm eff}(\phi(1020)K^0_S)$	$(65.1^{+8.7}_{-8.7})^\circ$
4	$\phi_1^{\rm eff}(f_0(980)K^0_S)$	$(36.9^{+10.9}_{-9.6})^\circ$
	$\phi_1^{\rm eff}(\phi(1020)K^0_S)$	$(44.9^{+13.2}_{-13.6})^\circ$

$B^0 \rightarrow K^0_S K^+ K^-$ Fit Results

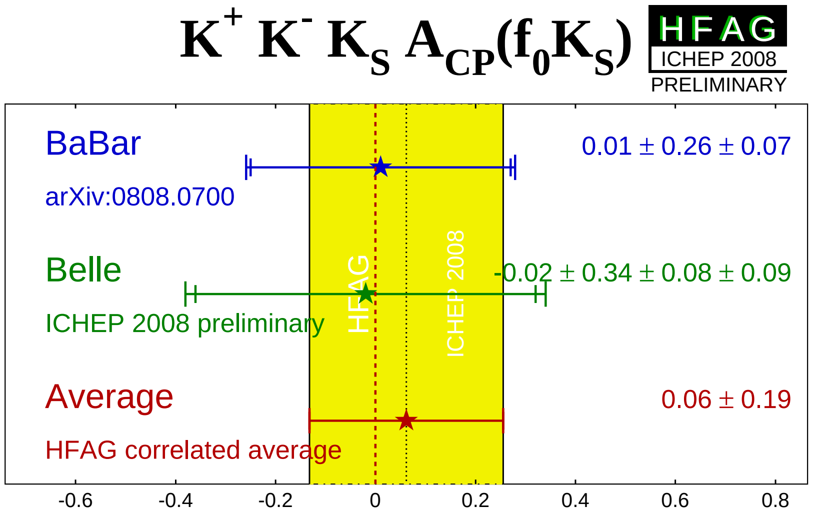
Belle

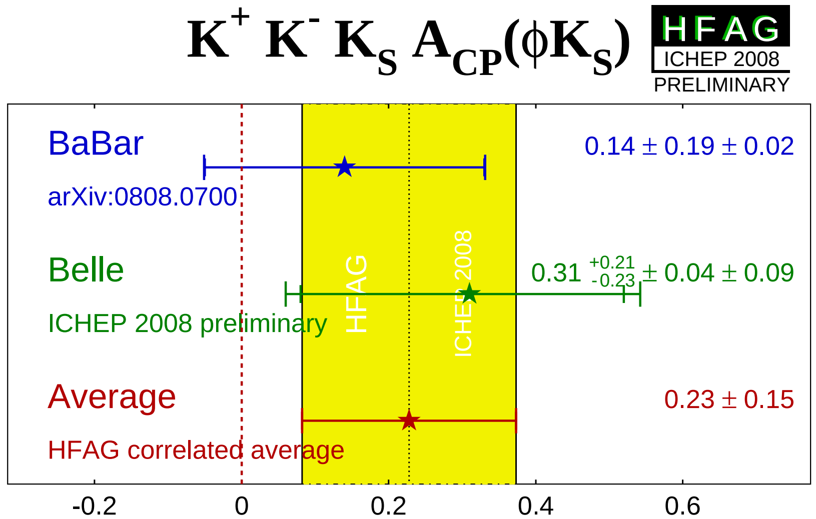
Solution 1

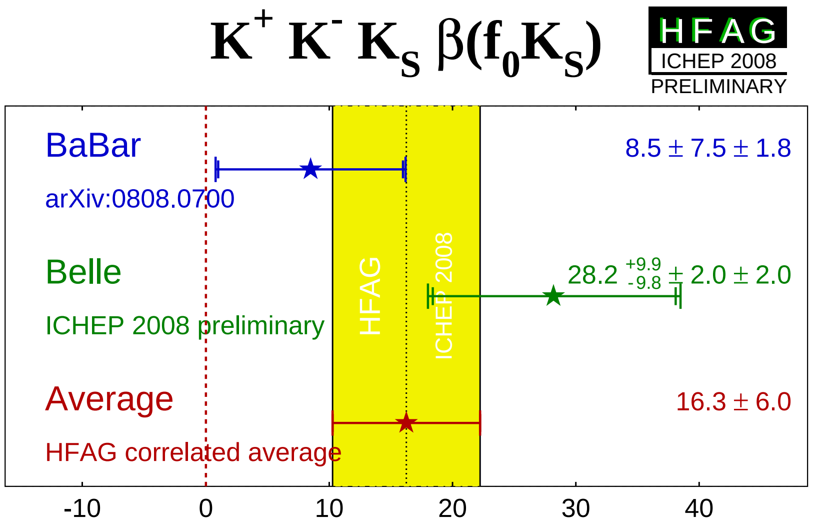
$$
\begin{aligned}
\mathcal{A}_{CP}(f_0(980)K^0_S) &= -0.02 \pm 0.34 \pm 0.08 \pm 0.09 \\
\phi_1^{\rm eff}(f_0(980)K^0_S) &= (28.2^{+9.8}_{-9.9} \pm 2.0 \pm 2.0)^\circ \\
\mathcal{A}_{CP}(\phi(1020)K^0_S) &= +0.31^{+0.21}_{-0.23} \pm 0.04 \pm 0.09 \\
\phi_1^{\rm eff}(\phi(1020)K^0_S) &= (21.2^{+9.8}_{-10.4} \pm 2.0 \pm 2.0)^\circ
\end{aligned}
$$

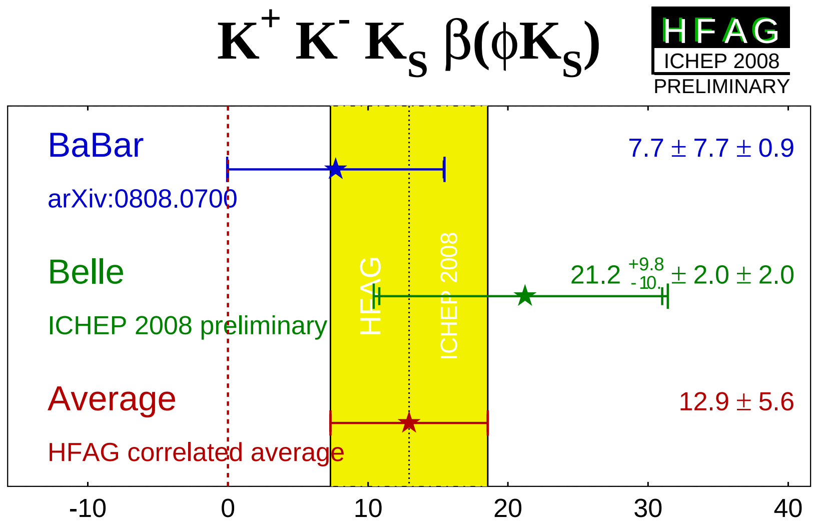
The first error is statistical, the second systematic, the third the model uncertainty

$B^0 \to K^0_S K^+ K^-$ Summary









Summary

$B^0 \to K_S^0 \pi^+ \pi^-$

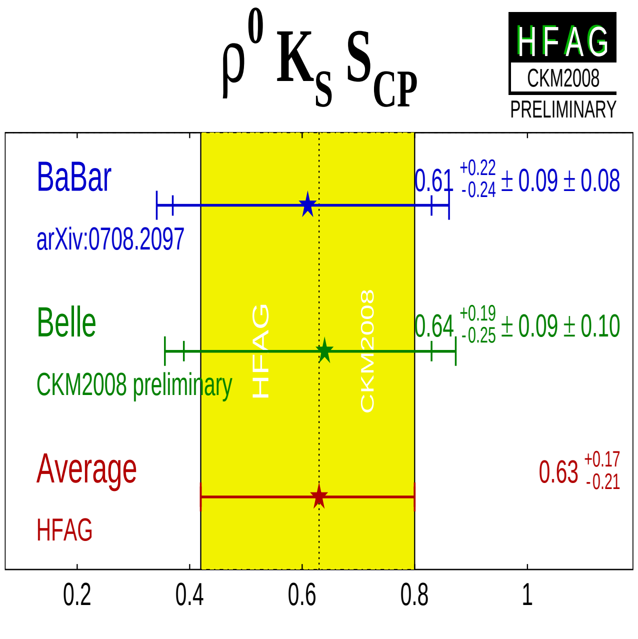
No evidence for direct CP violation

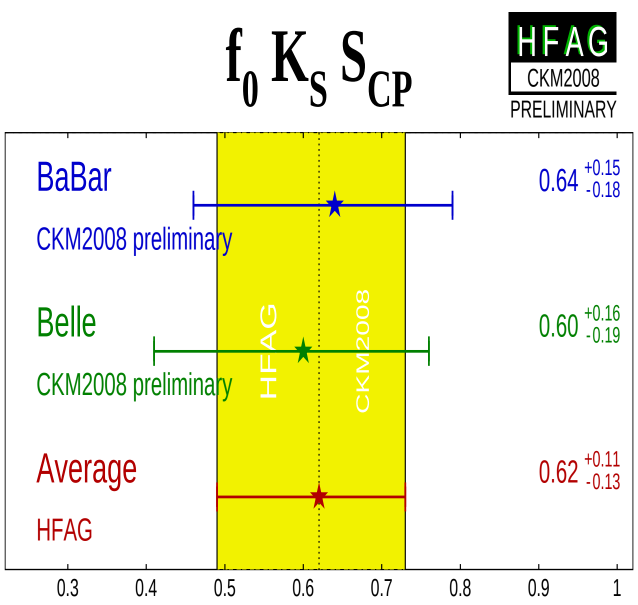
Indirect CP consistent with $b \to c\bar{c}s$

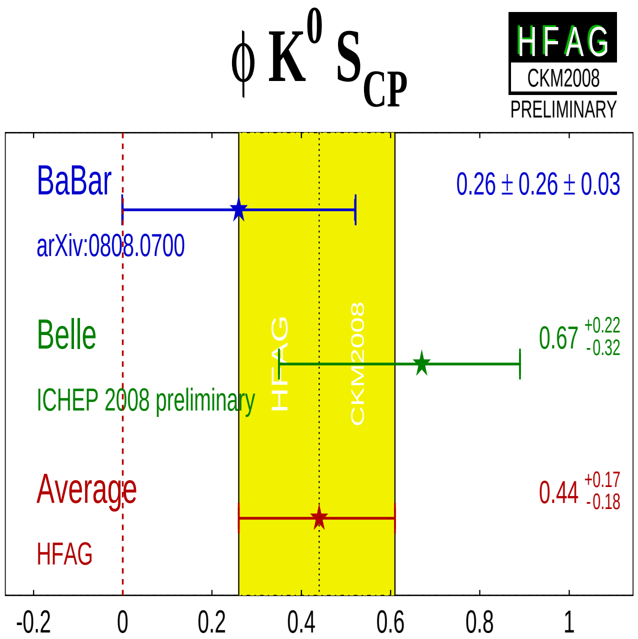
$B^0 \to K_S^0 K^+ K^-$

No evidence for direct CP violation

Indirect CP consistent with $b \to c\bar{c}s$







Backup

Previous Results

Belle

$B^0 \to \phi(1020) K_S^0$

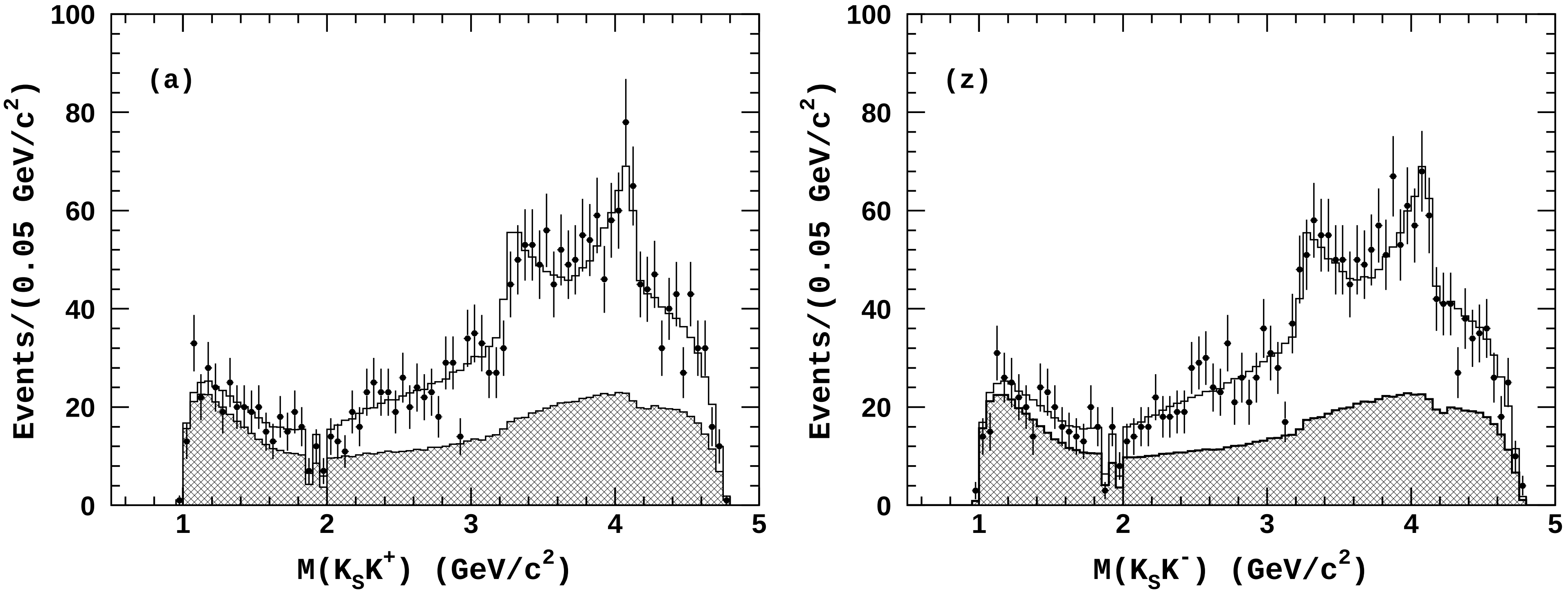
$$\mathcal{A}_{CP}(\phi(1020)K_S^0) = +0.07 \pm 0.15\ (\text{stat}) \pm 0.05\ (\text{syst})$$
$$\mathcal{S}_{CP}(\phi(1020)K_S^0) = +0.50 \pm 0.21\ (\text{stat}) \pm 0.06\ (\text{syst})$$

$B^0 \to f_0(980) K_S^0$

$$\mathcal{A}_{CP}(f_0(980)K_S^0) = -0.15 \pm 0.15\ (\text{stat}) \pm 0.07\ (\text{syst})$$
$$\mathcal{S}_{CP}(f_0(980)K_S^0) = 0.18 \pm 0.23\ (\text{stat}) \pm 0.11\ (\text{syst})$$

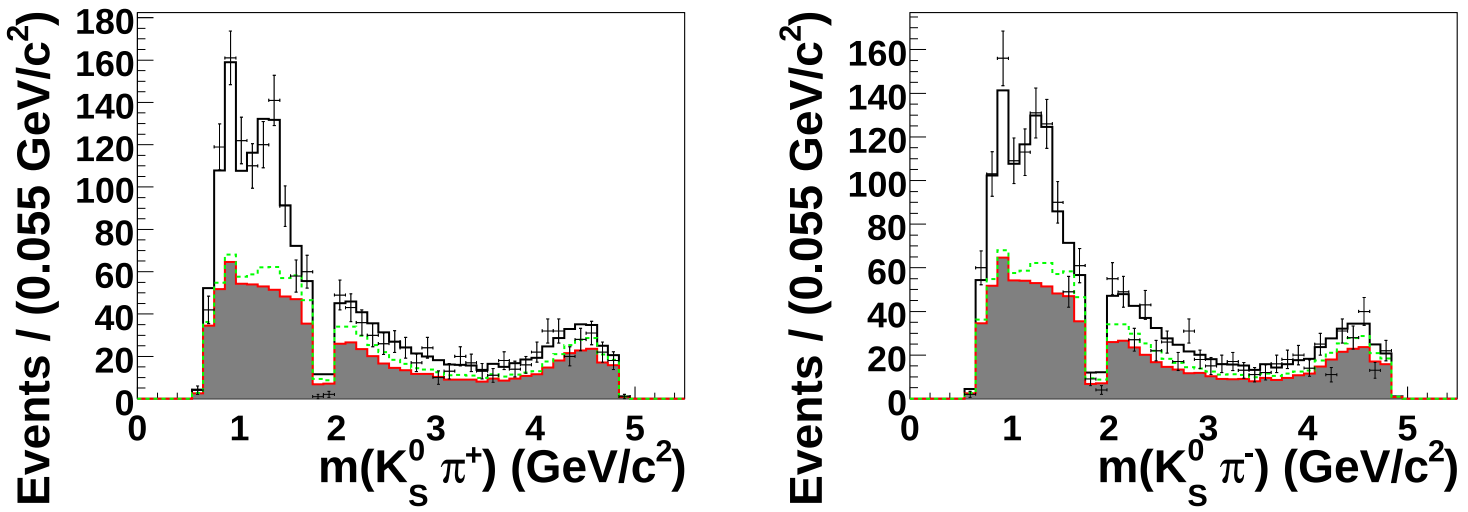
$B^0 \to K_S^0 K^+ K^-$ Mass Projections

Belle



$B^0 \to K_S^0 \pi^+ \pi^-$ Mass Projections

Belle



Model Uncertainty

Belle

$B^0 \to K_S^0 K^+ K^-$

$e^{-\alpha s} \to \frac{i\alpha}{s+i\alpha}$

$e^{-\alpha s} \to \frac{\alpha}{s+\alpha}$

$B^0 \to K_S^0 \pi^+ \pi^-$

$e^{-\alpha s} \to \frac{i\alpha}{s+i\alpha}$

$e^{-\alpha s} \to s^{-\alpha}$

Include $K_2^*(1430)$, $K_0^*(1680)$, $\omega(782)$, $\rho^0(1450)$, $\rho^0(1700)$ and $f_0(1710)$