S. Simula

INFN - Roma 3

CKM 2008

University of Rome “La Sapienza”

September 9-13, 2008

Determinations of V_{us} from unquenched lattice QCD simulations

Outline

- extraction of V_{us} from $K_{\ell 2}$ / $\pi_{\ell 2}$ and $K_{\ell 3}$ decays (no hyperons)
- hadronic quantities needed: f_K / f_π and $f_+(0)$
- lattice calculations: present status (Lattice ’08)
- new (preliminary) ETMC result on $f_+(0) = 0.956 \pm 0.006$ ($N_f = 2$)
- Callan-Treiman relation on the lattice
- discussion of systematic errors and future perspectives

on behalf of the

European Twisted Mass Collaboration

Motivations

- V_{us} is a fundamental parameter of the **Standard Model** playing a central role in the **CKM matrix**

- high precision test of **the CKM unitarity** (lepton-quark universality) thanks to large amount of new data [BNL-E865, KLOE, KTEV, ISTRA+, NA48] and new lattice QCD simulations:

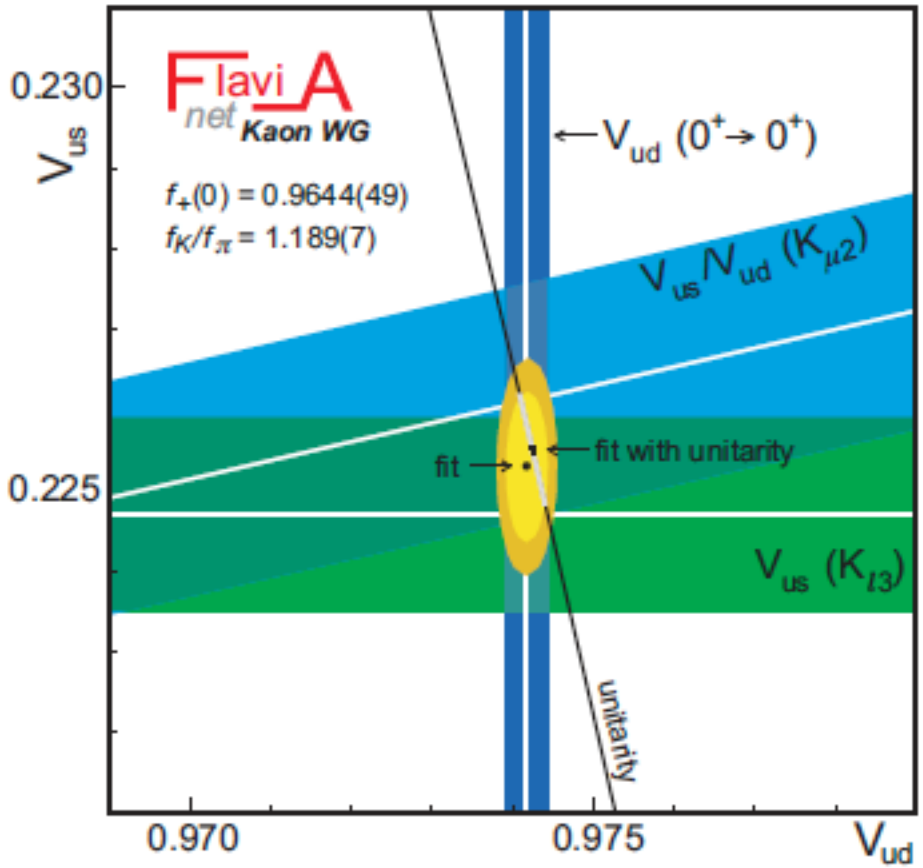
nuclear SFT	⇨	$\|V_{ud}\| = 0.97417 \pm 0.00026 \quad (\sim 0.03\%)$	Hardy&Towner '07
$K_{\ell 3}$ and $K_{\ell 2}$ decays	⇨	$\|V_{us}\| = 0.2253 \pm 0.0009 \quad (\sim 0.4\%)$	FlaviAnet '07
incl./excl. B decays	⇨	$\|V_{ub}\| = 0.00395 \pm 0.00035 \quad (\sim 9\%)$	PDG '08

CKM unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + O\left[\frac{M_W^2}{\Lambda_{NP}^2}\right] = 0.9998 \pm 0.0007 \quad (\sim 0.1\%)$$

*** $\delta|V_{us}|^2 \sim \delta|V_{ud}|^2 \;\Rightarrow\;$ the error from V_{us} is no longer the dominant uncertainty

*** effective scale of new physics $\Lambda_{NP} > 1$ TeV

* Vus comes from experiment and unquenched lattice QCD simulations



$K_{\ell 3}$ decays

$|V_{us}| = 0.2246\ (12)$ (~ 0.5%)

using

$f_+(0) = 0.9644\ (49)$ (~ 0.5%) [RBC/UKQCD Coll. '08]

$K_{\ell 2}$ decays

$|V_{us} / V_{ud}| = 0.2321\ (15)$ (~ 0.6%)

using

$f_K / f_\pi = 1.189\ (7)$ (~ 0.6%) [HPQCD Coll. '08]

*** the determinations of V_{us} from $K_{\ell 2}$ and $K_{\ell 3}$ decays are dominated by theoretical uncertainties

*** main topics of this talk: $f_+(0)$ and f_K / f_π with their uncertainties

V_{us} from $K_{\ell 2}$ decays

$$\frac{\Gamma\left(K \to \mu\bar{\nu}(\gamma)\right)}{\Gamma\left(\pi \to \mu\bar{\nu}(\gamma)\right)} \longrightarrow \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} = 0.2760\ (6) \quad [0.22\%]$$

[Marciano '04]

[PDG '06]

* $\frac{f_K}{f_\pi} = 1 + O\left(m_s - m_\ell\right)$ no AG-like theorem $\Rightarrow$ f_K / f_π - 1 ~ 0.2

* to match the experimental error one needs $\delta[f_K / f_\pi - 1] \sim 1\%$ [challenging!]

* on the lattice f_{PS} can be computed from 2-point correlation functions (with PS meson at rest)

$$C_{A_0 P}(t) \equiv \int d\vec{x} \left\langle A_0(x) P(0) \right\rangle \xrightarrow[t \to \infty]{} \frac{1}{2M_{PS}} \langle 0 | \bar{q}\gamma_0\gamma_5 q' | PS \rangle \langle PS | \bar{q}'\gamma_5 q | 0 \rangle e^{-M_{PS}t}$$

$$\langle 0 | \bar{q}\gamma_0\gamma_5 q' | PS \rangle \longrightarrow \sqrt{2} M_{PS} f_{PS}$$

<u>**sources of systematic errors**</u>

- finite size effects: small but visible (f_π only) $\Rightarrow$ resummed Lüscher formula (beyond 1-loop) [Colangelo et al. '05]

- discretization effects: small but visible $\Rightarrow$ continuum limit

- extrapolation/interpolation to the physical point (mainly to the physical pion)

- non-perturbative renormalization constants: absent for f_K / f_π

SU(3) vs "SU(2)" ChPT at NLO

SU(3): [Gasser&Leutwyler '85]

$$\chi(M^2) = M^2 \log\left(\frac{M^2}{\mu^2}\right)$$

$$f_\pi = f_3 \left\{ 1 - \frac{1}{(4\pi f_3)^2}\left[2\chi(M_\pi^2) + \chi(M_K^2)\right] + 8(L_4 + L_5)\frac{M_\pi^2}{f_3^2} + 16 L_4 \frac{M_K^2}{f_3^2} \right\}$$

$$f_K = f_3 \left\{ 1 - \frac{1}{(4\pi f_3)^2}\left[\frac{3}{4}\chi(M_\pi^2) + \frac{3}{2}\chi(M_K^2) + \frac{3}{4}\chi(M_\eta^2)\right] + 8 L_4 \frac{M_\pi^2}{f_3^2} + 8(2L_4 + L_5)\frac{M_K^2}{f_3^2} \right\}$$

3 parameters: f_3, L_4 and L_5

"SU(2)": treat m_s as heavy and absorb its effects in the LECs ("HSChPT")

[RBC/UKCD '08]

$$f_\pi = f_2 \left[1 + \alpha\,\delta_K^2\right] \left\{ 1 - \frac{2}{(4\pi f_2)^2}\left[\chi(M_\pi^2) - \ell_4 M_\pi^2\right] \right\}$$

$$f_K = f_2^K \left[1 + \alpha^K \delta_K^2\right] \left\{ 1 - \frac{2}{(4\pi f_2)^2}\left[\frac{3}{8}\chi(M_\pi^2) - \ell_4^K M_\pi^2\right] \right\}$$

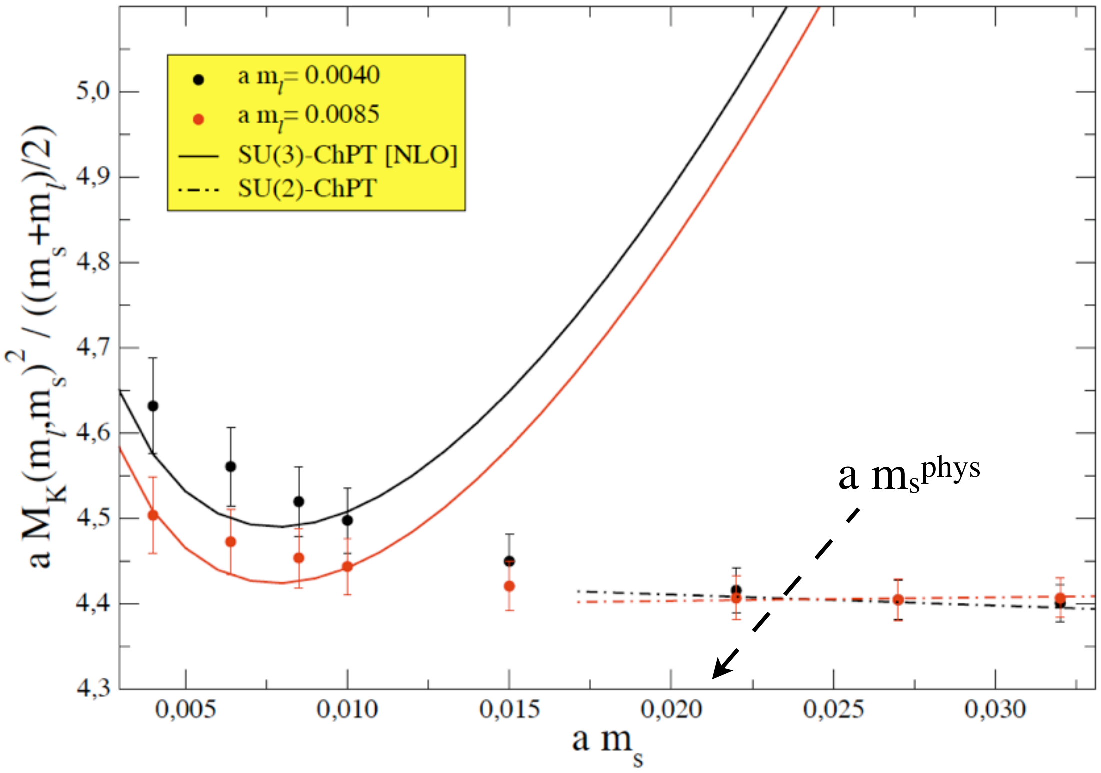
$$\delta_K^2 \propto m_s - m_s^{phys}$$

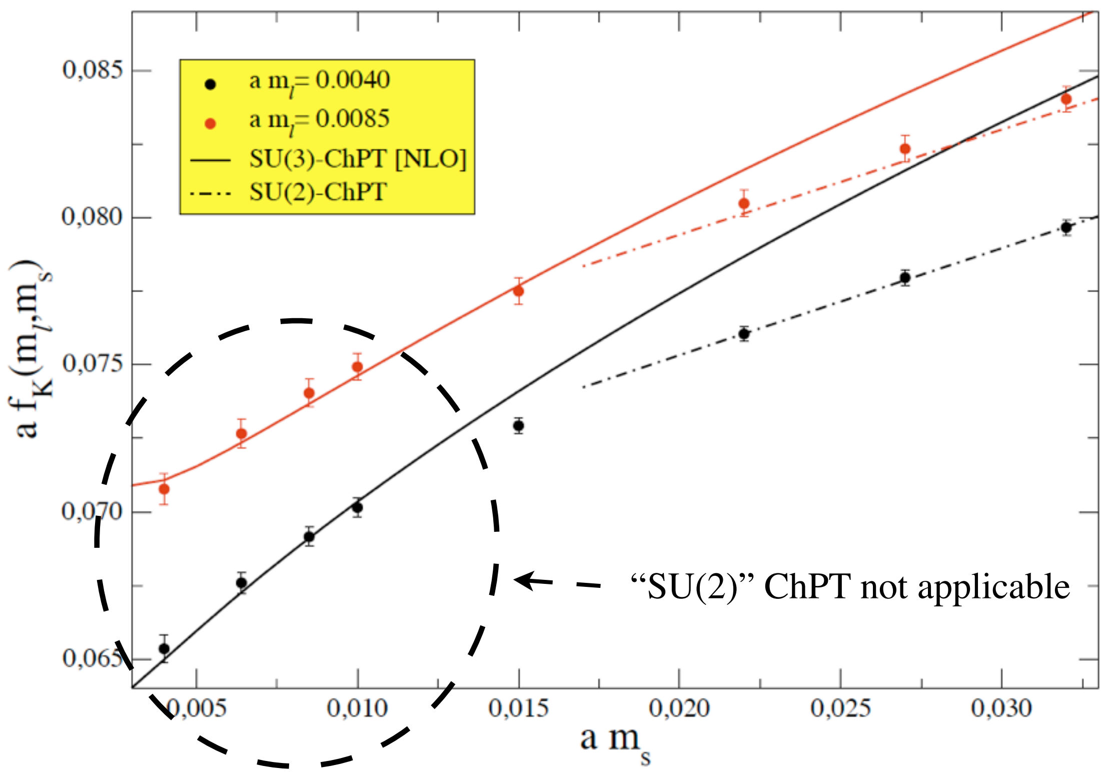
- low values of m_ℓ / m_s required
- less symmetry ⇒ less predictive power

$f_2 \neq f_3$

6 parameters: f_2, ℓ_4, α , f_2^K, ℓ_4^K, α^K

ETMC: Tarantino @ Lattice ‘08





* NLO SU(3) does not work for $m_s \sim m_s^{phys}$

* large NLO SU(3) corrections even at $m_\ell \sim m_\ell^{phys}$:

  $f_\pi^{phys} / f_3 \sim [f_K / f_\pi]^{phys}$

* within “SU(2)” ChPT LEC’s depend on m_s

* NLO “SU(2)”corrections are small at $m_\ell \sim m_\ell^{phys}$:

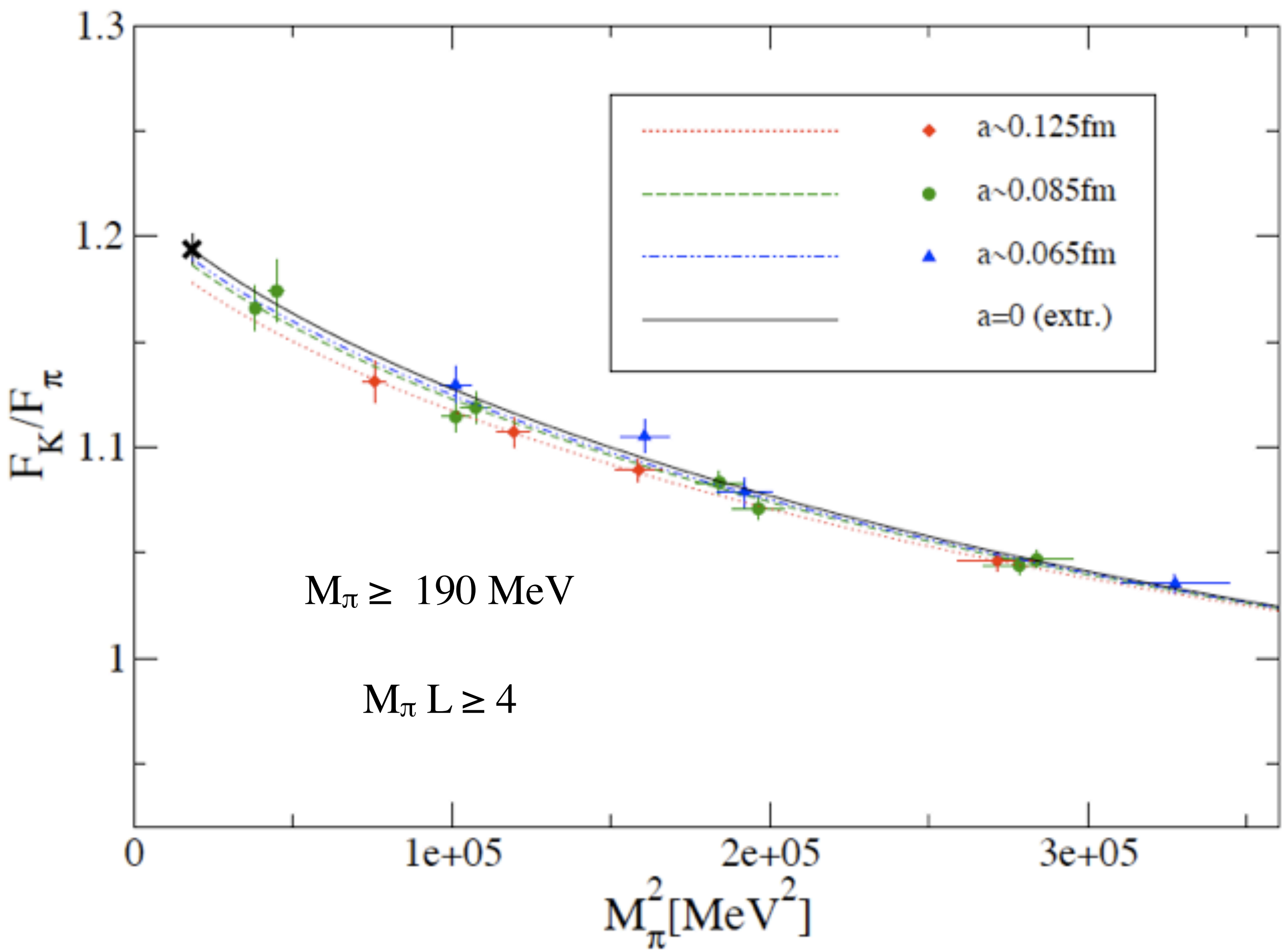
  $f_\pi^{phys} / f_2 < [f_K / f_\pi]^{phys}$

large strange sea quark effects

(no AG-like theorem)

same conclusions also from RBC/UKQCD and PACS-CS

BMW: Dürr @ Lattice '08

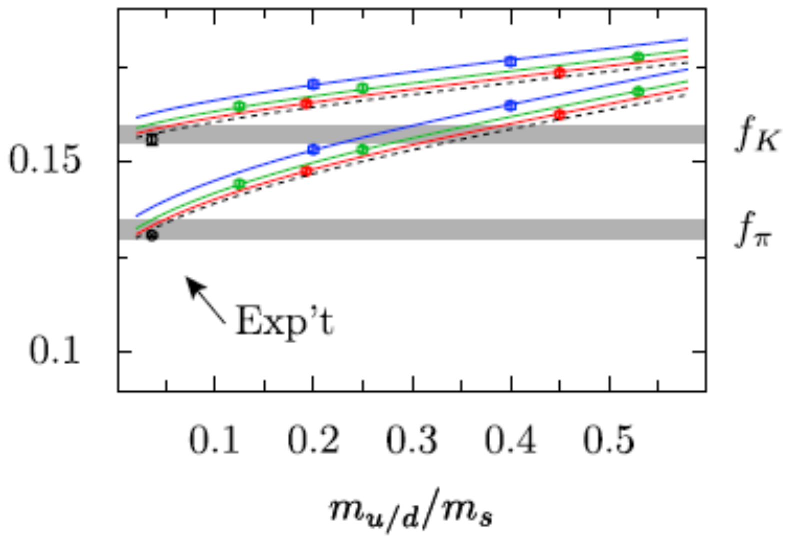


⟶ plot shows $\mathrm{data}(M_\pi^2, 2M_K^2{-}M_\pi^2) - \mathrm{fit}(M_\pi^2, 2M_K^2{-}M_\pi^2) + \mathrm{fit}(M_\pi^2, [2M_K^2{-}M_\pi^2]_{\mathrm{phys}})$.

⟶ f_K/f_π scales rather nicely [we have $a^2/\mathrm{fm}^2 = 0.0042, 0.0072, 0.0156$].

⟹ $f_K/f_\pi = 1.18(1)(1)$ at the physical m_{ud}, in the continuum, for infinite volume.

Staggered quarks ($N_f = 2 + 1$)

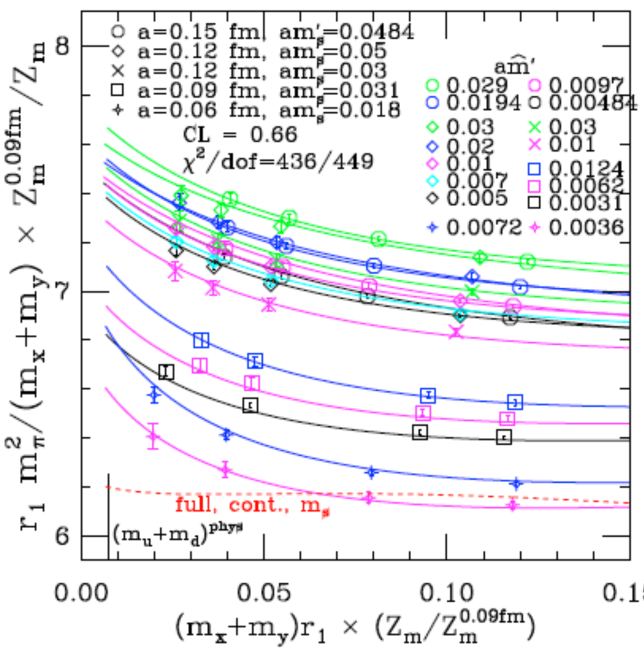


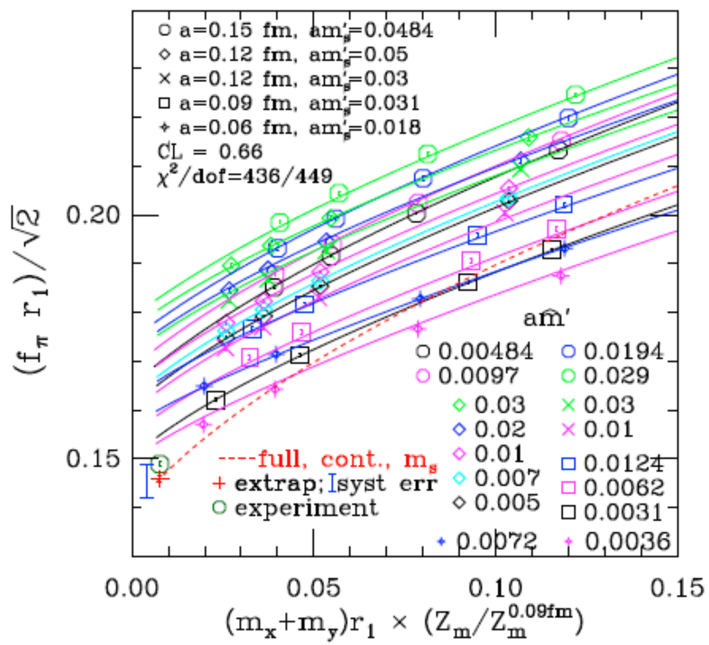
error budget

	f_K/f_π	f_K	f_π
r_1 uncerty.	0.3	1.1	1.4
a^2 extrap.	0.2	0.2	0.2
Finite vol.	0.4	0.4	0.8
$m_{u/d}$ extrap.	0.2	0.3	0.4
Stat. errors	0.2	0.4	0.5
m_s evoln.	0.1	0.1	0.1
m_d, QED, etc.	0.0	0.0	0.0
Total %	0.6	1.3	1.7

HPQCD '08

$$f_K/f_\pi = 1.189(2)(7)$$





- same gauge configurations
- different quark action
- different chiral extrapolations

MILC '07

$$f_K/f_\pi = 1.197(3)^{(+6)}_{(-13)}$$

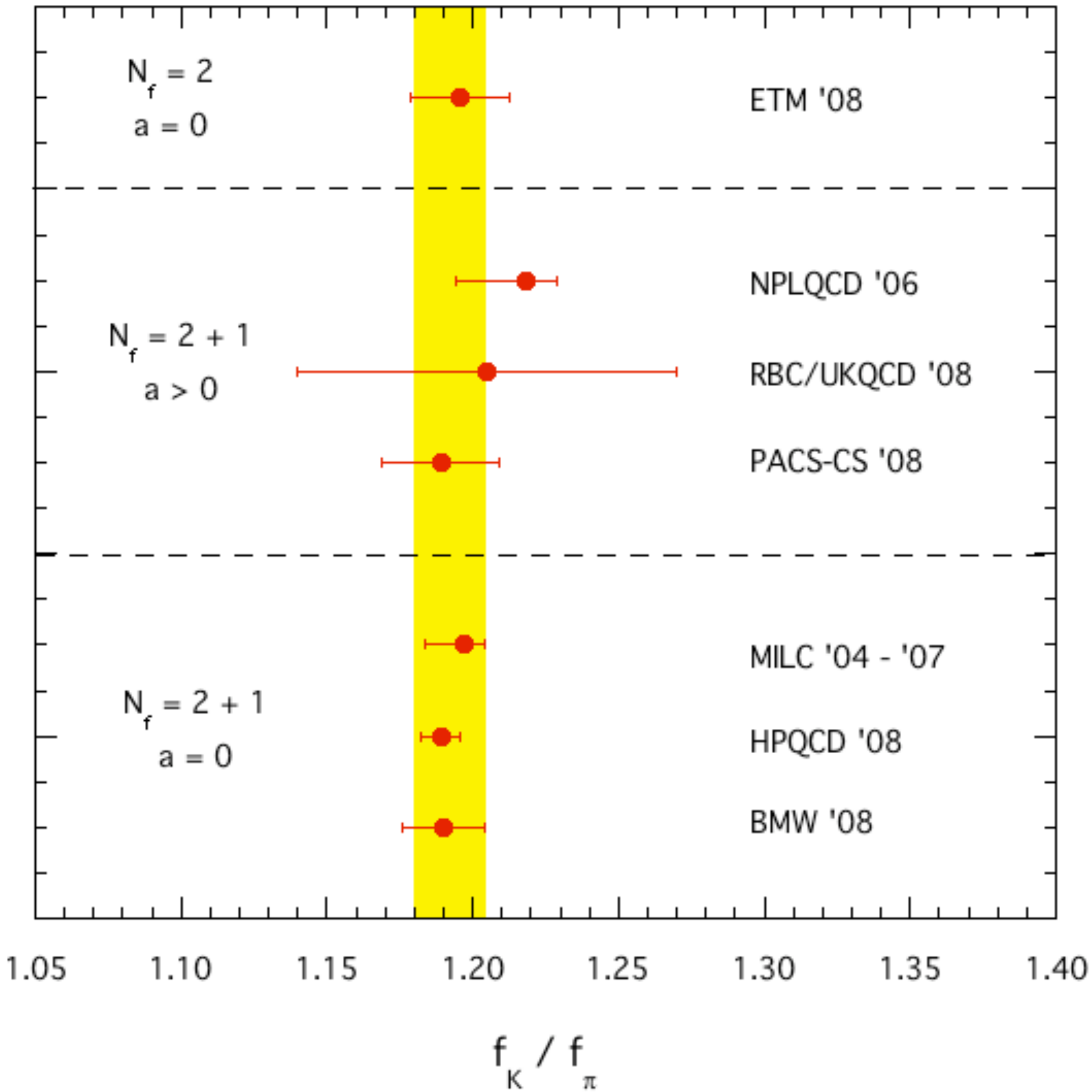
main concerns: 1) non-perturbative effects of the rooting trick
2) few details on the prior choices for Bayesian chiral extrapolations

f_K / f_π from lattice: unquenched simulations

Collaboration	N_f	fermions	a (fm)	M_π L	M_π (MeV)	f_K / f_π
MILC '04 - '07	2+1	SQ	≥ 0.06	≥ 4	≥ 240	$1.197(3)^{(+6)}_{(-13)}$
HPQCD '08	2+1	HISQ	≥ 0.09	≥ 4	≥ 250	1.189(2)(7)
BMW '08	2+1	SW	≥ 0.07	≥ 4	≥ 190	1.19(1)(1)
ETM '08	2	tmW	≥ 0.07	≥ 3	≥ 300	1.196(13)(11)
NPLQCD '06	2+1	DWF	0.13	≥ 4	≥ 290	$1.218(2)^{(+11)}_{(-24)}$
RBC/UKQCD '08	2+1	DWF	0.11	≥ 4	≥ 330	1.205(18)(62)
PACS-CS '08	2+1	NP-SW	0.09	≥ 2	≥ 160	1.189(20)

FlaviAnet Lattice Averaging Group is going to provide lattice averages using a detailed color coding of simulations based on:

- publication status;
- action and algorithms;
- renormalization;
- chiral extrapolation;
- continuum extrapolation;
- finite size effects.



my average for $N_f = 2 + 1$

$$f_K / f_\pi = 1.192 \pm 0.012 \quad (\sim 1\%)$$

conservative estimate

$$f_K / f_\pi = 1.190 \pm 0.015$$

[Lellouch @ Lattice '08]

$N_f = 2$

$$f_K / f_\pi = 1.196 \pm 0.017$$

small effects of quenching m_s

V_{us} from $K_{\ell 2}$ decays:

$$|V_{us}| = 0.2256\ (23) \quad (\sim 1\%)$$

using $|V_{ud}| = 0.97417\ (26)$

~ 1.5 times the uncertainty from $K_{\ell 3}$ decays

$$\langle \pi(p_\pi)|\hat{V}_\mu|K(p_K)\rangle = f_+(q^2)(p_K+p_\pi)_\mu + f_-(q^2)(p_\pi - p_K)_\mu$$

V_{us} from $K_{\ell 3}$ decays

$$f_0(q^2) = f_+(q^2) + f_-(q^2)\, q^2/(M_K^2 - M_\pi^2)$$

scalar f.f. vector f.f.: $f_+(0) = f_0(0)$

the same as in $K_{\ell 2}$ decays

* from experiment: $\Gamma(K \to \pi \ell \bar{\nu}_\ell) \longrightarrow |V_{us}|\, f_+(0) = 0.21664\,(48)$ [0.22%] [FlaviAnet ’07]

* to match the experimental error one needs $\delta[f_+(0) - 1] \sim 5\%$ [factor ~ 5 larger than in $K_{\ell 2}$ decays]

* ChPT expansion of the vector form factor at zero momentum transfer, $f_+(0)$

$$f_+(0) = 1 + f_2 + f_4 + O(p^8) \xrightarrow[AG]{} 1 + O\left[(m_s - m_\ell)^2\right]$$

* NLO f_2 is independent of LECs, calculable in terms of M_K, M_π and f_π: $f_2^{phys} = -\,0.023$

* NNLO f_4 depends on $O(p^6)$ LECs [Post and Schilcher (‘01), Bijnens and Talavera (‘03)]

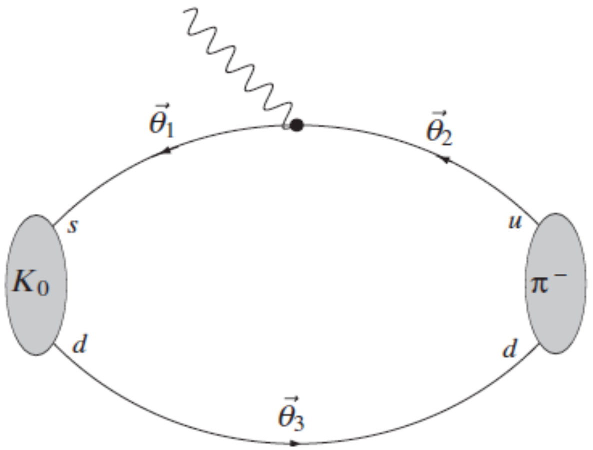
***** f_4 may be obtained from the slope and curvature of $f_0(q^2)$, but present data are not accurate enough *****

- from quark model [Leutwyler and Roos ’84]: $f_4 = -0.016 \pm 0.008 \Rightarrow f_+(0) = 0.961 \pm 0.008$ [used by PDG]

- from NNLO ChPT + 1/Nc [Cirigliano et al. ’06]: $f_4 = 0.007 \pm 0.012 \Rightarrow f_+(0) = 0.984 \pm 0.012$

- from NNLO ChPT + disp. rel. [Jamin et al. ’04]: $f_4 = -0.003 \pm 0.011 \Rightarrow f_+(0) = 0.974 \pm 0.011$

semileptonic form factors on the lattice



need of 2-point and 3-point correlators
(more expensive than f_K / f_π)

$$\vec{p}_K = \frac{2\pi}{L}\left(\vec{\theta}_3 - \vec{\theta}_1\right), \qquad \vec{p}_\pi = \frac{2\pi}{L}\left(\vec{\theta}_3 - \vec{\theta}_2\right),$$

$$\vec{q} = \vec{p}_K - \vec{p}_\pi = \frac{2\pi}{L}\left(\vec{\theta}_2 - \vec{\theta}_1\right)$$

2-point correlators: $$C_{\pi(K)}\left(t;\vec{p}\right) \equiv \sum_{x,z}\left\langle O_{\pi(K)}\left(\vec{x},t_x\right) O^\dagger_{\pi(K)}\left(\vec{z},t_z\right)\right\rangle \cdot \delta_{t,t_x-t_z}\, e^{-i\vec{p}\cdot(\vec{x}-\vec{z})}$$

$$\xrightarrow[t\to\infty]{} \frac{Z_{\pi(K)}}{2E_{\pi(K)}\left(\vec{p}\right)} e^{-E_{\pi(K)}\left(\vec{p}\right)\cdot t}$$

3-point correlators: $$C_\mu^{K\pi}\left(t,t';\vec{p}_K,\vec{p}_\pi\right) \equiv \sum_{x,y,z}\left\langle O_\pi\left(\vec{y},t_y\right)\hat{V}_\mu\left(\vec{x},t_x\right) O^\dagger_K\left(\vec{z},t_z\right)\right\rangle \cdot \delta_{t,t_x-t_z}\delta_{t',t_y-t_z} \cdot e^{-i\vec{p}_K\cdot(\vec{x}-\vec{z})}\, e^{i\vec{p}_\pi\cdot(\vec{x}-\vec{y})}$$

$$\xrightarrow[\substack{t\to\infty \\ t'-t\to\infty}]{} \frac{\sqrt{Z_K Z_\pi}}{4E_K\left(\vec{p}_K\right)E_\pi\left(\vec{p}_\pi\right)}\left\langle \pi\left(p_\pi\right)\middle|\hat{V}_\mu\middle|K\left(p_K\right)\right\rangle e^{-E_K\left(\vec{p}_K\right)\cdot t}\, e^{-E_\pi\left(\vec{p}_\pi\right)\cdot(t'-t)}$$

*** suitable ratios of 3-point / 2-point correlators** $\longrightarrow$ $\left\langle \pi\left(p_\pi\right)\middle|\hat{V}_\mu\middle|K\left(p_K\right)\right\rangle$

* local interpolating PS fields and vector current: $O_K = \bar{d}\gamma_5 s, \quad O_\pi = \bar{d}\gamma_5 u, \quad \hat{V}_\mu = Z_V\, \bar{s}\gamma_\mu u$

lattice methodology [started by $SPQ_{cd}R$ coll. '04]

1. **high-precision evaluation** (better than percent level) of the scalar form factor f_0 at $q^2_{max} = (M_K - M_\pi)^2$, obtained using a suitable ratio of 3-point correlation functions

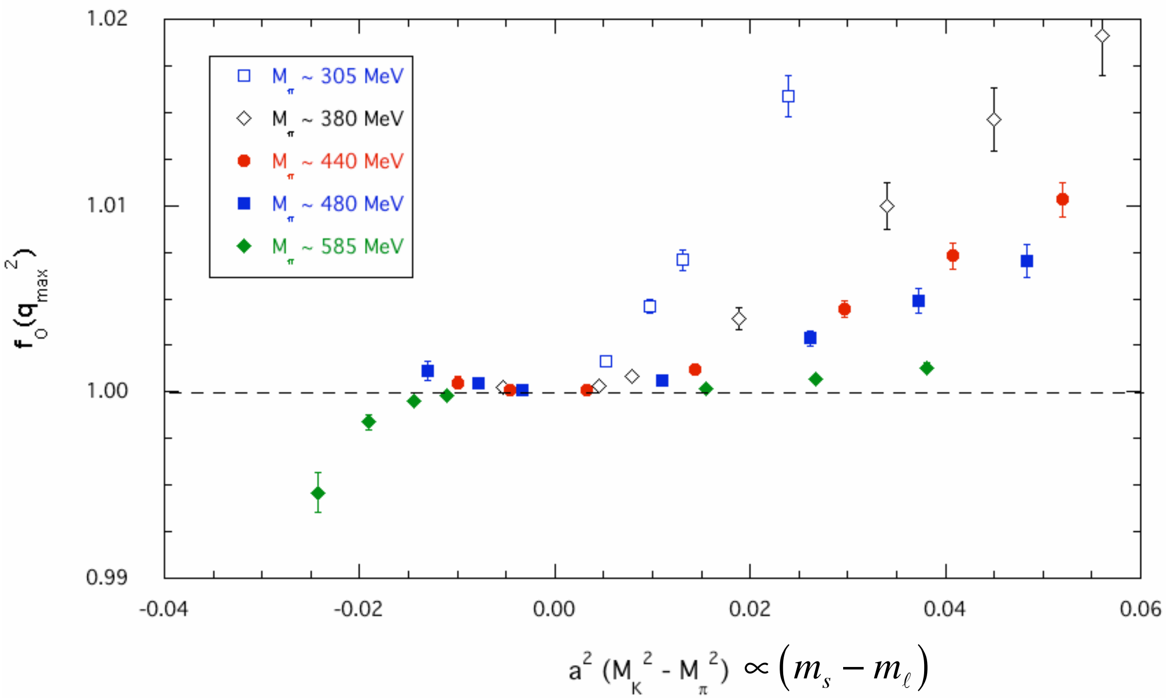
2. **extrapolation to $q^2 = 0$** to get $f_+(0) = f_0(0)$ (determination of the scalar slope λ_0)

3. **subtraction of the leading chiral logs** [study of $\Delta f = f_+(0) - 1 - f_2$] and **extrapolation to physical masses**

ETMC '08

β	a (fm)	Run	$am_\ell = am_{sea}$	V • T	M_π (MeV)	$M_\pi L$	gauge confs
3.9	~ 0.087	R_1	0.0030	$32^3 \bullet 64$	~ 265	≥ 3.7	240
		R_{2a}	0.0040	$32^3 \bullet 64$	~ 305	≥ 4.3	240
		R_{2b}	0.0040	$24^3 \bullet 48$	~ 310	≥ 3.3	480
		R_3	0.0064	$24^3 \bullet 48$	~ 380	≥ 4.0	240
		R_4	0.0085	$24^3 \bullet 48$	~ 440	≥ 4.7	240
		R_{5a}	0.0100	$24^3 \bullet 48$	~ 480	≥ 5.1	240
		R_6	0.0150	$24^3 \bullet 48$	~ 585	≥ 6.2	240
4.05	~ 0.069	R_{5b}	0.0080	$32^3 \bullet 64$	~ 480	≥ 5.4	240

$\beta = 3.9$: a m_s = {0.015, 0.022, 0.027, 0.032} around a $m_s^{phys} \sim 0.021$



$$\frac{\langle\pi|V^0|K\rangle\langle K|V^0|\pi\rangle}{\langle\pi|V^0|\pi\rangle\langle K|V^0|K\rangle} = \frac{(M_K + M_\pi)^2}{4M_K M_\pi}\left[f_0(q_{max}^2)\right]^2$$

< 0.1% at small SU(3) breakings

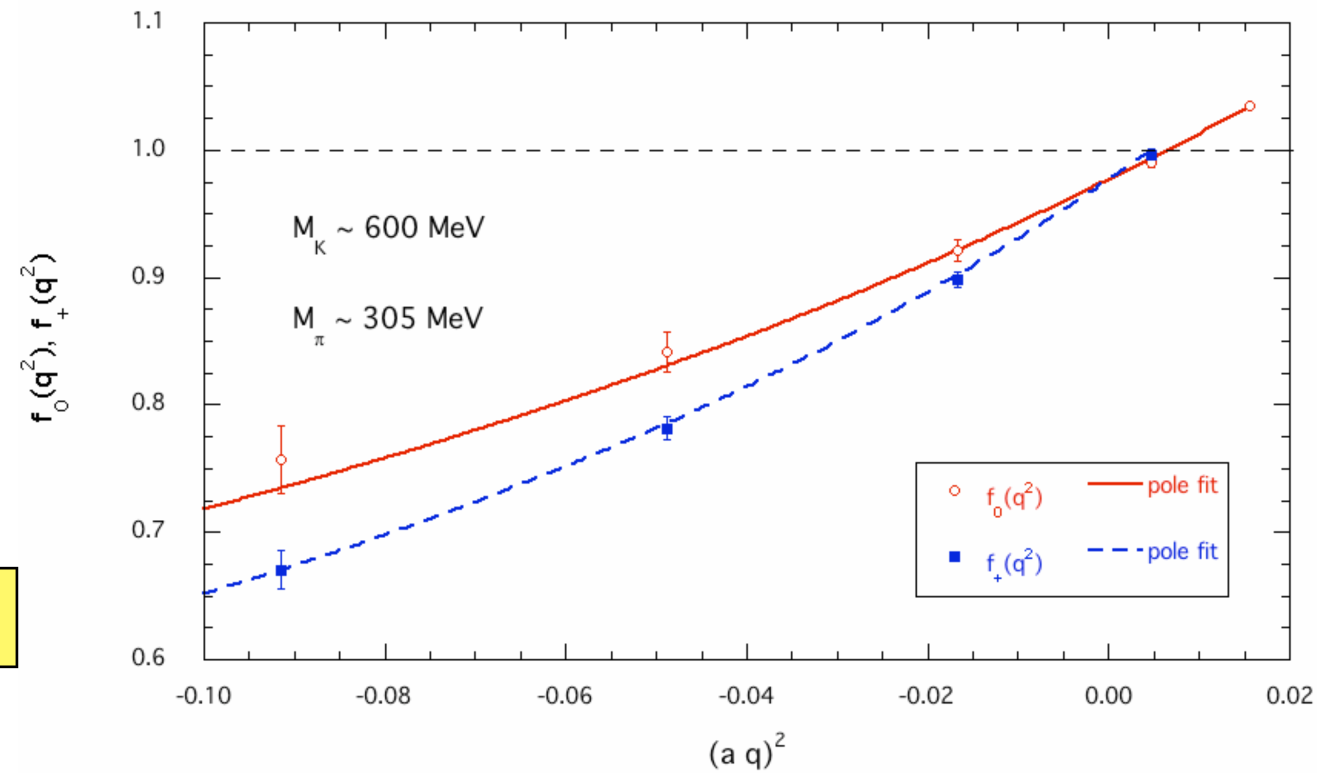
still < 0.25 % at larger SU(3) breakings

pole fit

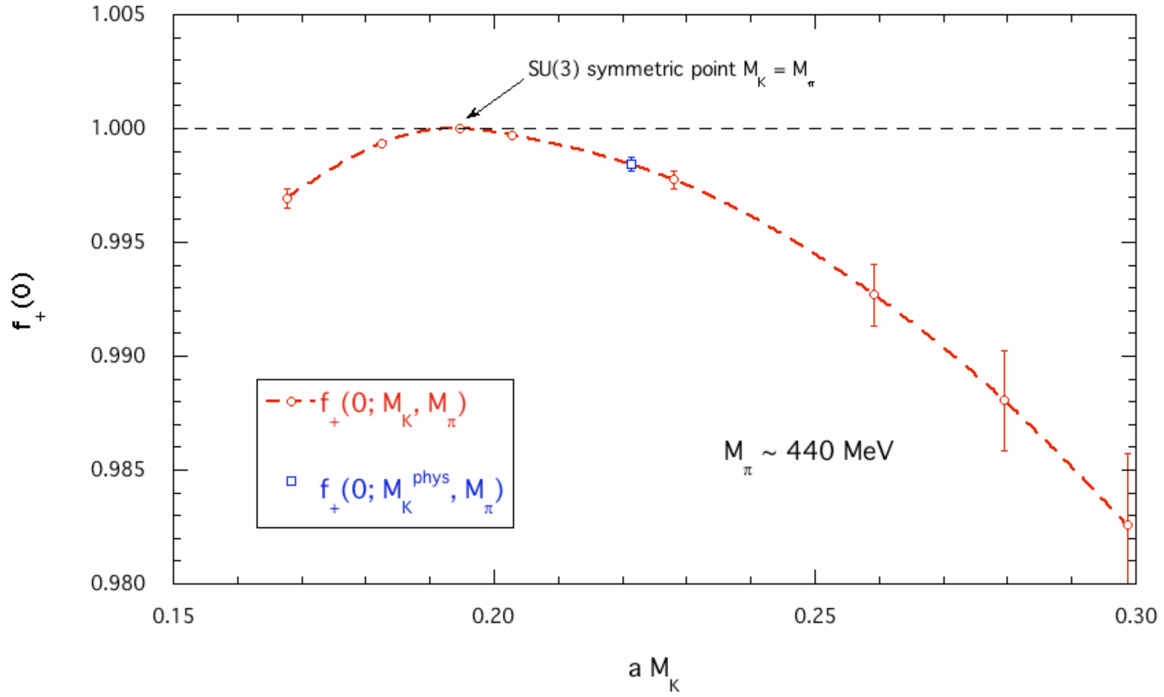
$$f_+(q^2) = f_+(0)/(1 - \lambda_+ q^2)$$

$$f_0(q^2) = f_+(0)/(1 - \lambda_0 q^2)$$

extraction of $f_+(0; M_K, M_\pi)$



* check of extrapolation to q^2=0: direct access to $f_+(0)$ using non-periodic BC's [UKQCD '07]



interpolation to M_K^{phys}

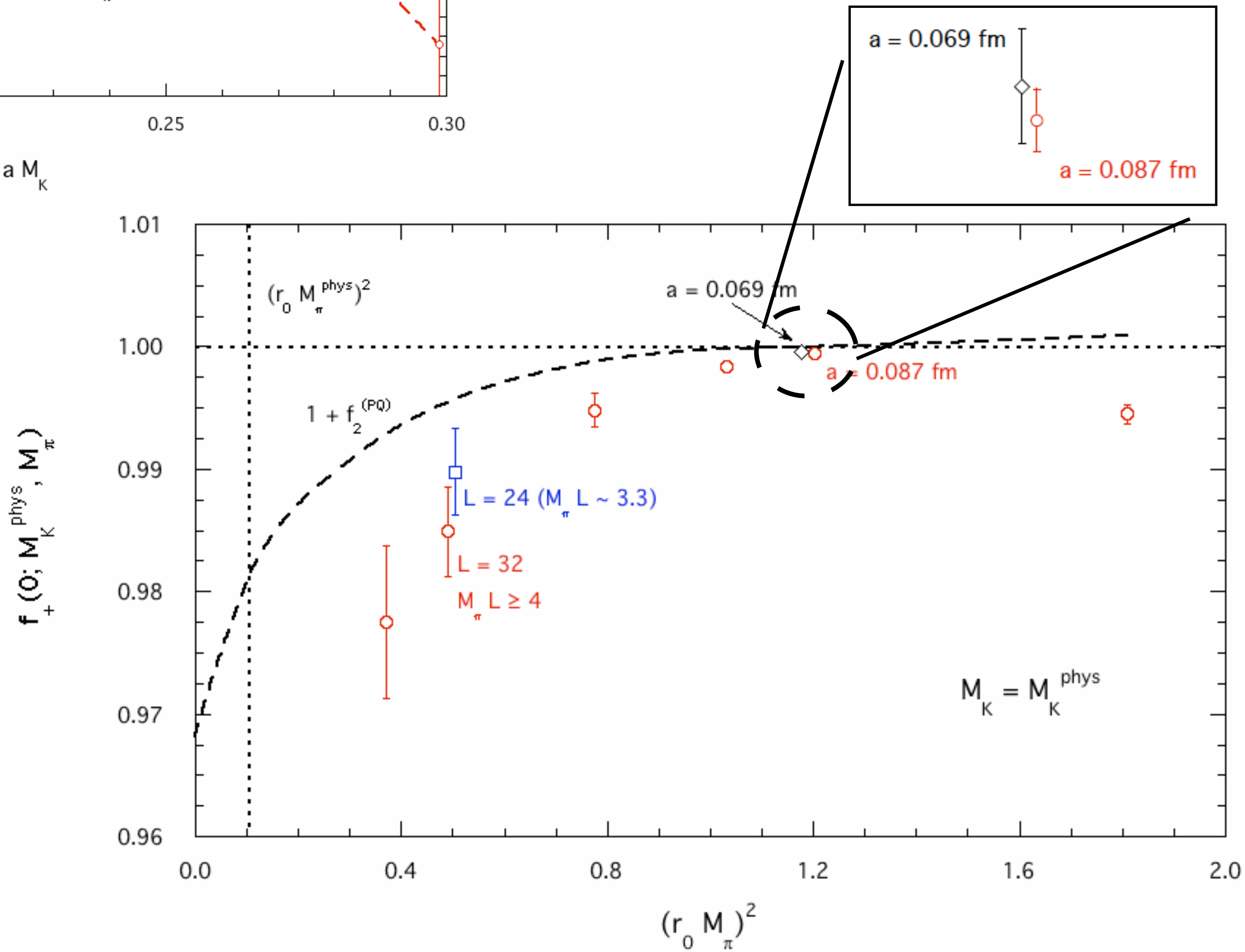
using a = 0.087 fm [ETMC '07]

well under control

$$f_2 \approx \Delta f = f_4 + O(p^8)$$

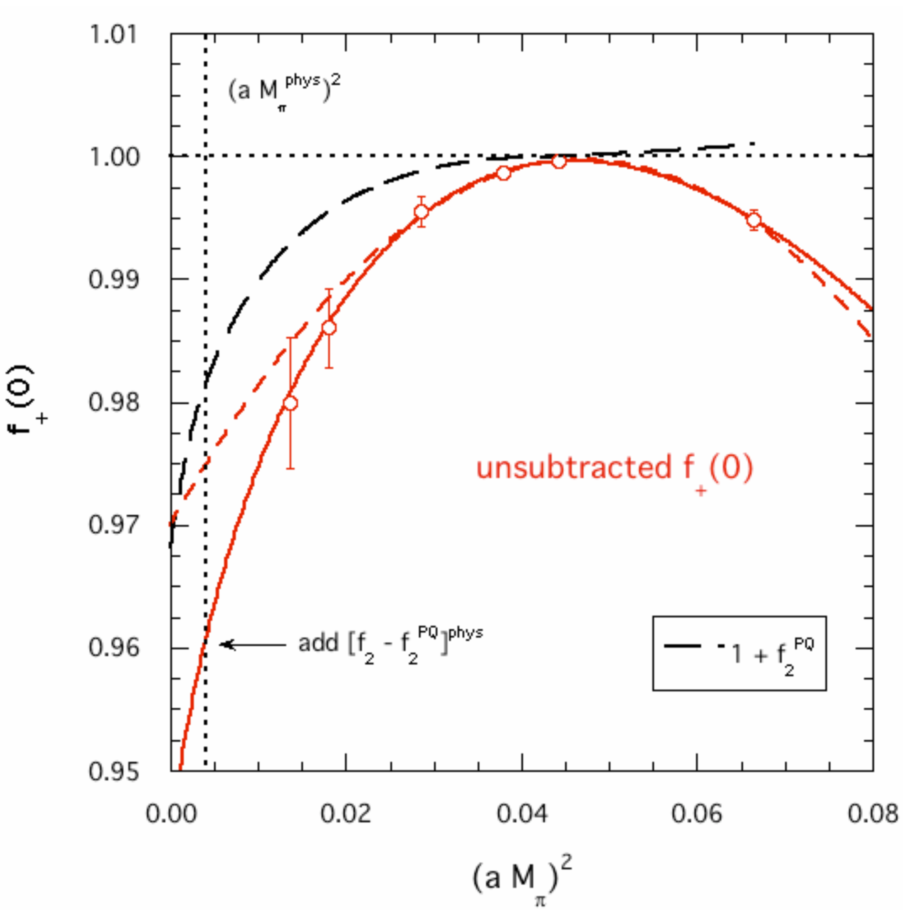
- small discretization effects

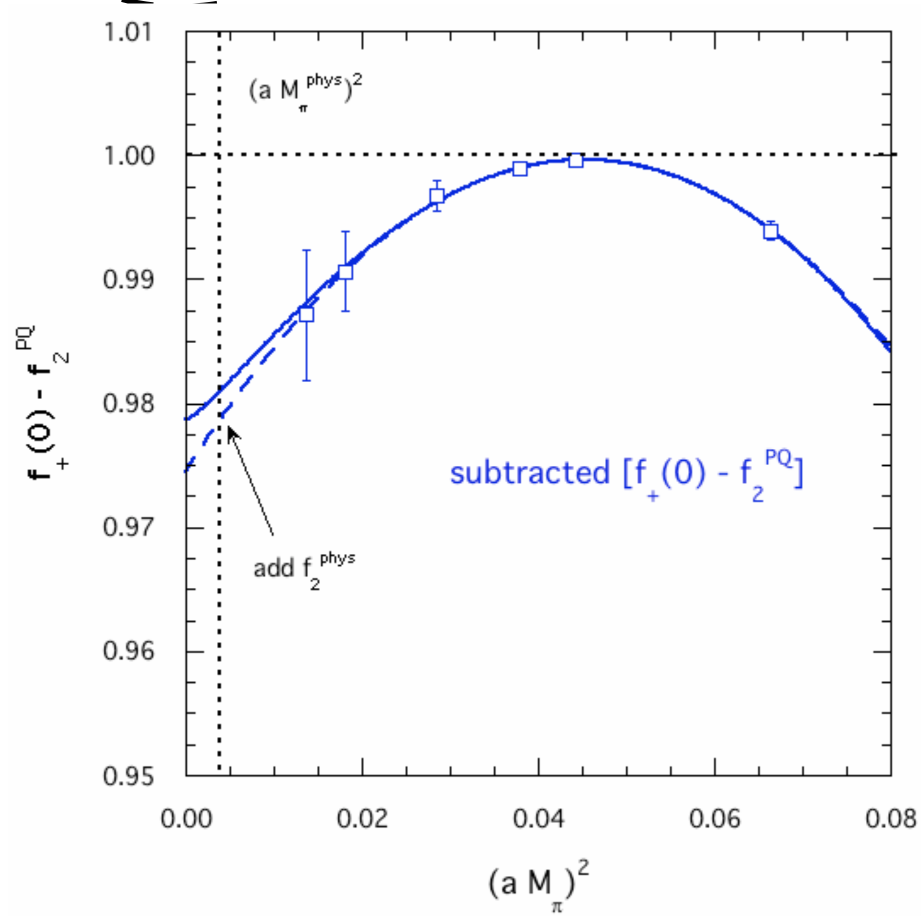
- small FSE when $M_\pi L \geq 4$



$$f_2^{PQ} = -\frac{2M_K^2 + M_\pi^2}{32\pi^2 f_\pi^2} + \frac{M_K^2\left(4M_K^2 - M_\pi^2\right)\log\left(2 - M_\pi^2/M_K^2\right)}{64\pi^2 f_\pi^2\left(M_K^2 - M_\pi^2\right)} + \frac{3M_K^2 M_\pi^2 \log\left(M_\pi^2/M_K^2\right)}{64\pi^2 f_\pi^2\left(M_K^2 - M_\pi^2\right)} \longrightarrow -\frac{\left(M_K^2 - M_\pi^2\right)^3}{96\pi^2 f_\pi^2 M_K^4} + O\left[\left(M_K^2 - M_\pi^2\right)^4\right]$$

[Becirevic et al. '06]





– – – – $A_0 + A_1 M_\pi^2 + A_2 M_\pi^4$

——— $A_0 + A_1 M_\pi^2 + A_2 M_\pi^4 + A_3 M_\pi^2 \log\left(M_\pi^2\right)$

* dominant chiral logs from f_2^{PQ}

* smooth extrapolation for [f_4 + ...]

error budget PRELIMINARY

* statistical error: 0.003

* finite size effect: 0.002

* discretization effects: 0.003

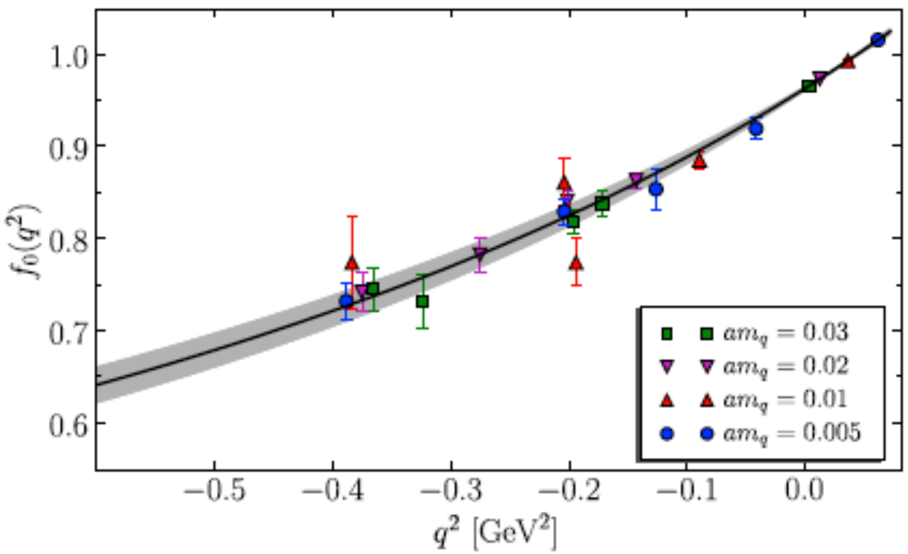
* q^2-dependence and chiral extrapolation: 0.004

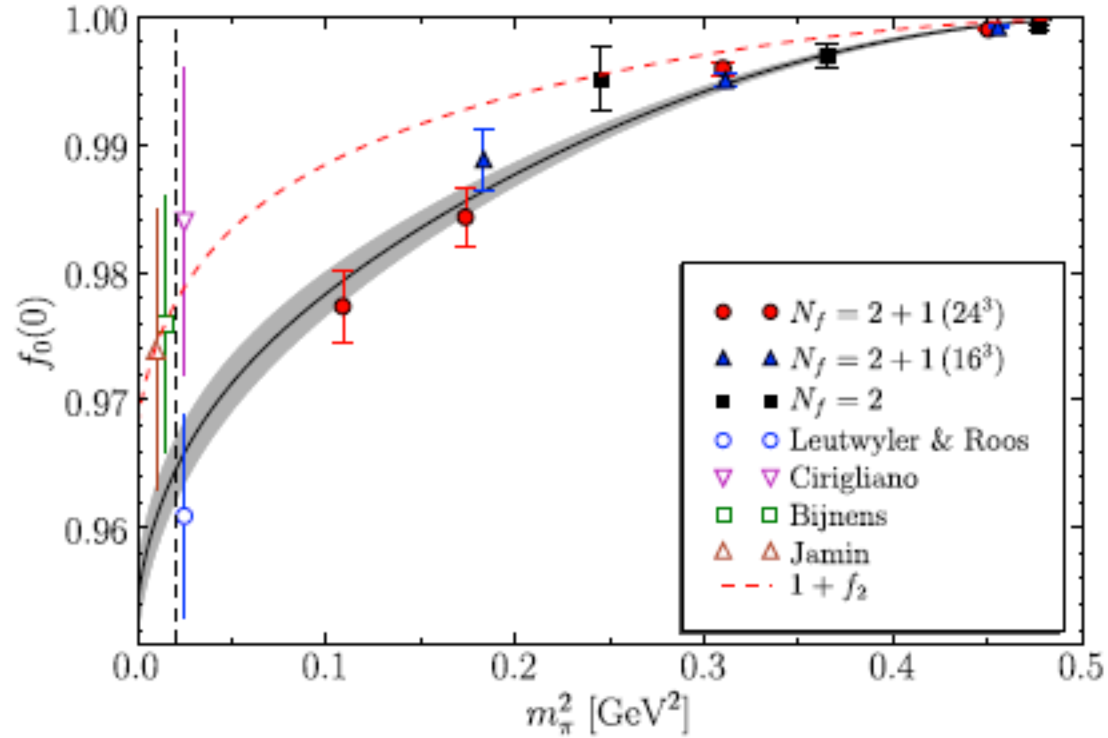
$$f_+(0) = 0.956 \pm 0.003_{stat.} \pm 0.005_{syst.} = 0.956 \pm 0.006$$

PRELIMINARY

main concerns: continuum limit and quenching of m_s

RBC/UKQCD '08





- $N_f = 2 + 1$
- Iwasaki gauge action [a = 0.114(2) fm]
- Domain Wall Fermions [$am_{res} = 0.00315$]
- volumes: (1.83 fm)^3 and (2.74 fm)^3
- $M_\pi \geq 330$ MeV
- $m_s \sim 1.15\, m_s^{phys}$

combined chiral and q^2 fits

$$f_0(q^2, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1(m_K^2 + m_\pi^2))}{1 - q^2/(M_0 + M_1(m_K^2 + m_\pi^2))^2},$$

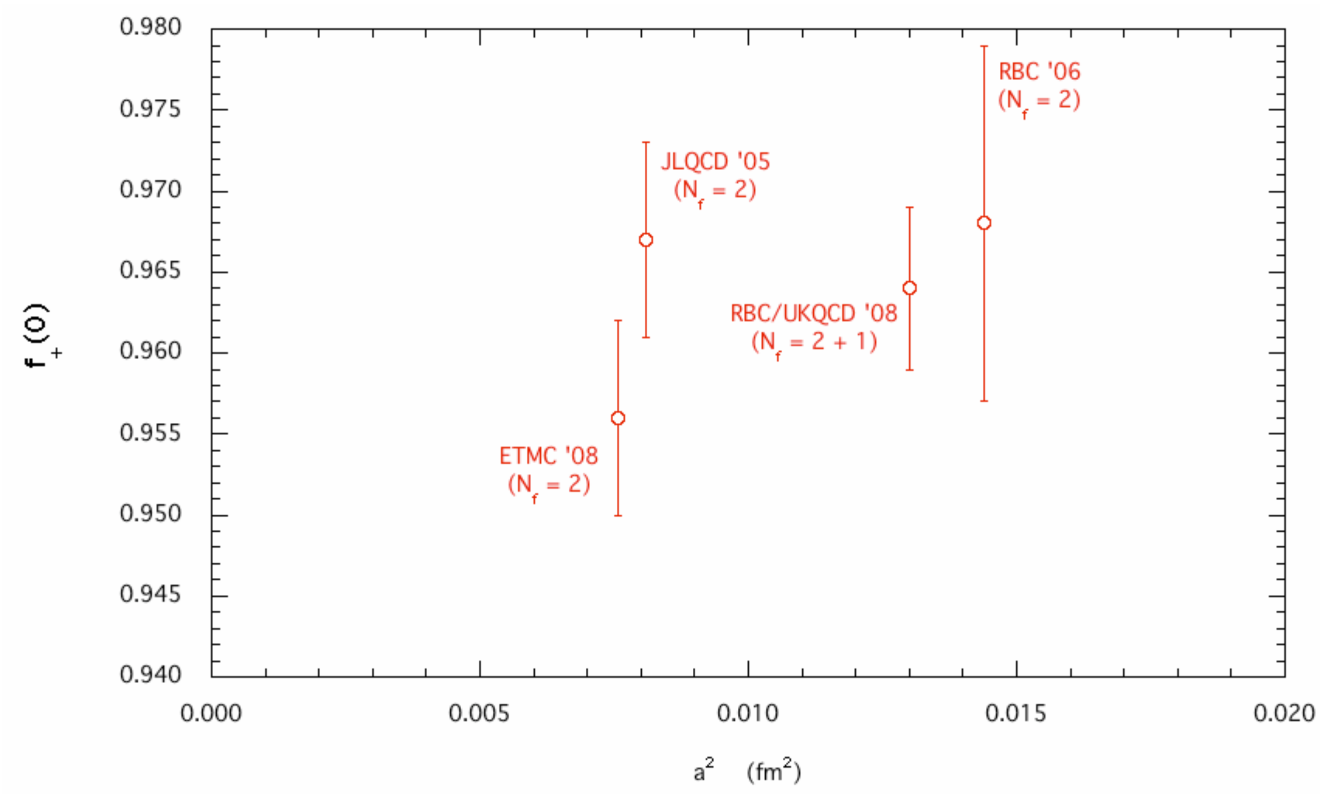
$$f_+(0) = 0.9644\,(33)_{stat.}\,(34)_{extrap.}\,(14)_{discr.} = 0.9644\,(49)$$

4% of $[f_+(0) - 1]$

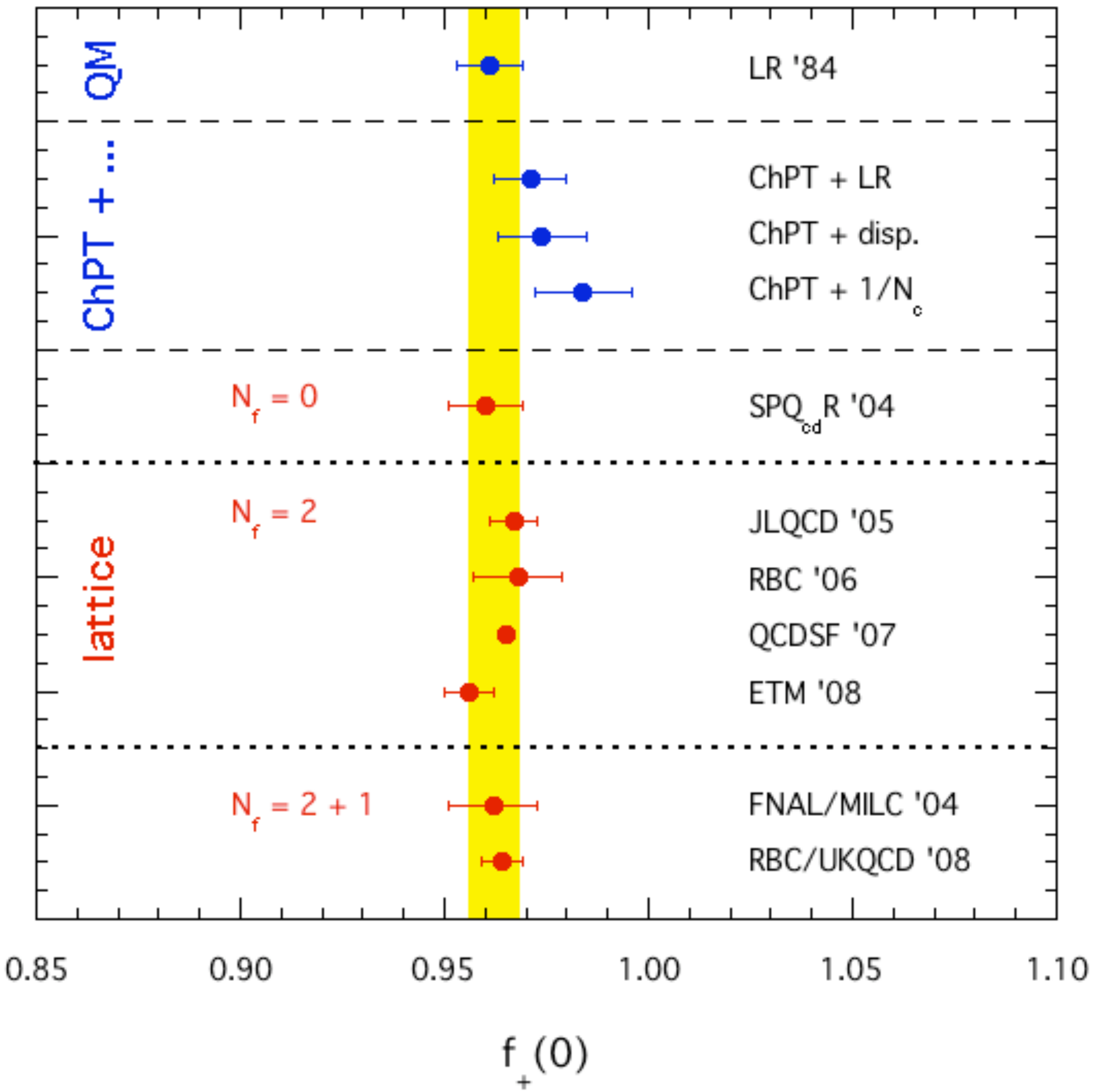
- main concerns: coarse lattice spacing and chiral fit of $f_+(0)$

$f_+(0)$ from lattice

Collaboration	N_f	fermions	a (fm)	M_π L	M_π (MeV)	$f_+(0)$
RBC/UKQCD '08	2+1	DWF	0.11	≥ 4	≥ 330	0.9644(33)(37)
FNAL/MILC '04	2+1	SQ				0.962(6)(9)
ETM '08	2	tmW	0.09	≥ 4	≥ 260	0.956(3)(5)
QCDSF '07	2	NP-SW	0.08	≥ 6	≥ 590	0.9647(15)(?)
RBC '06	2	DWF	0.12	≥ 6	≥ 490	0.968(9)(6)
JLQCD '05	2	NP-SW	0.09	≥ 5	≥ 550	0.967(6)
$SPQ_{cd}R$ '04	0	NP-SW	0.08	≥ 5	≥ 500	0.960(5)(7)



general concern: a systematic study of the scaling property of $f_+(0)$ is still lacking



my lattice average

$$f_+(0) = 0.962 \pm 0.006 \qquad (\sim 0.6\%)$$

$$f_+(0) = 0.964 \pm 0.005$$

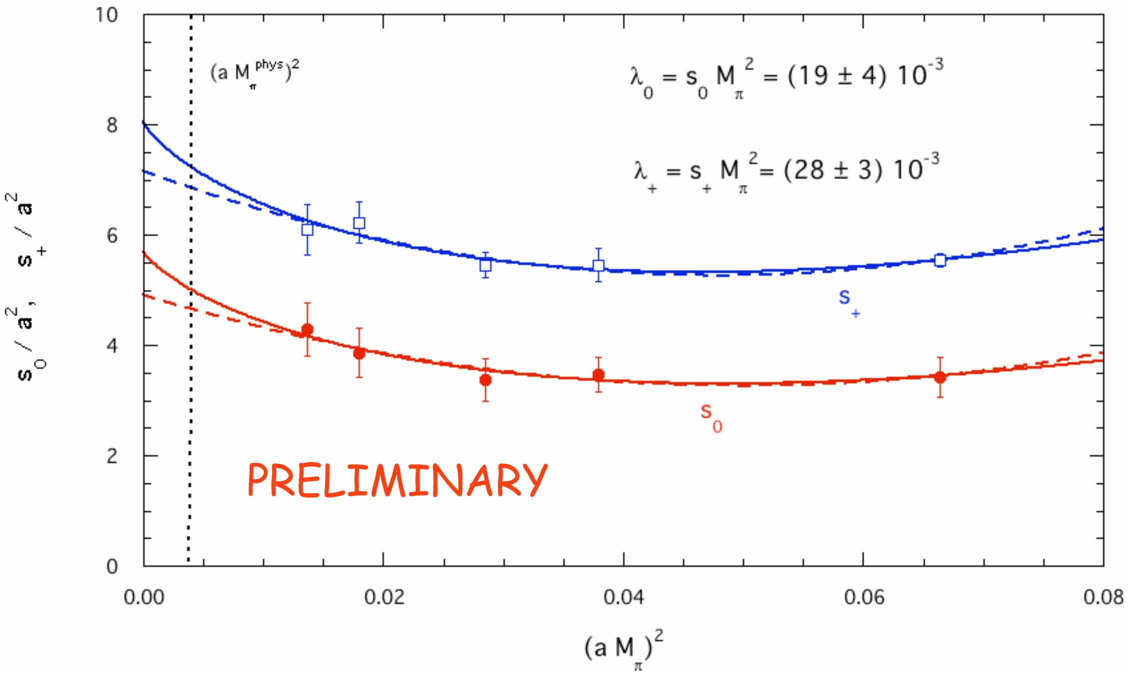
[Lellouch @ Lattice '08]

V_{us} from $K_{\ell 3}$ decays: $|V_{us}| = 0.2252\ (15) \quad (\sim 0.7\%)$

V_{us} from $K_{\ell 2}$ decays: $|V_{us}| = 0.2256\ (23) \quad (\sim 1\%)$

weighted average

$|V_{us}| = 0.2253\ (13) \quad (\sim 0.6\%)$



slopes of $f_+(q^2)$ and $f_0(q^2)$

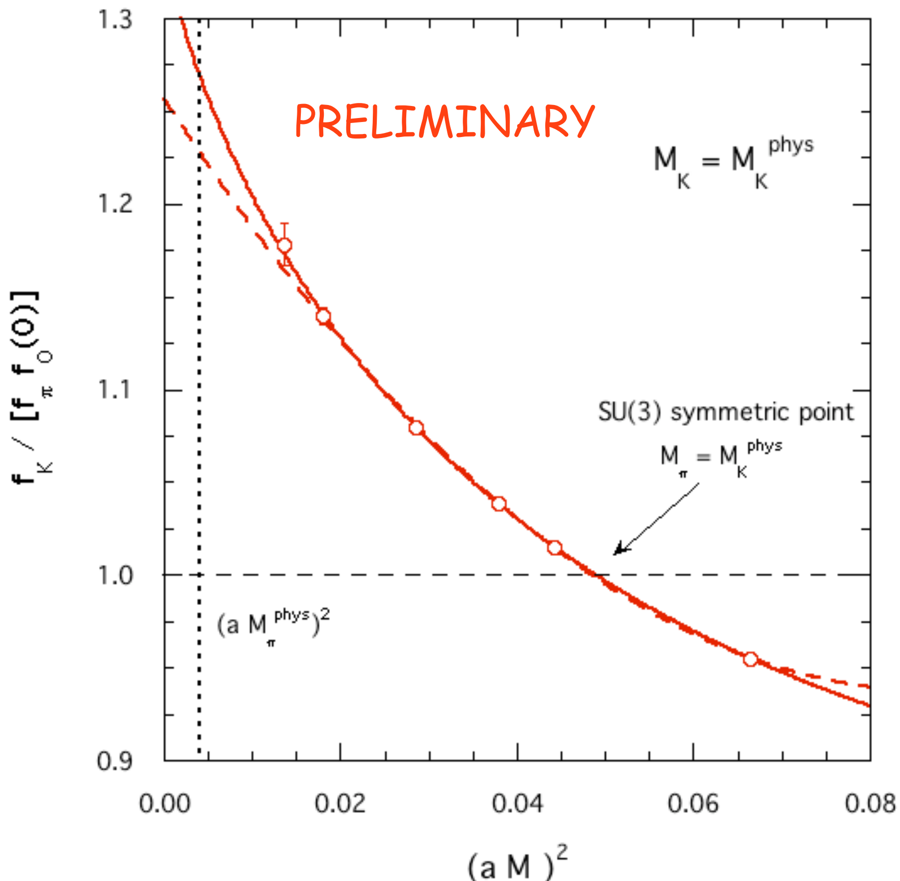
exp.: KTeV, KLOE, ISTRA+ and NA48

$$\lambda_+ = (25.2 \pm 0.9)\,10^{-3}$$
$$\lambda_0 = (13.4 \pm 1.2)\,10^{-3}$$

[FlaviAnet '07]

$$\lambda_0 = (9.5 \pm 1.4)\,10^{-3}$$

[NA48 only]

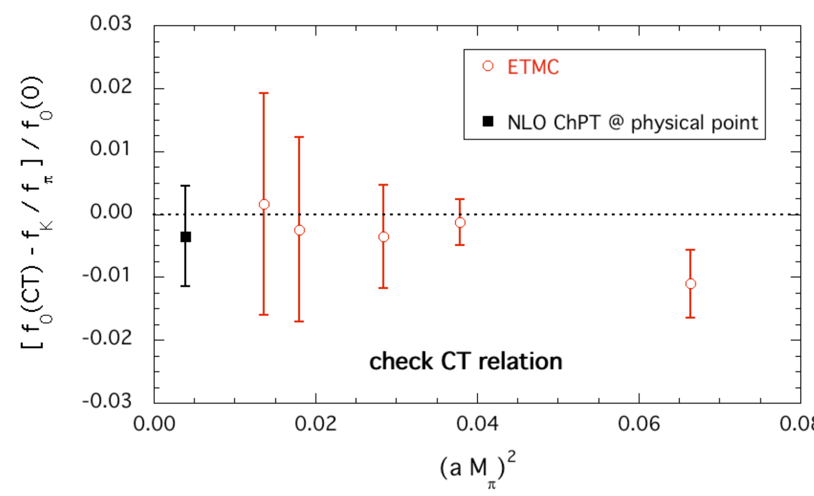


Callan-Treiman relation

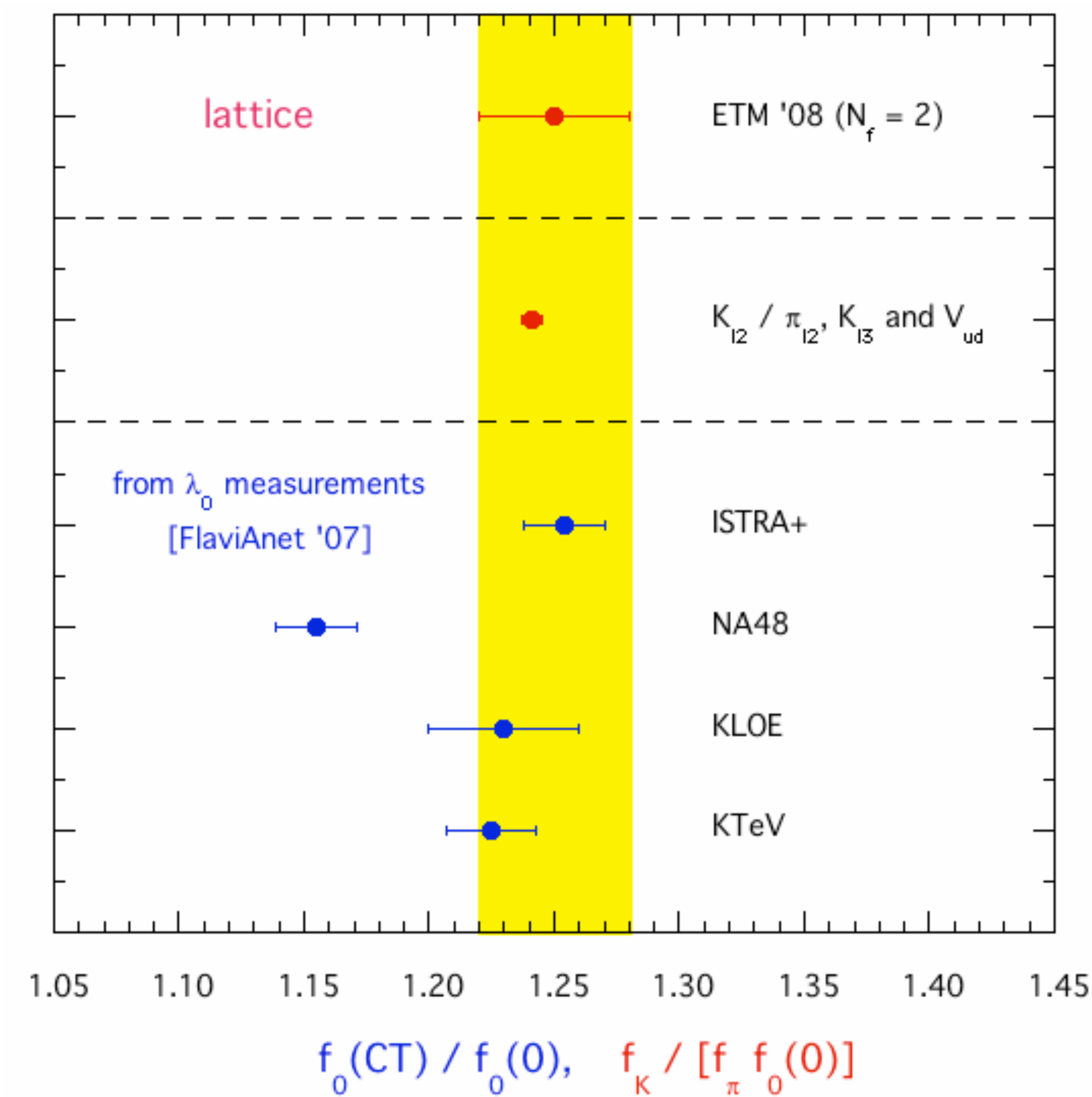
$$\frac{f_0\left(q^2 = M_K^2 - M_\pi^2\right)}{f_0(0)} = \frac{f_K}{f_\pi\ f_0(0)} + O(m_\ell)$$

1.25 ± 0.03

[ETM '08]



Callan-Treiman relation



SUMMARY

sources of uncertainties	f_K / f_π	$f_+(0)$
finite size effects	under control for $M_\pi L \geq 4$	under control for $M_\pi L \geq 4$
discretization effects	$O[a^2(m_s - m_\ell)\ \Lambda_{QCD}]$	$O[a^2(m_s - m_\ell)^2]$ (thanks to AG theorem)
continuum extrapolation	available	not yet **(but mandatory)**
momentum dependence	absent	under control
chiral extrapolation	depend on LECs at NLO	NLO known (AG protected) smooth extrapolation possible

present precision on the deviation from unity	~ 6%	~ 10%
improvement required to match exp. error	factor ~ 5	factor ~ 2

↑ **more promising**

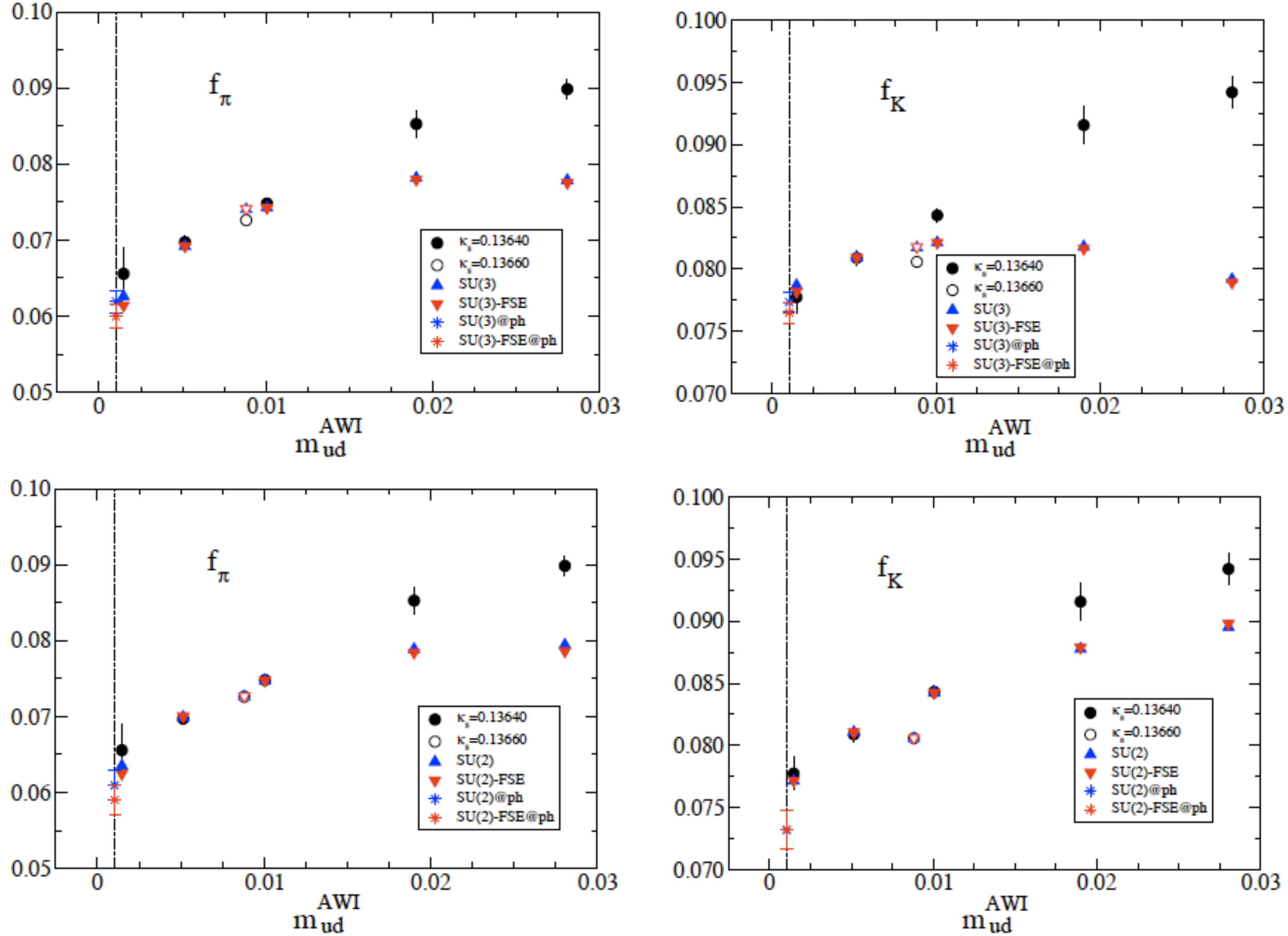
$$M_\pi \approx 200\ MeV \xrightarrow[M_\pi L \geq 4]{} L \geq 4\ fm \xrightarrow[a \approx 0.05\ fm]{} L/a \geq 80$$

BACKUP SLIDES

PACS-CS ’08: M_π (MeV) = 156, 296, 385, 411, 570, 702

NLO SU(3)

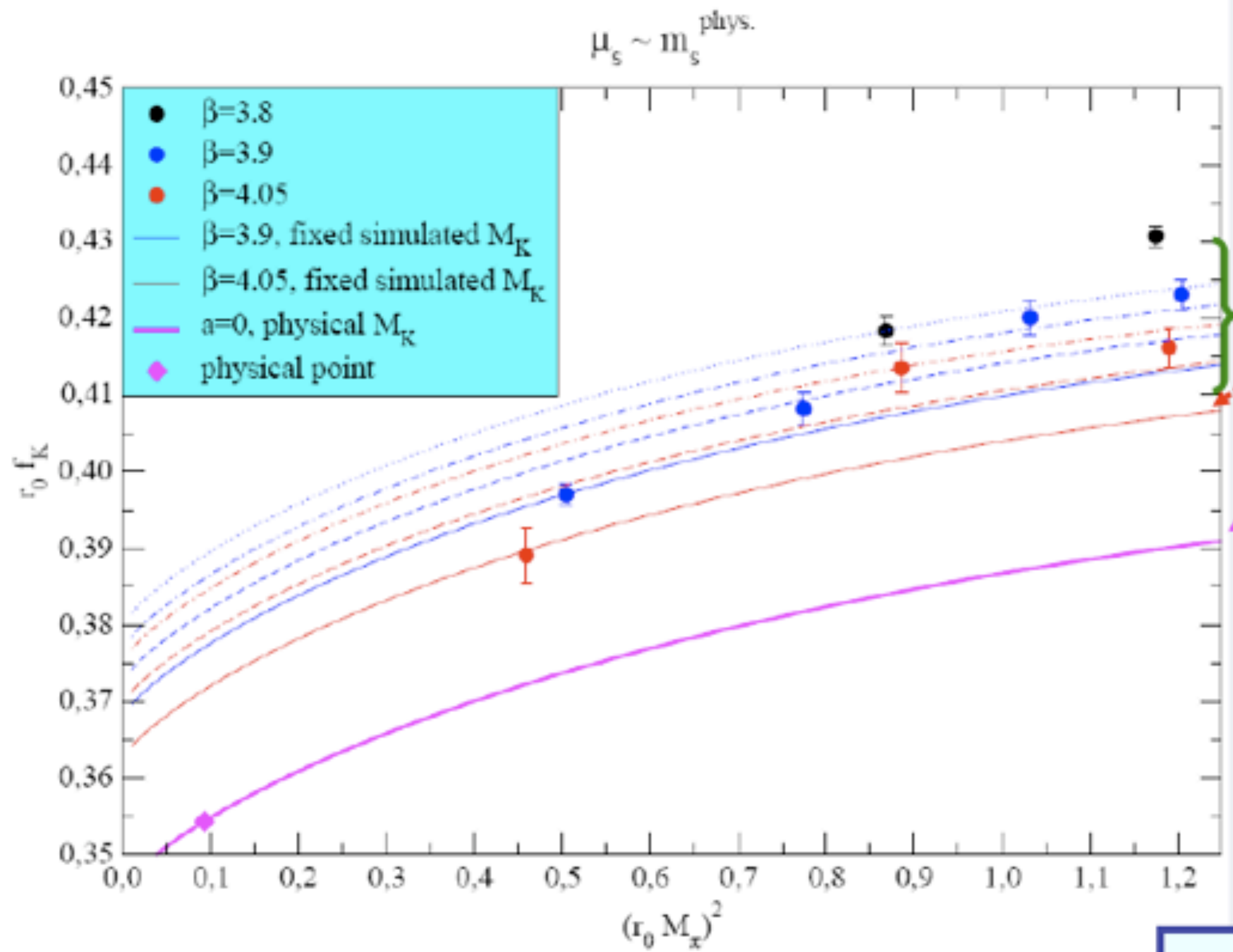
NLO “SU(2)”



* together with RBC/UKQCD and ETM, evidence that NLO SU(3) does not work well for $m_s \sim m_s^{phys}$;

* within SU(3) large NLO corrections even at $m_\ell \sim m_\ell^{phys}$: $f_\pi^{phys} / F_3 \sim [f_K / f_\pi]^{phys}$

Chiral Behaviour: dependence of f_K on m_π^2



Finite size effects (from 1-loop ChPT) [D. Becirevic, G. Villadoro, hep-lat/0506004] turn out to be negligible in the kaon region

- Data with $\mu_s \sim m_s^{phys.}$
- Curves at simulated β and M_K
- Curve with a=0, physical M_K

$M_\pi \geq$ 300 MeV

The interpolation to the physical M_K is well described by a linear dependence in M_K^2

twisted boundary conditions

* momenta are discretized on the lattice according to the choice of the BC's

L = 32 a
1/a ~ 2.5 GeV
2π/L ~ 0.5 GeV

* periodic BC's: $\psi\left(x+\hat{e}_j L\right)=\psi(x) \longrightarrow p_j = n_j \frac{2\pi}{L} \quad \left(n_j = 0,1,2,\ldots\right)$

* twisted BC's: $\psi\left(x+\hat{e}_j L\right)=e^{2\pi i\theta_j}\psi(x) \longrightarrow \tilde{p}_j = \theta_j \frac{2\pi}{L} + n_j \frac{2\pi}{L}$

shift of the "zero" momentum
by a continuous variable

* quark with arbitrary momentum $\frac{2\pi}{L}\vec{\theta}$ [Bedaque '04, Roma-ToV '04]

$$\sum_y D(x,y)S(x,y) = \delta_{x,z} \longrightarrow \sum_y D^{\vec{\theta}}(x,y)S^{\vec{\theta}}(y,z) = \delta_{x,z}$$

rephasing: $U_\mu^{\vec{\theta}}(x) = e^{2\pi i a\theta_\mu/L}\,U_\mu(x)$

* very useful to study the momentum dependence of $f_0(q^2)$ and $f_+(q^2)$

* more expensive: an inversion of the Dirac equation for each value of $\vec{\theta}$

* the all-to-all quark propagator S(x, y)

S(0, y) = point-to-all propagator

- formidable task: involve 12·V·T inversions of the Dirac equation

*** stochastic techniques:** ensemble of N_s stochastic sources $\eta^r(x)$ (color, spin and flavor labels dropped)

$$\begin{cases} \left[\eta^r(x)\right]^* \eta^r(x) = 1 \\ \lim_{N_s\to\infty} \frac{1}{N_s}\sum_{r=1}^{N_s}\left[\eta^r(x)\right]^* \eta^r(y) = \delta_{x,y} \end{cases}$$

- define: $\phi^r(x) \equiv \sum_y S(x,y)\cdot\eta^r(y) \longrightarrow \sum_y D(x,y)\phi^r(y) = \eta^r(x)$

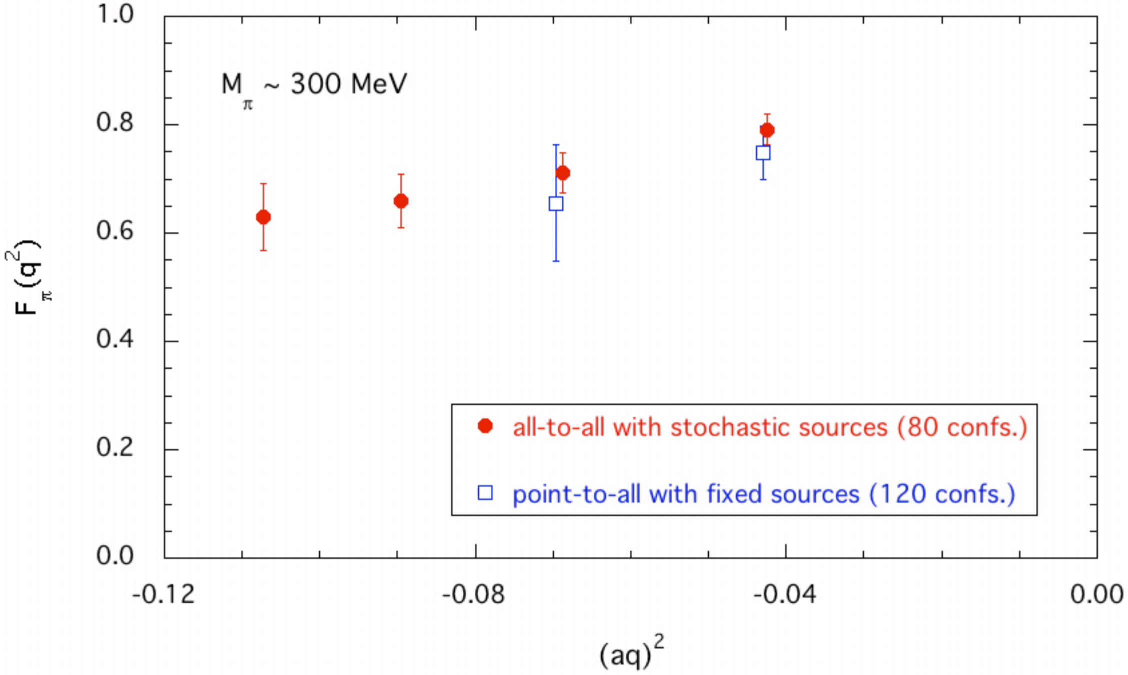
$$\frac{1}{N_s}\sum_{r=1}^{N_s}\phi^r(x)\left[\eta^r(y)\right]^* = S(x,y) + \text{noise} \qquad signal/noise \propto 1/\sqrt{V/N_s}$$

*** one-end-trick [UKQCD ’06]**

$$\frac{1}{N_s}\sum_{r=1}^{N_s}\Gamma\phi^r(x)\Gamma'\left[\phi^r(z)\right]^\dagger = \sum_y \Gamma S(x,y)\Gamma'\gamma_5 S(y,z)\gamma_5 + \text{noise}$$

$$signal/noise \propto V/\left[V/\sqrt{N_s}\right] \approx \sqrt{N_s}$$

ETMC (S.S. @ Lattice '07)



new approach:
all-to-all with stochastic sources

"traditional" approach:
point-to-all with fixed sources

$$\vec{q} = \frac{2\pi}{L}\{(1,0,0),(1,1,0),(1,1,1),(2,0,0)\}$$

same computational cost: → 80 confs. / → 120 confs.

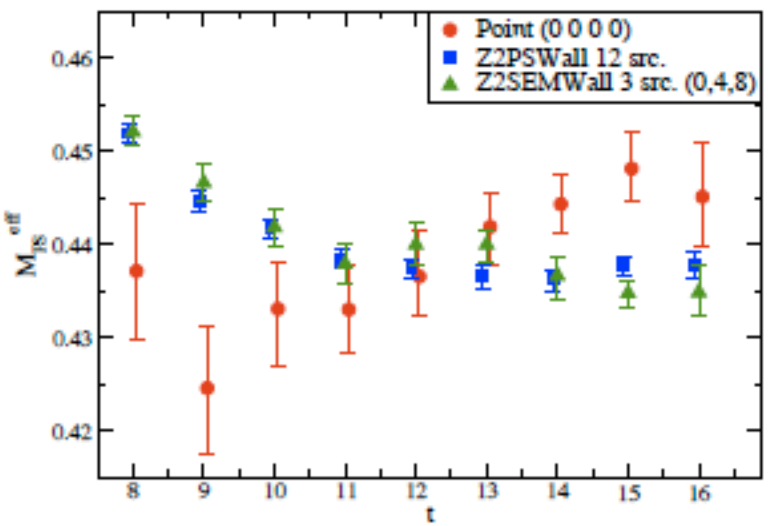


FIG. 4: Pseudoscalar effective mass plots at a fixed cost of 4704 inversions of the Dirac matrix.

arXiv: 0804.1501

RBC/UKQCD '08: detailed study of stochastic sources for 2- and 3-point correlation functions

$K_{\ell 3}$ decays

* **ETMC and RBC/UKQCD are using stochastic all-to-all propagators**

* **ETMC employs twisted BCs**