# Inclusive Rare Semileptonic B Decays: SM & Beyond

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CKM08 - Roma

⚠ = discussion point

# Effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD\times QED} + 4G_F\sqrt{2}V_{ts}^*V_{tb}\left[\sum_{i=1}^{10} C_i(\mu)P_i + \underbrace{\sum_{i=3}^{6} C_{iQ}(\mu)P_{iQ} + C_b(\mu)P_b}_{\text{for QED corrections}}\right]$$

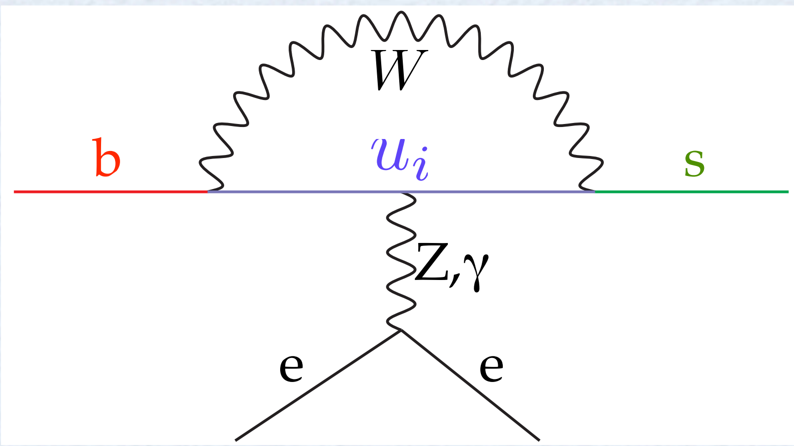
$$\begin{aligned}
P_1 &= (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L), & P_4 &= (\bar{s}_L\gamma_\mu T^a b_L)\sum(\bar{q}\gamma^\mu T^a q),\\
P_2 &= (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L), & P_5 &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L)\sum(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q),\\
P_3 &= (\bar{s}_L\gamma_\mu b_L)\sum(\bar{q}\gamma^\mu q), & P_6 &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L)\sum(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q)
\end{aligned}$$

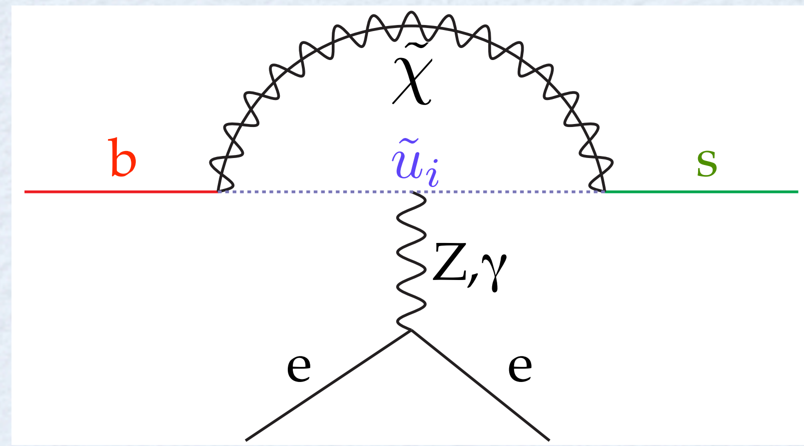
$$\begin{aligned}
P_7 &= e16\pi^2 m_b(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu}, & P_9 &= (\bar{s}_L\gamma_\mu b_L)\sum(\bar{l}\gamma^\mu l),\\
P_8 &= g16\pi^2 m_b(\bar{s}_L\sigma^{\mu\nu}T^a b_R)G^a_{\mu\nu}, & P_{10} &= (\bar{s}_L\gamma_\mu b_L)\sum(\bar{l}\gamma^\mu\gamma_5 l)
\end{aligned}$$

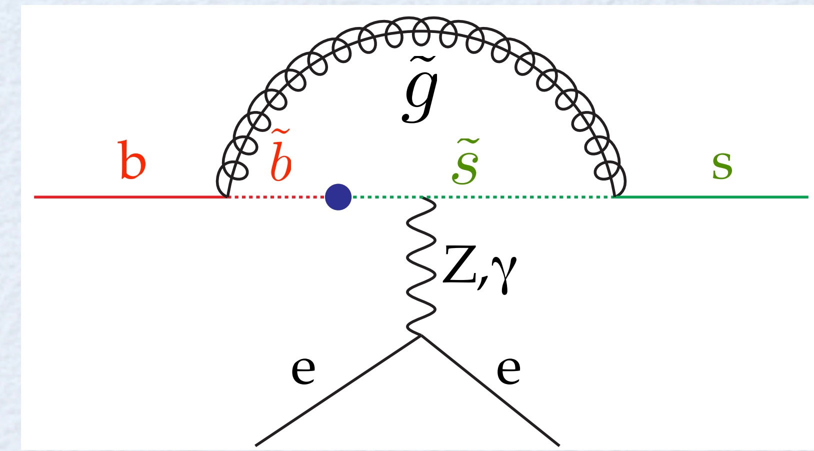
$$\begin{aligned}
P_{3Q} &= (\bar{s}_L\gamma_\mu b_L)\sum Q_q(\bar{q}\gamma^\mu q), & P_{5Q} &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L)\sum Q_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q),\\
P_{4Q} &= (\bar{s}_L\gamma_\mu T^a b_L)\sum Q_q(\bar{q}\gamma^\mu T^a q), & P_{6Q} &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L)\sum Q_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q),
\end{aligned}$$

$$P_b = 112\left[(\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L)(\bar{b}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} b) - 4(\bar{s}_L\gamma_\mu b_L)(\bar{b}\gamma^\mu b)\right]$$

# WHAT CAN WE LEARN?




SM

$V_{tb}\ V_{ts}$

MFV

$V_{tb}\ V_{ts}$

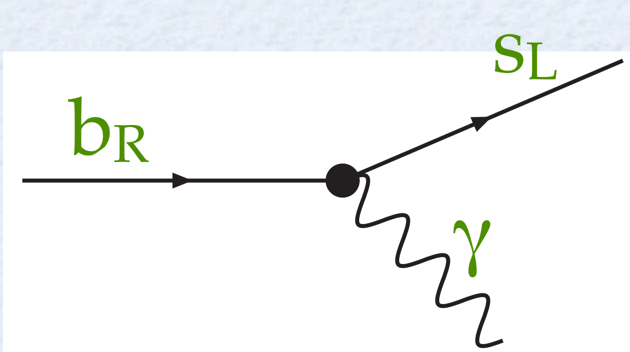
mass scale

generic

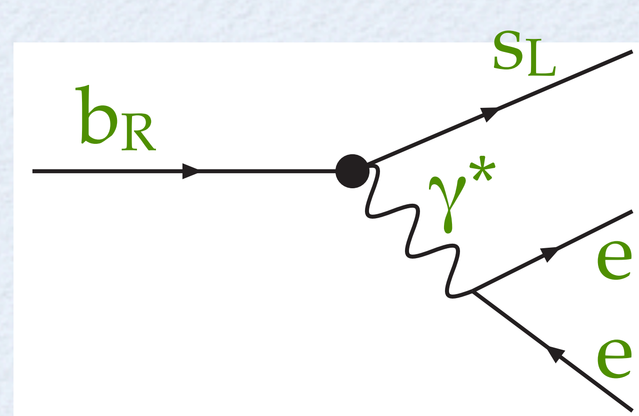
$\delta_{bs}$

mass scale

- Complementary to b→sγ:




vs

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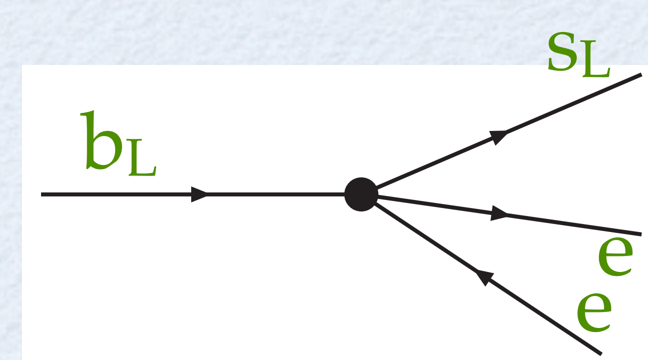
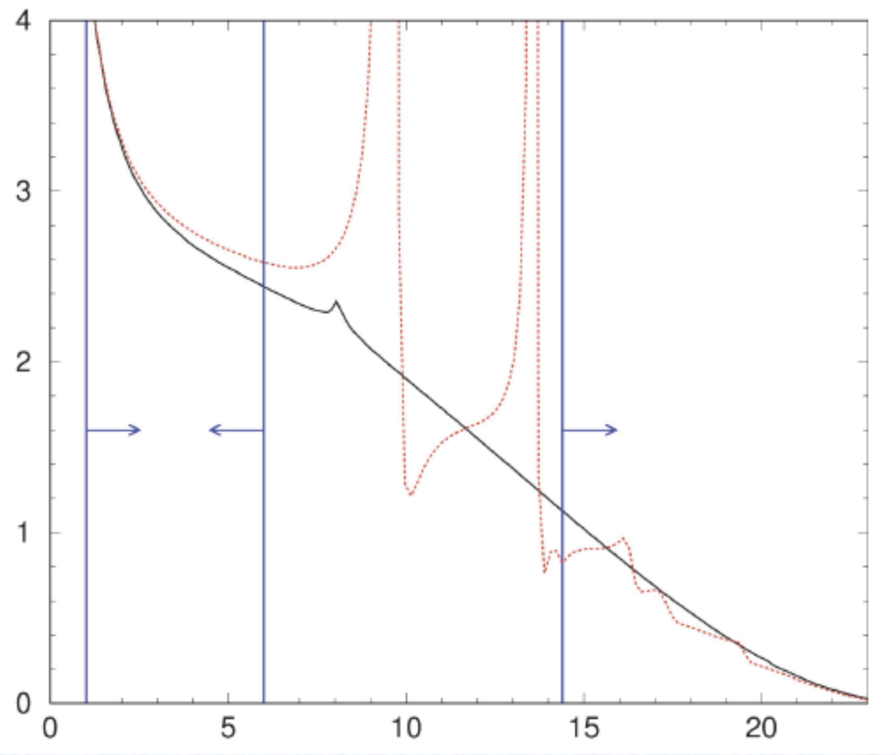


# $Q^2$ cuts

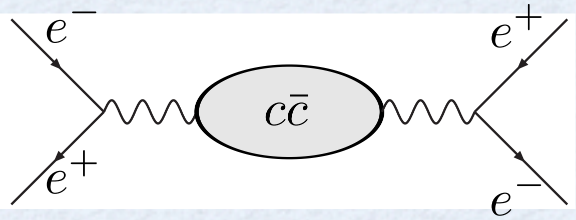
- Quark-hadron duality breaks down when the rate is dominated by charmonium resonances: $B \to X_s(J/\psi, \psi') \to X_s \ell^+ \ell^-$

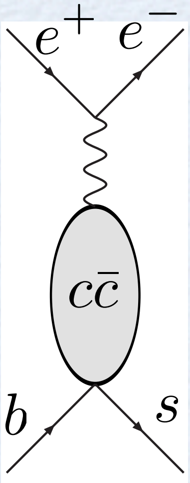
- Three regions:
  - 0.04 GeV² < q² < 1 GeV²
  - ***1 GeV² < q² < 6 GeV²***
  - ***q² > 14.4 GeV²***

dominated by the photon pole (b→sγ)




- Model using data [Krüger,Sehgal]

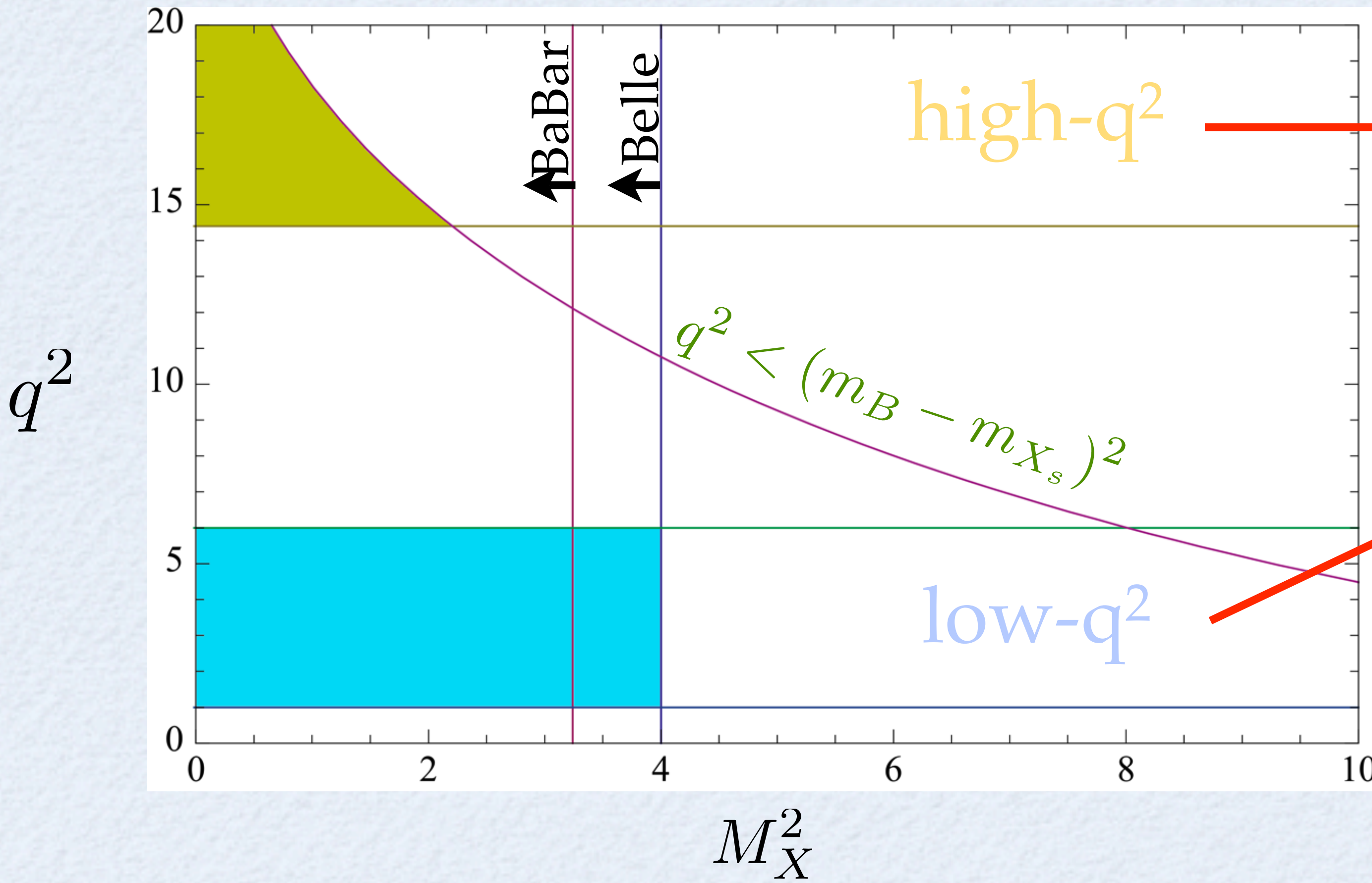



Away from the resonances expansion in $1/m_c^2$ performed

# $X_s$ cut

- MX cuts required to suppress the b → c l⁻ ν → s l⁻ l⁺ ν ν background




unaffected

non-perturbative effects dealt with using a Fermi motion model

- Correction factor added in experimental results [Ali, Hiller]
- New idea: use SCET to describe the $X_s$ system ($\Lambda^2 \ll p_{X_s}^2 \sim \Lambda m_b \ll m_b^2$)
- Reduce non-perturbative effects by considering: [Lee, Ligeti, Stewart, Tackmann]

$$\Gamma^{\rm cut}(B \to X_s \ell^+ \ell^-)/\Gamma^{\rm cut}(B \to X_u \ell \bar{\nu})$$ *[same $M_X$ cut]*

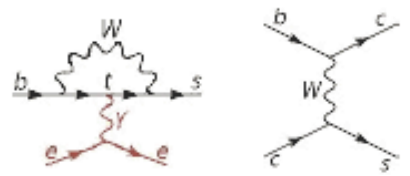
- ***Best if experimental results presented without correction***

# STATUS

$$\Gamma(\bar{B} \to X_s \ell^+ \ell^-) = \Gamma(b \to X_s \ell^+ \ell^-) + O\left(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}, \frac{\Lambda^2_{QCD}}{m_c^2}\right)$$

**QCD at NLO ($A_{LO}$,$A_{NLO}$)**

WC's:

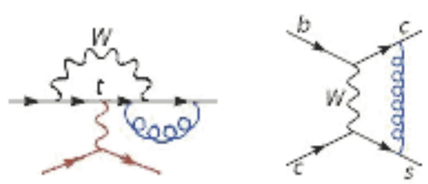


ME's:

Misiak
Buras,Münz

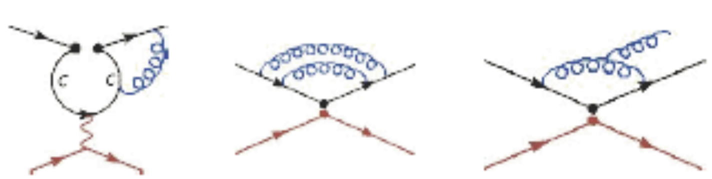
**QCD at NNLO ($A_{NNLO}$)**

WC's:



Bobeth,Misiak,Urban

ME's:



Asatryan,Asatrian,Greub,Walker
Ghinculov,Hurth,Isidori,Yao
Bobeth,Gambino,Gorbahn,Haisch

**QED at NLO ($A^{em}_{LO}$,$A^{em}_{NLO}$)**

WC's:

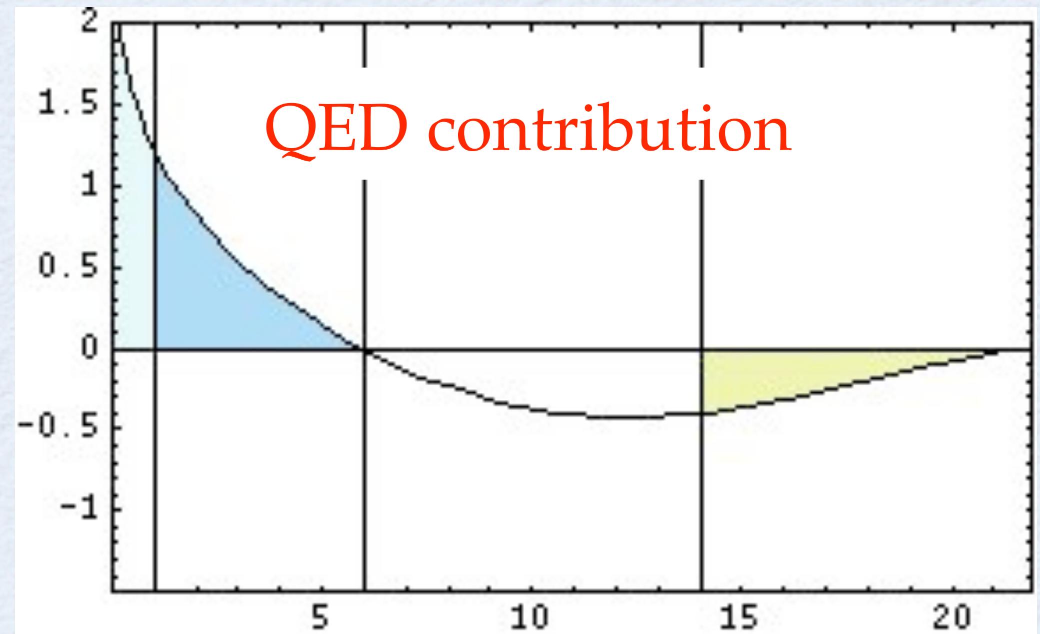
Bobeth,Gambino,Gorbahn,Haisch
Huber,Lunghi,Misiak,Wyler

ME's:

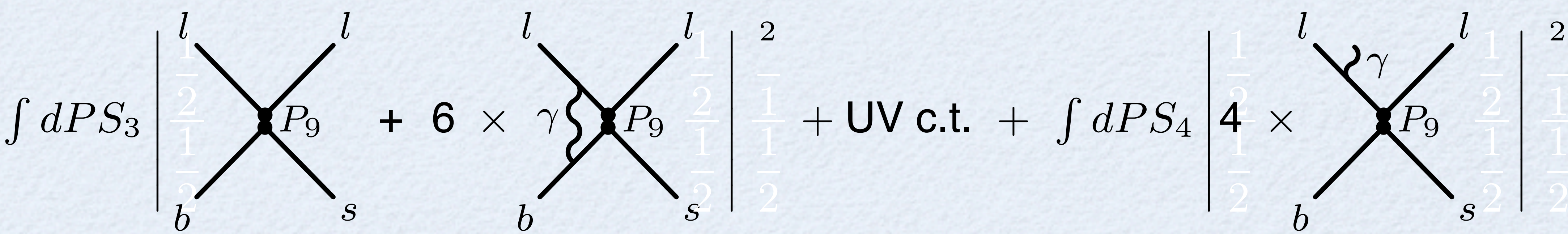
Huber,Lunghi,Misiak,Wyler

# QED effects

- The ***rate is proportional to*** $\boldsymbol{\alpha_{em}^2 (\mu^2)}$. Without QED corrections the scale μ is undetermined → 8% uncertainty




- The differential rate is not IR safe with respect to photon emission the results in the presence of a collinear logarithm, $\log(m_\ell/m_b)$

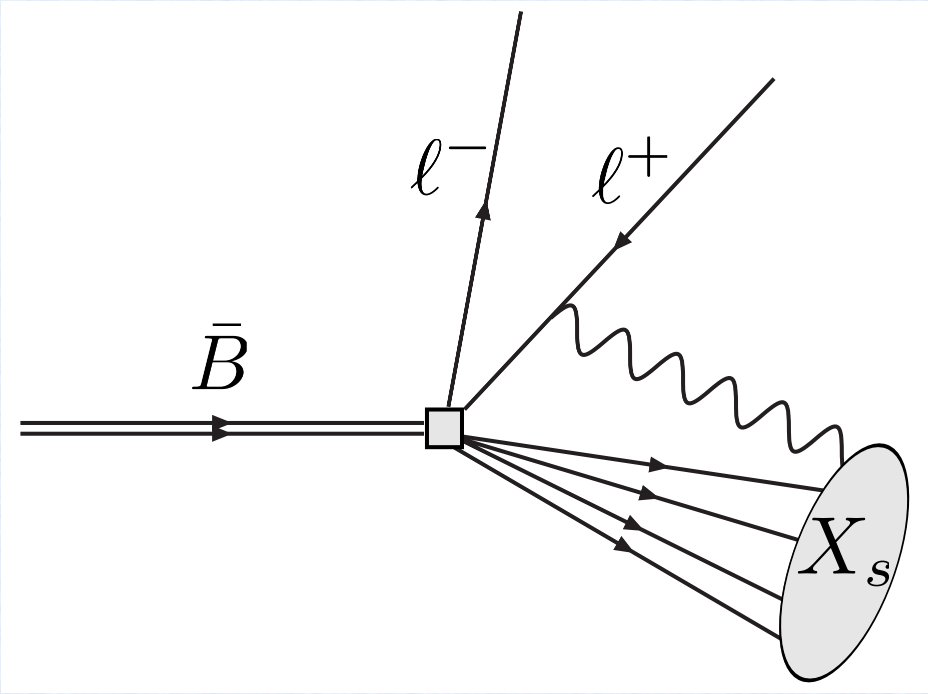



$$\text{virtual} = \frac{A_{\text{soft+collinear}}}{\epsilon^2} + \frac{B_{\text{collinear}} + B_{\text{soft}}}{\epsilon} + C$$

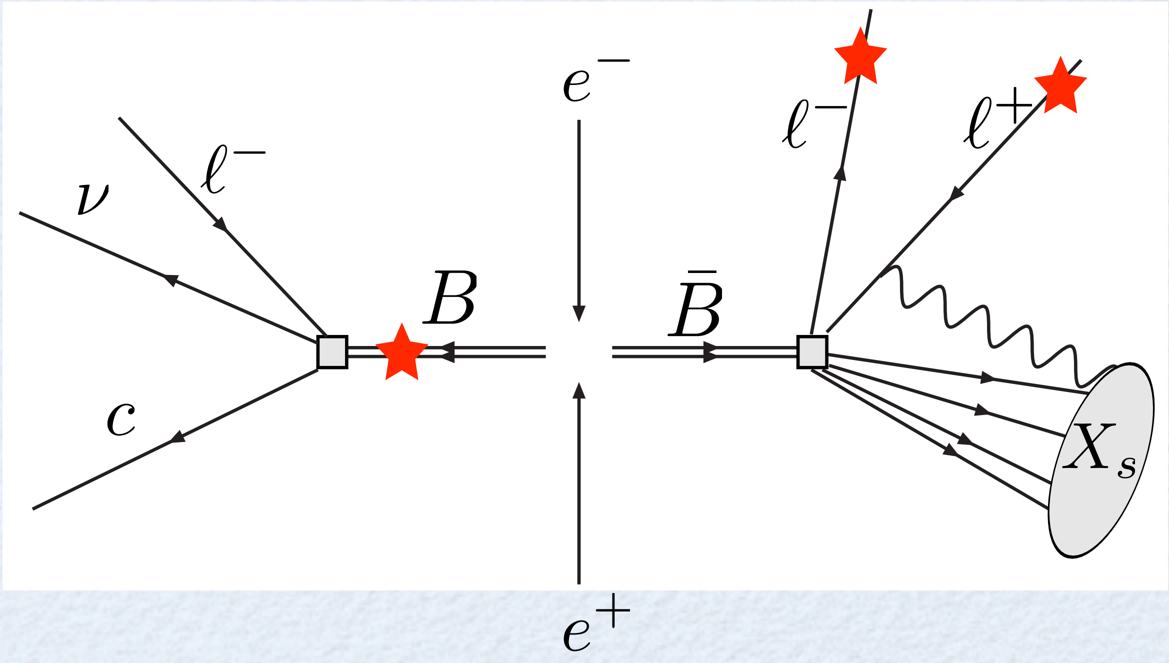
$$\text{real} = -\frac{A_{\text{soft+collinear}}}{\epsilon^2} + \frac{B'_{\text{collinear}} - B_{\text{soft}}}{\epsilon} + C'$$

# Collinear $\gamma$ vs experiments

- *Theory:*
  include all bremsstrahlung photons into the $X_s$ system.




- *Experiment (fully inclusive):*
  One B is identified; on the other side only the two leptons are reconstructed.



# Collinear $\gamma$ vs experiments

- *Experiment (sum over exclusive states)*

  at BaBar and Belle the $X_s$ system is reconstructed from a sum over exclusive states (*K plus up to 4 pions*). Momentum conservation is used to guarantee the *absence of energetic photons*

- *The collinear log present in the virtual corrections is not accompanied by the corresponding log in the real emission diagrams* and doesn't cancel even upon integration over the whole spectrum

- Exact theory prediction depends on details of the experimental analysis

- We urge our experimental colleagues to search and include energetic photons in the hadronic system (*open for discussion*)

# Renormalons

- *Cross sections expressed in terms of pole masses, are affected by renormalon ambiguities*
- These long distance effects are also responsible for the irreducible O(Λ) uncertainty in the bottom pole mass
- Can be removed switching to a short-distance mass (e.g. 1s)
- Tricky perturbative subtleties (Υ expansion)
- *Huge reward:* $m_{b,pole}$ uncertainties are almost completely removed

$$m_b^{1S} = (4.68 \pm 0.03)\mathrm{GeV} \qquad m_b^{\mathrm{pole}} \simeq (4.9 \pm 0.1)\mathrm{GeV}$$

# BR AND $A_{FB}$

- Differential decay width ( $\hat{s} = q^2/m_b^2$ ):

$$\frac{d\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} = \mathcal{B}(B \to X_c \ell \nu) \frac{\Gamma(B \to X_u \ell \nu)}{\Gamma(B \to X_c \ell \nu)} \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\Gamma(B \to X_u \ell \nu)}$$

$$\propto (4 + \frac{8}{\hat{s}}) \left| C_7^{eff} \right|^2 + (1 + 2\hat{s}) (\left| C_9^{eff} \right|^2 + \left| C_{10}^{eff} \right|^2) + 12 \,\mathrm{Re} \left( C_7^{eff} C_9^{eff*} \right)$$

- Forward-Backward asymmetry ( $z = \cos\theta_\ell$ ):

$$\mathcal{A}_{FB}(\hat{s}) \equiv \frac{d\mathcal{B}_{\ell\ell}/d\hat{s}(z > 0) - d\mathcal{B}_{\ell\ell}/d\hat{s}(z < 0)}{d\mathcal{B}_{\ell\ell}/d\hat{s}(z > 0) + d\mathcal{B}_{\ell\ell}/d\hat{s}(z < 0)} \propto \mathrm{Re} \left[ \left( 2C_7^{eff} + \hat{s} C_9^{eff} \right) C_{10}^* \right]$$

- New observables:

$$H_T \propto (1 - \hat{s})^2 \hat{s} \left[ \left( C_9 + \frac{2}{\hat{s}} C_7 \right)^2 + C_{10}^2 \right]$$

$$H_L \propto (1 - \hat{s})^2 \left[ (C_9 + 2C_7)^2 + C_{10}^2 \right]$$

$\Rightarrow$ Independent combinations of WC's

# INPUTS

| | |
|---|---|
| $\alpha_s(M_z) = 0.1189 \pm 0.0010$ [40] | $m_e = 0.51099892$ MeV |
| $\alpha_e(M_z) = 1/127.918$ | $m_\mu = 105.658369$ MeV |
| $s_W^2 \equiv \sin^2\theta_W = 0.2312$ | $m_\tau = 1.77699$ GeV |
| $\lvert V_{ts}V_{tb}/V_{cb}\rvert^2 = 0.962 \pm 0.002$ [41] | $m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)$ GeV [42] |
| $\lvert V_{ts}V_{tb}/V_{ub}\rvert^2 = (1.28 \pm 0.12) \times 10^2$ [41] | $m_b^{1S} = (4.68 \pm 0.03)$ GeV [31] |
| $BR(B \to X_c e\bar{\nu})_{\rm exp} = 0.1061 \pm 0.0017$ [43] | $m_{t,\rm pole} = (170.9 \pm 1.8)$ GeV [44] |
| $M_Z = 91.1876$ GeV | $m_B = 5.2794$ GeV |
| $M_W = 80.426$ GeV | $C = 0.58 \pm 0.01$ [31] |
| $\lambda_2^{\rm eff} = (0.12 \pm 0.02)$ GeV$^2$ | $\rho_1 = (0.06 \pm 0.06)$ GeV$^3$ [31] |
| $\lambda_1^{\rm eff} = (-0.243 \pm 0.055)$ GeV$^2$ [42] | $f_u^0 + f_s = (0 \pm 0.2)$ GeV$^3$ [24] |
| $f_u^0 - f_s = (0 \pm 0.04)$ GeV$^3$ [24] | $f_u^\pm = (0 \pm 0.4)$ GeV$^3$ [24] |

# Branching Ratio

- Theory [Huber,Lunghi,Misiak,Wyler; Huber,Hurth,Lunghi]:

$$\mathcal{B}_{\mu\mu}^{\rm low} = \Big[1.59 \pm 0.08_{\rm scale} \pm 0.06_{m_t} \pm 0.024_{C,m_c} \pm 0.015_{m_b} \pm 0.02_{\alpha_s(M_Z)}$$

$$\pm 0.015_{\rm CKM} \pm 0.026_{{\rm BR}_{sl}} \pm 0.08_{\alpha_s/m_b}\Big] \times 10^{-6} = (1.59 \pm 0.14) \times 10^{-6}$$

$$\mathcal{B}_{ee}^{\rm low} = (1.64 \pm 0.14) \times 10^{-6}$$

$$\mathcal{B}_{\mu\mu}^{\rm high} = 2.40 \times 10^{-7}\Big(1 + \left[{}^{+0.01}_{-0.02}\right]_{\mu_0} + \left[{}^{+0.14}_{-0.06}\right]_{\mu_b} \pm 0.02_{m_t} + \left[{}^{+0.006}_{-0.003}\right]_{C,m_c} \pm 0.05_{m_b}$$

$$+ \left[{}^{+0.0002}_{-0.001}\right]_{\alpha_s} \pm 0.002_{\rm CKM} \pm 0.02_{{\rm BR}_{sl}} \boxed{\pm 0.05_{\lambda_2} \pm 0.19_{\rho_1} \pm 0.14_{f_s} \pm 0.02_{f_u}} \pm 0.05_{\alpha_s/m_b}\Big)$$

$$= (2.40 \pm 0.7) \times 10^{-7}$$

$$\mathcal{B}_{ee}^{\rm high} = (2.1 \pm 0.6) \times 10^{-7}$$

The OPE is an expansion in $\Lambda_{\rm QCD}/(m_b - \sqrt{q^2})$

$$p_{X_s}^2 = (p_b - q)^2 = m_b^2 + q^2 - 2m_b q_0$$

$$< m_b^2 + q^2 - 2m_b\sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2$$

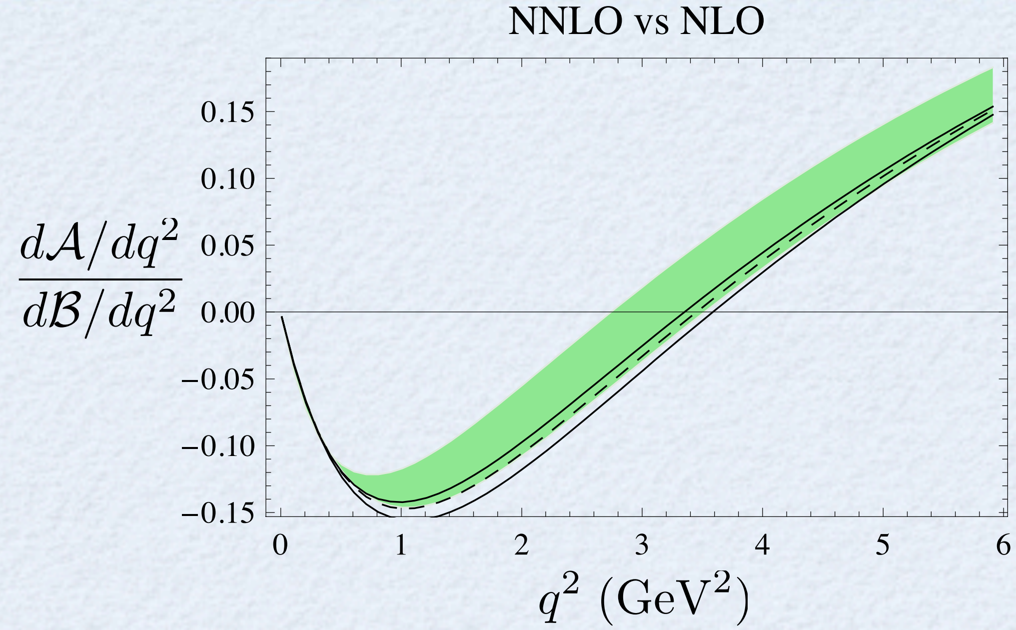
- Experiment [BaBar and Belle]:

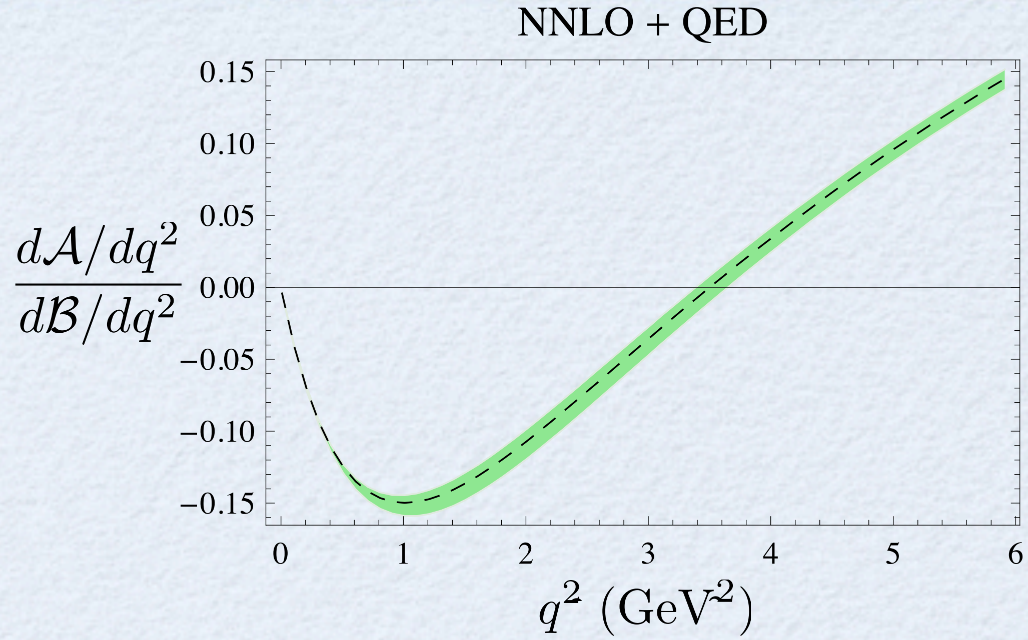
$$\mathcal{B}_{\ell\ell}^{\rm low} = (1.60 \pm 0.51) \times 10^{-6}$$

$$\mathcal{B}_{\ell\ell}^{\rm high} = (4.4 \pm 1.2) \times 10^{-7}$$

# LOW-Q²: FBA

[Huber,Hurth,Lunghi]




$$
\begin{aligned}
(q_0^2)_{\mu\mu} &= \left[3.50 \pm 0.10_{\rm scale} \pm 0.002_{m_t} \pm 0.04_{m_c,C} \pm 0.05_{m_b} \pm 0.03_{\alpha_s(M_Z)} \pm 0.001_{\lambda_1} \pm 0.01_{\lambda_2}\right] \mathrm{GeV}^2 \\
&= (3.50 \pm 0.12)\, \mathrm{GeV}^2 \\
(q_0^2)_{ee} &= (3.38 \pm 0.11)\, \mathrm{GeV}^2
\end{aligned}
$$

- Integrated observables:

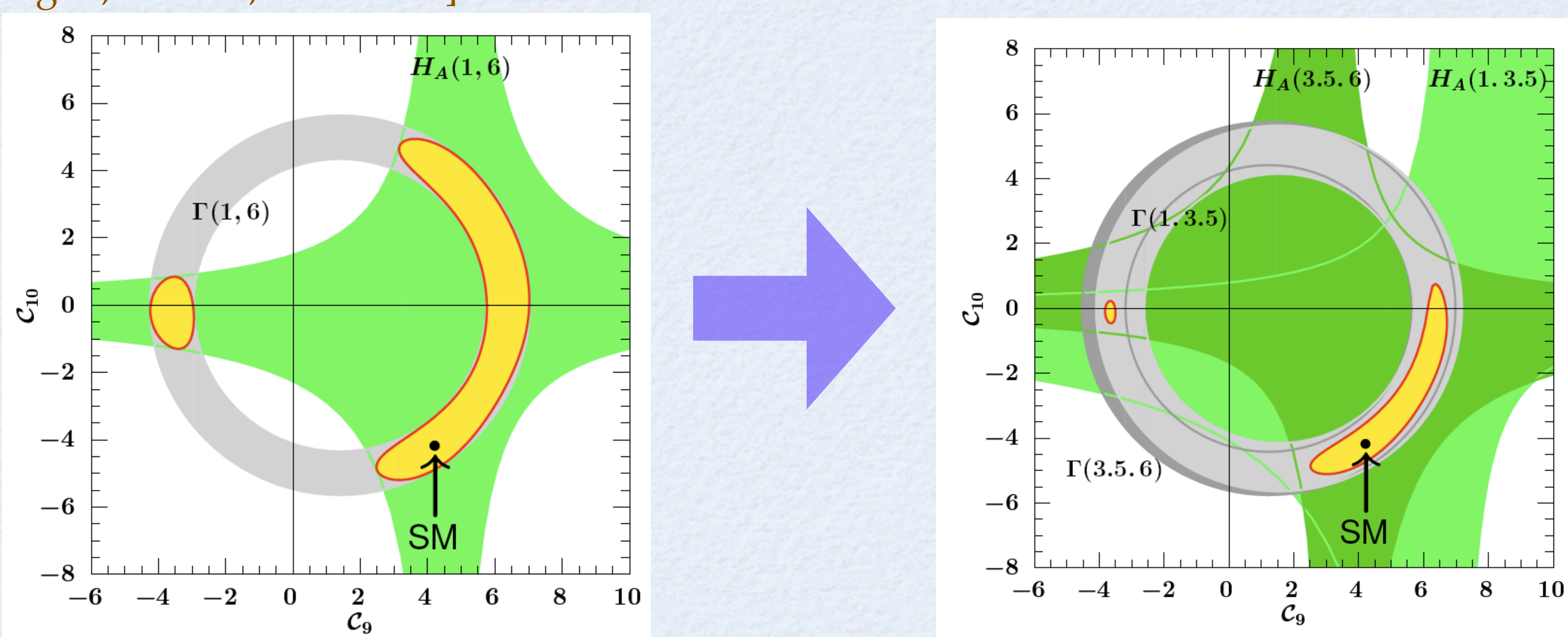
| Bin 1 ($q^2 \in [1, 3.5]\mathrm{GeV}^2$) | Bin 2 ($q^2 \in [3.5, 6]\mathrm{GeV}^2$) | $\mathrm{low} - \mathrm{q}^2$ ($q^2 \in [1, 6]\mathrm{GeV}^2$) |
|---|---|---|
| $(\bar{\mathcal{A}}_{\mu\mu})_{\rm bin1} = [-9.1 \pm 0.9]\%$ | $(\bar{\mathcal{A}}_{\mu\mu})_{\rm bin2} = [7.8 \pm 0.8]\%$ | $(\bar{\mathcal{A}}_{\mu\mu})_{\rm low} = [-1.5 \pm 0.9]\%$ |
| $(\bar{\mathcal{A}}_{ee})_{\rm bin1} = [-8.1 \pm 0.9]\%$ | $(\bar{\mathcal{A}}_{\mu\mu})_{\rm bin2} = [8.3 \pm 0.6]\%$ | $(\bar{\mathcal{A}}_{\mu\mu})_{\rm low} = [-0.9 \pm 0.9]\%$ |

# Low-$Q^2$: New Observables

$$\frac{d^2\Gamma}{dq^2dz} = \frac{3}{8}\Big[(1+z^2)H_T(q^2) + 2zH_A(q^2) + 2(1-z^2)H_L(q^2)\Big]$$

$$\frac{d\Gamma}{dq^2} = H_T + H_L \qquad\qquad \frac{d\mathcal{A}_{FB}}{dq^2} = \frac{3}{4}H_A$$

- Importance of splitting the Forward-Backward asymmetry in two bins:
[Lee,Ligeti,Stewart,Tackmann]




[Toy analysis: data extrapolated at 1 ab$^{-1}$, $C_7 < 0$ taken from $b \to s\gamma$]

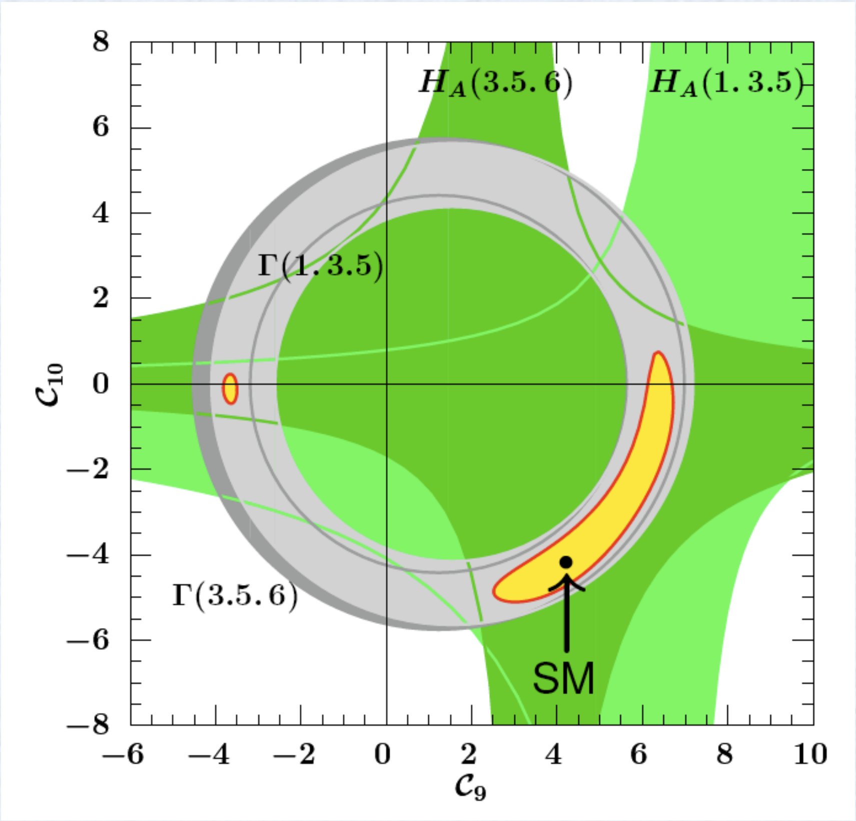
# Low-$q^2$: new observables

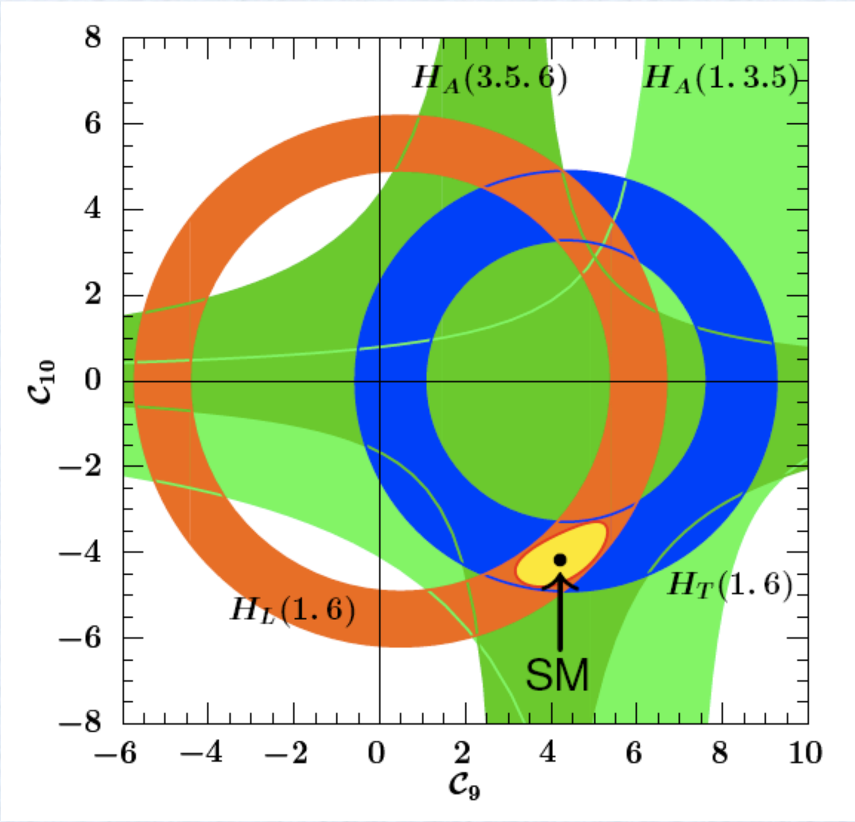
$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8}\Big[(1+z^2)H_T(q^2) + 2zH_A(q^2) + 2(1-z^2)H_L(q^2)\Big]$$

$$\frac{d\Gamma}{dq^2} = H_T + H_L \qquad \frac{d\mathcal{A}_{FB}}{dq^2} = \frac{3}{4}H_A$$

- Importance of splitting the $H_T$ and $H_L$:

[Lee,Ligeti,Stewart,Tackmann]




[Toy analysis: data extrapolated at 1 ab$^{-1}$, $C_7 < 0$ taken from $b \to s\gamma$]

# High-$q^2$: Reducing the errors

- New idea: normalize the decay width to the semileptonic B→$X_u$lν rate with the ***same dilepton invariant mass cut:***

$$\mathcal{R}(s_0) = \frac{\displaystyle\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\, \frac{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{\mathrm{d}\hat{s}}}{\displaystyle\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\, \frac{\mathrm{d}\Gamma(\bar{B}^0 \to X_u \ell \nu)}{\mathrm{d}\hat{s}}}$$

[Ligeti,Tackmann]

- ***Impact of non-perturbative*** $1/m_b^2$ ***and*** $1/m_b^3$ ***power corrections drastically reduced***

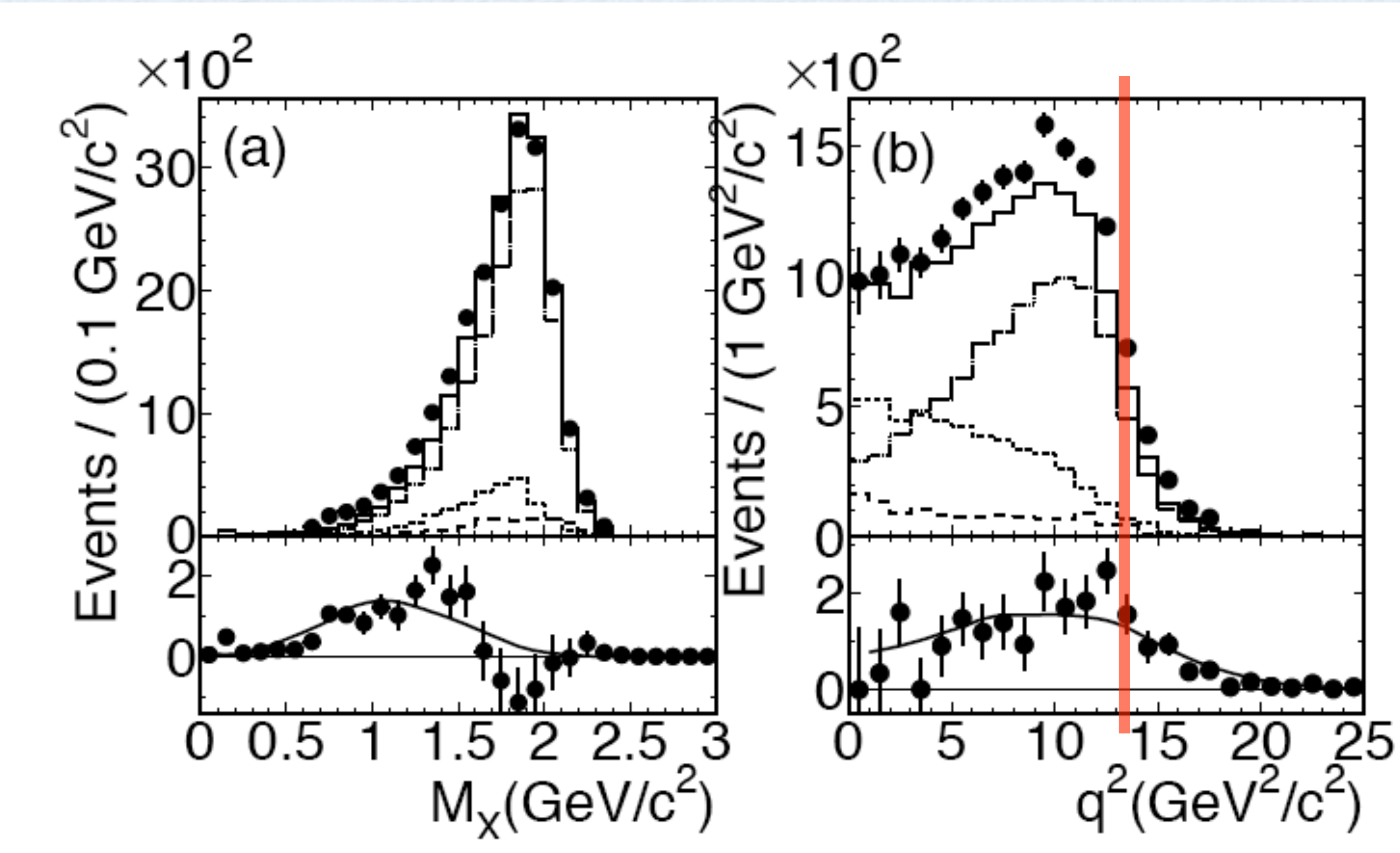
- In the high-$q^2$ region we find:

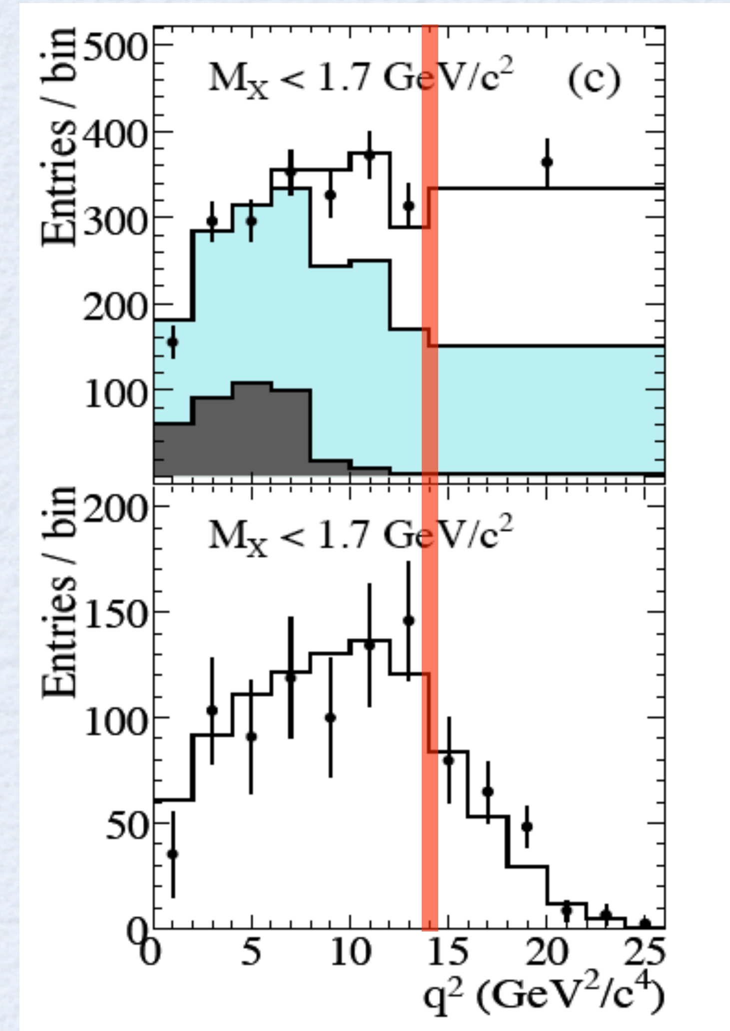
$$\begin{aligned}\mathcal{R}(14.4\mathrm{GeV}^2) &= 2.29 \times 10^{-3} \Big(1 \pm 0.04_{\mathrm{scale}} \pm 0.02_{m_t} \pm 0.01_{C,m_c} \pm 0.006_{m_b} \pm 0.005_{\alpha_s} \pm 0.09_{\mathrm{CKM}} \\ &\quad \pm 0.003_{\lambda_2} \pm 0.05_{\rho_1} \pm 0.03_{f_u^0+f_s} \pm 0.05_{f_u^0-f_s}\Big) \\ &= 2.29 \times 10^{-3}(1 \pm 0.13)\end{aligned}$$

[Huber,Hurth,Lunghi]

- ***The largest source of uncertainty is $V_{ub}$***

# High-$Q^2$: Reducing the Errors



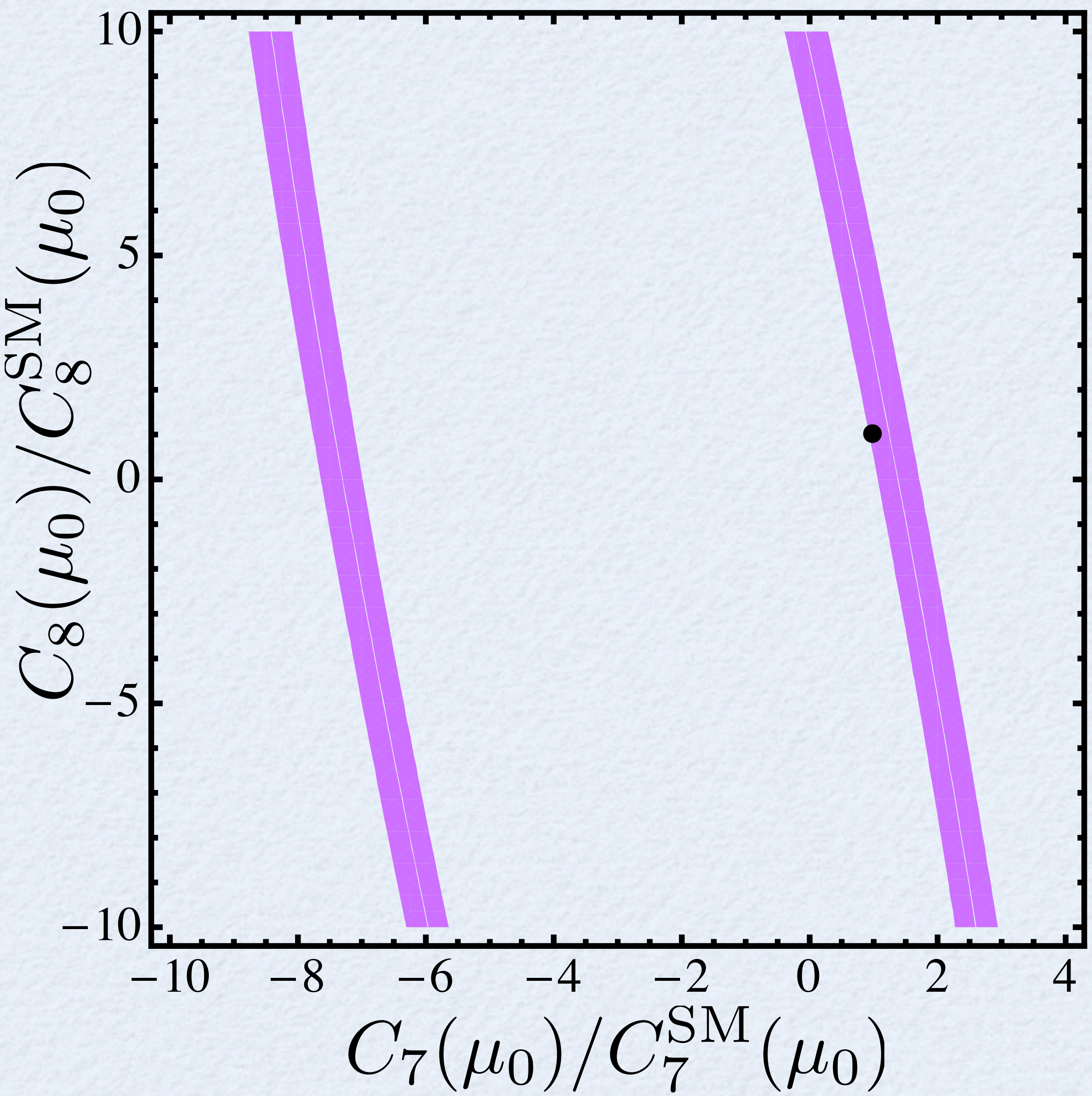

[Belle, 87 $fb^{-1}$, hep-ex/0311048]

[BaBar, 383 m $\Upsilon$, arXiv:0708.3702]

- Experiments already positioned to measure $B \to X_u l \nu$ with a $q^2$ cut
- Separation of $B^0$ and $B^+$ is important to control WA contributions

# Model independent analysis

- Use $B \rightarrow X_s\gamma$ to constrain $C_7$ and $C_8$:




Theory:

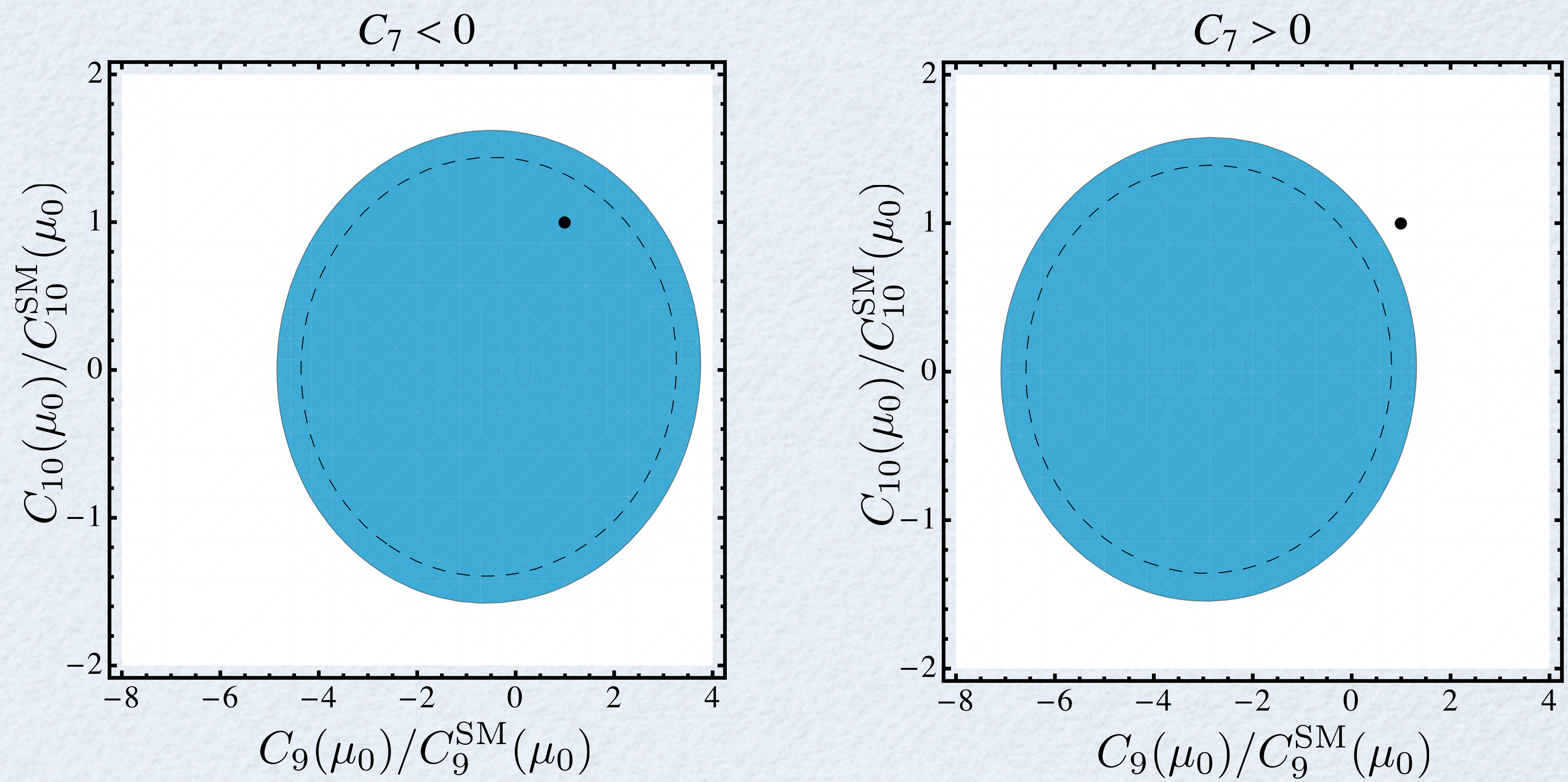
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\mathrm{SM}} = (3.15 \pm 0.23) \times 10^{-4}$$

Experiment:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\mathrm{exp}} = (3.52 \pm 0.25) \times 10^{-4}$$

# Model independent analysis

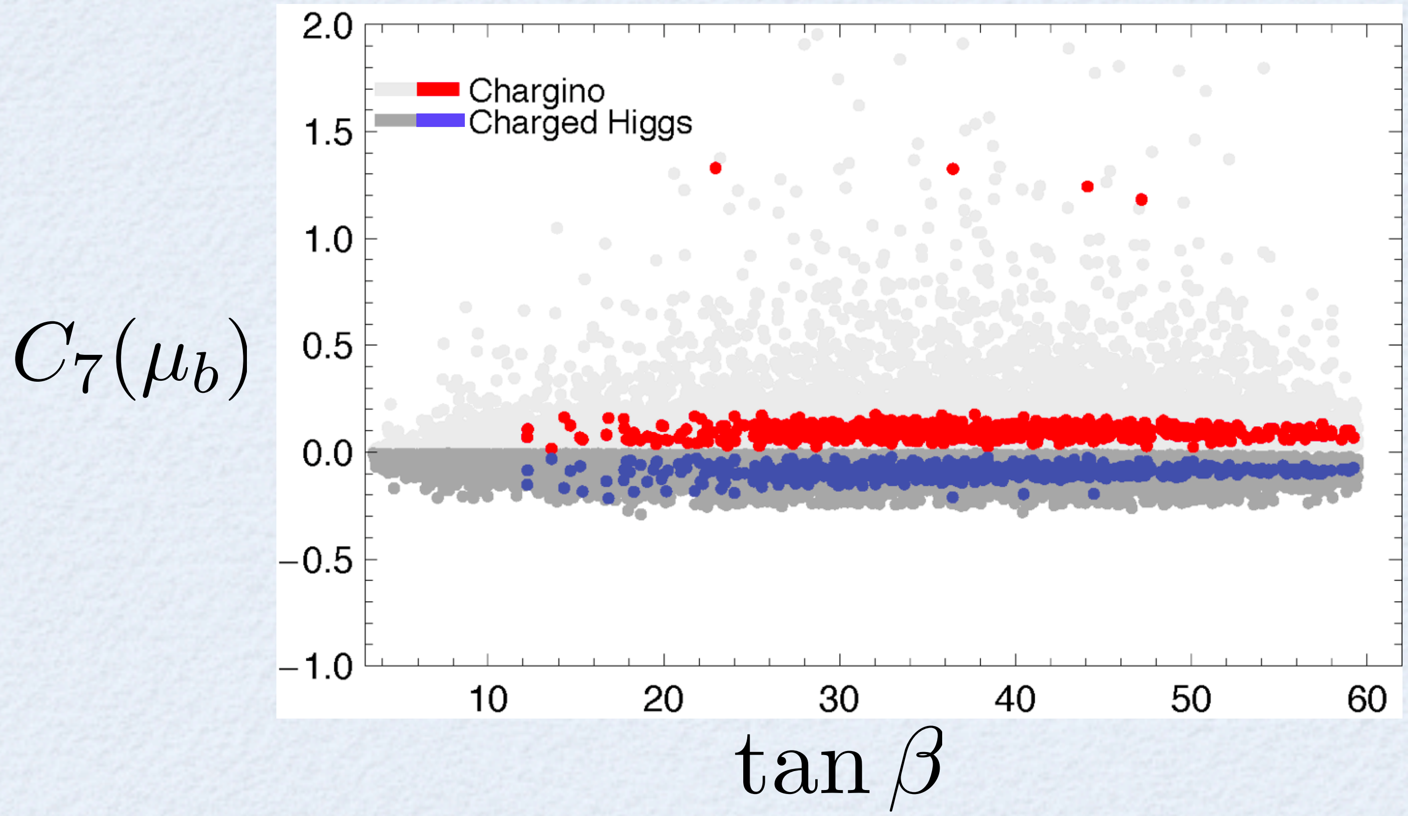
- Use $C_7$ and $C_8$ from $B \to X_s\gamma$ to constrain $C_9$ and $C_{10}$




- $C_7 > 0$ requires sizable contributions to $C_9$ and $C_{10}$
- Reversing the sign of $C_7$ we obtain $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) = (3.11 \pm 0.22) \times 10^{-6}$ hence the SM sign is favored at the 2.7σ level
  [Gambino,Haisch,Misiak]

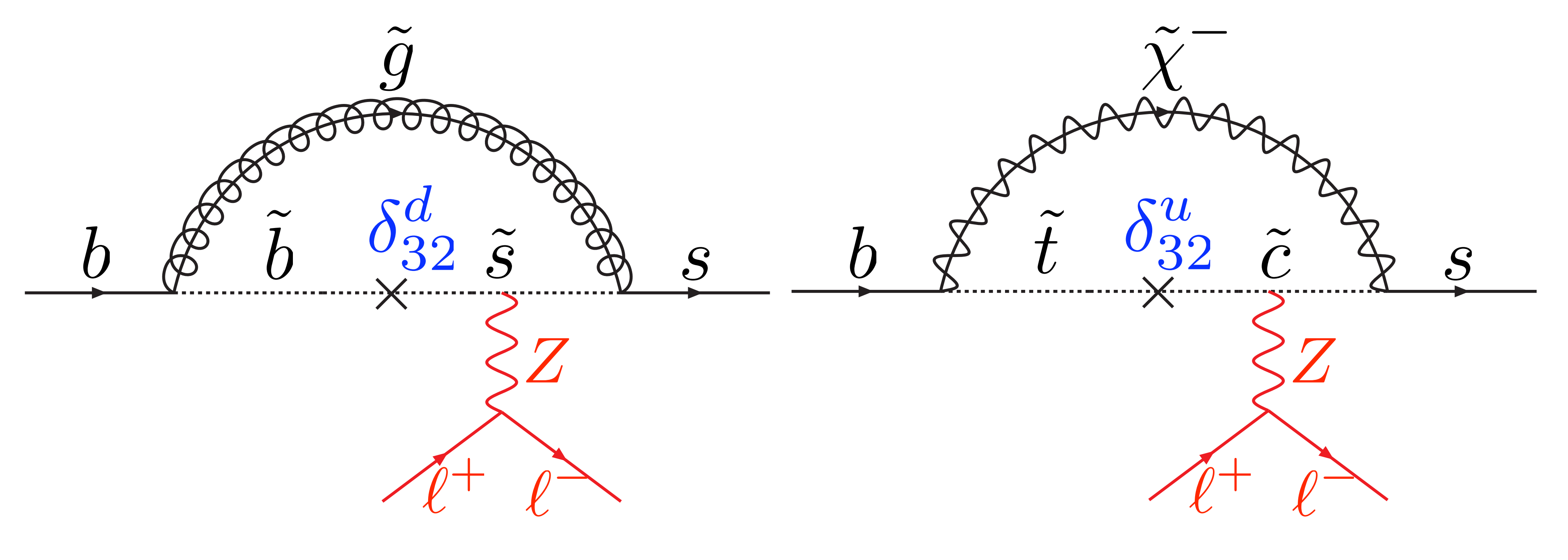
# MFV SUSY

- Computing aid: *Spheno* for the RGE of the MSSM and *MicrOMEGAs* for the relic dark matter density
- Effects on C9 and C10 are tiny: $\left|C_{9,10}(\mu_0)/C_{9,10}^{\rm SM}(\mu_0)\right| < 0.1$
- b→sγ shapes the surviving parameter space:



# SUSY: MIA analysis

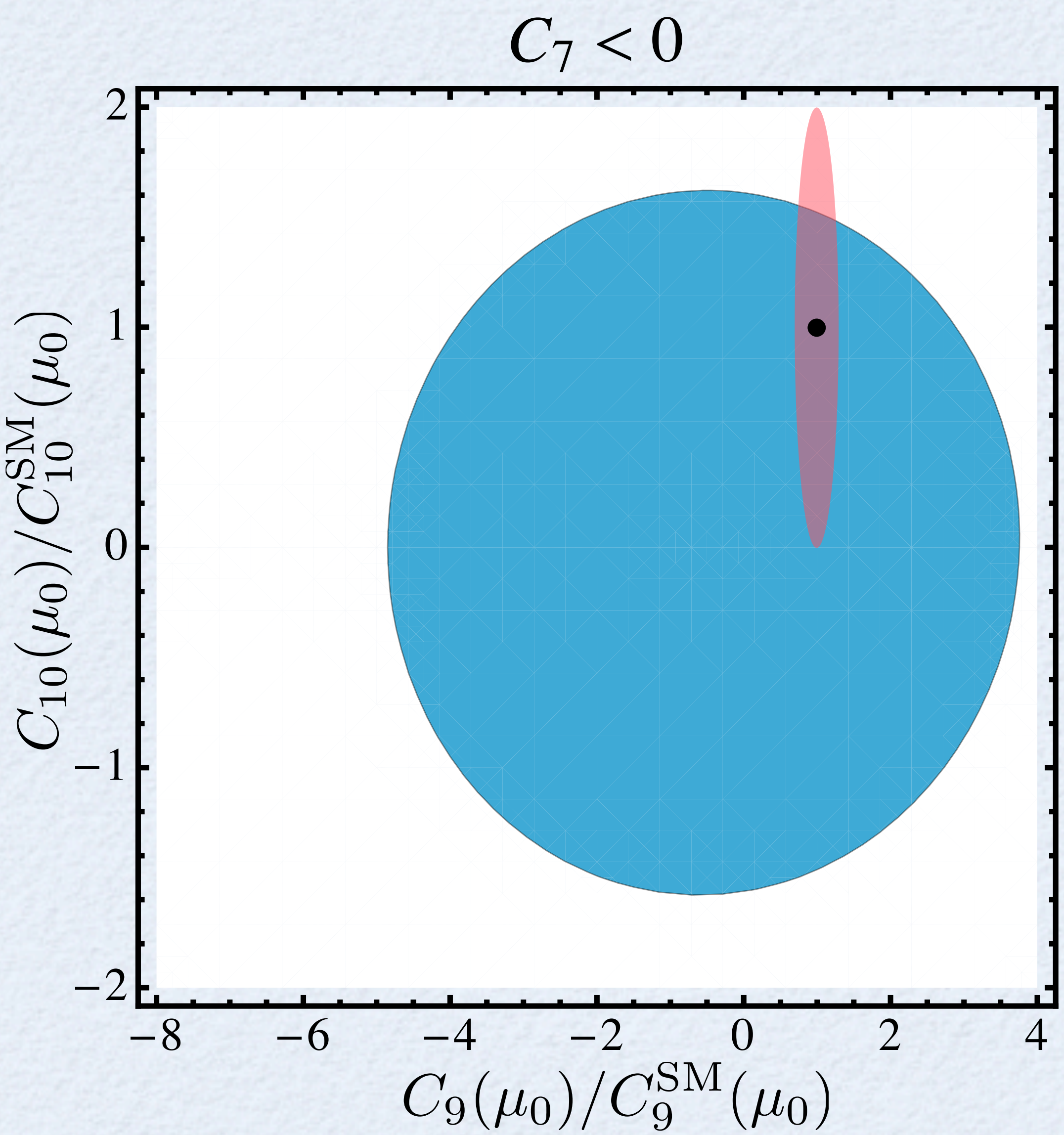
- In the most general MSSM, gluino and chargino diagrams can lead to huge contributions to the semileptonic operators:

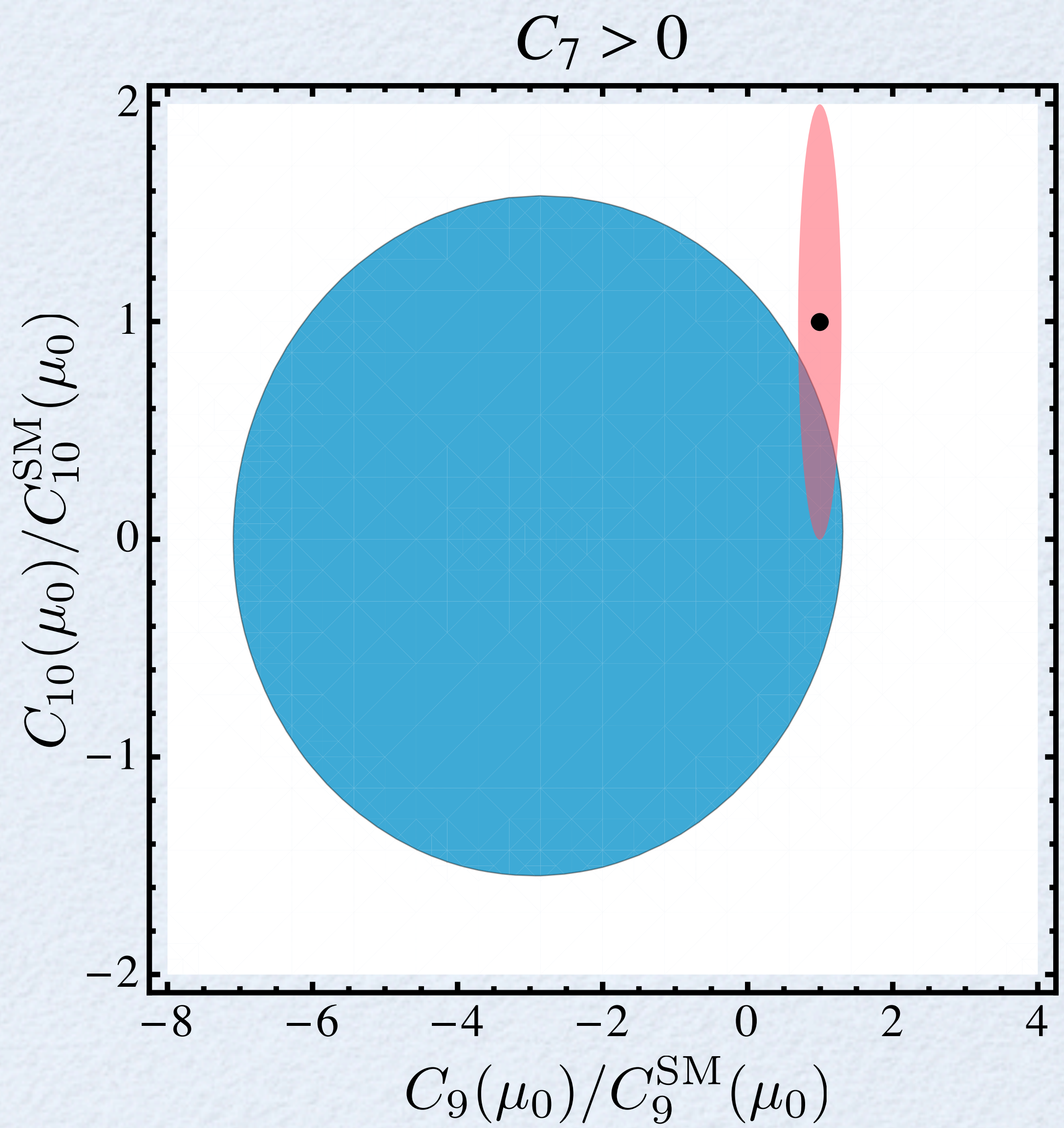



$$0.7 < C_9(\mu_0)/C_9^{\rm SM}(\mu_0) < 1.3$$

$$0 < C_{10}(\mu_0)/C_{10}^{\rm SM}(\mu_0) < 2$$

# SUSY: MIA analysis

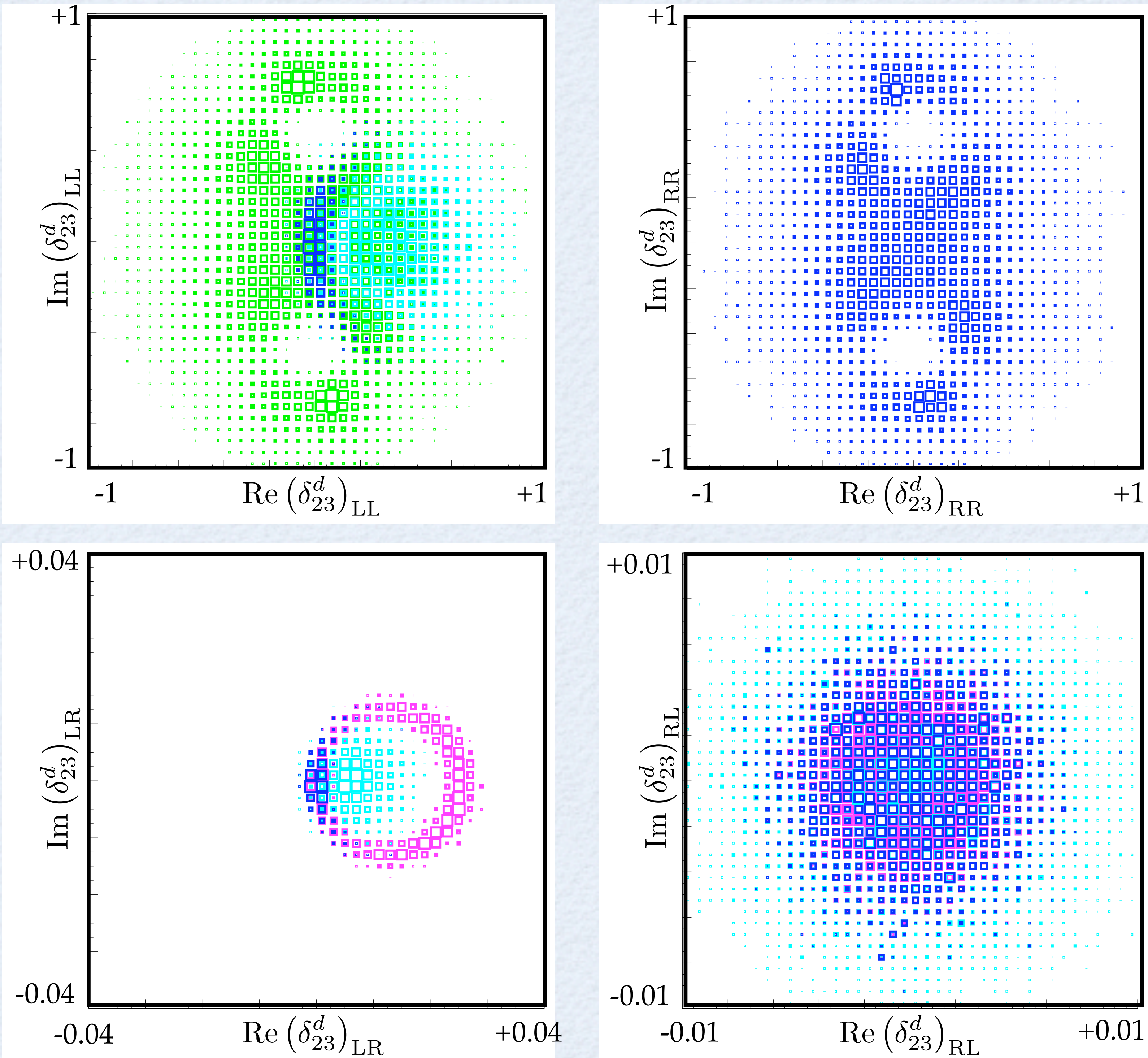



- The $C_7 > 0$ scenario is viable (with some degree of fine tuning)
- More than one mass insertion present at the same time

# SUSY: MIA ANALYSIS

- Constraints on (23) mass insertions in the down sector
  [Ciuchini,Silvestrini]



# Final Messages

- Experiments cordially encouraged ***not to correct for $X_s$ cut***
- Treatment of ***collinear photons***?
- Split the ***FB-asymmetry into two bins*** ([1,3.5]GeV$^2$ and [3.5,6] GeV$^2$)
- Measure separately ***$H_L$ and $H_T$***: $\Gamma \propto (1+z^2)\, H_T + 2\, z\, H_A + 2\, (1-z^2)\, H_L$
- Normalize the BR in the high-$q^2$ region to ***$B^0 \to X_u l\nu$ with the same $q^2$ cut***
- Relevant only to constrain ***non-MFV new physics models***