Exclusive $b \to s\,\ell^+\ell^-$

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Outline

1. $B \to K^* \ell^+ \ell^-$
   - amplitudes and rates
   - factorization
   - numerical predictions
   - warnings
   - brief comment on new physics
2. other modes (briefly)
3. discussion
4. backup

$B \to K^* \ell^+ \ell^-$: Amplitudes and rates

doubly differential decay rate:

e.g. [Lee/Ligeti/Stewart/Tackmann 06]

$$\frac{d^2\Gamma}{dq^2\, d\cos\theta} = \frac{3}{8}\left[(1+\cos^2\theta)\, H_T(q^2) + 2\cos\theta\, H_A(q^2) + 2\,(1-\cos^2\theta)\, H_L(q^2)\right] .$$

- $H_T(q^2)$ determines rate for transversely polarized K^*
- $H_L(q^2)$ determines rate for longitudinally polarized K^*
- $H_A(q^2)$ determines lepton forward-backward asymmetry

helicity and transversity amplitudes:

[Krüger/Matias 05]

$$H_T(q^2) = |A_{\perp\, L}|^2 + |A_{\perp\, R}|^2 + |A_{\parallel\, L}|^2 + |A_{\parallel\, R}|^2 ,$$

$$H_L(q^2) = |A_{0\, L}|^2 + |A_{0\, R}|^2 ,$$

$$H_A(q^2) = 2\,\mathrm{Re}\left[A_{\parallel\, R}\, A^*_{\perp\, R} - A_{\parallel\, L}\, A^*_{\perp\, L}\right] .$$

Following [Krüger/Matias 05] and [Beneke/Feldmann/Seidel 01,04], the transversity amplitudes (relevant for an angular analysis of $B \to K^*(K\pi)\ell\ell$) can be written as

$$A_{\perp\, L,R} \propto \left[(C_9 \mp C_{10}) \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} \mathcal{T}_1(q^2) \right],$$

$$A_{\parallel\, L,R} \propto \left[(C_9 \mp C_{10}) \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2} \mathcal{T}_2(q^2) \right],$$

$$A_{0\, L,R} \propto \left[(C_9 \mp C_{10}) \left\{ \frac{A_1(q^2)}{m_B - m_{K^*}} - \frac{m_B^2 - q^2}{m_B^2} \frac{A_2(q^2)}{m_B + m_{K^*}} \right\} + \frac{2m_b}{m_B^2} \left\{ \mathcal{T}_2(q^2) - \frac{m_B^2 - q^2}{m_B^2} \mathcal{T}_3(q^2) \right\} \right],$$

(some terms $m_{K^*}^2/m_B^2$ neglected, kinematic normalization factors can be found in [Krüger/Matias])

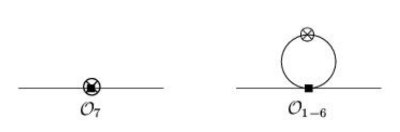
- $C_{9,10}$: short-distance Wilson coefficients for $b \to s\ell\ell$ (to be tested against NP)
- $V, A_{1,2}$: vector/axial-vector $B \to K^*$ transition form factors (sum rules/lattice)
- $\mathcal{T}_i(q^2)$: factorizable and <u>non-factorizable</u> effects from virtual photons, via $\mathcal{O}_{1-8}$ (QCDF/SCET)

factorization

- "naive" factorization:

$\otimes$: photon insertion

$$\langle B | \quad | K^* \gamma^* \rangle$$



$$\mathcal{T}_1(q^2) \simeq C_7^{\rm eff}\, T_1(q^2) + Y(q^2; C_{1-6})\, \frac{q^2}{2m_b}\, \frac{V(q^2)}{m_B + m_{K^*}}\,,$$

$$\mathcal{T}_2(q^2) \simeq C_7^{\rm eff}\, T_2(q^2) + Y(q^2; C_{1-6})\, \frac{q^2}{2m_b}\, \frac{A_1(q^2)}{m_B - m_{K^*}}\,,$$

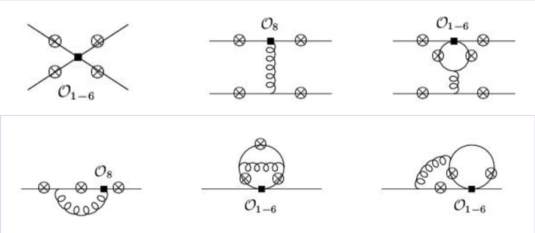
$$\mathcal{T}_3(q^2) \simeq C_7^{\rm eff}\, T_3(q^2) + Y(q^2; C_{1-6}) \left[\frac{m_B - m_{K^*}}{2m_b}\, A_2(q^2) - \frac{m_B + m_{K^*}}{2m_b}\, A_1(q^2) \right],$$

- only additional non-perturbative input: tensor form factors $T_i(q^2)$
- quark-loop function $Y(q^2)$ perturbative for $m_\rho^2 \ll q^2 \ll 4m_c^2$
  sometimes: $C_9 \to C_9^{\rm eff}(q^2) = C_9 + Y(q^2)$

factorization

- include annihilation and QCD corrections: [Beneke/Feldmann/Seidel 01]

$\langle B|$



$|K^*\gamma^*\rangle$

- also involve chromomagnetic penguin operator O_8^g
- leading power in $1/m_b$ can be calculated using QCDF/SCET (except for annihilation in $H_L(q^2)$)
- new non-perturbative input in spectator diagrams: light-cone wave functions

$$\begin{array}{l} \mathcal{T}_1(q^2) \simeq \frac{m_B^2}{m_B^2-q^2}\,\mathcal{T}_2(q^2) \\ \mathcal{T}_3(q^2) - \frac{m_B^2}{m_B^2-q^2}\,\mathcal{T}_2(q^2) \end{array} \simeq \left\{ \begin{array}{l} \xi_\perp(q^2,\mu)\, C_\perp(q^2,\mu) + \phi_B^\pm(\omega,\mu) \otimes \phi_{K^*}^\perp(u,\mu) \otimes T_\perp(\omega,u,\mu) \\ \xi_\parallel(q^2,\mu)\, C_\parallel(q^2,\mu) + \phi_B^\pm(\omega,\mu) \otimes \phi_{K^*}^\parallel(u,\mu) \otimes T_\parallel(\omega,u,\mu) \end{array} \right.$$

factorization

simplifications

- In the limit $m_b \to \infty$ and $q^2 \ll m_b^2$,
  number of independent $B \to K^*$ form factors reduces to 2: [Charles et al. 98]

$$\xi_\perp(q^2,\mu)\,, \quad \xi_\parallel(q^2,\mu)$$ [scale-/scheme-dependent (SCET)]

[→ forward-backward asymmetry zero, ...]

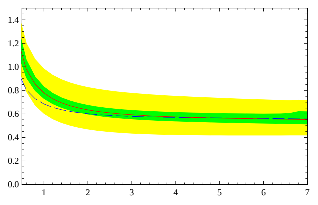
complications

- $1/m_b$ corrections induce new (partly unknown) non-perturbative parameters/functions. [→ isospin asymmetry, ...]
- theoretical working window : $1\ \text{GeV}^2 \lesssim q^2 \lesssim 6\ \text{GeV}^2$
  - dominance of C_7 at low q^2
  - presence of light and heavy vector meson resonances
  - non-factorizable effects in longitudinal rate

[in principle, also the region $4m_c^2 \ll q^2 \leq M_B^2$ can be addressed in HQET⊗ChPT]

rates

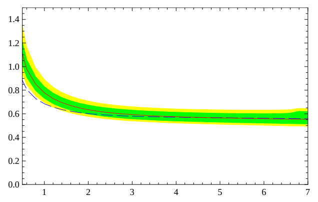
- using input values as in [Beneke/TF/Seidel 04] except for new life times from HFAG.
  (tensor form factor $\to \xi_\perp(0)$ from $B \to K^*\gamma$, all numbers preliminary)
- dashed line: LO approximation (incl. some tree-level $1/m_b$ corrections)
- yellow band: "all" uncertainties
- green band: w/o error on form factors and CKM input.

	$d\mathrm{Br}[B^0 \to K^{*0}\ell\ell]/dq^2$	partially integrated rate
total:		• Major uncertainty from longitudinal FF: ▸ normalize to $A_0(4\,\mathrm{GeV}^2) = 0.66$ (default in [BFS 04]) ▸ WARNING! recent update: $A_0(4\,\mathrm{GeV}^2) = 0.47$ [Ball/Zwicky 04] • theoretically prefered: $1\,\mathrm{GeV}^2 \le q^2 \le 6\,\mathrm{GeV}^2$ • BaBar window:* $0.1\,\mathrm{GeV}^2 \le q^2 \le 7.02\,\mathrm{GeV}^2$

* (just for comparison)

rates

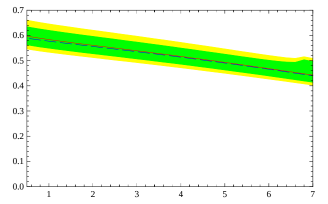
- using input values as in [Beneke/TF/Seidel 04] except for new life times from HFAG.
  (tensor form factor $\to \xi_\perp(0)$ from $B \to K^*\gamma$, all numbers preliminary)
- dashed line: LO approximation (incl. some tree-level $1/m_b$ corrections)
- yellow band: "all" uncertainties, but $A_0(q^2 = 4\ \mathrm{GeV}^2)$ factored out.
- green band: w/o error on form factors and CKM input.

	$d\mathrm{Br}[B^0 \to K^{*0}\ell\ell]/dq^2$	partially integrated rate
total:		$\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} dq^2$: $3.01^{+0.36}_{-0.28} \cdot 10^{-7} \times \left(\frac{A_0(4\ \mathrm{GeV}^2)}{0.66}\right)^2$ $\int_{0.1\ \mathrm{GeV}^2}^{7.02\ \mathrm{GeV}^2} dq^2$: $4.69^{+0.71}_{-0.53} \cdot 10^{-7} \times \left(\frac{A_0(4\ \mathrm{GeV}^2)}{0.66}\right)^2$ BABAR : $(2.6^{+1.1}_{-2.4} \pm 0.6) \cdot 10^{-7}$

* (just for comparison)

rates

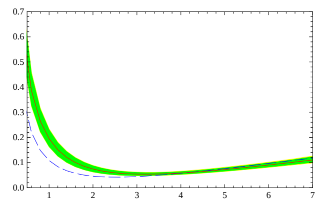
- using input values as in [Beneke/TF/Seidel 04] except for new life times from HFAG.
  (tensor form factor $\to \xi_\perp(0)$ from $B \to K^*\gamma$, all numbers preliminary)
- dashed line: LO approximation (incl. some tree-level $1/m_b$ corrections)
- yellow band: "all" uncertainties, but $A_0(q^2 = 4\ \mathrm{GeV}^2)$ factored out.
- green band: w/o error on form factors and CKM input.

	$d\mathrm{Br}[B^0 \to K^{*0}\ell\ell]/dq^2$	partially integrated rate
long.:		$\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} dq^2 \ : \ 2.63^{+0.28}_{-0.23} \cdot 10^{-7} \times \left(\frac{A_0(4\ \mathrm{GeV}^2)}{0.66}\right)^2$ $\int_{0.1\ \mathrm{GeV}^2}^{7.02\ \mathrm{GeV}^2} dq^2 \ : \ 3.63^{+0.40}_{-0.32} \cdot 10^{-7} \times \left(\frac{A_0(4\ \mathrm{GeV}^2)}{0.66}\right)^2$

* (just for comparison)

rates

- using input values as in [Beneke/TF/Seidel 04] except for new life times from HFAG.
  (tensor form factor $\to \xi_\perp(0)$ from $B \to K^*\gamma$, all numbers preliminary)
- dashed line: LO approximation (incl. some tree-level $1/m_b$ corrections)
- yellow band: "all" uncertainties, but $\xi_\perp(q^2 = 4\ \mathrm{GeV}^2)$ factored out.
- green band: w/o error on form factors and CKM input.

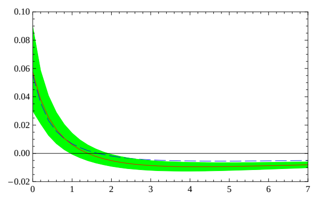
	$d\mathrm{Br}[B^0 \to K^{*0}\ell\ell]/dq^2$	partially integrated rate
trans.:		$\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} dq^2 \ : \ 0.38^{+0.06}_{-0.05} \cdot 10^{-7} \times \left(\frac{\xi_\perp(4\ \mathrm{GeV}^2)}{0.33}\right)^2$ $\int_{0.1\ \mathrm{GeV}^2}^{7.02\ \mathrm{GeV}^2} dq^2 \ : \ 1.05^{+0.18}_{-0.15} \cdot 10^{-7} \times \left(\frac{\xi_\perp(4\ \mathrm{GeV}^2)}{0.33}\right)^2$

*(just for comparison)

isospin asymmetry

- using input values as in [Beneke/TF/Seidel 04] except for new life times from HFAG.
  (tensor form factor $\to \xi_\perp(0)$ from $B \to K^* \gamma$, all numbers preliminary)
- dashed line: LO approximation (incl. some tree-level $1/m_b$ corrections)
- green band: all uncertainties.

$\frac{d\Gamma[B^0]/dq^2 - d\Gamma[B^\pm]/dq^2}{d\Gamma[B^0]/dq^2 + d\Gamma[B^\pm]/dq^2}$	partially integrated



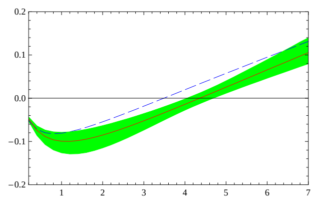
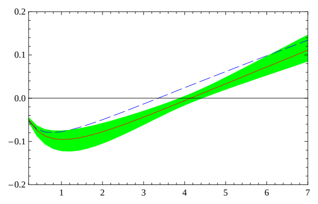
$\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} dq^2$: $0.007^{+0.003}_{-0.003}$

$\int_{0.1\,\mathrm{GeV}^2}^{7.02\,\mathrm{GeV}^2} dq^2$: $0.000^{+0.005}_{-0.006}$

[see also TF/Matias 03]

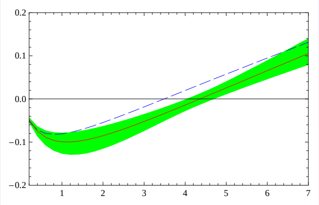
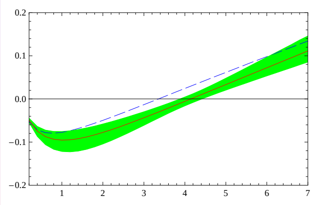
forward-backward asymmetry

- using input values as in [Beneke/TF/Seidel 04]
  (tensor form factor $\to \xi_\perp(0)$ from $B \to K^*\gamma$, all numbers preliminary)
- dashed line: LO approximation (incl. some tree-level $1/m_b$ corrections)
- green band: all parametric uncertainties. (form factor dependence decreases around q_0^2)

	$B^0 \to K^{*0}\ell\ell$	$B^\pm \to K^{*\pm}\ell\ell$
$\frac{(d\Gamma^F - d\Gamma^B)/dq^2}{(d\Gamma^F + d\Gamma^B)/dq^2}$:		
FBA zero q_0^2:	$4.36^{+0.33}_{-0.31}$ GeV^2	$4.15^{+0.27}_{-0.27}$ GeV^2
$\frac{\Gamma^F - \Gamma^B}{\Gamma^F + \Gamma^B}$	$\int_{1\text{ GeV}^2}^{6\text{ GeV}^2} dq^2$: $-0.033^{+0.014}_{-0.016}$ $\int_{0.1\text{ GeV}^2}^{7.02\text{ GeV}^2} dq^2$: $-0.023^{+0.011}_{-0.013}$	$\int_{1\text{ GeV}^2}^{6\text{ GeV}^2} dq^2$: $-0.025^{+0.011}_{-0.013}$ $\int_{0.1\text{ GeV}^2}^{7.02\text{ GeV}^2} dq^2$: $-0.021^{+0.010}_{-0.010}$

forward-backward asymmetry

- using input values as in [Beneke/TF/Seidel 04]
  (tensor form factor $\to \xi_\perp(0)$ from $B \to K^*\gamma$, all numbers preliminary)
- dashed line: LO approximation (incl. some tree-level $1/m_b$ corrections)
- green band: all parametric uncertainties. (form factor dependence decreases around q_0^2)

	$B^0 \to K^{*0}\ell\ell$	$B^\pm \to K^{*\pm}\ell\ell$
$\frac{(d\Gamma^F - d\Gamma^B)/dq^2}{(d\Gamma^F + d\Gamma^B)/dq^2}$:		
FBA zero q_0^2:	$4.36^{+0.33}_{-0.31}$ GeV^2	$4.15^{+0.27}_{-0.27}$ GeV^2
$\frac{\Gamma^F - \Gamma^B}{\Gamma^F + \Gamma^B}$	$\int_{1\,\text{GeV}^2}^{q_0^2} dq^2$: $-0.062^{+0.018}_{-0.023}$ $\int_{q_0^2}^{6\,\text{GeV}^2} dq^2$: $+0.033^{+0.016}_{-0.015}$	$\int_{1\,\text{GeV}^2}^{q_0^2} dq^2$: $-0.058^{+0.016}_{-0.021}$ $\int_{q_0^2}^{6\,\text{GeV}^2} dq^2$: $+0.036^{+0.016}_{-0.013}$

warnings

- systematic uncertainties from (partly) neglected $1/m_b$ corrections!
- how precise are the form factor estimates from sum rules/lattice?

  is it better to extract tensor form factors from experimental data on $B \to K^* \gamma$ or from sum rules/lattice ?
- how reliable are the phenomenological estimates for light-cone wave functions?
- how much do vector meson poles influence the intermediate q^2 region?
- part(!) of perturbative uncertainty could be reduced by resumming logs in SCET.

  [Ali/Kramer/Zhu, hep-ph/0601034]
- ...

new physics

- minimal flavour violation:
  SM operators, but different values for Wilson coefficients. ($\surd$)
- generic flavour violation:
  new operators with different chirality structure and/or new CP phases.
  ($\rightarrow$ formulas to be generalized $\rightarrow$ see backup)

$\Rightarrow$ difficult to establish deviations from SM via (partially integrated) rates!
(form factor uncertainties $\oplus$ non-factorizable effects)

$\Rightarrow$ concentrate on asymmetries (forward-backward, isospin, CP, angular)

other modes (briefly)

- $B \to K\ell^+\ell^-$:
  - $R_K = \Gamma[B \to K\mu\mu]/\Gamma[B \to Kee]$ [Bobeth/Hiller/Piranishvili, 0709.4174]
- $B_s \to \mu^+\mu^-$:
  - enhanced by large powers of $\tan\beta$ in type-II 2HDM
    [see previous talks; also Nierste arXiv:0807.3733]
- $B_s \to \phi\ell^+\ell^-$:
  - NP (model-independent) $\oplus$ naive factorization [Yilmaz, 0806.0269]
  - numerical analysis incl. non-factorizable effects [Beneke/TF/Seidel, in preparation]
- ...
- $B \to \rho\ell^+\ell^-$: [Beneke/TF/Seidel, 04]
  - different CKM hierarchy $\to$ potentially larger isospin and CP effects
  - useful cross-check to test factorization approach

- What else should appear in the write-up?

$\longrightarrow$ open for discussion ...

Right-handed operators $\mathcal{O}'_{7,9,10}$ [according to Krüger/Matias]

$$A_{\perp\,L,R} \propto \left[(C_9 \mp C_{10}) \, \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} \, \mathcal{T}_1(q^2) \right] , \qquad C_{7,9,10} \to C_{7,9,10} + C'_{7,9,10}$$

$$A_{\parallel\,L,R} \propto \left[(C_9 \mp C_{10}) \, \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2} \, \mathcal{T}_2(q^2) \right] , \qquad C_{7,9,10} \to C_{7,9,10} - C'_{7,9,10}$$

$$A_{0\,L,R} \propto \left[(C_9 \mp C_{10}) \left\{ \frac{A_1(q^2)}{m_B - m_{K^*}} - \frac{m_B^2 - q^2}{m_B^2} \, \frac{A_2(q^2)}{m_B + m_{K^*}} \right\} + \frac{2m_b}{m_B^2} \left\{ \mathcal{T}_2(q^2) - \frac{m_B^2 - q^2}{m_B^2} \, \mathcal{T}_3(q^2) \right\} \right] , \qquad C_{7,9,10} \to C_{7,9,10} - C'_{7,9,10}$$

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$$A_{\parallel\, L,R} \propto \left[(C_9 \mp C_{10}) \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2} \mathcal{T}_2(q^2) \right], \qquad C_{7,9,10} \to C_{7,9,10} - C'_{7,9,10}$$

$$A_{0\, L,R} \propto \left[(C_9 \mp C_{10}) \left\{ \frac{A_1(q^2)}{m_B - m_{K^*}} - \frac{m_B^2 - q^2}{m_B^2} \frac{A_2(q^2)}{m_B + m_{K^*}} \right\} + \frac{2m_b}{m_B^2} \left\{ \mathcal{T}_2(q^2) - \frac{m_B^2 - q^2}{m_B^2} \mathcal{T}_3(q^2) \right\} \right], \qquad C_{7,9,10} \to C_{7,9,10} - C'_{7,9,10}$$

- $C_{9,10}{}'$: short-distance Wilson coefficients for $b \to s_{L/R} \ell\ell$
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