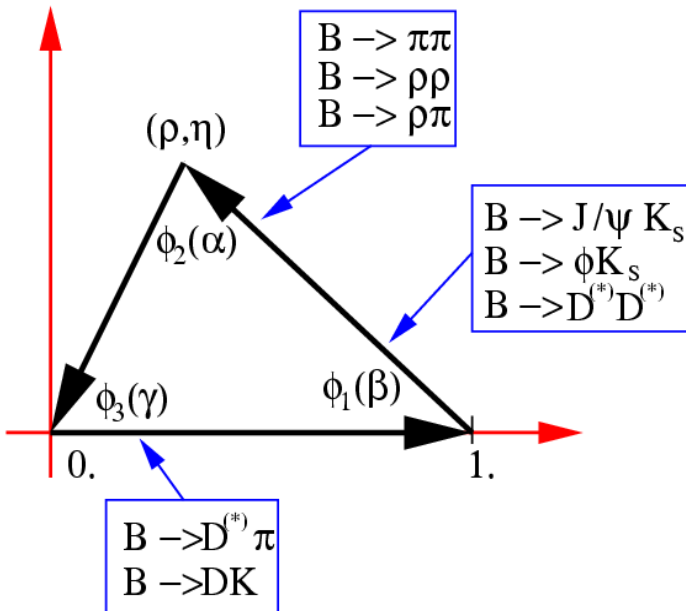
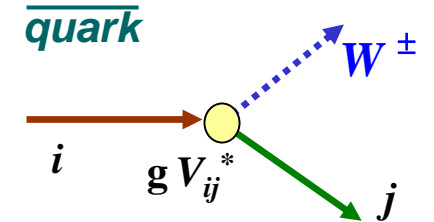
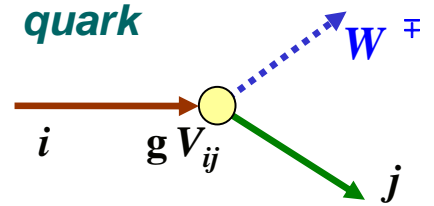

Measurements of φ_3/γ at *Belle*

Alex BONDAR
Budker Institute of Nuclear Physics
Novosibirsk, Russia

CKM matrix and ϕ_3/γ

Cabibbo-Kobayashi-Maskawa quark mixing matrix (CKM):

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \sim e^{i\phi_3}$$



CKM unitarity leads to triangle in complex plane:

$$\{i=1, j=3\}: V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\Rightarrow \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

Costraints on CKM parameters

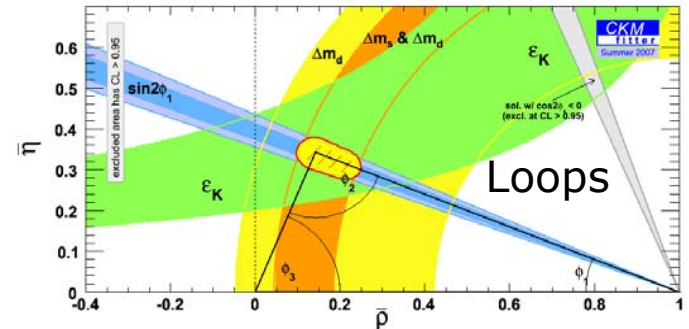
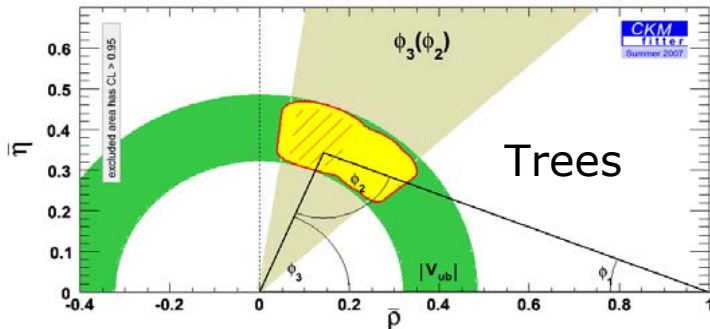
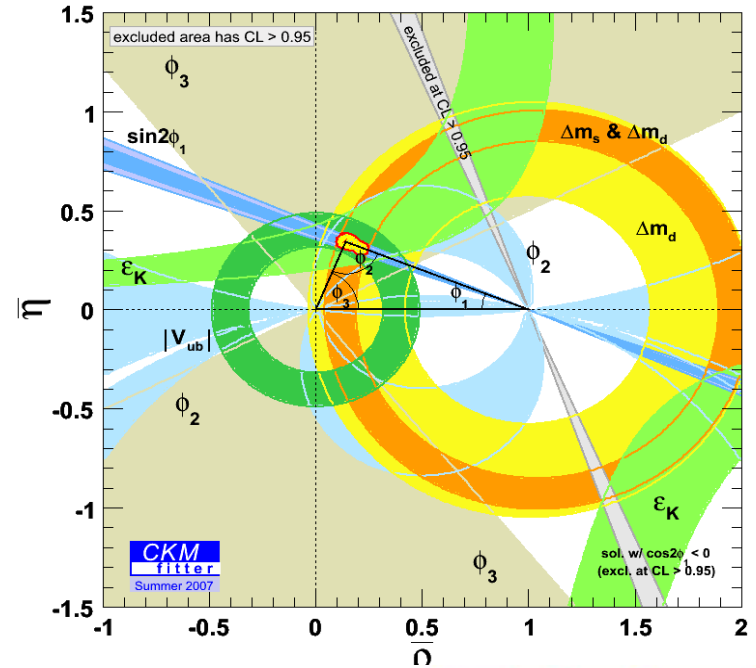
Direct angle measurements

[CKMfitter world averages, 2007]:

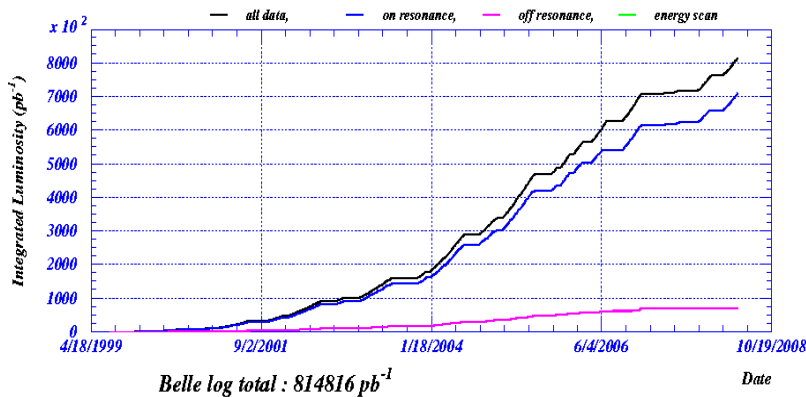
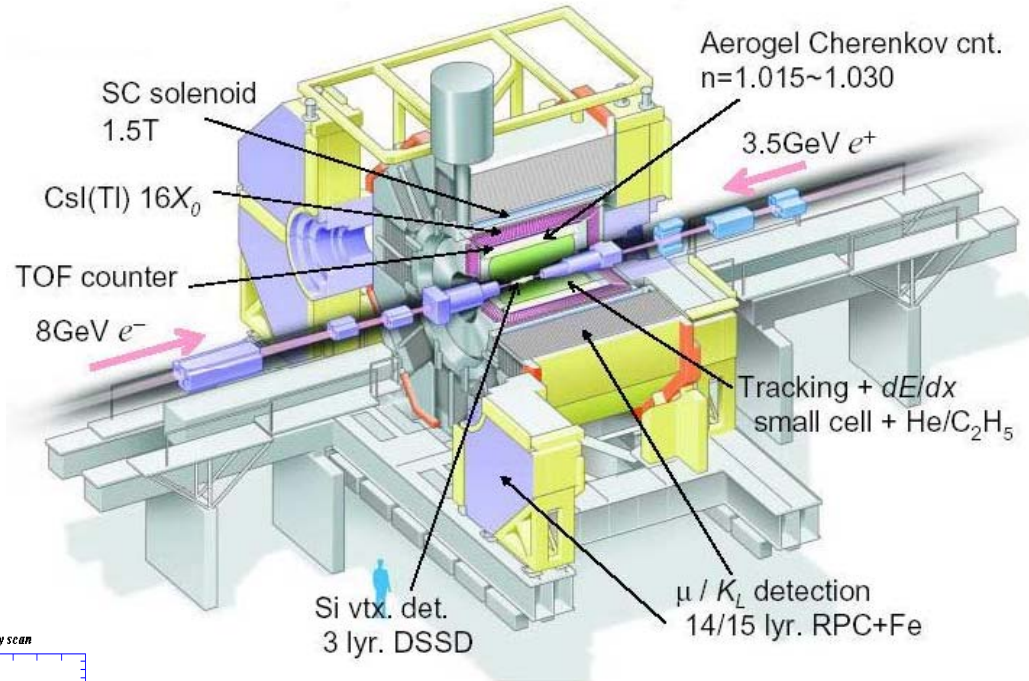
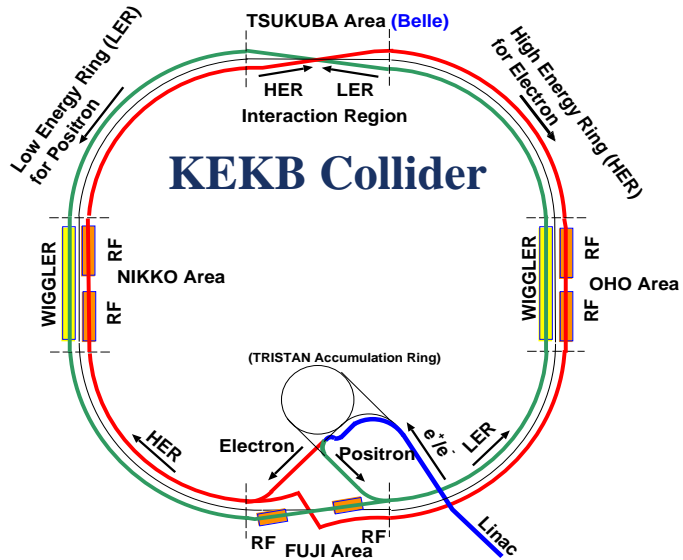
- $\varphi_1/\beta = 21.5 \pm 1.0^\circ$ ($B \rightarrow J/\psi K^0$)
- $\varphi_2/\alpha = 88 \pm 6^\circ$ ($B \rightarrow \rho\rho$)
- $\varphi_3/\gamma = 77 \pm 30^\circ$ ($B \rightarrow DK$)

[BaBar (SLAC) , Belle (KEK)]

Good agreement so far.



KEKB and Belle detector



3.5 GeV e^+ & 8 GeV e^- beams

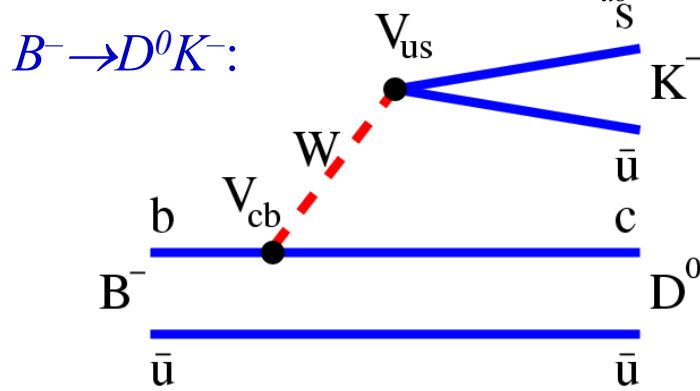
3 km circumference, 11 mrad crossing angle

$L = 1.7 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ (world record)

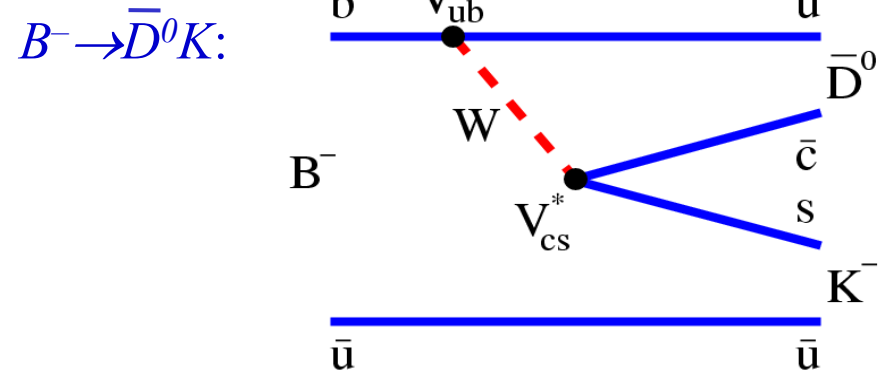
$\int L dt \sim 820 \text{ fb}^{-1}$ @ $Y(4S)$ +off($\sim 10\%$)

$B^+ \rightarrow D^0 K^+$ decay

CP violation enters the Standard Model as a complex phase \Rightarrow only observable in the interference. To measure φ_3/γ , use interference on V_{ub} and V_{cb} amplitudes:



$$A_1 \sim V_{cb} V_{us}^* \sim A \lambda^3$$



$$A_2 \sim V_{ub} V_{cs}^* \sim A \lambda^3 (\rho - i\eta) \sim e^{-i\varphi_3}$$

Amplitudes interfere if D^0 and \bar{D}^0 decay into the same final state $|D^0\rangle + re^{i\theta} |\bar{D}^0\rangle$

Relative phase: $\theta = -\varphi_3 + \delta$ ($B^- \rightarrow DK^-$), $\theta = \varphi_3 + \delta$ ($B^+ \rightarrow DK^+$)

includes weak (φ_3/γ) and strong (δ) phase.

Magnitude of CP violation is determined by the ratio of the two amplitudes:

$$r_B = \left| \frac{A(B^- \rightarrow D^0 K^-)}{A(B^- \rightarrow \bar{D}^0 K^-)} \right| = \left| \frac{V_{ub}^* V_{cs}}{V_{cb}^* V_{us}} \right| \times [\text{color supp.}]$$

Methods of φ_3/γ measurement

- Based on $B \rightarrow DK$. Different D^0 modes can be used:
 - GLW (CP eigenstates: $D^0 \rightarrow \pi\pi, KK, K\phi, K\omega$)
Belle: 275M BB pairs ($B \rightarrow D_{CP}K, B \rightarrow D_{CP}^*K$: [PRD 73 051106 \(2006\)](#))
BaBar: 382M BB pairs ($B \rightarrow D_{CP}K$: [arXiv:0802.4052](#),
 $B \rightarrow D_{CP}^*K$: [arXiv:0807.2408](#), $B \rightarrow D_{CP}K^*$: [PRD 72 071103 \(2005\)](#))
 - ADS (CF and DCS states: $D^0 \rightarrow K\pi, K\pi\pi$)
Belle: 657M BB pairs ($B \rightarrow DK$: [arXiv:0804.2063](#))
BaBar: 232M BB pairs ($B \rightarrow DK, B \rightarrow D^*K, B \rightarrow DK^*$ with $D \rightarrow K\pi$: [PRD 72 032004 \(2005\)](#))
 - Dalitz (multibody states: $D^0 \rightarrow K\pi\pi, KKK, \pi\pi\pi$)
Belle: 657M BB pairs ($B \rightarrow DK, B \rightarrow D^*K$ with $D^* \rightarrow D\pi^0, D^0 \rightarrow K_S\pi^+\pi^-$: [arXiv:0803.3375](#))
BaBar: 382M BB pairs ($B \rightarrow DK, B \rightarrow D^*K$ with $D^* \rightarrow D\pi^0, D\gamma,$
 $D^0 \rightarrow K_S\pi^+\pi^-, K_S K^+ K^-$: [arXiv:0804.2089](#))
371M BB pairs ($B^0 \rightarrow D^0 K^{*0}$: [Moriond EW 2008](#))
- Based on B^0 decays (measurement of $2\varphi_1 + \varphi_3$)
 - Belle: 386M BB pairs ($B \rightarrow D^*\pi$ partial, $B \rightarrow D^{(*)}\pi$ full rec.: [PRD 73 092003 \(2006\)](#))
 - BaBar: 232M BB pairs ($B \rightarrow D^*\pi$ partial: [PRD 71 112003 \(2005\)](#),
 $B \rightarrow D^{(*)}\pi, D\rho$ full rec.: [PRD 73 111101 \(2006\)](#))
 - 347M BB pairs ($B \rightarrow DK\pi$ time-dependent Dalitz: [arXiv:0712.3469](#))

GLW analysis

M. Gronau, D. London, D. Wyler PLB **253**, 483 (1991); PLB **265**, 172 (1991)

CP eigenstate of D -meson is used (D_{CP}).

CP-even: $D_1 \rightarrow K^+ K^-, \pi^+ \pi^-,$ CP-odd: $D_2 \rightarrow K_S \pi^0, K_S \omega, K_S \phi, K_S \eta \dots$

Double ratio:

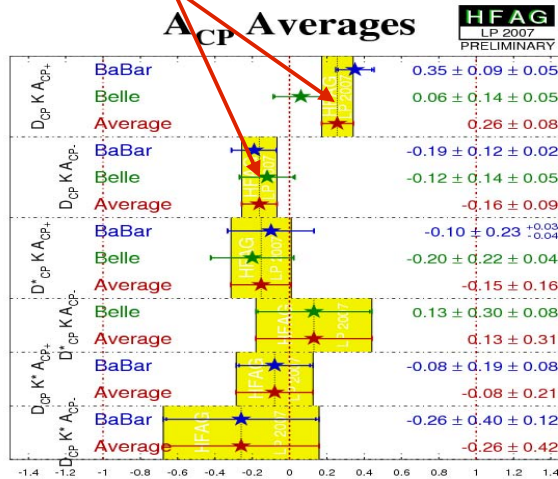
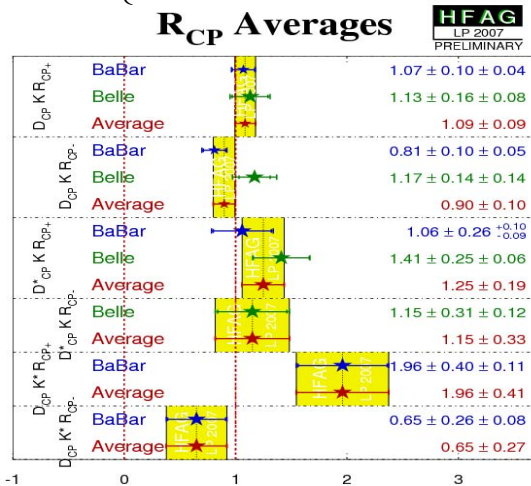
$$R_{1,2} = \frac{Br(B \rightarrow D_{1,2} K) / Br(B \rightarrow D_{1,2} \pi)}{Br(B \rightarrow D^0 K) / Br(B \rightarrow D^0 \pi)} = 1 + r_B^2 + 2r_B \cos \delta' \cos \varphi_3$$

CP-asymmetry:

$$A_{1,2} = \frac{Br(B^+ \rightarrow D_{1,2} K^+) - Br(B^- \rightarrow D_{1,2} K^-)}{Br(B^+ \rightarrow D_{1,2} K^+) + Br(B^- \rightarrow D_{1,2} K^-)} = \frac{2r_B \sin \delta' \sin \varphi_3}{R_{1,2}}$$

$$\delta' = \begin{cases} \delta & \text{for } D_1 \\ \delta + \pi & \text{for } D_2 \end{cases}$$

$A_{1,2}$ have opposite signs



Alternative set of variables:

$$x_{\pm} = r_B \cos(\delta \pm \varphi_3) = \frac{R_1(1 \mp A_1) - R_2(1 \mp A_2)}{4}$$

$$r_B^2 = \frac{R_1 + R_2 - 2}{2}$$

GLW analysis

Belle collaboration, 275M BB pairs [PRD **73**, 051106 (2006)]

$B \rightarrow DK$, $B \rightarrow D^*K$ modes ($D^* \rightarrow D\pi^0$ only)

- Cut on m_{ES} , PID, event shape variables
- ΔE fit to extract yields

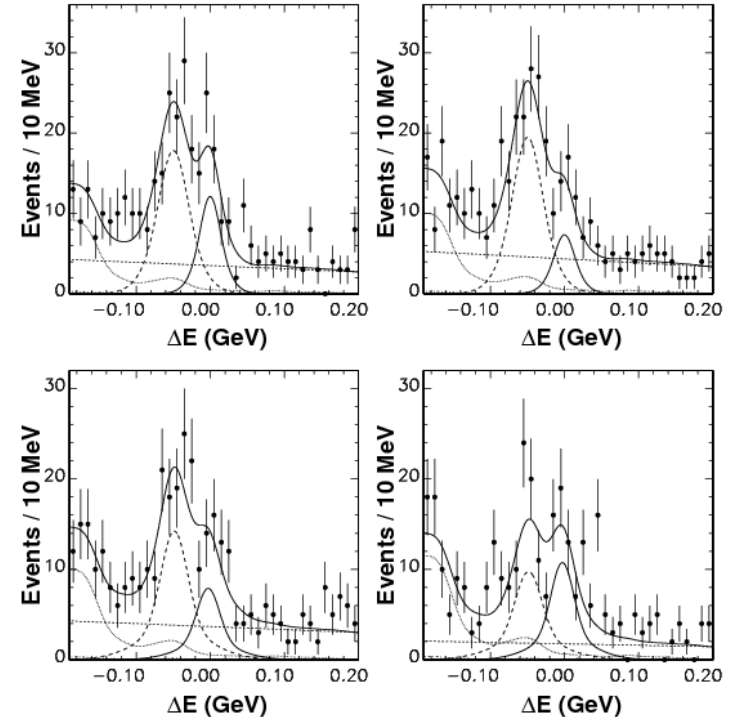
A_1	$+0.06 \pm 0.14 \pm 0.05$
A_2	$-0.12 \pm 0.14 \pm 0.05$
R_1	$1.13 \pm 0.16 \pm 0.08$
R_2	$1.17 \pm 0.14 \pm 0.14$

A_1^*	$-0.20 \pm 0.22 \pm 0.04$
A_2^*	$+0.13 \pm 0.30 \pm 0.08$
R_1^*	$1.41 \pm 0.25 \pm 0.06$
R_2^*	$1.15 \pm 0.31 \pm 0.12$

The same result expressed in Cartesian variables:

x_+	$-0.06 \pm 0.08 \pm 0.05$
x_-	$+0.04 \pm 0.08 \pm 0.04$
r^2	$0.15 \pm 0.11 \pm 0.08$

x_+^*	$+0.17 \pm 0.15 \pm 0.04$
x_-^*	$-0.04 \pm 0.15 \pm 0.04$
r^{*2}	$0.28 \pm 0.20 \pm 0.07$

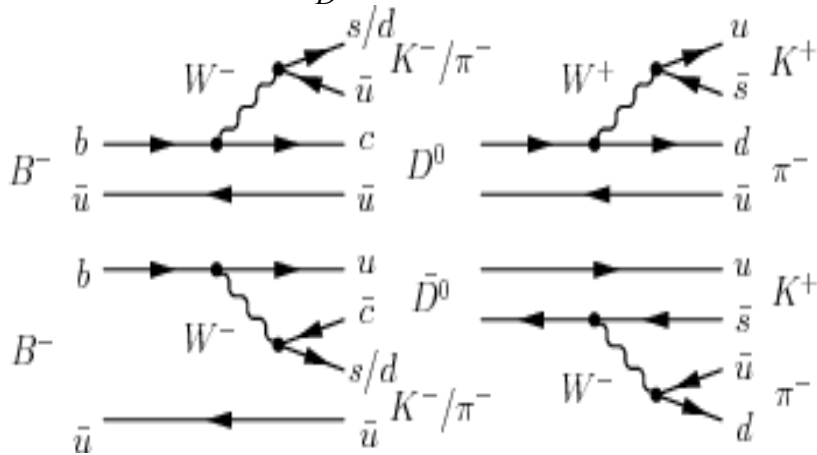


ADS analysis

D. Atwood, I. Dunietz and A. Soni, PRL **78**, 3357 (1997)

Enhance magnitude of CP violation by using Doubly Cabibbo-suppressed D decays

e.g. $B^- \rightarrow [K^+\pi^-]_D K^-$:

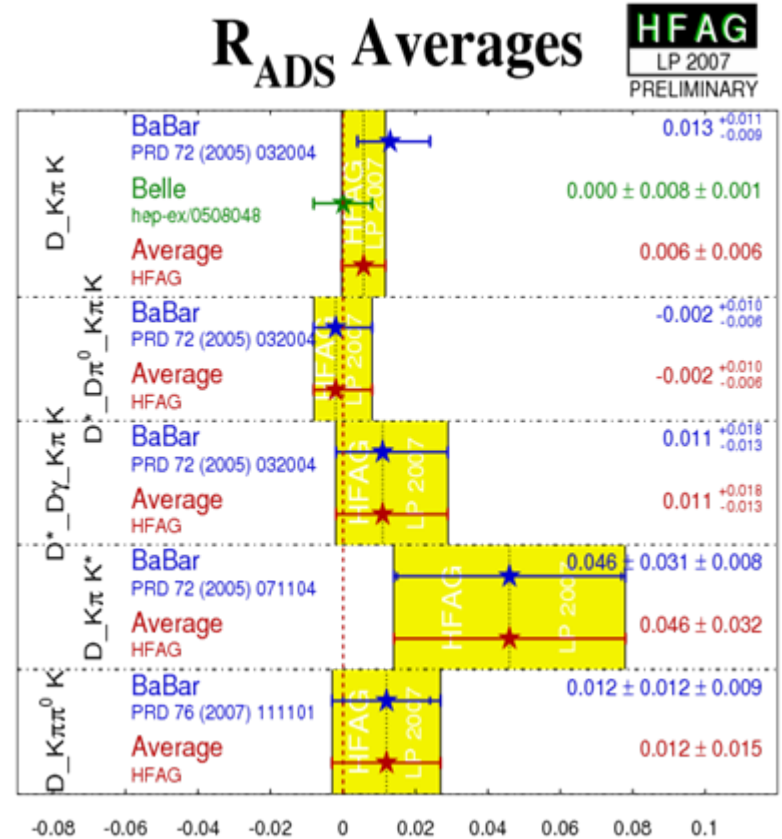


Observable:

$$R_{ADS} = \frac{Br(B \rightarrow D_{\text{supp}} K)}{Br(B \rightarrow D_{\text{fav}} K)} = r_B^2 + r_D^2 + 2r_B r_D \cos \varphi_3 \cos \delta$$

where

$$\delta = \delta_B + \delta_D, \quad r_D = \left| \frac{\Lambda(D^0 \rightarrow K^- \pi^+)}{\Lambda(D^0 \rightarrow K^+ \pi^-)} \right| = 0.0578 \pm 0.0008$$



ADS analysis

Belle collaboration, 657M BB pairs [[arXiv: 0804:2063, submitted to PRD\(RC\)](#)]
 $B^- \rightarrow [K^- \pi^+]_D K^-$ (suppressed) and $B^- \rightarrow [K^+ \pi^-]_D K^-$ (favored) modes are selected.

- Cut on M_{bc} , M_D , PID likelihood, event shape
- Fit ΔE to extract signal yield

$$R_{ADS} = (8.0^{+6.3}_{-5.7} \text{ } ^{+2.0}_{-2.8}) \times 10^{-3}$$

CP asymmetry:

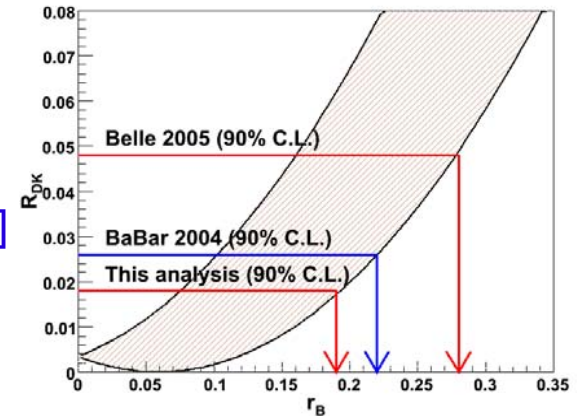
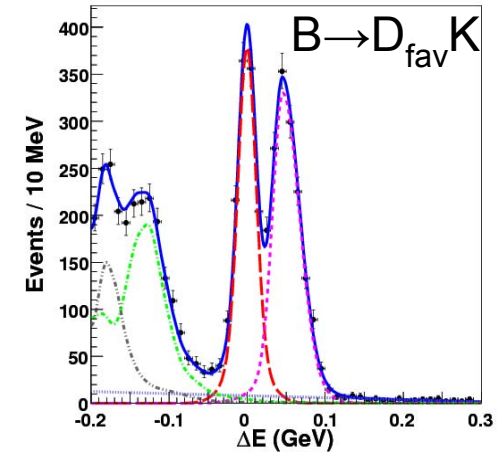
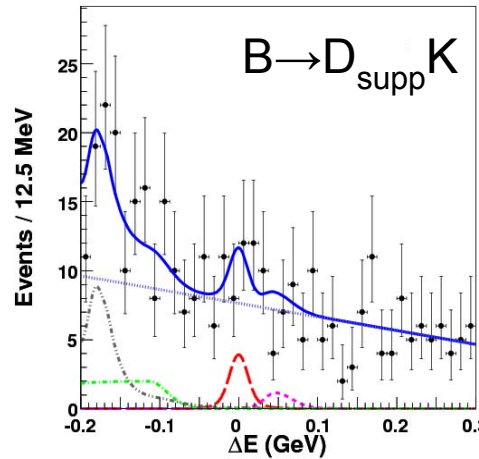
$$A_{ADS} = -0.13^{+0.98}_{-0.88} \pm 0.26$$

$$r_B < 0.19 \text{ at } 90\% \text{ CL}$$

(with the conservative assumption $\cos \varphi_3 \cos \delta = -1$)

Using CLEO measurement $\delta_D = (22^{+11}_{-12} \text{ } ^{+9}_{-11})^\circ$ [[arXiv: 0802:2268](#)]

and φ_3, δ_B measurements from Dalitz analysis, tighter r_B constraint can be obtained.



Dalitz analysis method

A. Giri, Yu. Grossman, A. Soffer, J. Zupan, PRD **68**, 054018 (2003)

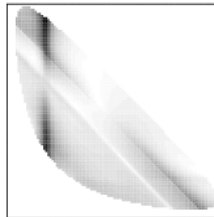
A. Bondar, Proc. of Belle Dalitz analysis meeting, 24-26 Sep 2002.

$$|D^0\rangle + re^{i\theta} |\bar{D}^0\rangle$$

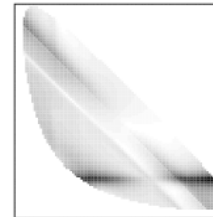
Using 3-body final state, identical for D^0 and \bar{D}^0 : $K_S\pi^+\pi^-$.

Dalitz distribution density: $dp(m_{K_S\pi^+}^2, m_{K_S\pi^-}^2) \sim |f_D|^2 dm_{K_S\pi^+}^2 dm_{K_S\pi^-}^2$

$$f_D(m_{K_S\pi^+}^2, m_{K_S\pi^-}^2) =$$



$$+ re^{i\delta \pm i\phi_3}$$



(assuming CP-conservation in D^0 decays)

If $f_D(m_{K_S\pi^+}^2, m_{K_S\pi^-}^2)$ is known, parameters $(\varphi_3/\gamma, r_B, \delta)$ are obtained from the fit to Dalitz distributions of $D \rightarrow K_S\pi^+\pi^-$ from $B^\pm \rightarrow DK^\pm$ decays

Need to know a complex form of the D^0 decay amplitude, but only $|f_D|^2$ is obtained from $D^* \rightarrow D\pi$

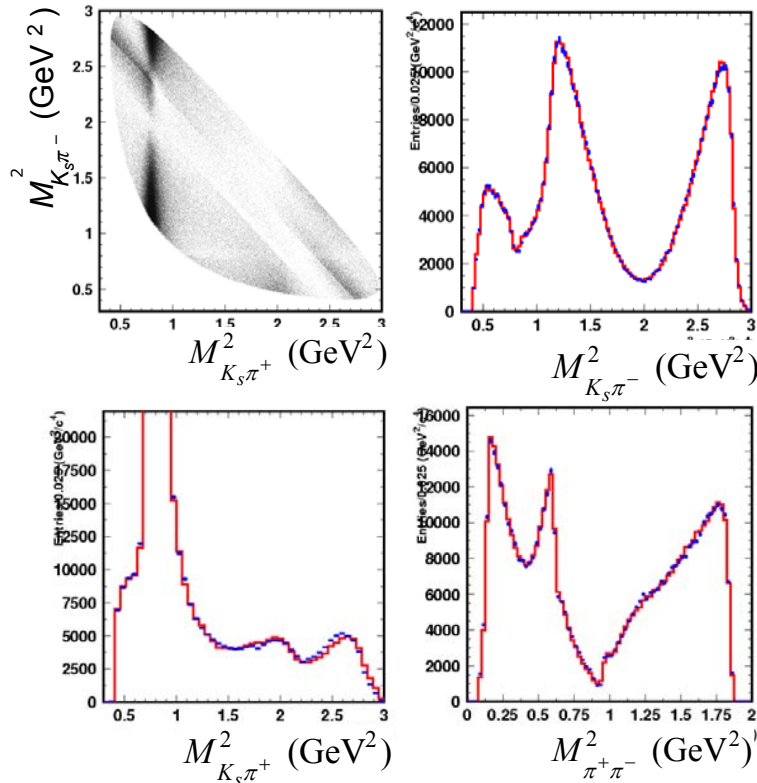
\Rightarrow Need to use model description, model uncertainty as a result

Dalitz: $K_S\pi^+\pi^-$ amplitude

Belle collaboration, 657M BB pairs [arXiv: 0803.3375]

[preliminary]

Isobar model is used as a baseline. K -matrix for systematics test.



Intermediate state	Amplitude	Phase, °
$K_S \sigma_1$	1.56 ± 0.06	214 ± 3
$K_S \rho(770)$	1 (fixed)	0 (fixed)
$K_S \omega$	0.0343 ± 0.0008	112.0 ± 1.3
$K_S f_0(980)$	0.385 ± 0.006	207.3 ± 2.3
$K_S \sigma_2$	0.20 ± 0.02	212 ± 12
$K_S f_2(1270)$	1.44 ± 0.04	342.9 ± 1.7
$K_S f_0(1370)$	1.56 ± 0.12	110 ± 4
$K_S \rho(1450)$	0.49 ± 0.08	64 ± 11
$K^*(892)^+\pi^-$	1.638 ± 0.010	133.2 ± 0.4
$K^*(892)^-\pi^+$	0.149 ± 0.004	325.4 ± 1.3
$K^*(1410)^+\pi^-$	0.65 ± 0.05	120 ± 4
$K^*(1410)^-\pi^+$	0.42 ± 0.04	253 ± 5
$K^*_0(1430)^+\pi^-$	2.21 ± 0.04	358.9 ± 1.1
$K^*_0(1430)^-\pi^+$	0.36 ± 0.03	87 ± 4
$K^*_2(1430)^+\pi^-$	0.89 ± 0.03	314.8 ± 1.1
$K^*_2(1430)^-\pi^+$	0.23 ± 0.02	275 ± 6
$K^*(1680)^+\pi^-$	0.88 ± 0.27	82 ± 17
$K^*(1680)^-\pi^+$	2.1 ± 0.2	130 ± 6
Nonresonant	2.7 ± 0.3	160 ± 5

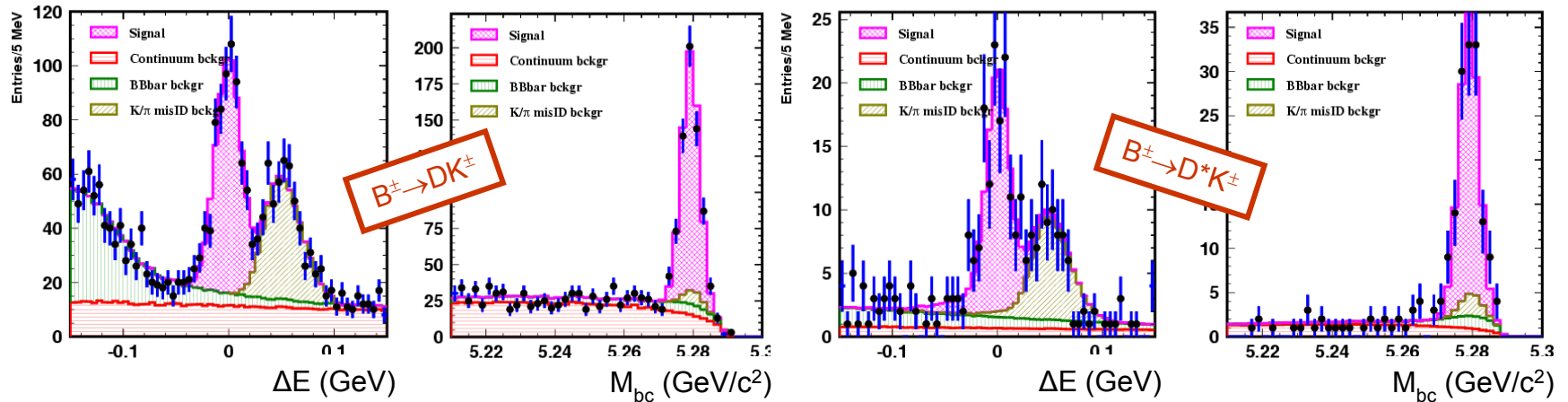
$\sigma_1(M=522 \pm 6 \text{ MeV}, \Gamma=453 \pm 10 \text{ MeV})$

$\sigma_2(M=1033 \pm 7 \text{ MeV}, \Gamma=88 \pm 7 \text{ MeV})$

Dalitz: signal selection

Belle collaboration, 657M BB pairs [arXiv: 0803:3375]

[preliminary]



- $|\Delta E| < 30$ MeV
- $M_{bc} > 5.27$ GeV/c²
- Continuum rejection variables $\cos\theta_{thr}$, “virtual calorimeter” Fisher discriminant: $|\cos\theta_{thr}| < 0.8$, $\mathcal{F} > -0.7$ in $(M_{bc}, \Delta E)$ fit to determine background composition.
- $|M_{ks\pi\pi} - M_D| < 11$ MeV/c²
- $144.9 < \Delta M < 145.9$ MeV/c² ($B \rightarrow D^* K$ only)

Whole range is used in Dalitz fit, included into likelihood.

756 events, 29% background ($B \rightarrow DK$).
 149 events, 20% background ($B \rightarrow D^* K, D^* \rightarrow D\pi^0$).

In “clean” signal region
 ($|\cos\theta_{thr}| < 0.8, \mathcal{F} > -0.7$)

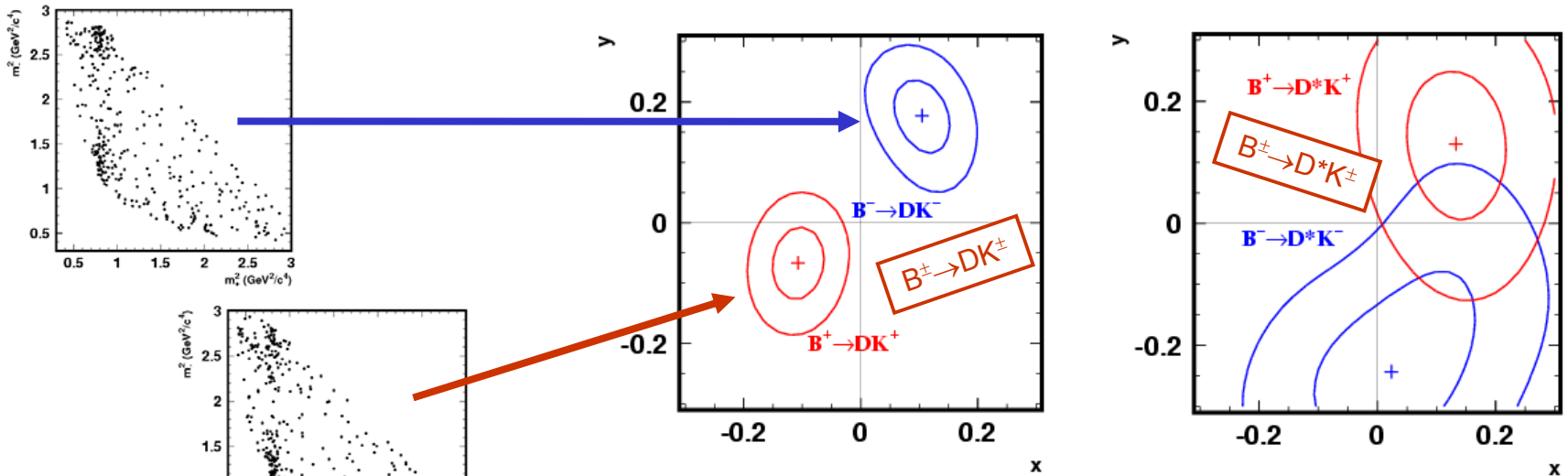
Dalitz: fit results

[preliminary]

Fit parameters are $x_{\pm} = r_B \cos(\pm\varphi_3 + \delta)$ and $y_{\pm} = r_B \sin(\pm\varphi_3 + \delta)$

Unbinned maximum likelihood fit with event-by-event background treatment

(ΔE , M_{bc} , $|\cos\theta_{thr}|$, F included into likelihood)



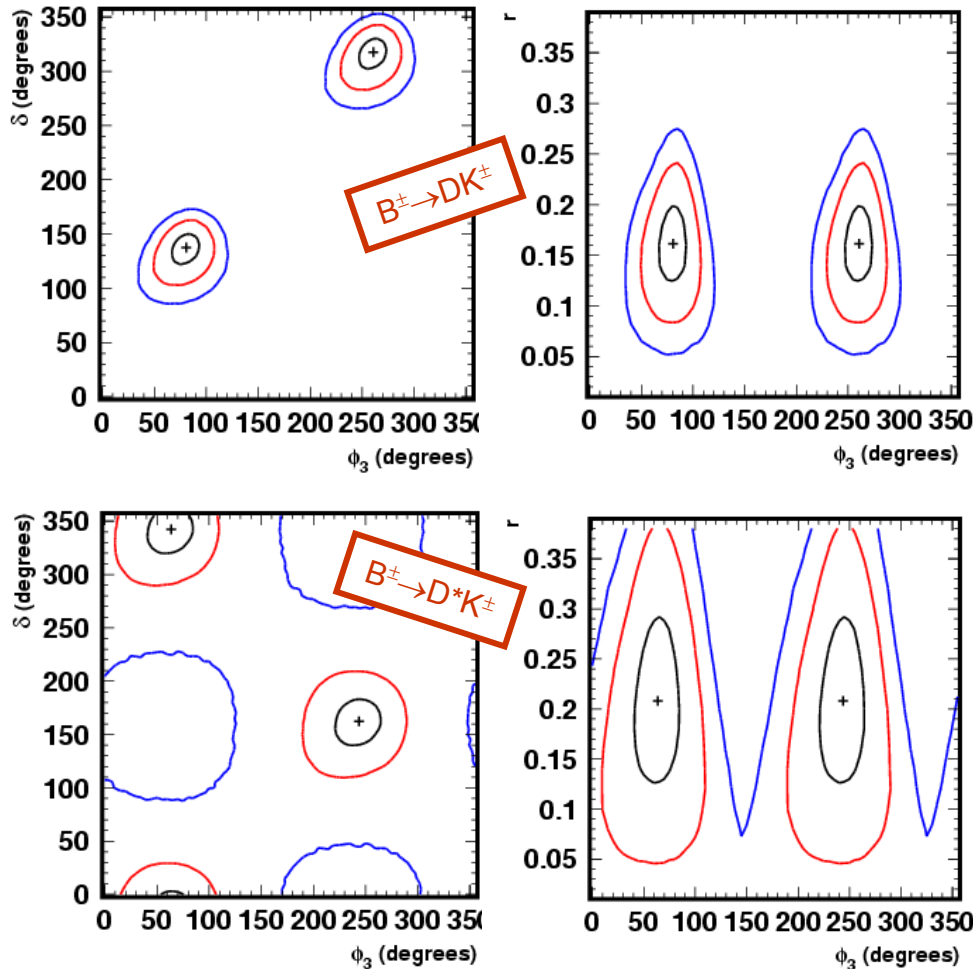
$x_- = +0.105 \pm 0.047 \pm 0.011$
 $y_- = +0.177 \pm 0.060 \pm 0.018$
 $x_+ = -0.107 \pm 0.043 \pm 0.011$
 $y_+ = -0.067 \pm 0.059 \pm 0.018$

$x_- = +0.024 \pm 0.140 \pm 0.018$
 $y_- = -0.243 \pm 0.137 \pm 0.022$
 $x_+ = +0.133 \pm 0.083 \pm 0.018$
 $y_+ = +0.130 \pm 0.120 \pm 0.022$

Errors are statistical and experimental systematic. Model error not included.

Dalitz: fit results

[preliminary]



$B^\pm \rightarrow DK^\pm$ only:

$$\varphi_3 = 81_{-15}^{+13} \pm 5^\circ (\text{syst}) \pm 9^\circ (\text{model})$$

$B^\pm \rightarrow D^*K^\pm$ only:

$$\varphi_3 = 64_{-23}^{+21} \pm 4^\circ (\text{syst}) \pm 9^\circ (\text{model})$$

$B^\pm \rightarrow DK^\pm, B^\pm \rightarrow D^*K^\pm$ combined:

$$\varphi_3 = 76_{-13}^{+12} \pm 4^\circ (\text{syst}) \pm 9^\circ (\text{model})$$

$$r_{DK} = 0.16 \pm 0.04 \pm 0.01(\text{syst}) \pm 0.05(\text{model})$$

$$r_{D^*K} = 0.21 \pm 0.08 \pm 0.01(\text{syst}) \pm 0.05(\text{model})$$

$$\delta_{DK} = 136_{-16}^{+14} \pm 4^\circ (\text{syst}) \pm 23^\circ (\text{model})$$

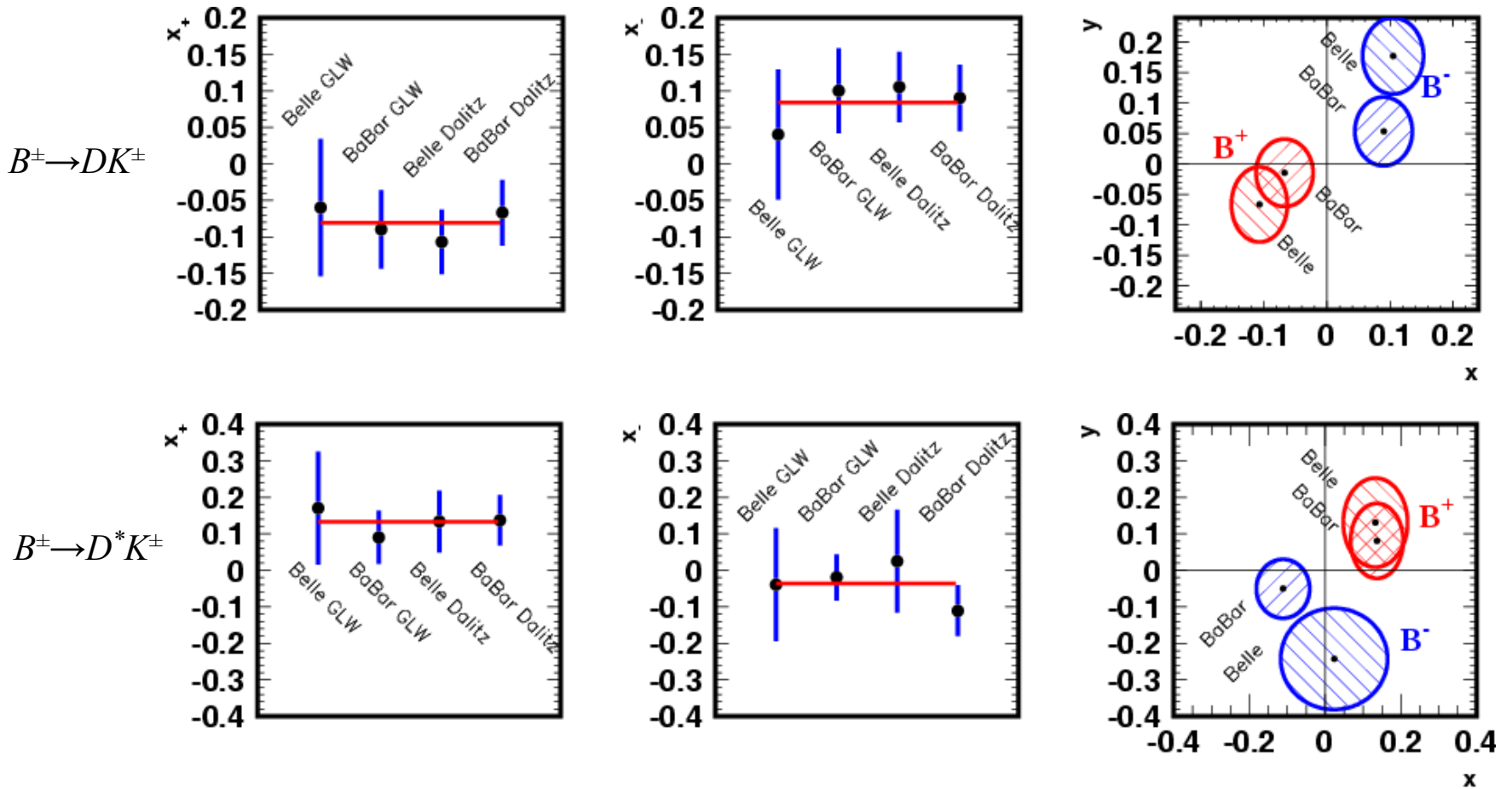
$$\delta_{D^*K} = 343_{-22}^{+20} \pm 4^\circ (\text{syst}) \pm 23^\circ (\text{model})$$

Model error estimate is the same as in previous (385M BB) analysis.

Stat. CL of CPV: $(1-5.5 \cdot 10^{-4})$ or 3.5σ !

Comparison of measurements

GLW and Dalitz methods in Cartesian variables (x,y):



Summary

- Many new measurements related to φ_3/γ appeared in 2008:
 - BaBar GLW, Belle ADS and $D^*\pi$ partial rec. updates,
 - Belle Dalitz update with $D^0 \rightarrow K_S \pi^+ \pi^-$
 - BaBar Dalitz update with $D^0 \rightarrow K_S \pi^+ \pi^-$ and new $D^0 \rightarrow K_S K^+ K^-$.
- Combining all B-factory results, there is strong evidence of CP violation in $B \rightarrow DK$.
Good agreement between different measurements, both in r_B and γ/φ_3
- r_B is shown to be significantly non-zero.
Can predict future sensitivity with confidence.
- φ_3/γ statistical precision is now comparable to D^0 model uncertainty.
Model-independent Dalitz analysis using charm data (CLEO) will allow to obtain a more reliable result.

Dalitz: systematic errors

Systematic errors for (x,y) in units of 10^{-2}

	B→DK		B→D*K	
	x	y	x	y
Signal shape	0.66	1.16	0.89	1.22
Continuum background shape	0.34	1.10	0.94	1.63
bb background shape	0.13	0.72	0.77	0.28
Background fractions	0.17	0.29	0.52	0.80
Efficiency shape	0.51	0.15	0.86	0.21
Total	1.08	1.78	1.81	2.22

Model uncertainty calculated in terms of (φ_3, r_B, δ):

$$\Delta\varphi_3 = 9^\circ$$

$$\Delta r_B = 0.05$$

$$\Delta\delta = 23^\circ$$

Comparison of measurements

“Naïve” world averages:

χ^2 fit using GLW, ADS and Dalitz observables (assuming Gaussian errors).

CL in a number of σ calculated as

$$CL = \sqrt{-2 \ln \frac{pdf}{pdf_{\max}}}$$

Systematic error is *not* included
Model error is *not*.

$r_B < 0.19$ (90% CL) – ADS

$r_B < 0.13$ (90% CL) – ADS with fixed $\varphi_3, \delta_B, \delta_D$

$r_B = 0.118^{+0.025}_{-0.022}$ – All Dalitz

$r_B = 0.103^{+0.018}_{-0.015}$ – All measurements

$\gamma / \varphi_3 = (80 \pm 13)^\circ$ – All DK

$\gamma / \varphi_3 = (81^{+9}_{-11})^\circ$ – All DK + D^*K

