CKM-angle γ from charged B decays at BABAR

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for the BABAR Collaboration,

CKM 2008, Roma (Italia), Sept. 9-13.

$(\bar{\rho}, \bar{\eta})$

α

$$\gamma \equiv arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

β

$(0, 0)$ $(0, 1)$

Why is it difficult to measure γ ?

See T. Gershon's talk

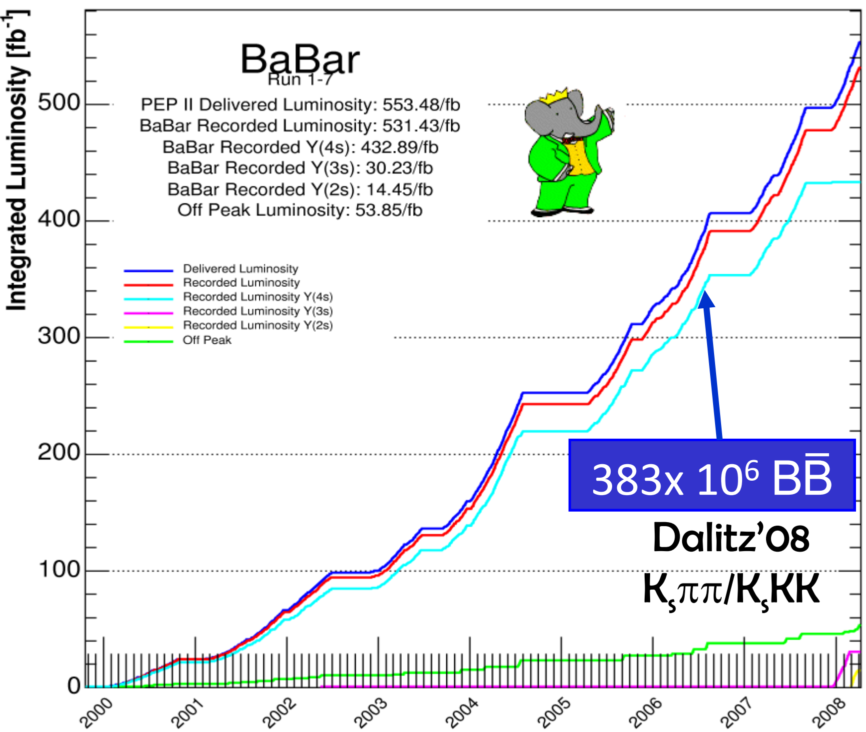
➔ Measurement of γ using direct CP violation (interference [$b \to c \Leftrightarrow b \to u$]):

- 3 various $B^- \to \tilde{D}^{(*)0} K^{(*)-}$ charged decays (no time dependence): DK, D^*K, and DK^*.
- Size of CPV is limited by the size of $|A_{ub}/A_{cb}|$ amplitudes ratio: 3 $r^{(*)}_{(s)B}$ nuisance parameters (~5-30% ?).

➔ 3 methods that need a lot of B mesons:

Same $\tilde{D}^0 \equiv [D^0/\bar{D}^0]$ final state

- GLW: $\tilde{D} \equiv$CP-eigenstate: many modes, but small asymmetry.
- ADS: $\tilde{D} \equiv$DCS: large asymmetry, but very few events.
- GGSZ: $\tilde{D} \equiv$Dalitz: better than a mixture of ADS+GLW $\Rightarrow$ large asymmetry in some regions, but strong phases varying other the Dalitz plane.



➔ As of ICHEP'08 $\gamma[D^{(*)0}K^{(*)-}] = (67^{+32}_{-25})^\circ$ CKMFitter WA

$\Rightarrow$ much less precise than α & β CKM angles.

➔ Not all the statistics used by BABAR for the results shown here only ~80% (excludes final Run6@2007 $\Upsilon(4S)$ dataset → $N_{tot.}$~465x10⁶ $B\bar{B}$)

$\Rightarrow$ updates expected soon ! (+ final reprocessing).

➔ All of the measurements presented are and will be statistics limited.

GLW method : $B^- \to \tilde{D}^{(*)0}[\text{CP-eigenstate}]_D K^{(*)-}$ and observables from $B^\pm$ yields

- Theoretically very clean to determine γ
- Relatively small BFs ~10^{-6} (including sec. BFs) STATISTICS LIMITED !

  $\Rightarrow$ small CP asymmetry (r_B)
- Reconstruct D meson in CP-eigenstates (accessible to D^0 and $\bar{D}^0$), & in many modes (normalize to $D^{(*)0}$ flavour state decays ($K^-\pi^+$)):
  - CP-even (CP+) $\equiv D_+$: K^+K^-, $\pi^+\pi^-$
  - CP-odd (CP-) $\equiv D_-$: $K^0_S\pi^0$, $K^0_S\omega[\pi\pi\pi^0]$, $K^0_S\phi[KK]$

  Use channels D^{*0} to $D^0\pi^0/D^0\gamma$ and K^{*-} to $K^0_S\pi^-$

- **ratio of BFs:** (CP eigenstate/flavor state) (double ratios normalized with $D^{(*)0}\pi^-$ for systematic cancellations)

$$R_{CP\pm} \equiv \frac{\Gamma(B^- \to D_\pm K^-) + \Gamma(B^+ \to D_\pm K^+)}{[\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \bar{D}^0 K^+)]/2} = 1 + r_B^2 \pm 2\, r_B \cos(\delta_B)\cos(\gamma)$$

weak sensitivity to $r_B^2 \ll 1$

- **direct CPV ($B^+ \leftrightarrow B^-$):**

$$A_{CP\pm} \equiv \frac{\Gamma(B^- \to D_\pm K^-) - \Gamma(B^+ \to D_\pm K^+)}{\Gamma(B^- \to D_\pm K^-) + \Gamma(B^+ \to D_\pm K^+)} = \frac{\pm 2\, r_B \sin(\delta_B)\sin(\gamma)}{R_{CP\pm}}$$

8 fold-ambiguities on γ

GLW : $B^- \to D^0_{CP} K^-$

PRD 77, 111102(R) (June 2008)

382×10^6 $B\bar{B}$

- Selection based on m_{ES} and event shape variables.
- Extended max. likelihood fit to ΔE and Cherenkov angle θ_C of the prompt track.
- Use of $B^- \to D^0\pi^-$ as normalization channel and control sample, and D^0 mass side-bands..
- No $D^0 \to K^0_S\phi$ mode (GGSZ $D^0 \to K^0_S K^+K^-$)

$N_{CP+}=239\pm21$
$N_{CP-}=235\pm21$
$N_{K\pi}=1872\pm51$

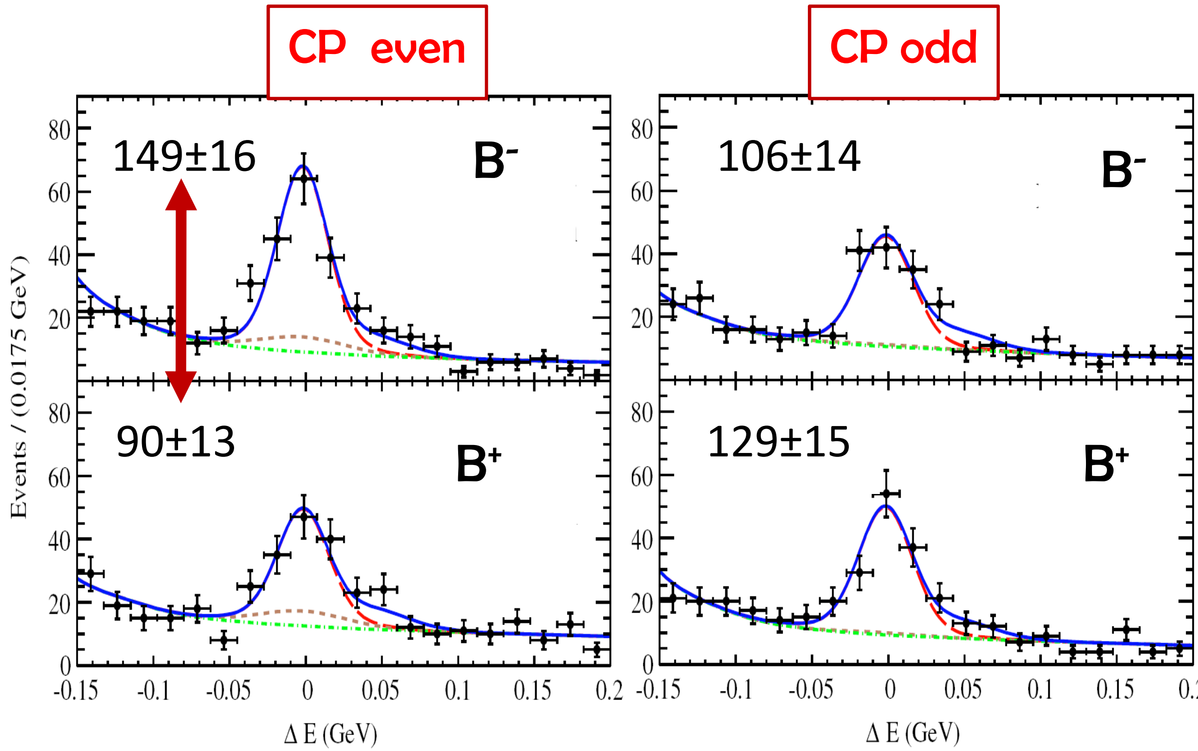
±stat.±syst.

$$A_{CP+} = 0.27 \pm 0.09 \pm 0.04$$
$$A_{CP-} = -0.09 \pm 0.09 \pm 0.02$$
$$R_{CP+} = 1.06 \pm 0.10 \pm 0.05$$
$$R_{CP-} = 1.03 \pm 0.10 \pm 0.05$$

$$x_+ = -0.09 \pm 0.05 \pm 0.02$$
$$x_- = +0.10 \pm 0.05 \pm 0.03$$
$$r_B^2 = +0.05 \pm 0.07 \pm 0.03$$

$$x_\pm = r_B\cos(\delta_B \pm \gamma) = \frac{R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-})}{4}$$

$$r_B^2 = \frac{R_{CP+} + R_{CP-} - 2}{2}$$



1. Direct CPV at 2.8 σ for CP+ decays
2. Not enough sensitivity to γ, but :

- most precise GLW measurement.
- $x_\pm$ compatible with GGSZ and as precise.

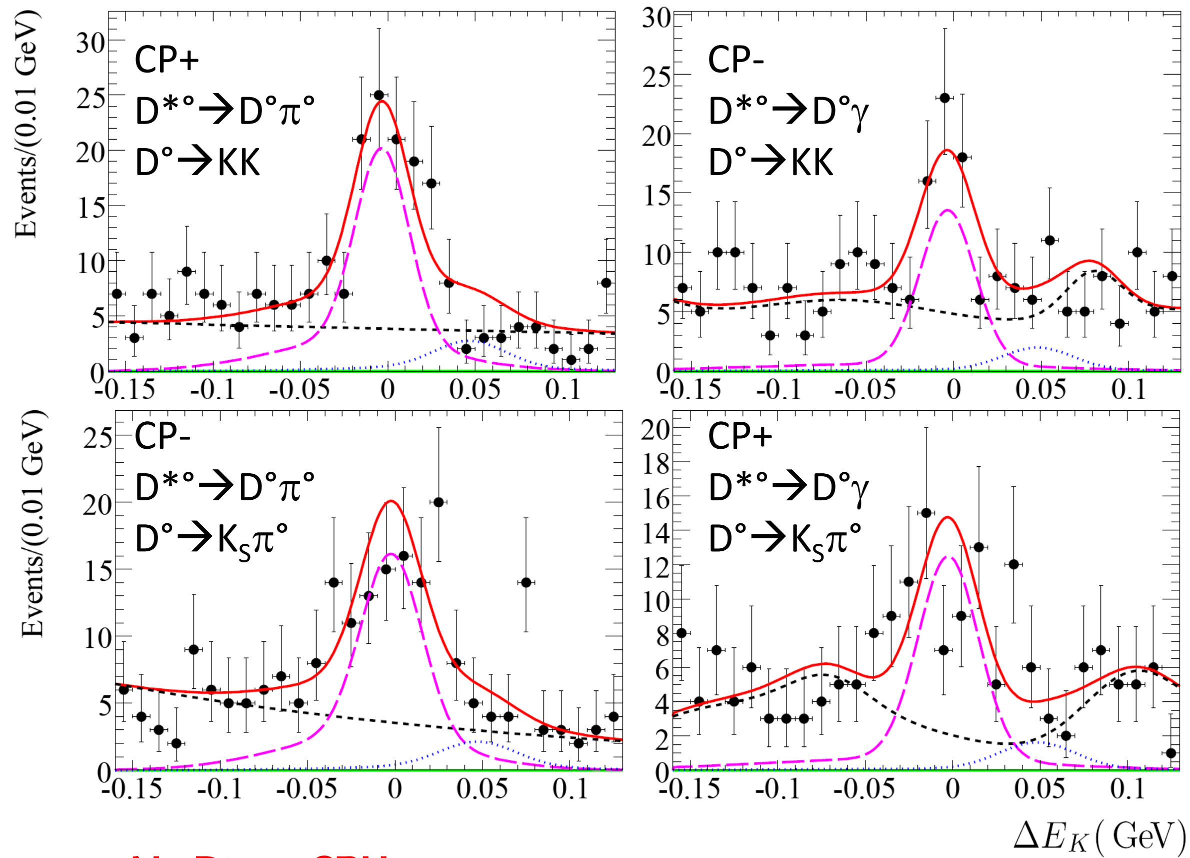
GLW : $B^- \rightarrow D^{*0}_{CP}[\rightarrow D^0\pi^0/\gamma]K^-$

arXiv:0807.2408 hep/ex
(July 2008), submitted to PRD

383×10^6 $B\bar{B}$

- Selection based on m_{ES} and event shape variables.
- Extended max. likelihood fit to ΔE and dE/dx + Cherenkov PID of fast track.
- Use of $B^- \rightarrow D^{*0}\pi^-$ as normalization channel and control sample and D^0 mass side-bands.
- D^{*0} CP flips for $D^0\pi^0$ and $D^0\gamma$ same D^0 final states (PRD 70, 091503 (2004))

N_{CP+}=244±22
N_{CP-}=225±23
$N_{K\pi}$=1410±57



±stat.±syst.

$$\begin{aligned} A^*_{CP+} &= -0.11 \pm 0.09 \pm 0.01 \\ A^*_{CP-} &= 0.06 \pm 0.10 \pm 0.02 \\ R^*_{CP+} &= 1.31 \pm 0.13 \pm 0.04 \\ R^*_{CP-} &= 1.10 \pm 0.12 \pm 0.04 \end{aligned}$$

$$\begin{aligned} x^*_+ &= +0.09 \pm 0.07 \pm 0.02 \\ x^*_- &= -0.02 \pm 0.06 \pm 0.02 \\ r^{*\,2}_B &= +0.22 \pm 0.09 \pm 0.03 \end{aligned}$$

($K^0_S\phi$ removed for Cartesian coords.)

1. No Direct CPV seen.
2. Not enough sensitivity to γ, but :

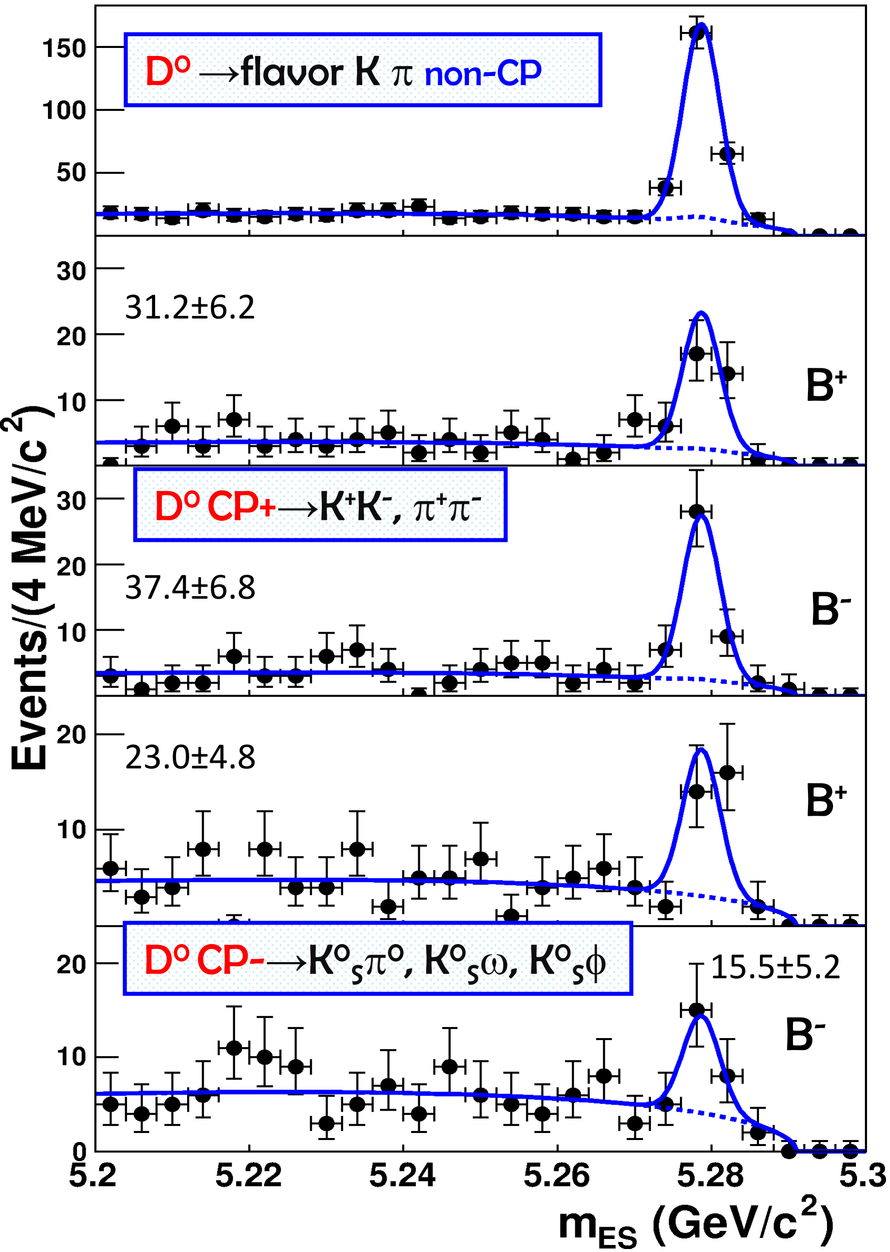
- most precise D*K GLW measurement.
- $x^*_\pm$ compatible with GGSZ and as precise, r^*_B expected large.

379x10^6 $B\bar{B}$

GLW : $B^- \rightarrow D^0_{CP} K^{*-}[\rightarrow K^0_S \pi^-]$

NEW

PRELIMINARY



- Selection re-optimized and event shape in Neural Net wrt 2005 (higher eff'cy, bkg. rejection, and stat.)
- (peaking)-bkg. from ΔE and mD^0 sidebands.
- CP+ pollution for $K^0_S(K^+K^-)$non-ϕ & $K^0_S(\pi^+\pi^-\pi^0)$non-ω measured in the data.
- Vary by 2π the δ strong phases from S-wave $K\pi$ pairs in $K^{*-}[K^0_S\pi^-]$ decays.

$N_{CP+}=68.6\pm9.2$
$N_{CP-}=38.5\pm7.0$
$N_{K\pi}=231\pm17$

±stat.±syst.

$$A^s_{CP+} = 0.09 \pm 0.13 \pm 0.05$$
$$A^s_{CP-} = -0.23 \pm 0.21 \pm 0.07$$
$$R^s_{CP+} = 2.17 \pm 0.35 \pm 0.09$$
$$R^s_{CP-} = 1.03 \pm 0.27 \pm 0.13$$

$$x_{s+} = 0.18 \pm 0.14 \pm 0.05$$
$$x_{s-} = 0.38 \pm 0.14 \pm 0.05$$

1. No Direct CPV seen.
2. Precision improved wrt 2005, results confirmed, world's only measurement.

ADS method : $B^- \to \tilde{D}^{(*)0}[K^+\pi^-]_D K^{(*)-}$ and observables from $B^\pm$ yields

- Same idea as for GLW, same final state in different $\tilde{D}^0$ [$\bar{D}^0$/D^0] states:

$[K^+\pi^-]_D K^-$: Doubly-Cabibbo-Suppressed (DCS) decays instead of CP-eigenstate.

- Small BFs(~10^{-6}), but amplitudes ~ comparable in size: expect larger CPV!
- Count B candidates with opposite sign K !

2 observables

- ratio of BFs: (Wrong Sign $D^0 \to K^+\pi^-$/Right Sign $D^0 \to K^-\pi^+$)

$$\mathrm{R_{ADS}} \equiv \frac{\Gamma([K^+\pi^-]K^-) + \Gamma([K^-\pi^+]K^+)}{\Gamma([K^-\pi^+]K^-) + \Gamma([K^+\pi^-]K^+)} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)$$

good sensitivity to $r^2_B > r^2_D$

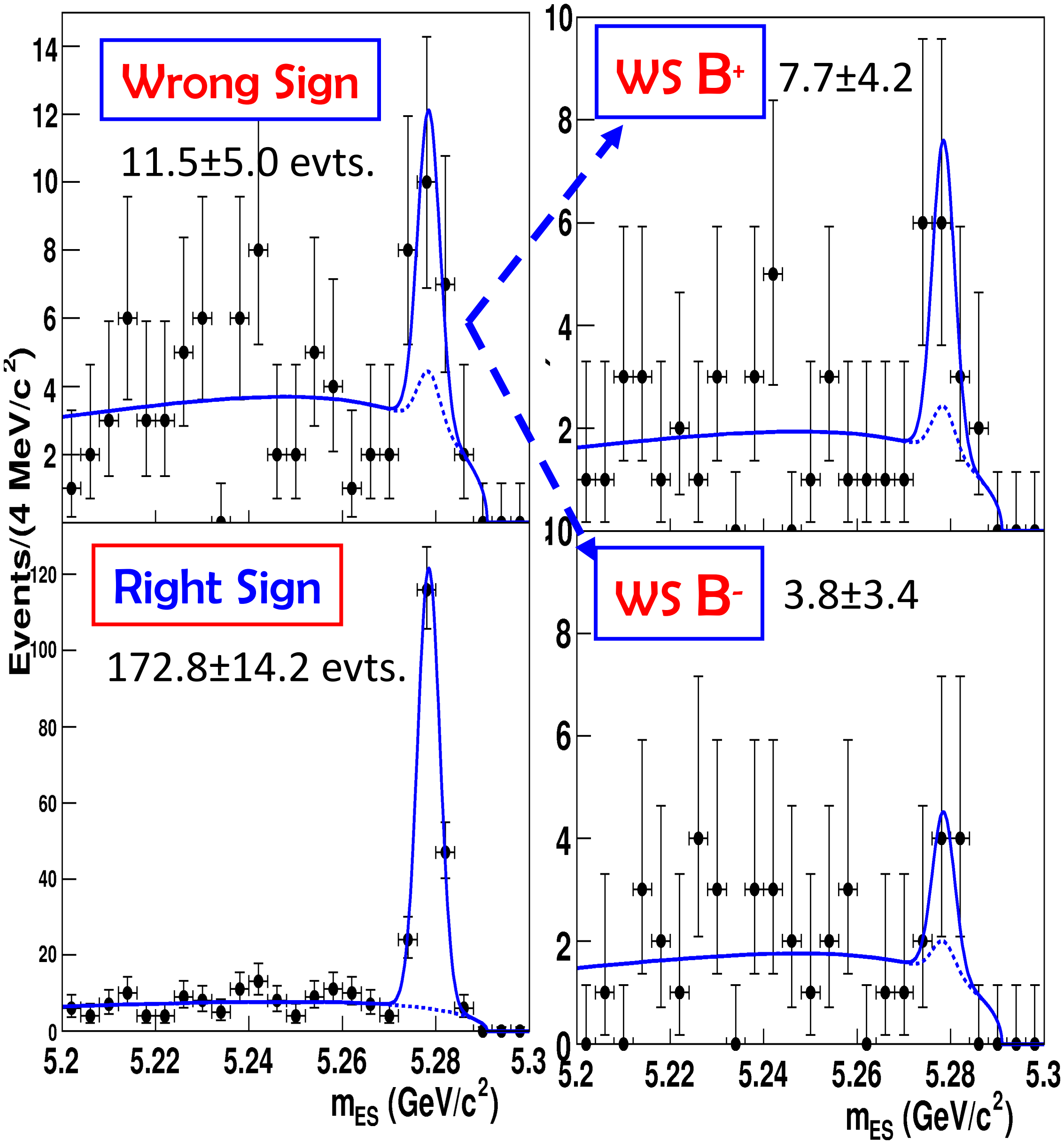
- direct ACPV: from $B^+ \leftrightarrow B^-$ direct asymmetry in yield if enough ADS events seen.

$$\mathrm{A_{ADS}} \equiv \frac{\Gamma[K^+\pi^-]K^-) - \Gamma([K^-\pi^+]K^+)}{\Gamma[K^+\pi^-]K^-) + \Gamma([K^-\pi^+]K^+)} = \frac{2\, r_B\, r_D \sin(\delta_B + \delta_D)\sin(\gamma)}{\mathrm{R_{ADS}}}$$

379x10^6 $B\bar{B}$

ADS : $B^- \to D^0[K^+\pi^-]\ K^{*-}[\to K^0_S\pi^-]$

NEW
PRELIMINARY



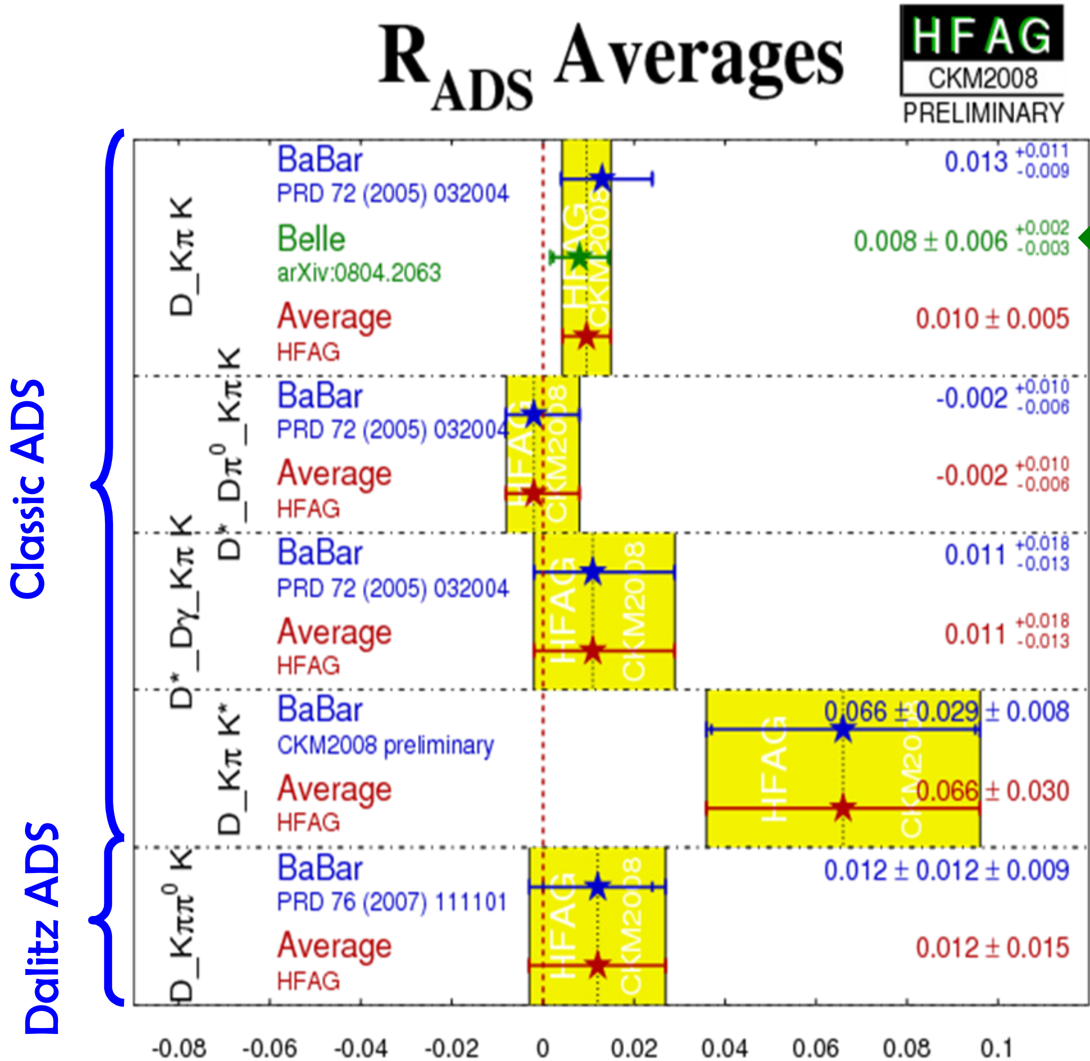
- 11.5±5.0 WS ADS signal events.
- Accounts for $(K^0_S\,\pi^-)$non-K^* with S-waves pairs, directly determined from the data.
- Syst.: detector asymmetry, peaking bkg., and same final state bkgd. ($K^+\pi^-K^{*-}$ charmless bkg. ...).

±stat.±syst.

$$A^S_{ADS} = -0.34 \pm 0.45 \pm 0.16$$
$$R^S_{ADS} = 0.066 \pm 0.029 \pm 0.010$$

1. No Direct CPV seen.
2. Better precision wrt 2005 result, again unique in the world.

ADS : status as of CKM'08



arXiv:0804.2063 [hep/ex]
(March 2008) sub. to PRD(RC)

675x10^6 $B\bar{B}$ pairs (no significant ADS signal)

PRD 72, 032004 (2005)

232x10^6 $B\bar{B}$

NEW

379x10^6 $B\bar{B}$

PRD 76, 111101(R) (2007)

226x10^6 $B\bar{B}$

➔ With current statistics is not possible to constraint r_B with R_{ADS} measurements alone (and almost nothing for A_{ADS}).

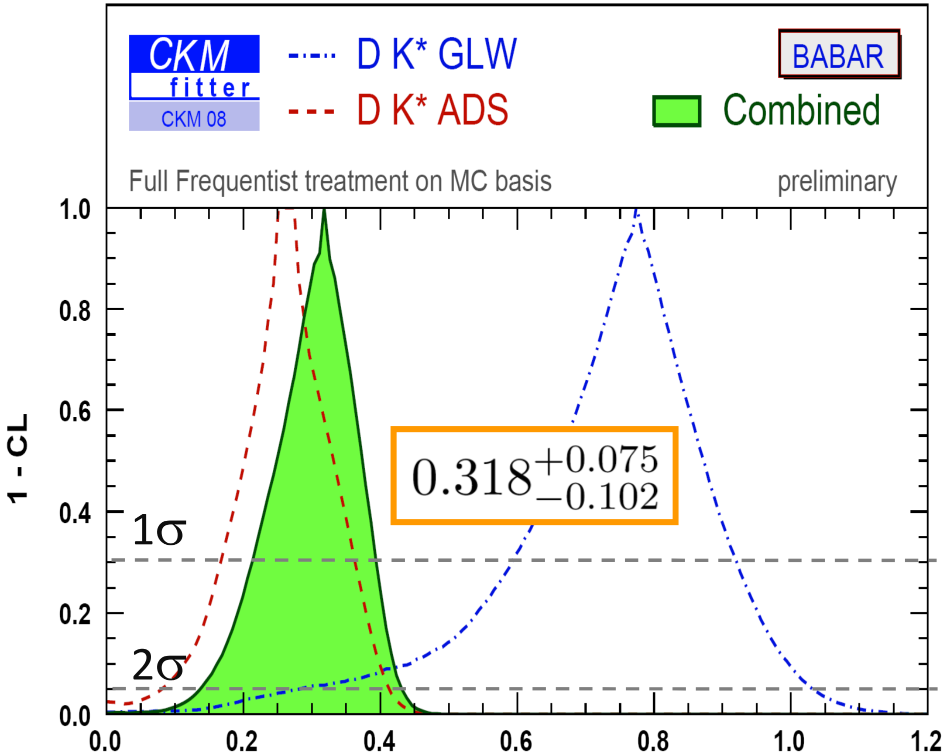
379x10⁶ $B\bar{B}$

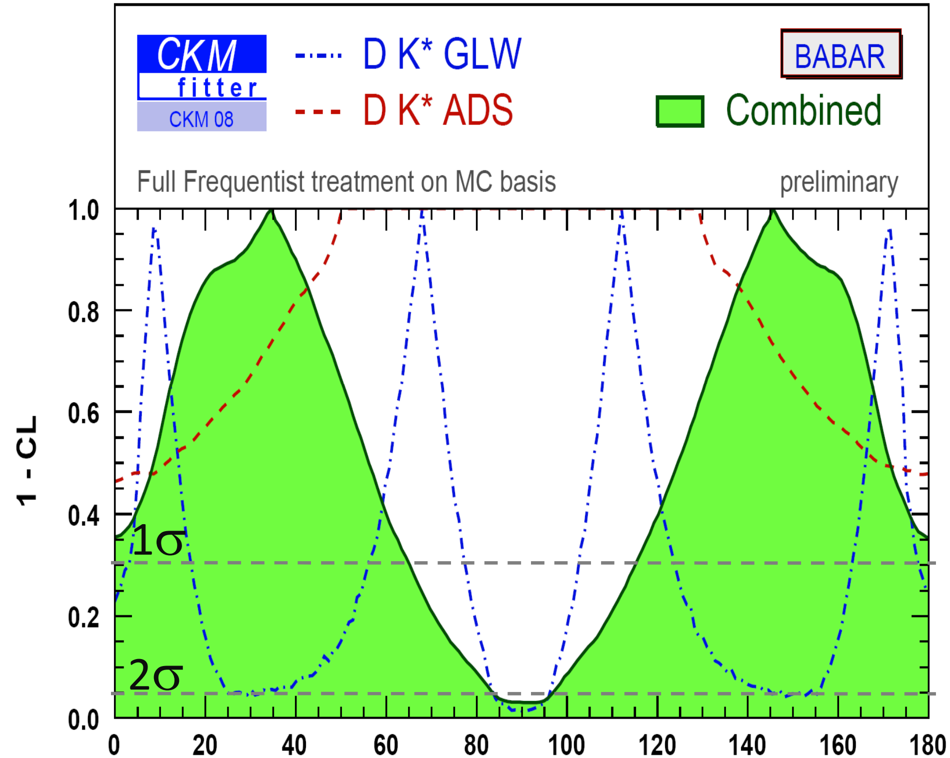
GLW/ADS : $B^- \to \tilde{D}^0 K^{*-}[\to K^0_S \pi^-]$

NEW

My frequentist (CKM-Fitter EPJ,C41,1 (2005)) approach to determine γ and $\kappa.r_{sB}$ from $\Delta\chi^2$ toy in ADS/GLW combination for $B^- \to D^0 K^{*-}[K^0_S\pi^-]$

$\gamma \in [0,\pi]$ & $(\delta_D+\delta_{sB}) \in [0,2\pi]$





$\kappa.r_{sB}$ in [0.14,0.43] @95% CL

(κ~(0.9±0.1) depending on K* selection, accounts for K* natural width and non 2 body $D^0K^0_S\pi^-$ B-decays
⇒no assumption (nature, number, strong phase...)

$\gamma \notin [84°,97°]$
(excluded @95% CL)

Dalitz GGSZ : $B^- \to \tilde{D}^{(*)0}[K^0_s h^+ h^-]_D K^{(*)-}$, $h=\pi/K$

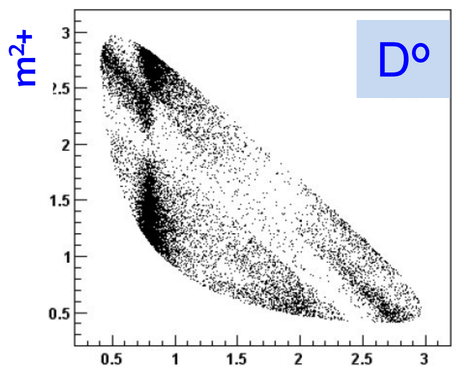
- $\tilde{D}^0 \to K^0_S[\pi^+\pi^-/K^+K^-]$ 3-body self conjug. final states accessible through many different decays. Only $(\pi/K)^\pm$: clean, efficient, and reasonable $BF([K^0_S\pi\pi]_D K^-) \approx 10^{-5}$ ($\times$10 D^0_{CP}, $BF(K^0_S\pi\pi)/BF(K^0_S KK)$~6).

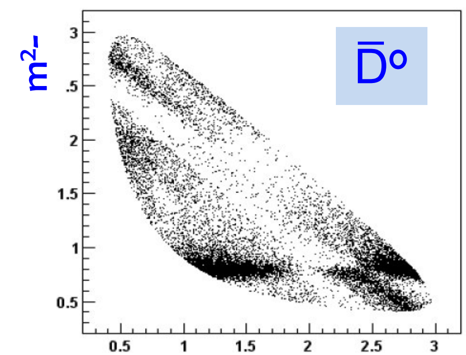
→ need Dalitz structure analysis : $D^0/\bar{D}^0$ decay amplitudes $\mathcal{A}_{D-/+}$ to separate interferences between resonances ⇒ precise modelization.

$m^2_\pm = m^2(K^0_s h^\pm)$

- schematic view of interference [$b \to c \Leftrightarrow b \to u$]:

$$\mathcal{A}(B^-) = |\mathcal{A}_B(D^0K^-)| \times \left(\text{[D}^0\text{ plot]} \times \mathcal{A}_{D-/+} + r_B e^{i(\delta_B - \gamma)} \text{[}\bar{D}^0\text{ plot]} \times \mathcal{A}_{D+/-} \right)$$





sensitivity varies strongly over Dalitz plane
⇒ model + mixture ADS+GLW

Simultaneous fit to $\tilde{D}^0 \to K^0_s h^+h^-$ Dalitz plot density of B^+/B^- data yields to extract from this density difference r_B's, δ_B's, and γ

- 2 fold ambiguity : $(\gamma,\delta_B) \to (\gamma+\pi,\delta_B+\pi)$
- No D^0 mixing, nor CPV therein D decays.

Dalitz GGSZ : $B^- \rightarrow \tilde{D}^{(*)0}\,[K^0_S h^+ h^-]_D K^{(*)-}$, $h=\pi/K$

- Extract γ from Dalitz-plot distribution of D^0 daughters using Cartesian coordinates:

$$\Gamma^{(*)}(B^{-/+})_{-/+} \propto |\mathcal{A}_{D-/+}|^2 + r^{(*)2}_{(s)B}\,|\mathcal{A}_{D+/-}|^2 + 2\lambda \times \left[x^{(*)}_{(s)-/+}\,\mathcal{R}e\{\mathcal{A}_{D-/+}\mathcal{A}^*_{D+/-}\} + y^{(*)}_{(s)-/+}\,\mathcal{I}m\{\mathcal{A}_{D-/+}\mathcal{A}^*_{D+/-}\}\right]$$

$$\begin{cases} x_\pm = r_B\cos(\delta_B \pm \gamma) \\ y_\pm = r_B\sin(\delta_B \pm \gamma) \\ r_B^2 = x_\pm^2 + y_\pm^2 \end{cases}$$

λ = +1 for $B\rightarrow D^0K, D^{*0}[D^0\pi^0], D^0K^*$

-1 for $B\rightarrow D^{*0}[D^0\gamma]K$

- Experimentally:

PRD78, 034023 (2008)

$383\times10^6\ B\bar{B}$

- $\mathcal{A}_{D-/+}$ determined from high stat./purity $c\bar{c}$ data control samples: $D^{*+}\rightarrow D^0\pi^+$, $D^0\rightarrow K^0_S h^+h^-$

  $\Rightarrow$New Dalitz models with coherent sum of quasi-2-body amplitudes.

  Note: additional phase of D^0 decays varying over the Dalitz plane ($\neq$ from ADS/GLW):

  model systematic uncertainty on γ...

- Eff'cy selections improved (+ optimized event shape treatment).
- max. likelihood fit (m_{ES}, ΔE, and evt. shape variable) to extract yields and PDF params.

  + using $B\rightarrow D^{(*)0}\pi^-$ and $D^0a^-_1$ control samples.

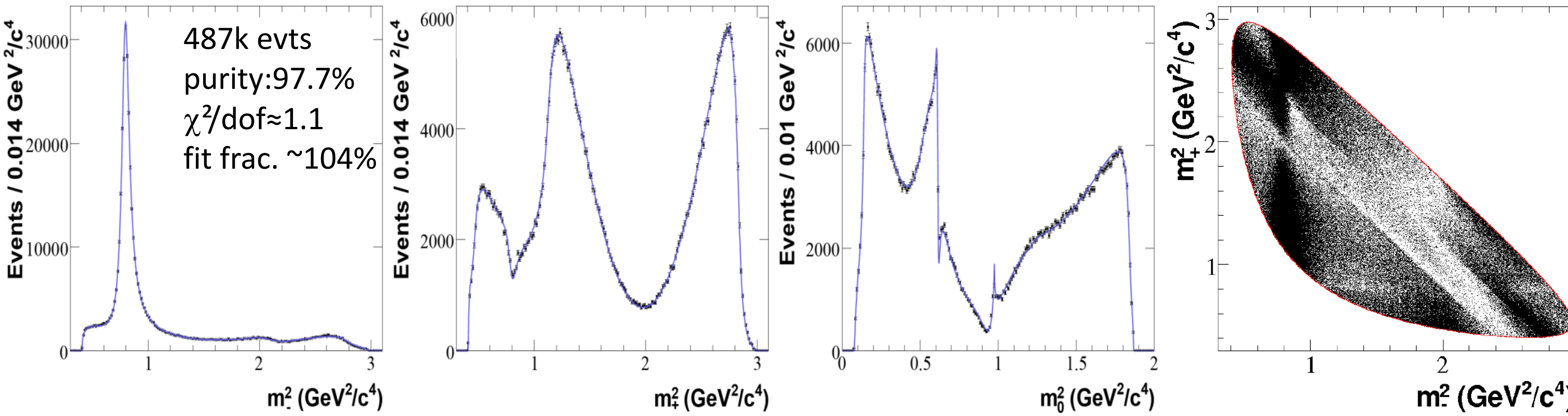
PRD78, 034023 (2008)

Dalitz analysis : $D^0/\bar{D}^0 \to K^0_S h^+h^-$, $h=\pi/K$

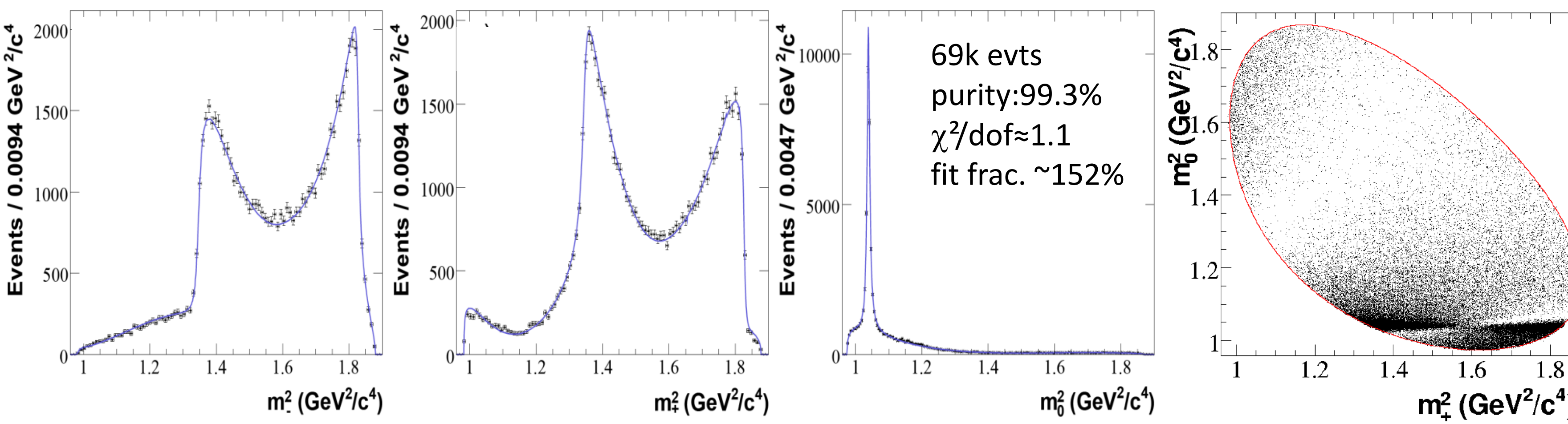
data (351 fb^{-1})

- $K^0_S\pi\pi$: 10 BW resonances (isobar) for P(dominant)-D waves + K-matrix for $\pi\pi$ (first >4σ evidence !) and $K\pi$ (LASS) S-waves (K-matrix formalism deals with broad, overlapping, multi-channels resonances).
- $K^0_S KK$: isobar model used for the first time ($K^0_S a_0(980)$ S-wave and $K^0_S\phi(1020)$ P-waves dominate).

$D \to K^0_S\pi^+\pi^-$



$D \to K^0_S K^+K^-$

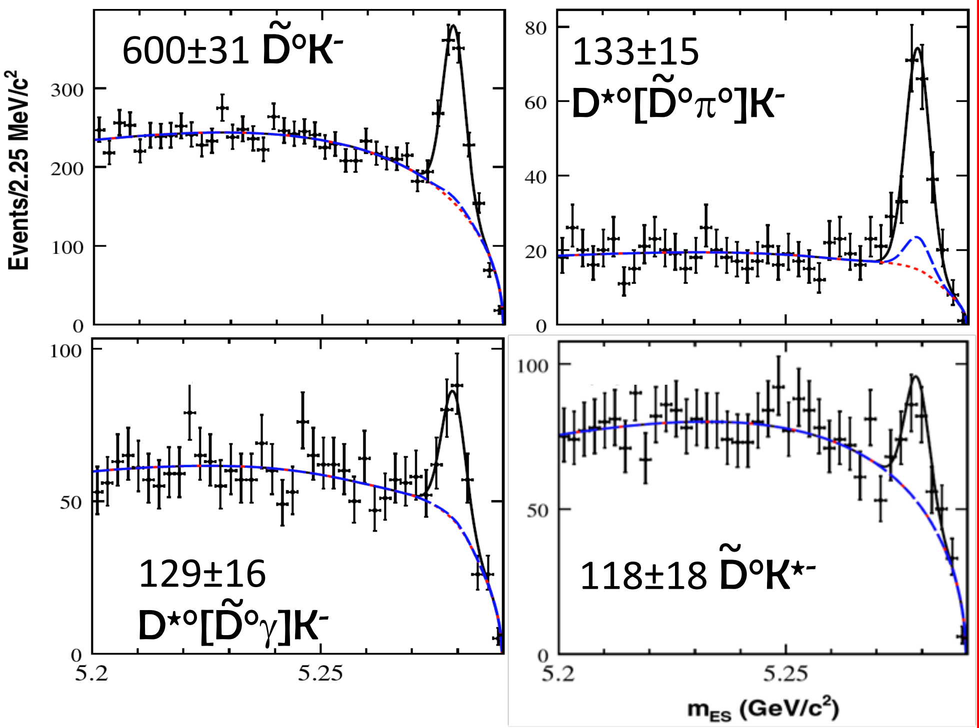


$B^- \rightarrow \tilde{D}^{(*)0}[K^0_s h^+ h^-]_D K^{(*)-}$: yields

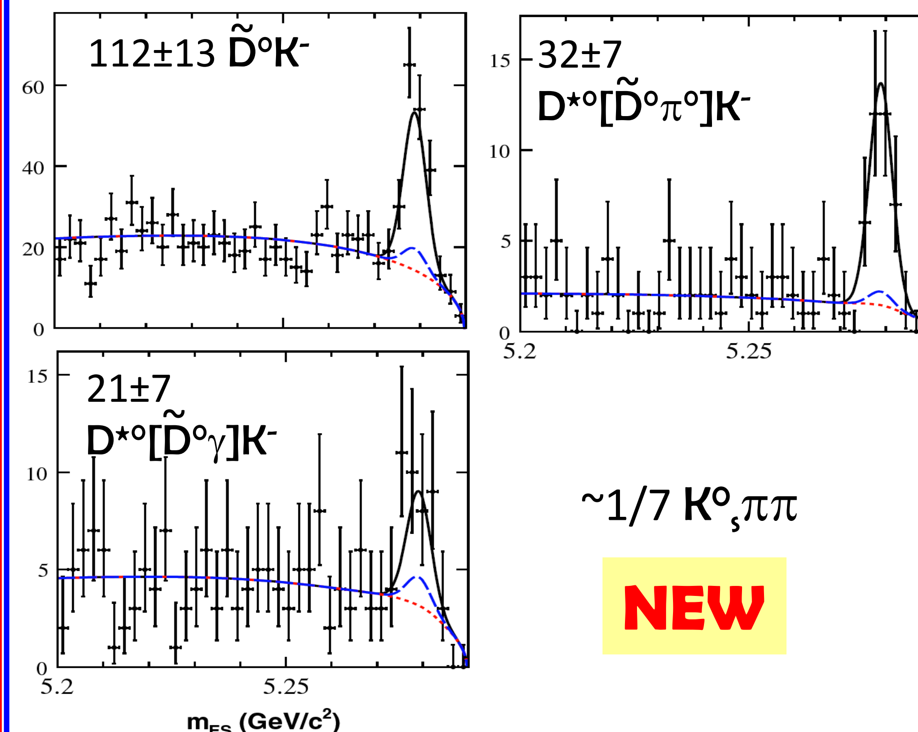
PRD78, 034023 (2008)

$383 \times 10^6\ B\bar{B}$

980 $D \rightarrow K^0_s \pi^+\pi^-$ events



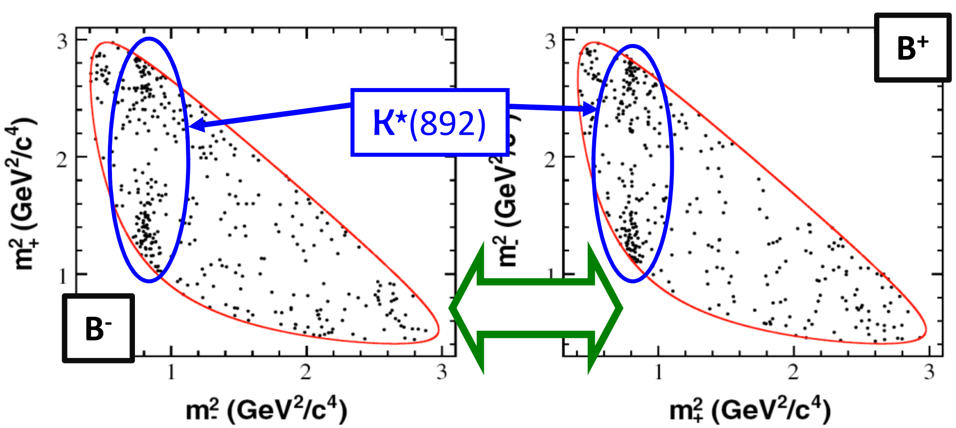
165 $D \rightarrow K^0_s K^+K^-$ events



~1/7 $K^0_s\pi\pi$

NEW

600±31 $\tilde{D}^0 K^-$



$B^- \Leftrightarrow B^+$ differences mean CPV

Fit results: 12 (3x4) Cartesian coordinates

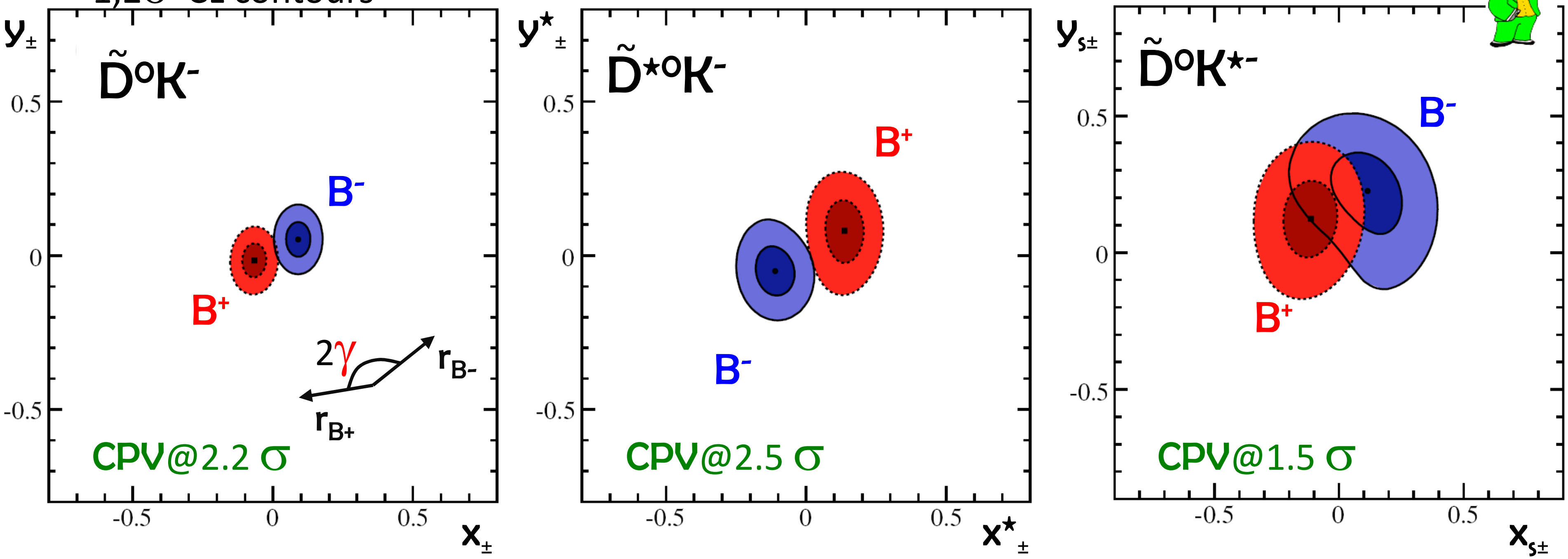
PRD78, 034023 (2008)

$383\times10^{6}\ B\bar{B}$

$z^{(*)}_{(s)\pm} \equiv (x^{(*)}_{(s)\pm}, y^{(*)}_{(s)\pm})$
$= (\mathcal{Re}, \mathcal{Im})\{r^{(*)}{}_{s_B} e^{i(\delta^{(*)}_{(s)B} \pm \gamma)}\}$

➔ Almost Gaussian behavior near physical bounds ($r_B \approx 0$), better than the 7 physics params.

1,2σ CL contours



$x-$	$0.090 \pm 0.043 \pm 0.015 \pm 0.011$
$y-$	$0.053 \pm 0.056 \pm 0.007 \pm 0.015$
$x+$	$-0.067 \pm 0.043 \pm 0.014 \pm 0.011$
$y+$	$-0.015 \pm 0.055 \pm 0.006 \pm 0.008$

x^*-	$-0.111 \pm 0.069 \pm 0.014 \pm 0.004$
y^*-	$-0.051 \pm 0.080 \pm 0.009 \pm 0.010$
x^*+	$0.137 \pm 0.068 \pm 0.014 \pm 0.005$
y^*+	$0.080 \pm 0.102 \pm 0.010 \pm 0.012$

$xs-$	$0.115 \pm 0.138 \pm 0.039 \pm 0.014$
$ys-$	$0.226 \pm 0.142 \pm 0.058 \pm 0.011$
$xs+$	$-0.113 \pm 0.107 \pm 0.028 \pm 0.018$
$ys+$	$0.125 \pm 0.139 \pm 0.051 \pm 0.010$

±stat. ±syst.(exp.) ±syst.(Dalitz model)

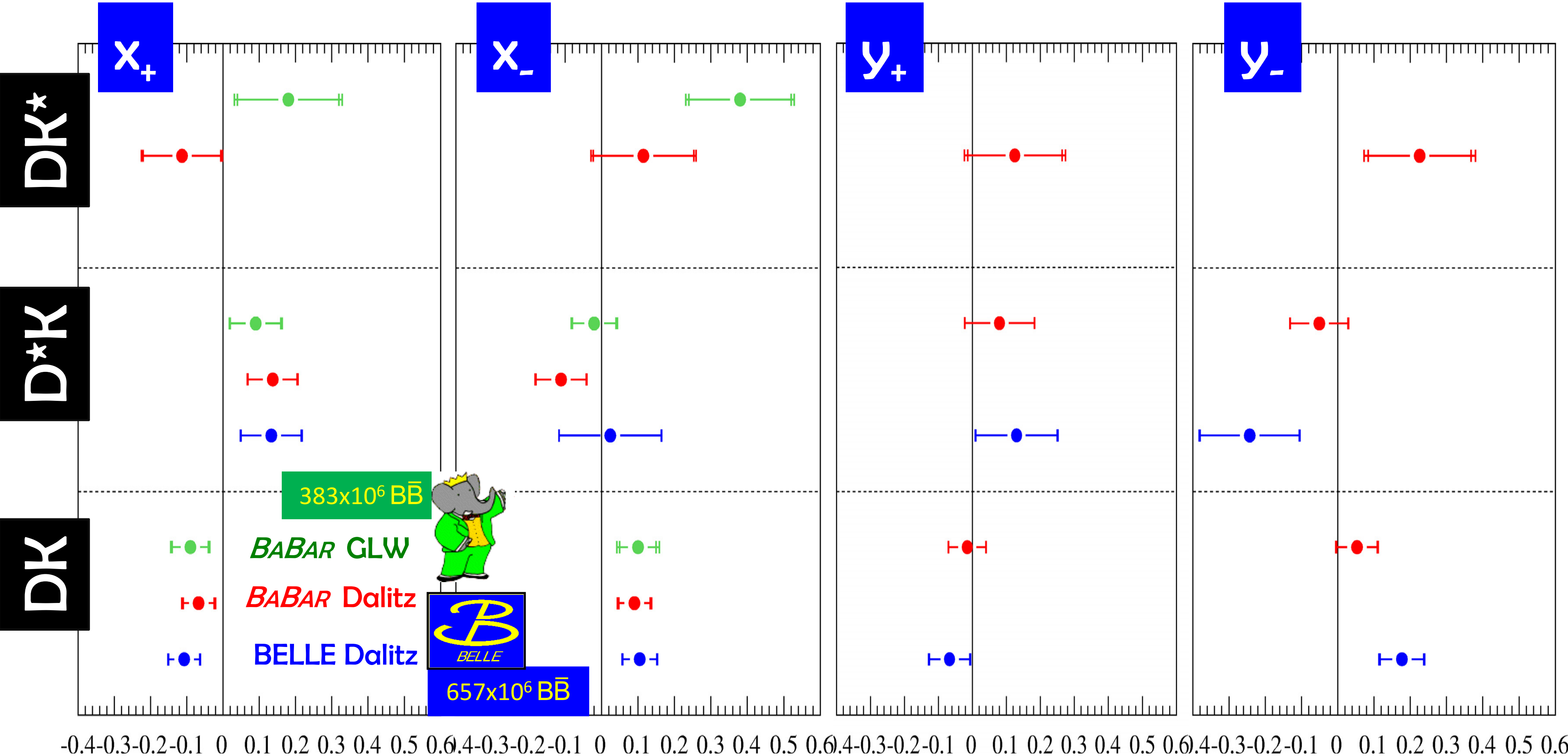
$|z_{B+} - z_{B-}| \neq 0$ ➔ direct CPV@ 3.0σ combined

Cartesian coordinates today world's Summary

Belle: A.Bondar's talk

$(x^{(*)}_{(s)\pm}, y^{(*)}_{(s)\pm}) = (\mathcal{Re}, \mathcal{Im})\{r^{(*)}_{s_B} e^{i(\delta^{(*)}_{(s)B} \pm \gamma)}\}$

$$x_{\pm} = \frac{R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-})}{4}$$



- Similar precision between BABAR and BELLE Dalitz, and BABAR GLW for $x_{\pm}$
- Overall good consistency: GLW helps in reducing uncertainties on γ within a global comb.

Dalitz : γ results for $B^- \to \tilde{D}^{(*)0} K^{(*)-}$

PRD78, 034023 (2008)

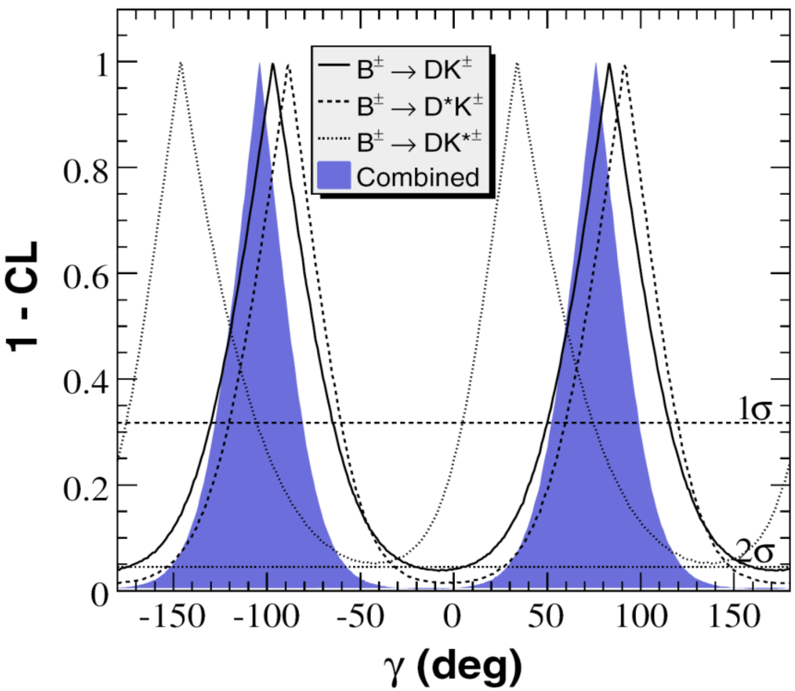
383×10^6 $B\bar{B}$

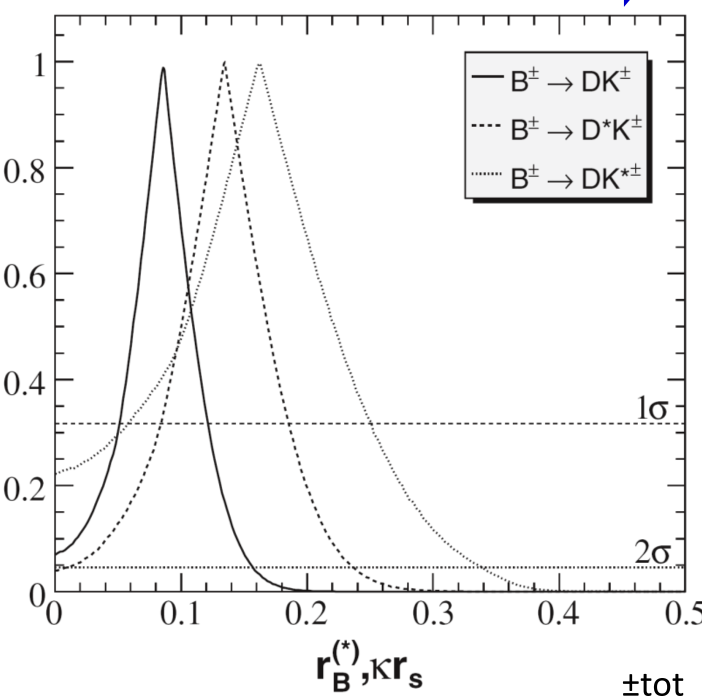
From measured CP parameters: $(x\pm, y\pm)$, $(x^*\pm, y^*\pm)$, $(x_s\pm, y_s\pm)$ perform combined fit to pseudo experiments : frequentist approach removing unphysical regions ($r_{B+} \neq r_{B-}$ ...)

extract 1D CL intervals

- r_B, r^*_B, r_{sB}
- δ_B, δ^*_B, δ_{sB}
- γ

Physics params





$$\gamma[\mathrm{mod}\ \pi] = (76^{+23}_{-24})^\circ \ \{5, 5\}^\circ$$

±tot {syst. exp., syst. model}

$r_B(DK) = (8.6 \pm 3.5)\% \ \{1.0, 1.1\}\%$

$r_B(D^*K) = (13.5 \pm 5.1)\% \ \{1.1, 0.5\}\%$

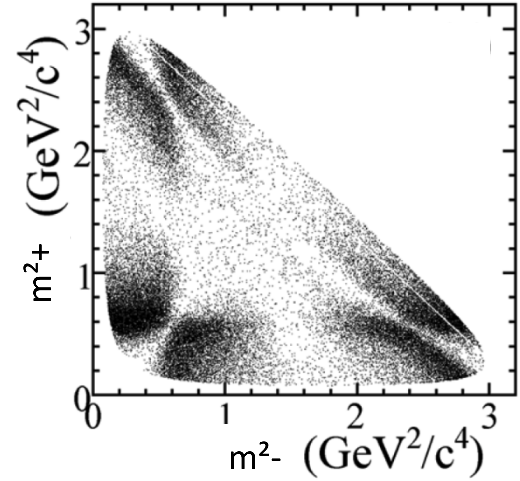
$\kappa r_B(DK^*) = (16.3^{+8.8}_{-10.5})\% \ \{3.7, 2.1\}\%$

➔ Statistics limited, small r_B ~10% favored (limits sensitivity to γ).

PRL99, 251801 (2007)

324×10^6 $B\bar{B}$

Dalitz for $B^- \to \tilde{D}^0[\pi^-\pi^+\pi^0]K^-$



➔ Compared to $\tilde{D}^0[K^0_S\pi\pi]K^-$:

- ~1/3 signal rate: (170±29) signal evts.
- larger background and different Dalitz struct. [$\rho(770\text{-}1700)\pi$, $f_0(980\text{-}1710)\pi$...].

➔ due to significant nonlinear correlations we use polar coordinates, instead of Cartesian, (and r_B, δ_B and γ), naturally (decay yields: $\Gamma(B^\pm) \propto 1+(\rho_\pm)^2-(x^0)^2$) defined as:

$$\rho_\pm \equiv \sqrt{(x_\pm - x^0)^2 + y_\pm^2}$$

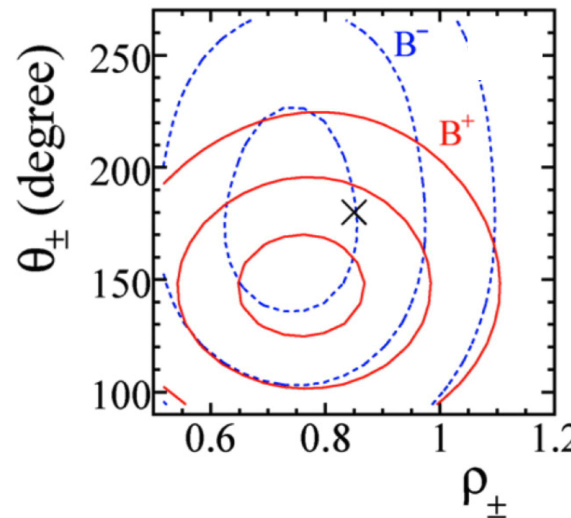
$$\theta_\pm \equiv \mathrm{atan}\left(\frac{y_\pm}{x_\pm - x^0}\right)$$

$$x^0 \equiv \int A_D(m^-, m^+)\bar{A}_D(m^+, m^-)dm^-dm^+ = 0.85$$

±stat.±syst.

$$\begin{aligned}\rho_- &= 0.72 \pm 0.11 \pm 0.06 \\ \theta_- &= (173 \pm 42 \pm 19)^\circ \\ \rho_+ &= 0.75 \pm 0.11 \pm 0.06 \\ \theta_+ &= (147 \pm 23 \pm 13)^\circ\end{aligned}$$

Dalitz model dominates for syst.



1, 2, 3σ

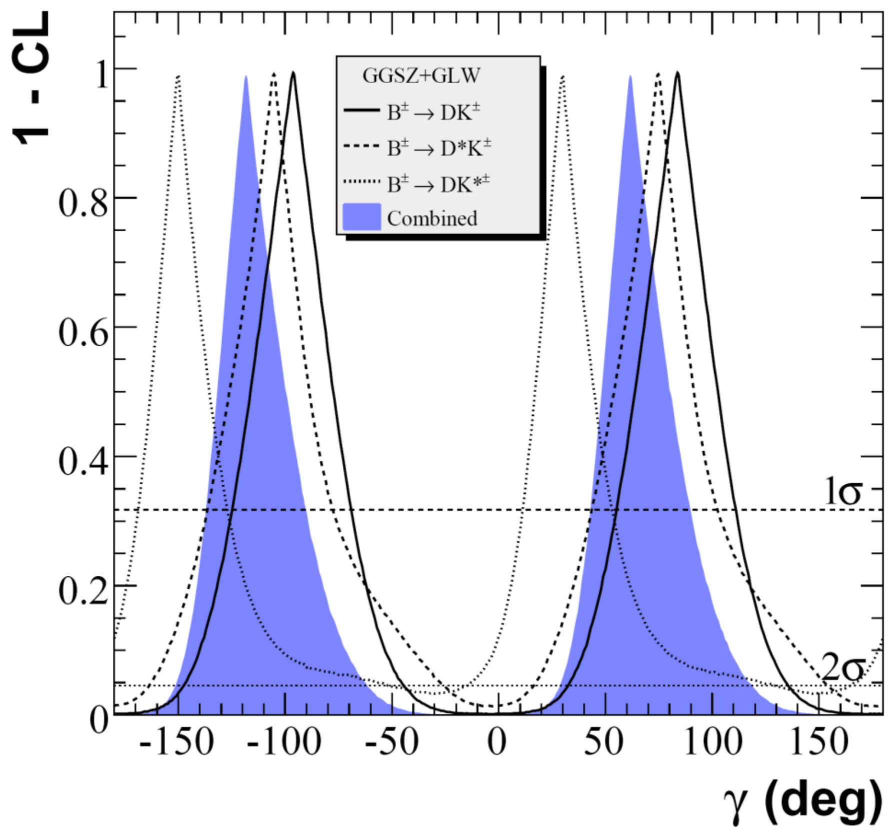
Extract weak constraints (@ 1σ):

$0.06 < r_B < 0.78$, $-30^\circ < \gamma < 80^\circ$,

and $-27^\circ < \delta_B < 78^\circ$

Conclusions and perspectives

- Measure γ at B-Factories ~ impossible mission few years ago ! ...
- Now it's possible, but we are not yet there to precision era!
- Using direct CPV and interference in charged $B^{\pm}$ decays to $\tilde{D}^{(*)0}K^{(*)\pm}$:
  - 3 clean theoretical methods ~ all the machinery in place $\Rightarrow$ Dalitz is still the most powerful
  - Need much more data/channels (r_B) $\Rightarrow$ wait for much more statistics & update with existing one !
  - Need model independent approach for Dalitz (input from CLEO-C) at higher stat.



BABAR GGSZ+GLW alone:

$\gamma = (62^{+28}_{-19})^{\circ},\ [29, 119]^{\circ}$

$r_B(DK) = (9.2^{+2.7}_{-2.8})\%,\ [3.5, 14.8]\%$

$r_B(D^*K) = (10.8^{+5.2}_{-4.1})\%,\ [2.0, 22.6]\%$

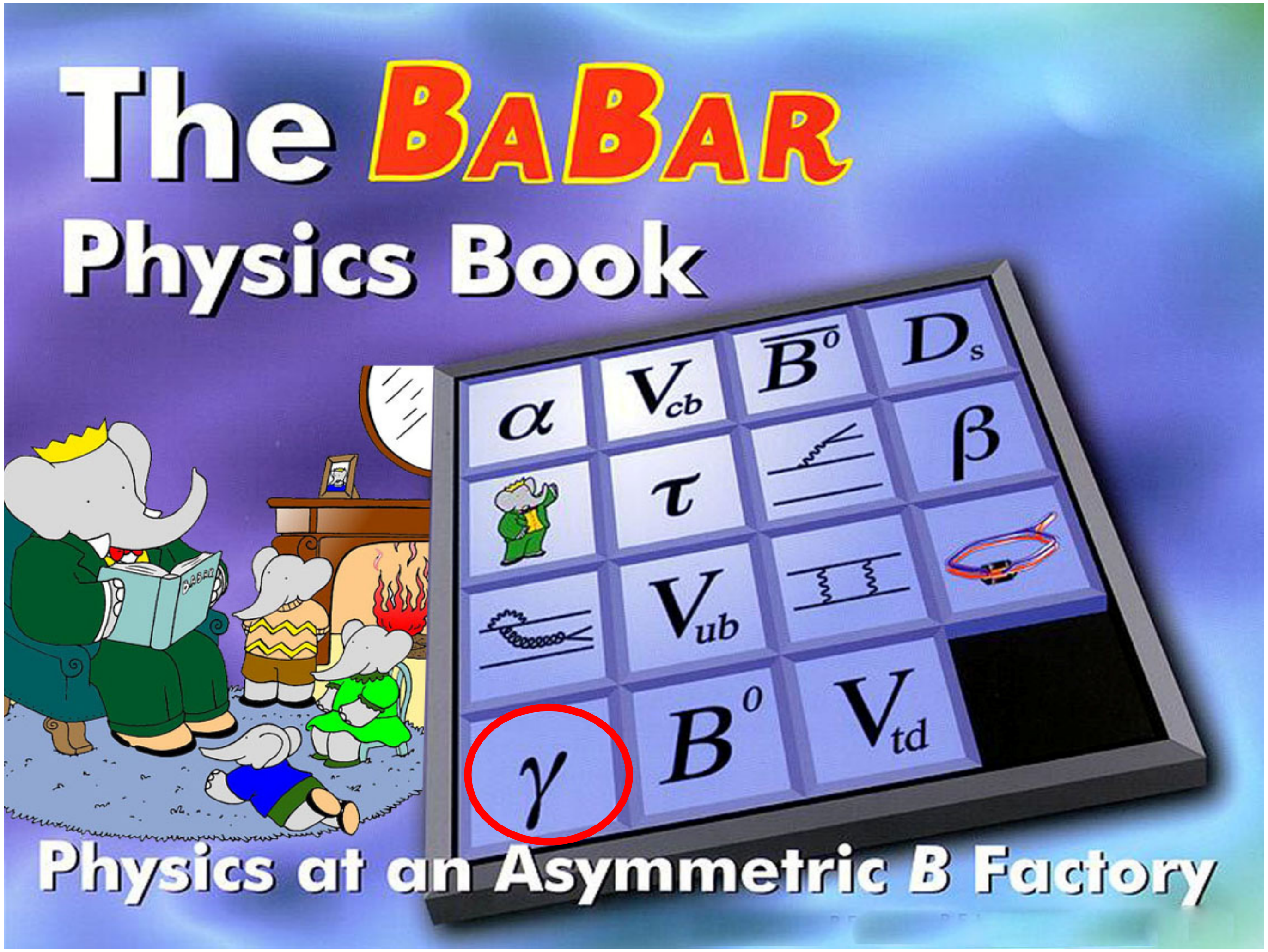
$\kappa r_B(DK^*) = (17.9^{+8.7}_{-9.6})\%,\ < 35.3\%$

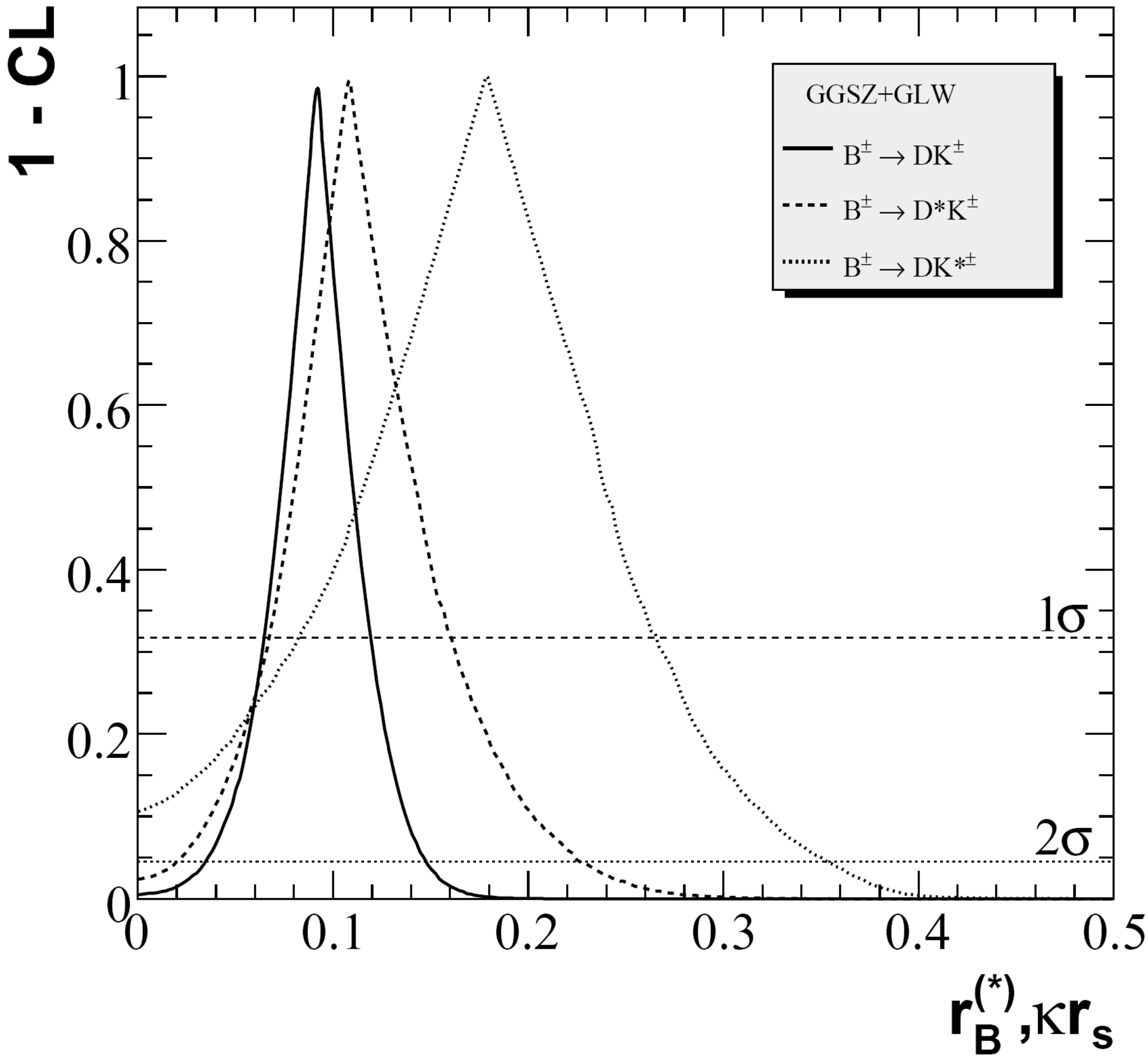
{1σ, @ 95% C.L.}

direct CPV@ 3.3σ

Courtesy average from N. Lopez-March, F. Martinez-Vidal

Backup slides

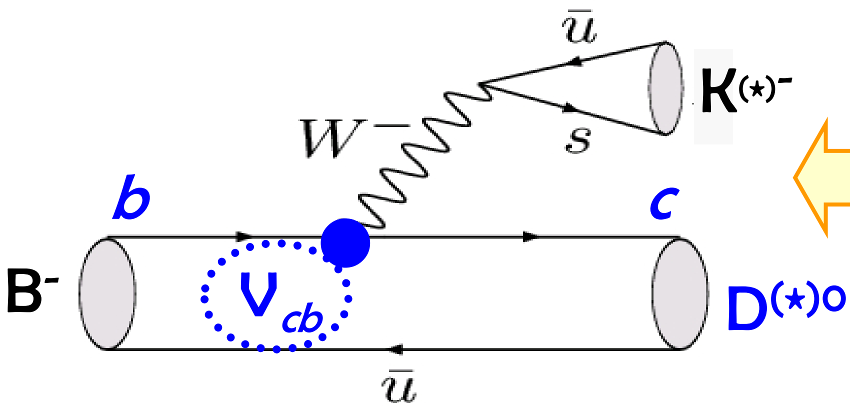



1 - CL
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
GGSZ+GLW
$B^\pm \to DK^\pm$
$B^\pm \to D^*K^\pm$
$B^\pm \to DK^{*\pm}$
1σ
2σ
$r_B^{(*)}, \kappa r_s$

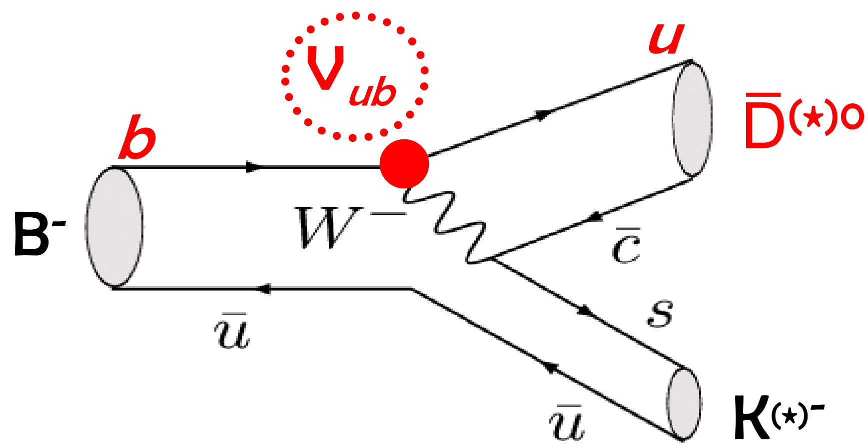
γ from interference in charged $B^- \to \tilde{D}^{(*)0} K^{(*)-}$ decays

Same $\tilde{D}^{(*)0} \equiv [\, D^{(*)0} / \bar{D}^{(*)0} \,]$ final states

Cabibbo-"favored"



(Cabibbo & color)-suppressed



$\boxed{A_{cb}(D^{(*)0}K^{(*)-}) \propto \lambda^3}$

$\boxed{A_{ub}(\bar{D}^{(*)0}K^{(*)-}) \propto \lambda^3 \sqrt{\bar{\eta}^2 + \bar{\rho}^2}\; e^{i(\delta_B - \gamma)}}$

relative strong & weak phases

$$A_{tot} = A_{cb} + A_{ub}$$
$$\propto [(1 \pm r_B\, e^{i(\delta_B - \gamma)})$$
$$\equiv (1 \pm z_-)]$$

(z: Cartesian coordinates if $D^0/\bar{D}^0$ is CP eigenstate)

Size of CP asymmetry: r_B is the critical parameter

$r_B \equiv |A/A| \sim 0.05\text{-}0.30$

Cabibbo and color suppression

PLB 557, 198(2003)

r_B small $\Rightarrow$ small experimental sensitivity to γ

(precision as $1/r_B$)

Methods to extract γ in $B^{\pm} \to \tilde{D}^{(*)0} K^{(*)\pm}$ decays

Use of direct CPV in $B^{\pm}$ [$b \to c \Leftrightarrow b \to u$] interference

➔ one (δ_B, r_B) pair of NUISANCE parameters for each 3 DK, D*K or DK* channel

$\Rightarrow$ 7 unknowns parameters to measure : γ, $(\delta_B, r_B)_{DK}$, $(\delta^*_B, r^*_B)_{D^*K}$, & $(\delta_{sB}, r_{sB})_{DK^*}$

Same $\tilde{D}^0 \equiv [\, D^0/\bar{D}^0 \,]$ ➔ various final states to enhance the V_{ub}/V_{cb} interference

3 theoretically "clean" methods (no penguins, no New Physics):

- $\tilde{D}^0_{CP} \equiv$[CP-eigenstate] : GLW — PLB253,483(1991) PLB265,172(1991)
- $D^0 \to [K^+\pi^-]$ & $\bar{D}^0 \to [K^-\pi^+]$ (Wrong Sign): ADS — PRL78,3257(1997) PRD63,036005(2001)
- $\tilde{D}^0 \equiv [K^0_S\pi^+\pi^-, K^0_S K^+K^-, \pi^+\pi^-\pi^0]$: Dalitz/GGSZ (not just counting). — PRL78,3257(1997) PRD68,054018(2003)

➔ Neglect D^0-$\bar{D}^0$ mixing and CPV therein (< or ~1%)

GLW method : $B^- \to \tilde{D}^{(*)0}[\text{CP-eigenstate}]_D K^{(*)-}$

- Theoretically very clean to determine γ (but 8 fold-ambiguities)
- Relatively small BFs ~10^{-6} (including sec. BFs) STATISTICS LIMITED !

  $\Rightarrow$ small CP asymmetry (r_B=?)
- Reconstruct D meson in CP-eigenstates (accessible to D^0 and $\bar{D}^0$), & in many modes (normalize to $D^{(*)0}$ flavour state decays ($K^-\pi^+$)):

  - CP-even (CP+) $\equiv D_+$: K^+K^-, $\pi^+\pi^-$
  - CP-odd (CP-) $\equiv D_-$: $K^0_S\pi^0$, $K^0_S\omega[\pi\pi\pi^0]$, $K^0_S\phi[KK]$

  Use channels D^{*0} to $D^0\pi^0/D^0\gamma$ and K^{*-} to $K^0_S\pi^-$

Schematic view:

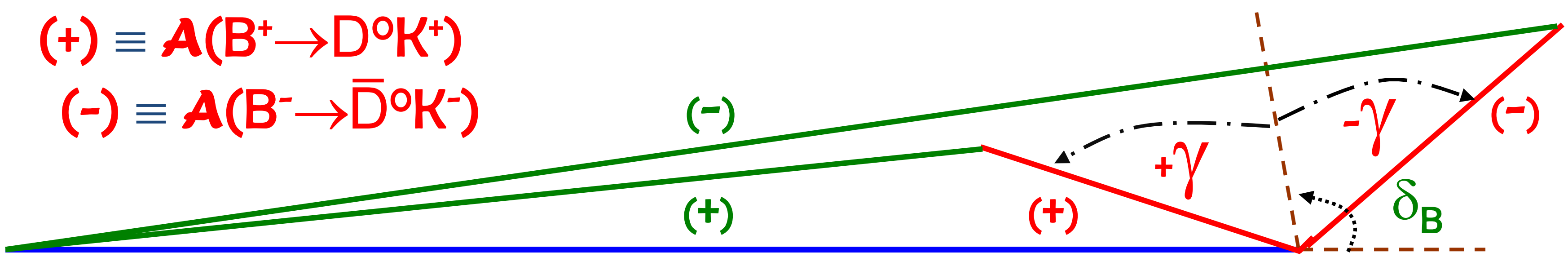
$\mathcal{A}_{tot} = \mathcal{A}_{cb} + \mathcal{A}_{ub}$

$(\pm) \equiv \sqrt{2}\ \mathcal{A}(B^\pm \to D_{CP}K^\pm)$

$(+) \equiv \mathcal{A}(B^+ \to D^0K^+)$

$(-) \equiv \mathcal{A}(B^- \to \bar{D}^0K^-)$

Here we plot: γ~60°, δ_B~100°,
and $r_B \equiv |\mathcal{A}/\mathcal{A}|$~0.25 (very optimistic)



$\mathcal{A}(B^+ \to \bar{D}^0K^+) = \mathcal{A}(B^- \to D^0K^-)$

GLW : observables and Cartesian coordinates

- ratio of BFs:

$$R_{CP\pm} = 1 + r_B^2 \pm 2\, r_B \cos(\delta_B) \cos(\gamma)$$

weak sensitivity to $r_B^2 \ll 1$

- direct CPV ($B^+ \leftrightarrow B^-$):

$$A_{CP\pm} = \frac{\pm 2\, r_B \sin(\delta_B) \sin(\gamma)}{R_{CP\pm}}$$

→ 8 fold ambiguities

→ 3 observables are independent

($A_{CP+} R_{CP+} = - A_{CP-} R_{CP-}$)

and 3 unknowns (r_B, γ, δ_B)

Let's define the Cartesian coordinates $(x_\pm, y_\pm) = (\mathcal{Re}, \mathcal{Im})\, z_\pm$

(another parametrisation, naturally related to the decay amplitudes $A_{ub}(B^\pm)$):

$$x_\pm = r_B \cos(\delta_B \pm \gamma) = \frac{R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-})}{4}$$

$$y_\pm = r_B \sin(\delta_B \pm \gamma)$$ $\Rightarrow$ Not accessible from GLW observables

So that:

$$r_B^2 = \frac{R_{CP+} + R_{CP-} - 2}{2} = x_\pm^2 + y_\pm^2$$

GLW : $B^- \rightarrow D^0_{(CP)} K^-$

PRD 77, 111102(R) (June 2008)

382×10^6 $B\bar{B}$

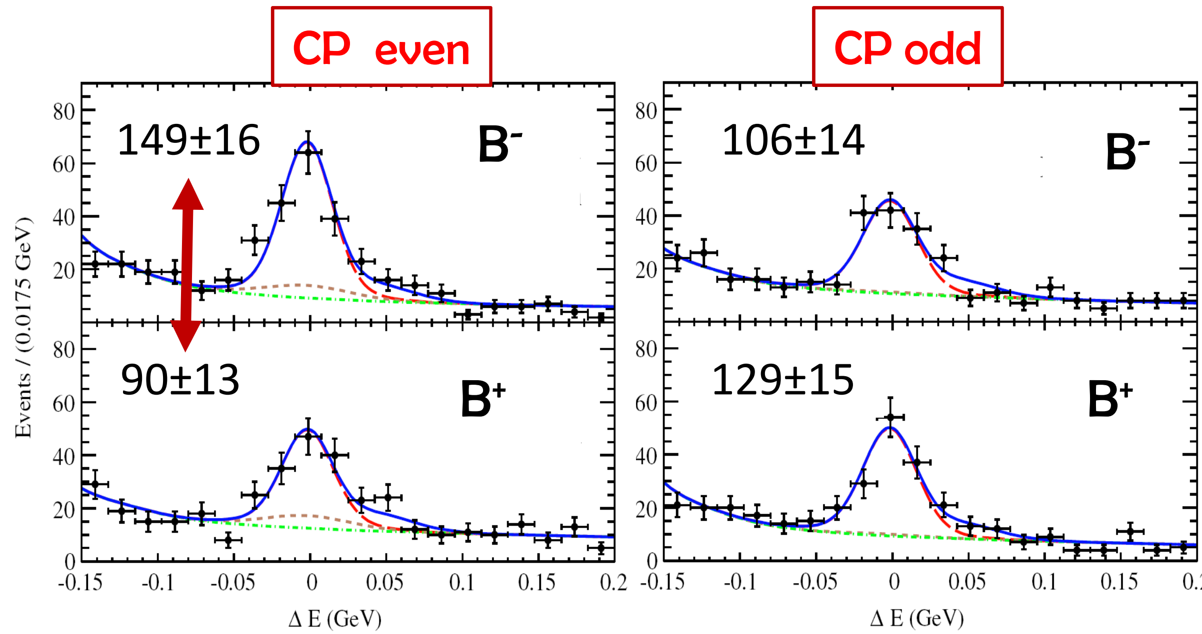
- Selection based on m_{ES} and event shape variables.
- Extended max. likelihood fit to ΔE and Cherenkov angle θ_C of the prompt track.
- Use of $B^- \rightarrow D^0\pi^-$ as normalization channel and control sample, and D^0 mass side-bands..
- No $D^0 \rightarrow K^0_S\phi$ mode (no overlap with GGSZ $D^0 \rightarrow K^0_S K^+ K^-$ measurement)
- Accounts for CP+ contamination for $K_S\omega(\pi\pi\pi^0)$ from helicity meas. in data, detector charge asymmetry, $DK/D\pi$ eff'cy ≠, peaking background, PDFs, R_{CP} from double ratio $R\pm/R$ ...

$N_{CP+}=239\pm21$
$N_{CP-}=235\pm21$
$N_{K\pi}=1872\pm51$

±stat.±syst.

$$\begin{aligned} A_{CP+} &= 0.27 \pm 0.09 \pm 0.04 \\ A_{CP-} &= -0.09 \pm 0.09 \pm 0.02 \\ R_{CP+} &= 1.06 \pm 0.10 \pm 0.05 \\ R_{CP-} &= 1.03 \pm 0.10 \pm 0.05 \end{aligned}$$

$$\begin{aligned} x_+ &= -0.09 \pm 0.05 \pm 0.02 \\ x_- &= +0.10 \pm 0.05 \pm 0.03 \\ r_B^2 &= +0.05 \pm 0.07 \pm 0.03 \end{aligned}$$



1. Direct CPV at 2.8 σ for CP+ decays
2. Not enough sensitivity to γ, but :

- most precise GLW measurement.
- $x_\pm$ compatible with GGSZ and as precise.

GLW : $B^- \to D^{*0}_{(CP)} K^-$, $D^{*0} \to D^0\pi^0/\gamma$

arXiv:0807.2408 hep/ex (July 2008), submitted to PRD

$383\times10^6\ B\bar{B}$

- Selection based on m_{ES} and event shape variables.
- Extended max. likelihood fit to ΔE and dE/dx + Cherenkov PID of fast track.
- Use of $B^- \to D^{*0}\pi^-$ as normalization channel and control sample and D^0 mass side-bands.
- D^{*0} CP flips for $D^0\pi^0$ and $D^0\gamma$ same D^0 final states (PRD 70, 091503 (2004))
- Accounts for CP+ contamination for $K^0_S\omega(\pi\pi\pi^0)$ & $K^0_S\phi(KK)$ (S waves), detector charge asymmetry, peaking background, PDFs, $D^*\pi^-/\rho^-$ BFs, $\pi^0 \Leftrightarrow \gamma$ cross-feed, R_{CP} from double ratio ...

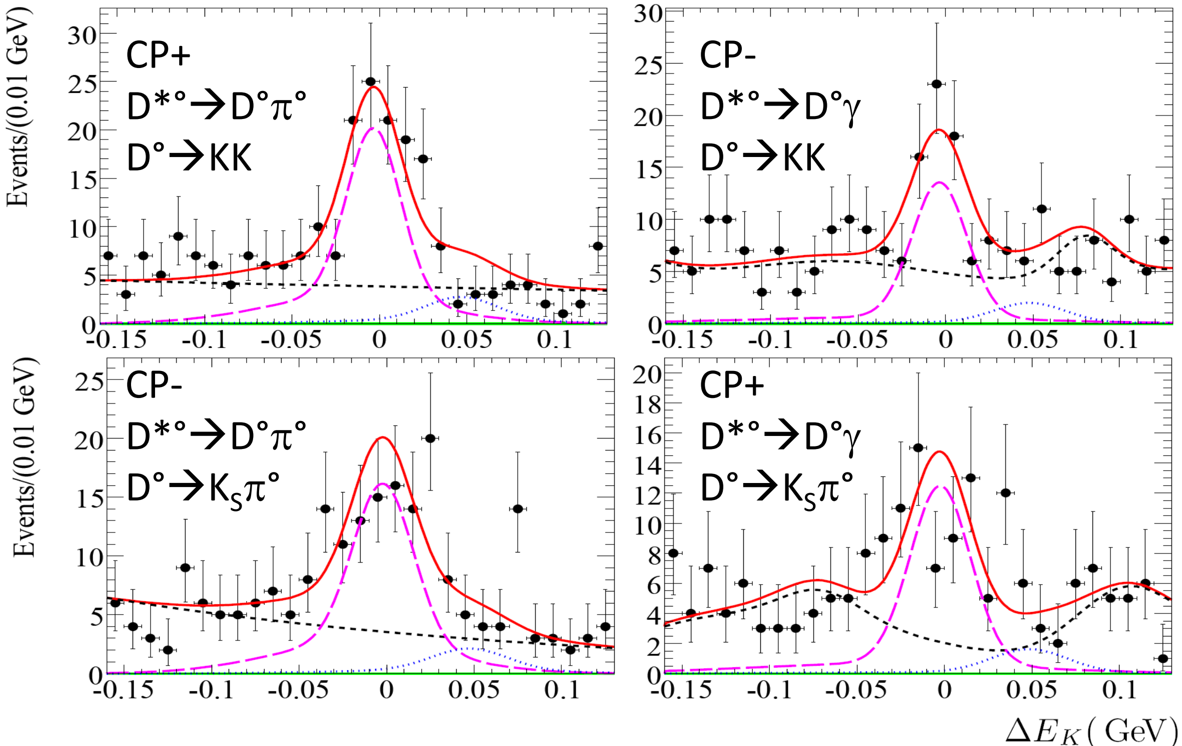
$N_{CP+}=244\pm22$
$N_{CP-}=225\pm23$
$N_{K\pi}=1410\pm57$

±stat.±syst.

$$A^*_{CP+} = -0.11 \pm 0.09 \pm 0.01$$
$$A^*_{CP-} = 0.06 \pm 0.10 \pm 0.01$$
$$R^*_{CP+} = 1.31 \pm 0.13 \pm 0.04$$
$$R^*_{CP-} = 1.10 \pm 0.12 \pm 0.04$$

$$x^*_+ = +0.09 \pm 0.07 \pm 0.02$$
$$x^*_- = -0.02 \pm 0.06 \pm 0.02$$
$$r^{*\,2}_B = +0.22 \pm 0.09 \pm 0.03$$

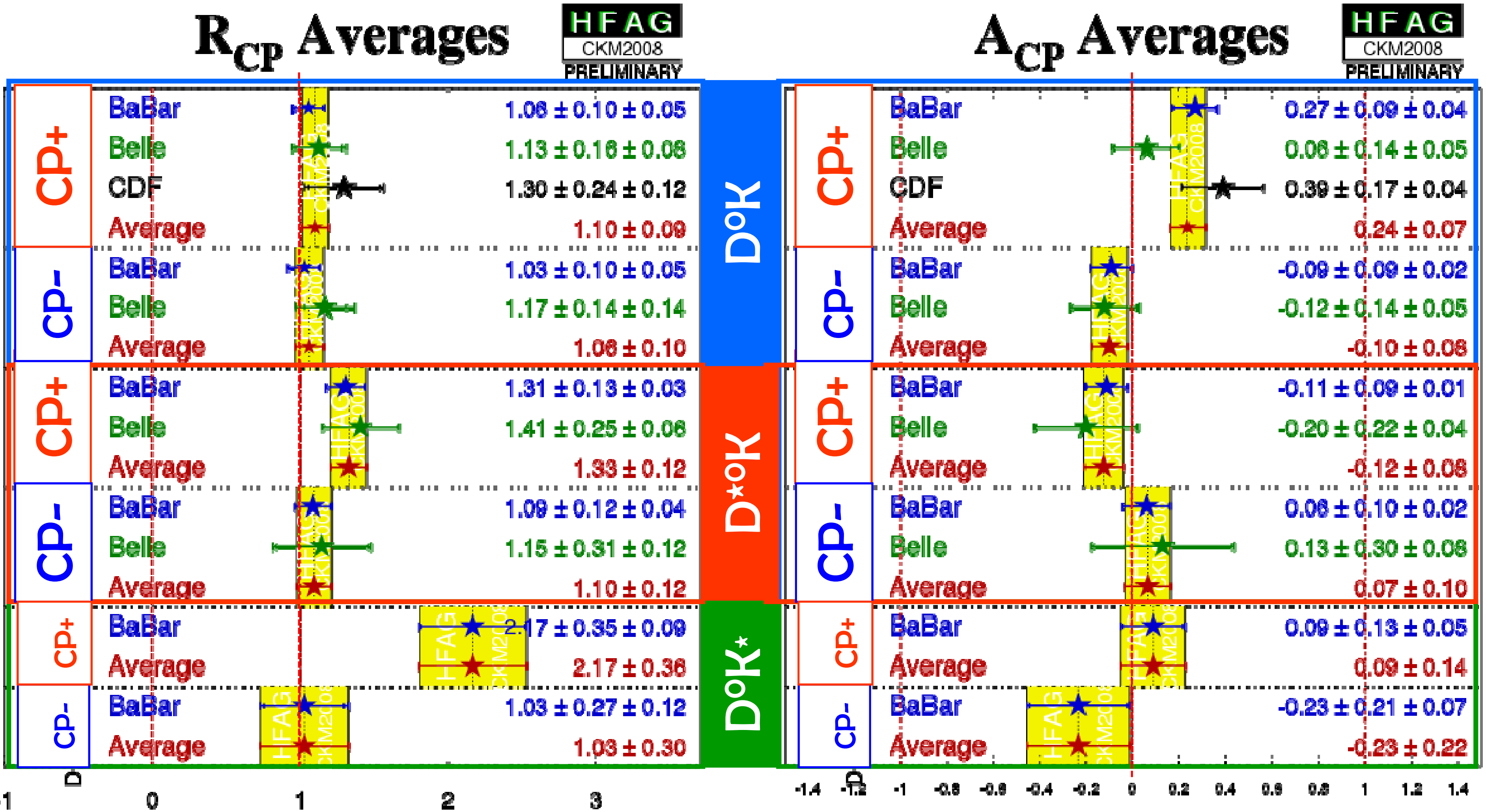
($K^0_S\phi$ removed for Cartesian coords.)



1. No Direct CPV seen.
2. Not enough sensitivity to γ, but :

- most precise D*K GLW measurement.
- $x^*_\pm$ compatible with GGSZ and as precise, r^*_B expected high.

GLW : status as of CKM08



➔ *BaBar* has the most precise GLW measurements (Belle uses 275x10⁶ $B\bar{B}$ wrt 383x10⁶), CDF is a new comer since FPCP'08. And DK*- from *BaBar* with 232x10⁶ $B\bar{B}$ only…

➔ With current statistics it is not possible to constraint γ with GLW measurements alone, but help significantly to improve global constraint on γ and r_B.

ADS method : $B^- \to \tilde{D}^{(*)0}[K^+\pi^-]_D K^{(*)-}$

- Same idea as for GLW, same final state in different $\tilde{D}^0$ $[\bar{D}^0/D^0]$ states:

$[K^+\pi^-]_D K^-$:Doubly-Cabibbo-Suppressed (DCS) decays instead of CP-eigenstate.

Suppressed (V_{ub})

$B^- \to \bar{D}^0 K^-$

$\hookrightarrow K^+\pi^-$

Favored (Right-Sign)

interference

Favored

$B^- \to D^0 K^-$

$\hookrightarrow K^+\pi^-$

DCS

Suppressed (Wrong-Sign)

- Small BFs($\sim 10^{-6}$), but amplitudes ~ comparable in size: expect larger CPV!
- Count B candidates with opposite sign K !

$$\mathcal{A}(B^- \to [K^+\pi^-]_D K^-) \propto r_B e^{i(\delta_B - \gamma)} + r_D e^{-i\delta_D}$$

$$r^2_D = \left|\frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)}\right|^2 = (0.365 \pm 0.021)\%$$

PLB 592,1 (PDG 2004)

δ_D : D decay strong phase unknown (scan all possible values).

ADS : observables

$$\mathcal{A}(B^- \to [K^+\pi^-]_D K^-) \propto r_B e^{i(\delta_B - \gamma)} + r_D e^{-i\delta_D}$$

$B^- \to B^+ \Rightarrow -\gamma \to +\gamma$, $K^- \leftrightarrow K^+$, $K^+ \leftrightarrow K^-$

2 observables

- ratio of BFs: (Wrong Sign $D^0 \to K^+\pi^-$/Right Sign $D^0 \to K^-\pi^+$)

$$\mathrm{R}_{\mathrm{ADS}} \equiv \frac{\Gamma([\mathbf{K}^+\pi^-]K^-) + \Gamma([\mathbf{K}^-\pi^+]K^+)}{\Gamma([K^-\pi^+]K^-) + \Gamma([K^+\pi^-]K^+)} = \mathbf{r}_{\mathbf{B}}^{\mathbf{2}} + \mathbf{r}_{\mathbf{D}}^{\mathbf{2}} + 2\mathbf{r}_{\mathbf{B}}\mathbf{r}_{\mathbf{D}} \cos(\delta_{\mathbf{B}} + \delta_D) \cos(\gamma)$$

good sensitivity to r_B^2

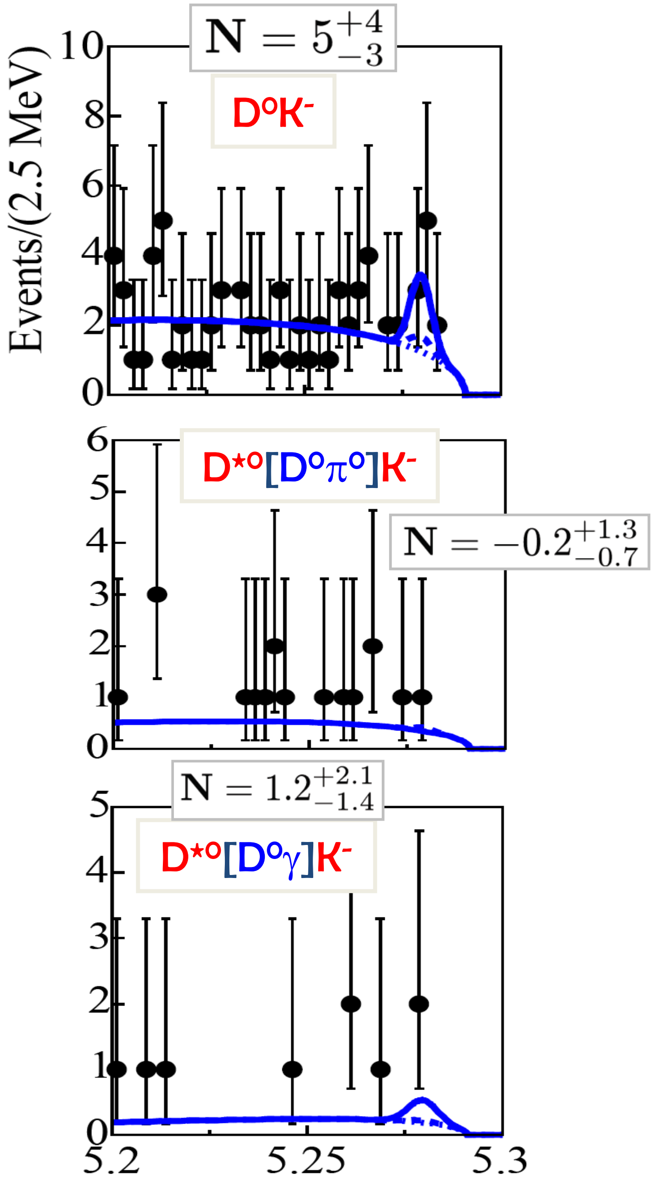
- direct ACPV: $B^+ \leftrightarrow B^-$ direct asymmetry in yield if enough events seen.

$$\mathrm{A}_{\mathrm{ADS}} \equiv \frac{\Gamma[\mathbf{K}^+\pi^-]K^-) - \Gamma([\mathbf{K}^-\pi^+]K^+)}{\Gamma[\mathbf{K}^+\pi^-]K^-) + \Gamma([\mathbf{K}^-\pi^+]K^+)} = \frac{2\ \mathrm{r_B}\ \mathrm{r_D}\ \sin(\delta_{\mathrm{B}} + \delta_{\mathrm{D}})\ \sin(\gamma)}{\mathrm{R}_{\mathrm{ADS}}}$$

ADS $D^{(*)0}[K^+\pi^-]K^-$ results

PRD 72, 032004 (2005)

$232\times10^6\ B\bar{B}$



No significant signal yet

$\Rightarrow$ only (Bayesian) limits on

R_{ADS} and $r^{(*)}_B$

(using: $|\cos(\delta^{(*)}_B+\delta_D)\cos\gamma|<1$

and upper limits for $\gamma=0°$, $\delta^{(*)}=180°$

or $\gamma=180°$, $\delta^{(*)}=0°$ worst case scenario maximal

$b\to c$ and $b\to u$ destructive interference, $\delta^{(*)}=\delta_D+\delta^{(*)}_B$)

	R_{ADS}	"r_B"
D^0K	<0.029 90% prob	r_B <0.23 90% prob
$D^{*0}K$	<0.023 ($D^{*0}\to D^0\pi^0$) <0.045 ($D^{*0}\to D^0\gamma$) 90% prob	$(r^*_B)^2<(0.16)^2$ 90% prob + Bondar & Gershon PRD70,091503(2004)

ADS $D^0[K^+\pi^-\pi^0]K^-$ results

PRD 76, 111101(R) (2007)

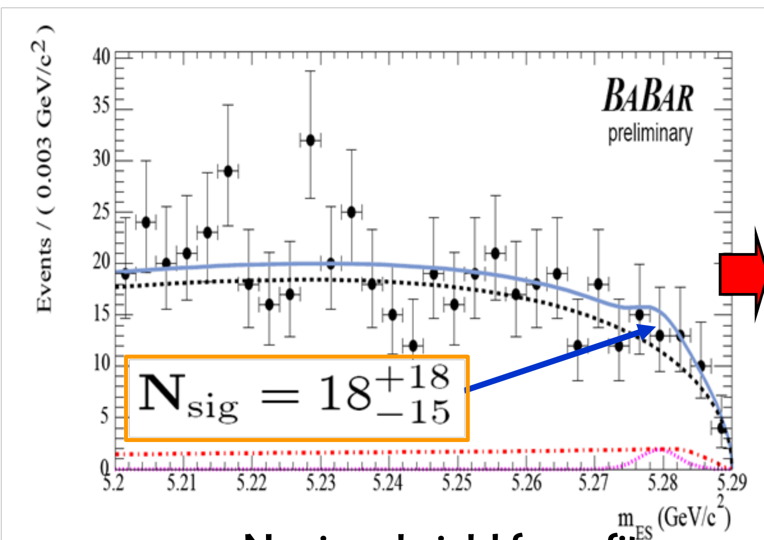
$226\times10^6\ B\bar{B}$

- Similar to previous analyses with DCS $D^0 \to K^+\pi^-\pi^0$
- Complication for γ extraction from $|A_D|$, δ_D varying across the D^0 Dalitz plane

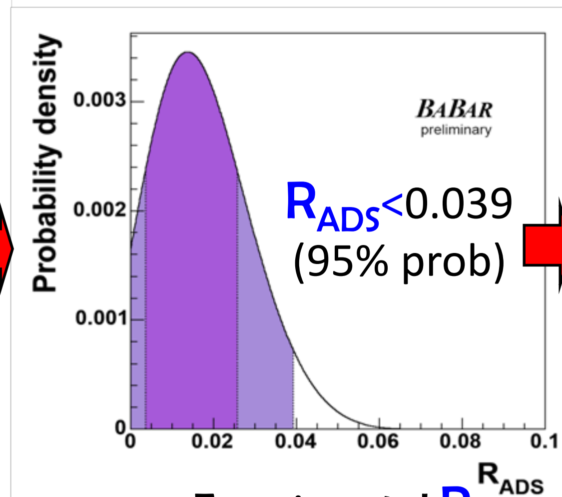
$$R_{ADS} \equiv r_B^2 + r_D^2 + 2 r_B r_D C\cos(\gamma) \qquad C = \frac{\int A_D(\vec{s})\bar{A}_D(\vec{s})\cos(\delta_D(\vec{s}) + \delta_B(\vec{s}))d\vec{s}}{\sqrt{\int |A_D(\vec{s})|^2 d\vec{s}}\sqrt{\int |\bar{A}_D(\vec{s})|^2 d\vec{s}}}$$

$$\vec{s} = (m^2_{K\pi}, m^2_{K\pi^0})$$

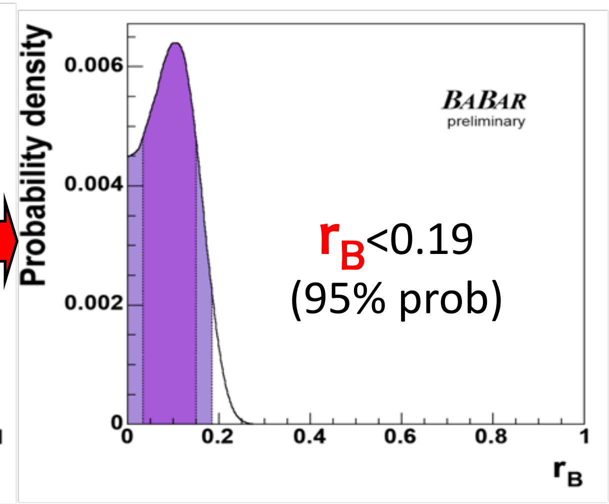
  - C is unknown, $|C|\le1$
  - $r_D^2 = (0.214\pm0.011)\%$ PRL 97, 221803 (2005)
  - Compared to $K\pi$: more background but higher BF and smaller r_D (better r_B sensitivity)
- Similar sensitivity to r_B (limit on R_{ADS} using $|C\cos\gamma|\le1$)



~No signal yield from fit



Experimental R_{ADS} integrated likelihood



Bayesian $L(r_B)$ (flat r_B and $|C\cos\gamma|$ prior PDFs)

Dalitz GGSZ : $B^- \to \tilde{D}^{(*)0}[K^0_s h^+ h^-]_D K^{(*)-}$, $h=\pi/K$

- $\tilde{D}^0 \to K^0_S[\pi^+\pi^-/K^+K^-]$ 3-body self conjug. final states accessible through many different decays. Only $(\pi/K)^\pm$: clean, efficient, and reasonable $BF([K^0_S\pi\pi]_D K^-) \approx 10^{-5}$ ($\times$10 D^0_{CP}, $BF(K^0_S\pi\pi)/BF(K^0_S KK)$~6).

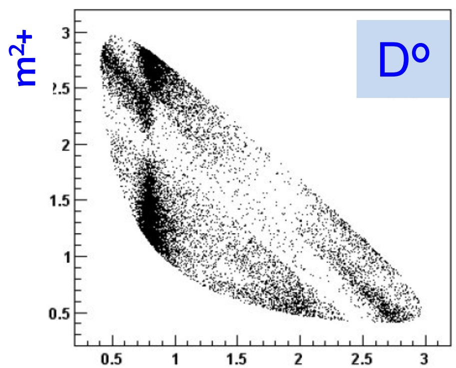
→ need Dalitz structure analysis : $D^0/\bar{D}^0$ decay amplitudes $\mathcal{A}_{D-/+}$ to separate interferences between resonances ⇒ precise modelization.

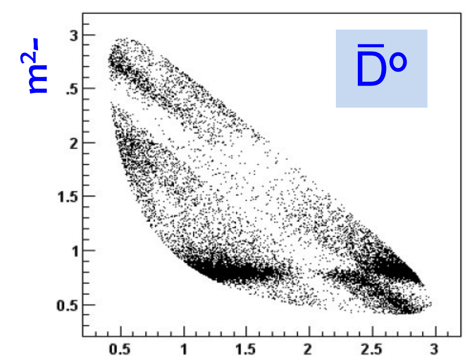
$m^2_\pm = m^2(K^0_s h^\pm)$

- schematic view of interference ($b \to c$) ⇔ ($b \to u$):

$B^- \to B^+ \to -\gamma \to +\gamma$, $m_- \leftrightarrow m_+$, $\bar{D}^0 \leftrightarrow D^0$

$$\mathcal{A}(B^-) = |\mathcal{A}_B(D^0K^-)| \times \left(\times \mathcal{A}_{D-/+} + r_B e^{i(\delta_B - \gamma)} \times \mathcal{A}_{D+/-} \right)$$





sensitivity varies strongly over Dalitz plane
⇒ model + mixture ADS+GLW

Simultaneous fit to $\tilde{D}^0 \to K^0_s h^+h^-$ Dalitz plot density of B^+/B^- data to extract from this density difference r_B's, δ_B's, and γ

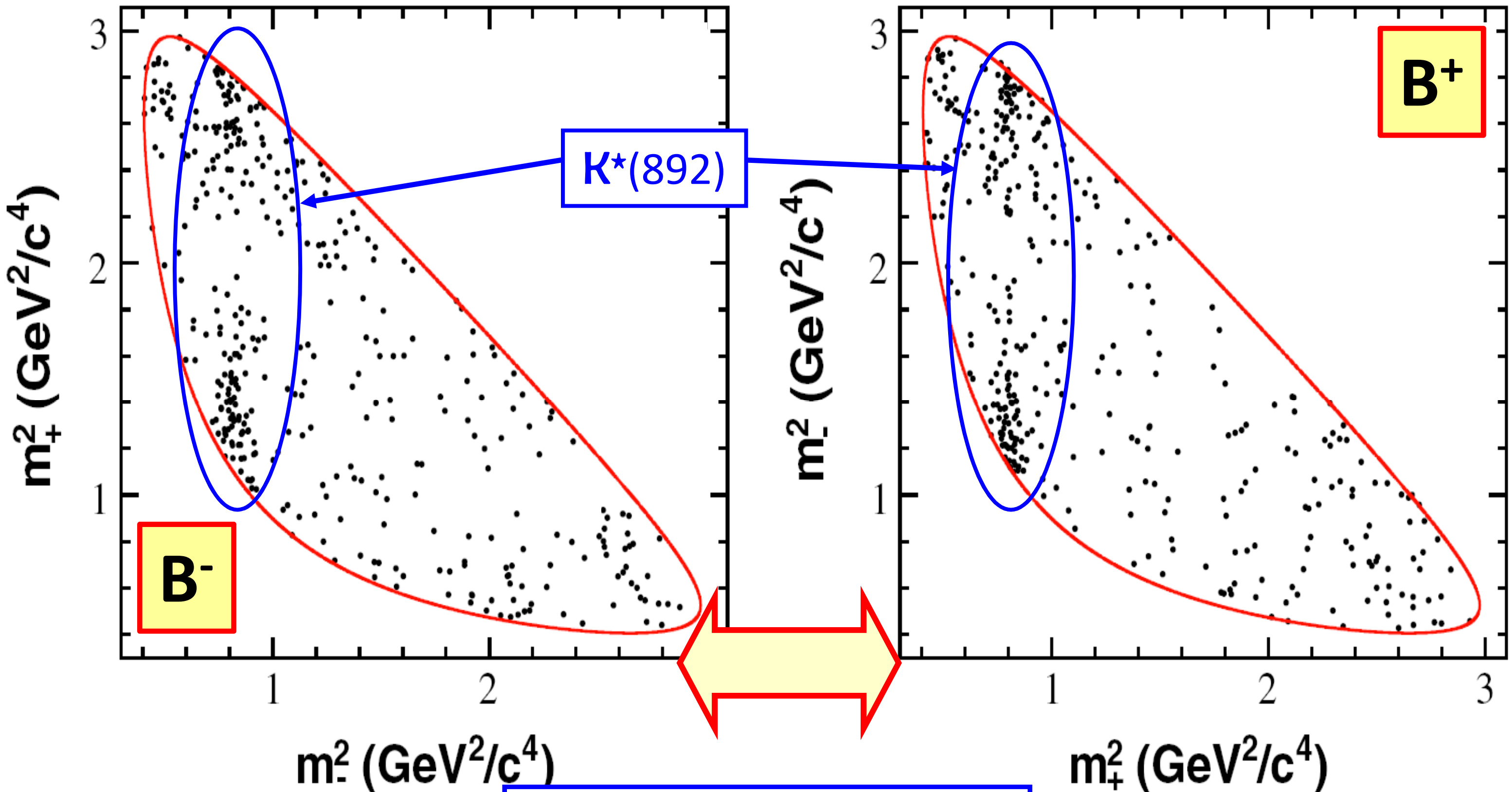
- No D^0 mixing, nor CPV in D decays.
- 2 fold ambiguity : $(\gamma,\delta_B) \to (\gamma+\pi, \delta_B+\pi)$

$B^- \to \tilde{D}^0[K^0_s\pi^+\pi^-]_D K^-$: Dalitz CPV ?

PRD78, 034023 (2008)

$383\times10^6\ B\bar{B}$

$600\pm31\ \tilde{D}^0K^-$



$B^- \Leftrightarrow B^+$ differences
mean CPV

Fit parameters $B^- \to \tilde{D}^{(*)0} K^{(*)-}$

PRD78, 034023 (2008)

$383 \times 10^6 \; B\bar{B}$

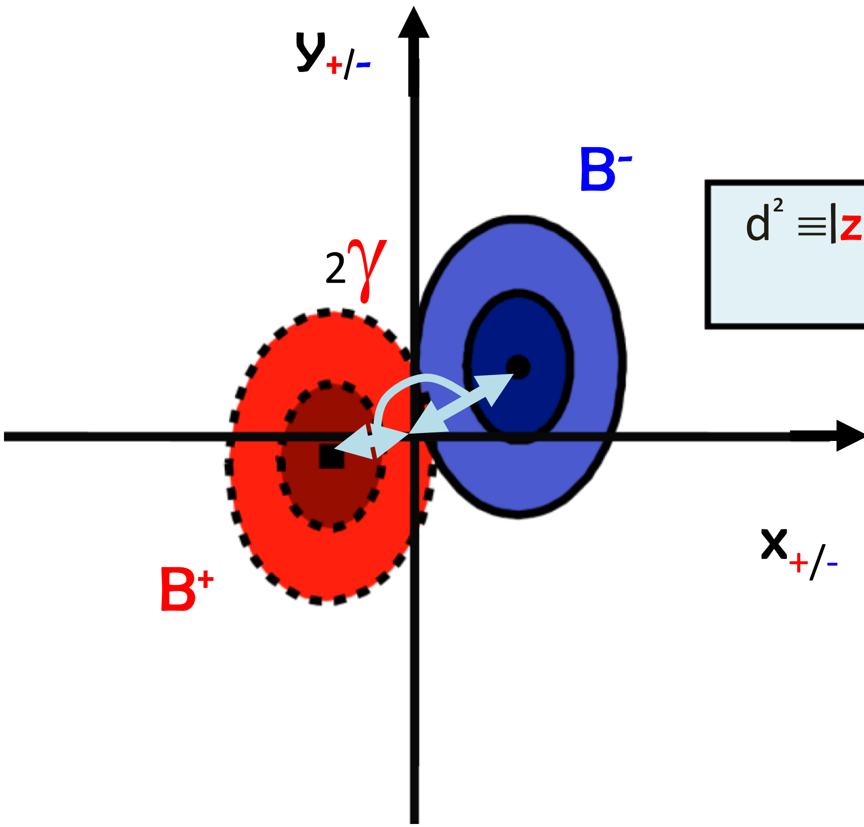
➔ 3 kinds of decays: 7 unknowns:

- r_B, r^*_B, r_{sB}
- δ_B, δ^*_B, δ_{sB}
- γ

Non Gaussian effects + biases : low stat. sample & low sensitivity near physical bound $r_B \sim 0$

Extraction of CP parameters from multivariable max. likelihood simultaneous fit to almost Gaussian and uncorrelated Cartesian coordinates 12=(3×4): $(x_\pm, y_\pm)$, $(x^*_\pm, y^*_\pm)$, & $(x_{s\pm}, y_{s\pm})$.

$$z^{(*)}_{(s)\pm} \equiv (x^{(*)}_{(s)\pm}, y^{(*)}_{(s)\pm}) = (\mathcal{Re}, \mathcal{Im})\{r^{(*)}{}_{s_B} e^{i(\delta^{(*)}_{(s)B} \pm \gamma)}\}$$



$$d^2 \equiv |z_{B+} - z_{B-}|^2 = r^2_{B+} + r^2_{B-} - 2\, r_{B+} r_{B-} \cos(2\gamma)$$

$d \neq 0 \Rightarrow$ size of direct CPV

for $\tilde{D}^0 K^{*-}$

$$(x_{s\pm}, y_{s\pm}) = (\mathcal{Re}, \mathcal{Im})\{\kappa . r s_B e^{i(\delta_{sB} \pm \gamma)}\}$$

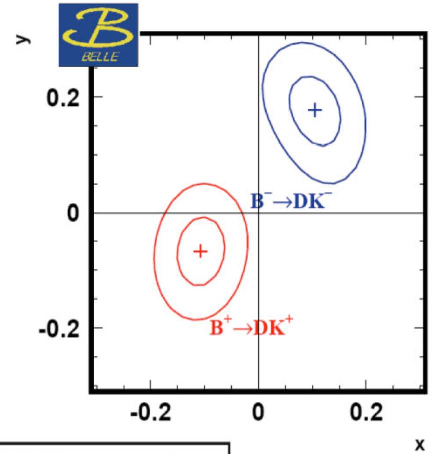
Gronau PLB557, 198(2003)

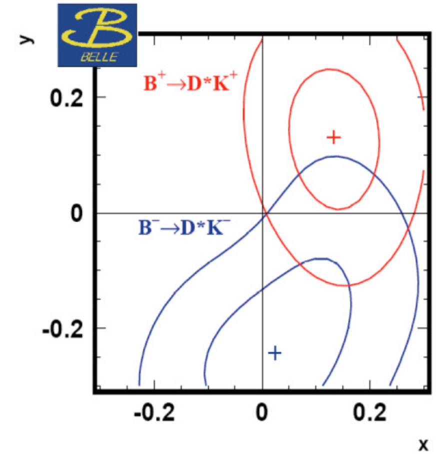
$\kappa \in [0,1]$: accounts for $(K^0_S \pi^-)$non-K^{*-} large natural width bckgd $\Rightarrow$ no assumptions on: nature, number, strong phases ...

Comparison with Belle

Belle preliminary arXiv:0803.3375 **657M BB**

Parameter	$B^+ \to DK^+$	$B^+ \to D^*K^+$
x_-	$+0.105 \pm 0.047 \pm 0.011$	$+0.024 \pm 0.140 \pm 0.018$
y_-	$+0.177 \pm 0.060 \pm 0.018$	$-0.243 \pm 0.137 \pm 0.022$
x_+	$-0.107 \pm 0.043 \pm 0.011$	$+0.133 \pm 0.083 \pm 0.018$
y_+	$-0.067 \pm 0.059 \pm 0.018$	$+0.130 \pm 0.120 \pm 0.022$

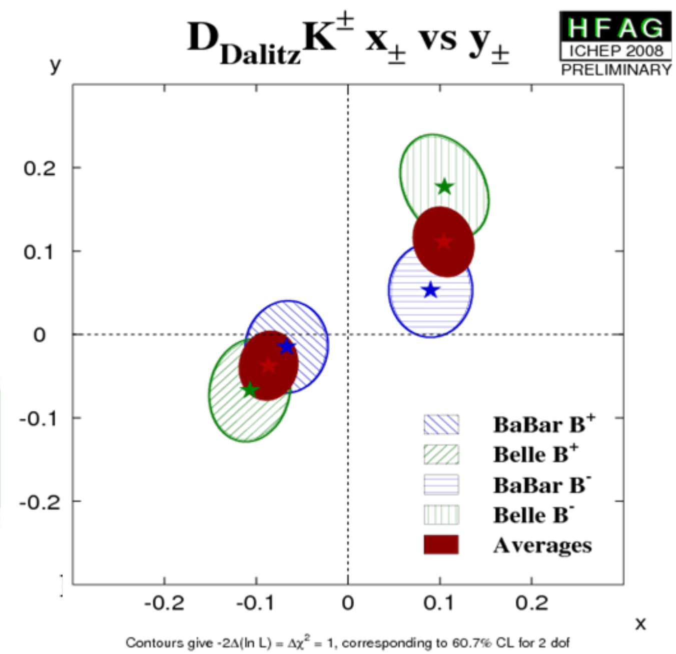




Parameter	1σ interval	2σ interval	Systematic error	Model uncertainty
ϕ_3	$76^{\circ}\,^{+12^{\circ}}_{-13^{\circ}}$	$49^{\circ} < \phi_3 < 99^{\circ}$	4°	9°
r_{DK}	0.16 ± 0.04	$0.08 < r_{DK} < 0.24$	0.01	0.05
r_{D^*K}	0.21 ± 0.08	$0.05 < r_{D^*K} < 0.39$	0.02	0.05
δ_{DK}	$136^{\circ}\,^{+14^{\circ}}_{-16^{\circ}}$	$100^{\circ} < \delta_{DK} < 163^{\circ}$	4°	23°
δ_{D^*K}	$343^{\circ}\,^{+20^{\circ}}_{-22^{\circ}}$	$293^{\circ} < \delta_{DK} < 389^{\circ}$	4°	23°

Parameters	$B^- \to \tilde{D}^0 K^-$
x_-	$0.090 \pm 0.043 \pm 0.015 \pm 0.011$
y_-	$0.053 \pm 0.056 \pm 0.007 \pm 0.015$
x_+	$-0.067 \pm 0.043 \pm 0.014 \pm 0.011$
y_+	$-0.015 \pm 0.055 \pm 0.006 \pm 0.008$

$$r_B = 0.086 \pm 0.035$$
$$r_B^* = 0.135 \pm 0.051$$



$$\gamma = (76^{+22}_{-23} \pm 5 \pm 5)^{\circ}$$

$$\gamma = (76^{+12}_{-13} \pm 4 \pm 9)^{\circ}$$

BaBar errors on x,y are comparable with Belle, but error on γ is worse:
"$1/r_B$ effect"

Main systematic errors

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Experimental syst.

- m_{ES}, ΔE, Fisher PDF's shapes
- Fractions of D^0 in the backg
- ***Background Dalitz shape***
- Efficiency in the Dalitz plot
- Cross feed($D^0\gamma$-$D^0\pi^0$)
- CPV in $D\pi$ and $B\bar{B}$ bkg
- Charge flavor correlation
- Non-K* decays ($B^- \to D^0 K_s^0 \pi$)

Dalitz model

- used alternative models for $K_s\pi\pi$ and K_sKK, where the resonances are described with different parametrizations or removed. (e.g: different solution of the K-matrix to describe $\pi\pi$ S-wave,different parametrization to describe $K_0^*(1430)$, ...)
- Dalitz efficiency
- Bkg Dalitz shape
- ... (see Diego Milanes talk)

Stat. error is dominant

Example:

$$x_- = 0.090 \pm \boxed{0.043} \pm \boxed{0.015} \pm \boxed{0.011}$$
$$y_- = 0.053 \pm \boxed{0.056} \pm \boxed{0.007} \pm \boxed{0.015}$$

Frequentistic method

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- Interpretation of results
  - With cartesian coordinates measured, **z**, we construct a multivariate gaussian PDF to relate it with the relevant parameters **p**

$$\mathcal{L}(\mathbf{z};\mathbf{p};V) = \frac{1}{(2\pi)^{n/2}\sqrt{|V|}} e^{-\frac{1}{2}(\mathbf{z}-\mathbf{z}^{(t)})^T V^{-1}(\mathbf{z}-\mathbf{z}^{(t)})} \equiv \frac{1}{(2\pi)^{n/2}\sqrt{|V|}} e^{-\frac{1}{2}\chi^2(\mathbf{z};\mathbf{p};V)}$$

  - The CL of a true parameter, μ, is calculate minimizing respect the other parameters, **q**. For each value of μ in its range the fit will return a $\chi^2_{\mathrm{min}}(\mu_0,\mathbf{q}_0)$.
  - In a 100% gaussian case, the CL is given by

$$\Delta\chi^2(\mu_0) = \chi^2_{\mathrm{min}}(\mu_0,\mathbf{q_0}) - \chi^2_{\mathrm{min}}$$

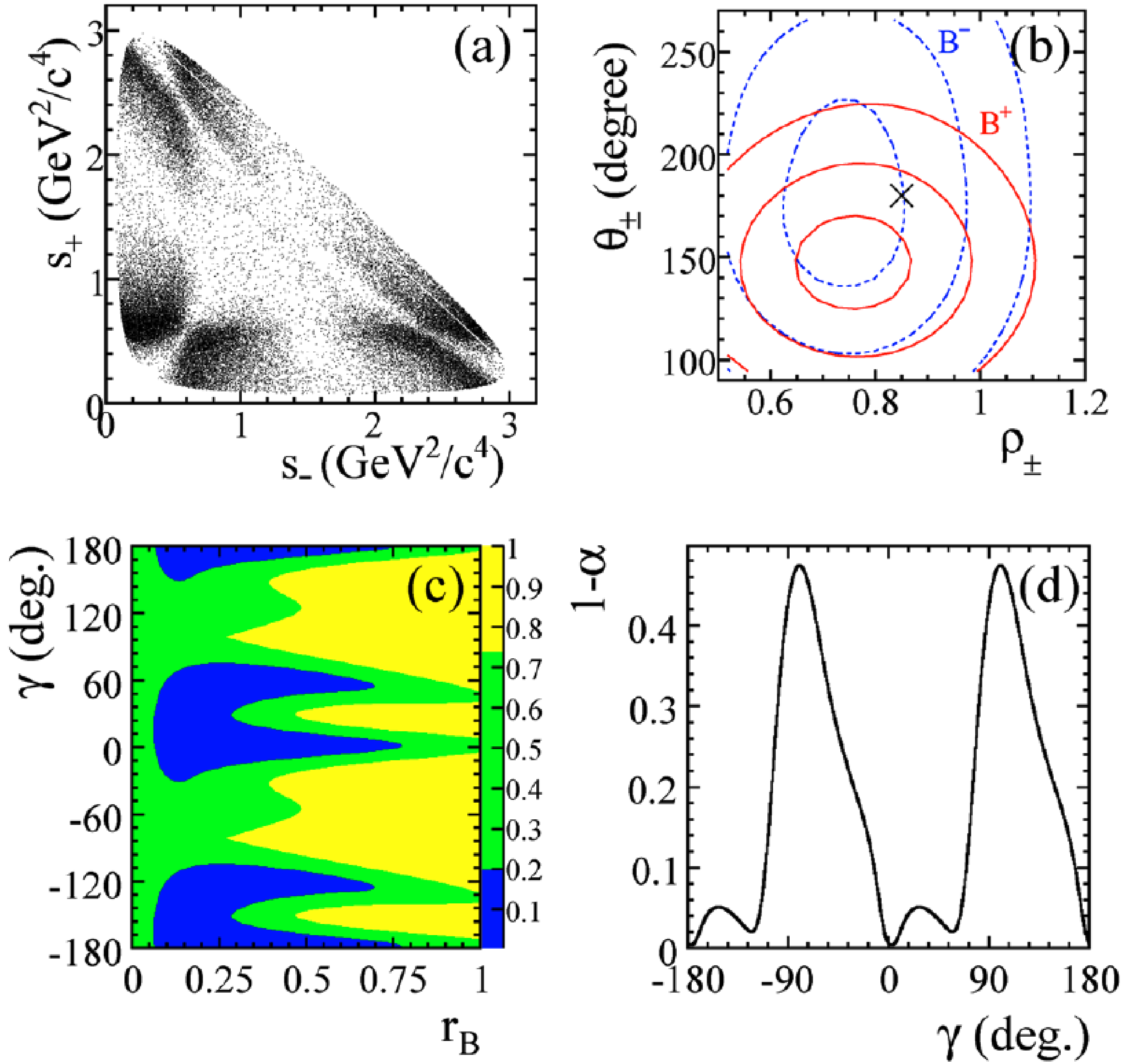
$$\mathrm{CL} = 1-\alpha = Prob(\Delta\chi^2(\mu_0),\nu=1) = \frac{1}{\sqrt{2^\nu}\Gamma(\nu/2)}\int_{\Delta\chi^2(\mu_0)}^{\infty} e^{-t/2}t^{\nu/2-1}dt$$

- In practice, use toy MC to evaluate CL

- Accounts for unphysical regions ($r_B+ \neq r_B-$) of parameter space (Feldman-Cousins)
- 1D intervals CL

Component	a_r	ϕ_r (deg)	Fraction (%)	
$K^*(892)^-$	1.740 ± 0.010	139.0 ± 0.3	55.7 ± 2.8	CA $K^*\pi$
$K_0^*(1430)^-$	8.2 ± 0.7	153 ± 8	10.2 ± 1.5	
$K_2^*(1430)^-$	1.410 ± 0.022	138.4 ± 1.0	2.2 ± 1.6	
$K^*(1680)^-$	1.46 ± 0.10	-174 ± 4	0.7 ± 1.9	
$K^*(892)^+$	0.158 ± 0.003	-42.7 ± 1.2	0.46 ± 0.23	DCS $K^*\pi$
$K_0^*(1430)^+$	0.32 ± 0.06	143 ± 11	< 0.05	
$K_2^*(1430)^+$	0.091 ± 0.016	85 ± 11	< 0.12	
$\rho(770)^0$	1	0	21.0 ± 1.6	$\pi\pi$ P,D waves
$\omega(782)$	0.0527 ± 0.0007	126.5 ± 0.9	0.9 ± 1.0	
$f_2(1270)$	0.606 ± 0.026	157.4 ± 2.2	0.6 ± 0.7	
β_1	9.3 ± 0.4	-78.7 ± 1.6		$\pi\pi$ S- wave (K-matrix)
β_2	10.89 ± 0.26	-159.1 ± 2.6		
β_3	24.2 ± 2.0	168 ± 4		
β_4	9.16 ± 0.24	90.5 ± 2.6		
$f_{11}^{\rm prod}$	7.94 ± 0.26	73.9 ± 1.1		
$f_{12}^{\prime\,\rm prod}$	2.0 ± 0.3	-18 ± 9		
$f_{13}^{\prime\,\rm prod}$	5.1 ± 0.3	33 ± 3		
$f_{14}^{\prime\,\rm prod}$	3.23 ± 0.18	4.8 ± 2.5		
$s_0^{\rm prod}$	-0.07 ± 0.03			
$\pi\pi$ S-wave			11.9 ± 2.6	
M (GeV/c^2)	1.463 ± 0.002			$K\pi$ S- wave
Γ (GeV/c^2)	0.233 ± 0.005			
F	0.80 ± 0.09			
ϕ_F	2.33 ± 0.13			
R	1			
ϕ_R	-5.31 ± 0.04			
a	1.07 ± 0.11			
r	-1.8 ± 0.3			

Component	a_r	ϕ_r (deg)	Fraction (%)
$K_S^0 a_0(980)^0$	1	0	55.8
$K_S^0 \phi(1020)$	0.227 ± 0.005	-56.2 ± 1.0	44.9
$K_S^0 f_0(1370)$	0.04 ± 0.06	-2 ± 80	0.1
$K_S^0 f_2(1270)$	0.261 ± 0.020	-9 ± 6	0.3
$K_S^0 a_0(1450)^0$	0.65 ± 0.09	-95 ± 10	12.6
$K^- a_0(980)^+$	0.562 ± 0.015	179 ± 3	16.0
$K^- a_0(1450)^+$	0.84 ± 0.04	97 ± 4	21.8
$K^+ a_0(980)^-$	0.118 ± 0.015	138 ± 7	0.7


(a)
s_+ (GeV^2/c^4)
s_- (GeV^2/c^4)
(b)
B^-
B^+
$\theta_\pm$ (degree)
$\rho_\pm$
(c)
γ (deg.)
r_B
(d)
1-α
γ (deg.)