Measuring weak phases using $B \to D^{*}V$ modes

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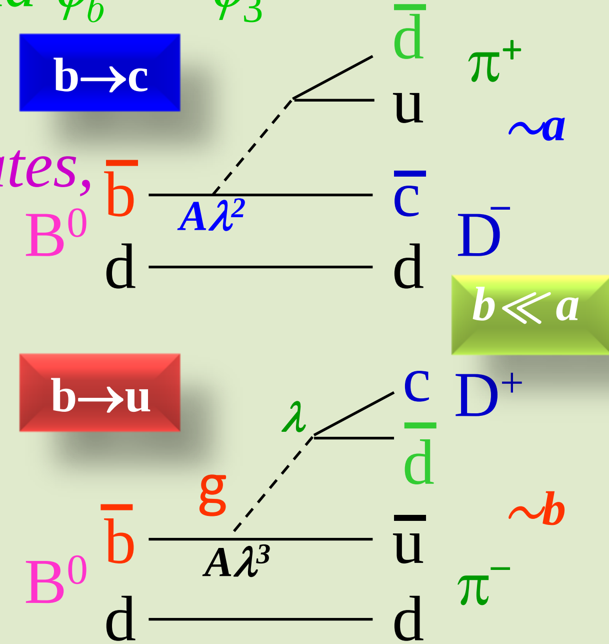
- $2\phi_1(\beta) + \phi_3(\gamma)$ *using* $B^0(t) \to D^{*+}\rho^-$
- $\phi_3(\gamma)$ *using* $B^{\pm} \to D^{*} K^{*\pm}$ *mode*

$(2\phi_1+\phi_3)$ Time dependence asymmetries in $D\pi$: Dunietz

- *Consider a final state "f" into which both B^0 and $\bar{B}^0$ decay.*
- *Only one amplitude contributes to each $B^0 \to f$ and $\bar{B}^0 \to f$.*
- *CPV can occur through interference of direct decay and mixing.*

$$A \equiv Amp(B^0 \to f) \quad = \quad a e^{i\delta^a} e^{i\phi_a} \; , \; \phi_a = 0 \text{ and } \phi_b = -\phi_3$$

$$A' \equiv Amp(\bar{B}^0 \to f) \quad = \quad b e^{i\delta^b} e^{i\phi_b} \; .$$



By measuring the time dependent decay rates,

$$\Gamma(B^0(t) \to f), \quad \Gamma(\bar{B}^0 \to f),$$
$$\Gamma(B^0(t) \to \bar{f}), \quad \Gamma(\bar{B}^0 \to \bar{f}),$$

determine the weak phase $(2\phi_1+\phi_3)$

For $f = D^-\pi^+$ mode

$$\frac{\Gamma(B^0 \to D^+\pi^-)}{\Gamma(B^0 \to D^-\pi^+)} \simeq \left| \frac{V_{ub}V_{cd}^*}{V_{cb}^*V_{ud}} \right|^2 \simeq 4 \times 10^{-4}$$

hard to measure $B^0 \to D^+\pi^-$

Atwood Dunietz Soni Method for ϕ_3

•Consider decay of $\bar{D}^0$ to doubly Cabibbo suppressed non-CP eigenstates and D^0 to the CP conjugate Cabibbo allowed mode.

•Leads to similar size of the interfering amplitudes of the B^+ decay. Larger asymmetries with such states.

$$B^+ \to D^0 K^+ \to [K^-\pi^+]_{D^0} K^+ \sim B^+ \to \bar{D}^0 K^+ \to [K^-\pi^+]_{\bar{D}^0} K^+$$

•Use knowledge of relevant Cabibbo allowed and suppressed D branching ratios

•Enough information even to solve for ϕ_3 as well as the difficult to measure $BR(B^+ \to D^0 K^+)$.

•Method very clean. No real problems. Need DCS D decay rates.

- Extend techniques to VV
- Multiple helicities offer advantages in measuring $(2\phi_1+\phi_3)$.
- VV modes as example of ADS method .

$2\phi_1 + \phi_3$ *using* $B^0(t) \to D^{*+}\rho^-$

Phys.Rev.Lett.85:1807,2000.

In the decay of $B \to V_1V_2$ *if* $V_1 \to P_1P_1'$ *and* $V_2 \to P_2P_2'$ *the decay amplitude may be expressed as*

$$A(B \to V_1V_2) \quad \propto \quad \left(A_0 \cos\theta_1 \cos\theta_2 + \frac{A_\parallel}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \cos\phi \right.$$

$$\left. -i\frac{A_\perp}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \sin\phi\right),$$

Using CPT invariance, the total decay amplitudes can be written as

$$\mathcal{A} = Amp(B^0 \to f) = A_0 g_0 + A_\parallel g_\parallel + i\, A_\perp g_\perp$$

$$\bar{\mathcal{A}} = Amp(\overline{B^0} \to \bar{f}) = \bar{A}_0 g_0 + \bar{A}_\parallel g_\parallel - i\, \bar{A}_\perp g_\perp$$

$$\mathcal{A}' = Amp(\overline{B^0} \to f) = A'_0 g_0 + A'_\parallel g_\parallel - i\, A'_\perp g_\perp$$

$$\bar{\mathcal{A}}' = Amp(B^0 \to \bar{f}) = \bar{A}'_0 g_0 + \bar{A}'_\parallel g_\parallel + i\, \bar{A}'_\perp g_\perp$$

g_λ *are the coefficients of the helicity amplitudes that depend purely on angles describing the kinematics.*

Time dependent decay rate

Time dependent decay of the neutral B meson to a final state f

$$\Gamma(B^0(t) \to f) = e^{-\Gamma t}\Big(\frac{|\mathcal{A}|^2 + |\mathcal{A}'|^2}{2} + \frac{|\mathcal{A}|^2 - |\mathcal{A}'|^2}{2}\cos(\Delta M t) - \mathrm{Im}(\frac{q}{p}\mathcal{A}^*\mathcal{A}')\sin(\Delta M t)\Big)$$

For VV mode a more complicated form

$$\Gamma(B^0(t) \to f) = e^{-\Gamma t}\sum_{\lambda \le \sigma}\Big(\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma}\cos(\Delta M t) - \rho_{\lambda\sigma}\sin(\Delta M t)\Big) g_\lambda g_\sigma$$

$$\frac{\Lambda_{\lambda\lambda} = |A_\lambda|^2 + |A'_\lambda|^2}{2}$$

$$\frac{\Sigma_{\lambda\lambda} = |A_\lambda|^2 - |A'_\lambda|^2}{2}$$

$$\rho_{ii} = \mathrm{Im}(\frac{q}{p}A_i^* A'_i)$$

$$\rho_{\perp\perp} = -\mathrm{Im}(\frac{q}{p}A_\perp^* A'_\perp)$$

3×3=9

$$i = \{0, \|\}$$

$$\Lambda_{\perp i} = -\mathrm{Im}(A_\perp A_i^* - A'_\perp A'^*_i) \qquad \Lambda_{\|0} = \mathrm{Re}(A_\| A_0^* + A'_\| {A'_0}^*)$$

$$\Sigma_{\perp i} = -\mathrm{Im}(A_\perp A_i^* + A'_\perp A'^*_i) \qquad \Sigma_{\|0} = \mathrm{Re}(A_\| A_0^* - A'_\| A'^*_0)$$

$$\rho_{\perp i} = \mathrm{Re}\Big(\frac{q}{p}[A_\perp^* A'_i + A_i^* A'_\perp]\Big) \qquad \rho_{\|0} = -\mathrm{Im}\Big(\frac{q}{p}[A_\|^* A'_0 + A_0^* A'_\|]\Big)$$

3×2=6 | 3×1=3

Interference terms

***Total 18 observables** in one decay mode*

Consider a final state f into which both B^0 and $\overline{B}^0$ decay e.g. f= $D^{*+}\rho^-$ Only one amplitude contributes to each $B^0 \to f$ and $\overline{B}^0 \to f$.

$$A_\lambda \equiv Amp(B^0 \to f)_\lambda = a_\lambda e^{i\delta_\lambda^a} e^{i\phi_a} ,$$
$$A'_\lambda \equiv Amp(\overline{B^0} \to f)_\lambda = b_\lambda e^{i\delta_\lambda^b} e^{i\phi_b} ,$$
$$\bar{A}'_\lambda \equiv Amp(B^0 \to \bar{f})_\lambda = b_\lambda e^{i\delta_\lambda^b} e^{-i\phi_b} ,$$
$$\bar{A}_\lambda \equiv Amp(\overline{B^0} \to \bar{f})_\lambda = a_\lambda e^{i\delta_\lambda^a} e^{-i\phi_a}$$

$\lambda = \{0, \|, \perp\}$,

$\phi_{a,b}$ are weak phases,

$\delta_\lambda^{a,b}$ are strong phases.

$\phi_a = 0 \quad \phi_b = -\phi_3$

For the conjugate process one has similarly

$$\Gamma(B^0(t) \to \bar{f}) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \Big(\bar{\Lambda}_{\lambda\sigma} + \bar{\Sigma}_{\lambda\sigma} \cos(\Delta M t) - \bar{\rho}_{\lambda\sigma} \sin(\Delta M t) \Big) g_\lambda g_\sigma$$

$\Lambda_{\lambda\sigma} \to \bar{\Lambda}_{\lambda\sigma}, \Sigma_{\lambda\sigma} \to \bar{\Sigma}_{\lambda\sigma}, \rho_{\lambda\sigma} \to \bar{\rho}_{\lambda\sigma}$ *if* $A_\lambda \to \bar{A}'_\lambda$ & $A'_\lambda \to \bar{A}$

$\Gamma(B^0(t) \to \bar{f})$ *gives 18 more observables.*

Cleanly extract weak phases. How??

- $\Lambda_{\lambda\lambda} = \bar{\Lambda}_{\lambda\lambda} = \frac{(a_\lambda^2 + b_\lambda^2)}{2}, \quad \Sigma_{\lambda\lambda} = -\bar{\Sigma}_{\lambda\lambda} = \frac{(a_\lambda^2 - b_\lambda^2)}{2}$

extract a_λ^2, ignore b_λ^2

- $$\begin{aligned} \Lambda_{\perp i} &= -\bar{\Lambda}_{\perp i} = b_\perp b_i \sin(\delta_\perp - \delta_i + \Delta_i) - a_\perp a_i \sin(\Delta_i), \\ \Sigma_{\perp i} &= \bar{\Sigma}_{\perp i} = -b_\perp b_i \sin(\delta_\perp - \delta_i + \Delta_i) - a_\perp a_i \sin(\Delta_i) \end{aligned}$$

where $\Delta_i \equiv \delta_\perp^a - \delta_i^a$ *and* $\delta_\lambda \equiv \delta_\lambda^b - \delta_\lambda^a$ *Solve* $a_\perp a_i \sin\Delta_i$

- *Use the coefficients of the sin(Δ m t) term*

CP phase $\phi = -2\phi_M + \phi_b - \phi_a$

$$\Rightarrow \quad 2b_\lambda \cos\delta_\lambda = \pm\frac{\rho_{\lambda\lambda} + \bar{\rho}_{\lambda\lambda}}{a_\lambda \sin\phi}, \quad 2b_\lambda \sin\delta_\lambda = \pm\frac{\rho_{\lambda\lambda} - \bar{\rho}_{\lambda\lambda}}{a_\lambda \cos\phi}$$

- *Terms involving interference of different helicity,*

$$\begin{aligned} \rho_{\perp i} &= -a_\perp b_i \cos(\phi + \delta_i - \Delta_i) - a_i b_\perp \cos(\phi + \delta_\perp + \Delta_i), \\ \bar{\rho}_{\perp i} &= -a_\perp b_i \cos(\phi - \delta_i + \Delta_i) - a_i b_\perp \cos(\phi - \delta_\perp - \Delta_i) \end{aligned}$$

- *Put all the information above together, extract weak phase ϕ*

$$\rho_{\perp i} + \bar\rho_{\perp i} = -\cot\phi\, a_i a_\perp \cos\Delta_i \left[\frac{\rho_{ii}+\bar\rho_{ii}}{a_i^2} - \frac{\rho_{\perp\perp}+\bar\rho_{\perp\perp}}{a_\perp^2}\right]$$

$$-a_i a_\perp \sin\Delta_i \left[\frac{\rho_{ii}-\bar\rho_{ii}}{a_i^2} + \frac{\rho_{\perp\perp}-\bar\rho_{\perp\perp}}{a_\perp^2}\right],$$

a_λ^2 determined from $\Lambda_{\lambda\lambda}$ and $\Sigma_{\lambda\lambda}$

$\rho_{\lambda\sigma}$ and $\bar\rho_{\lambda\sigma}$ measured quantities

$$\rho_{\perp i} - \bar\rho_{\perp i} = \tan\phi\, a_i a_\perp \cos\Delta_i \left[\frac{\rho_{ii}-\bar\rho_{ii}}{a_i^2} - \frac{\rho_{\perp\perp}-\bar\rho_{\perp\perp}}{a_\perp^2}\right]$$

$$-a_i a_\perp \sin\Delta_i \left[\frac{\rho_{ii}+\bar\rho_{ii}}{a_i^2} + \frac{\rho_{\perp\perp}+\bar\rho_{\perp\perp}}{a_\perp^2}\right]$$

$a_i\, a_\perp \sin\Delta_i$ obtained from $\Lambda_{\perp i}$ and $\Sigma_{\perp i}$

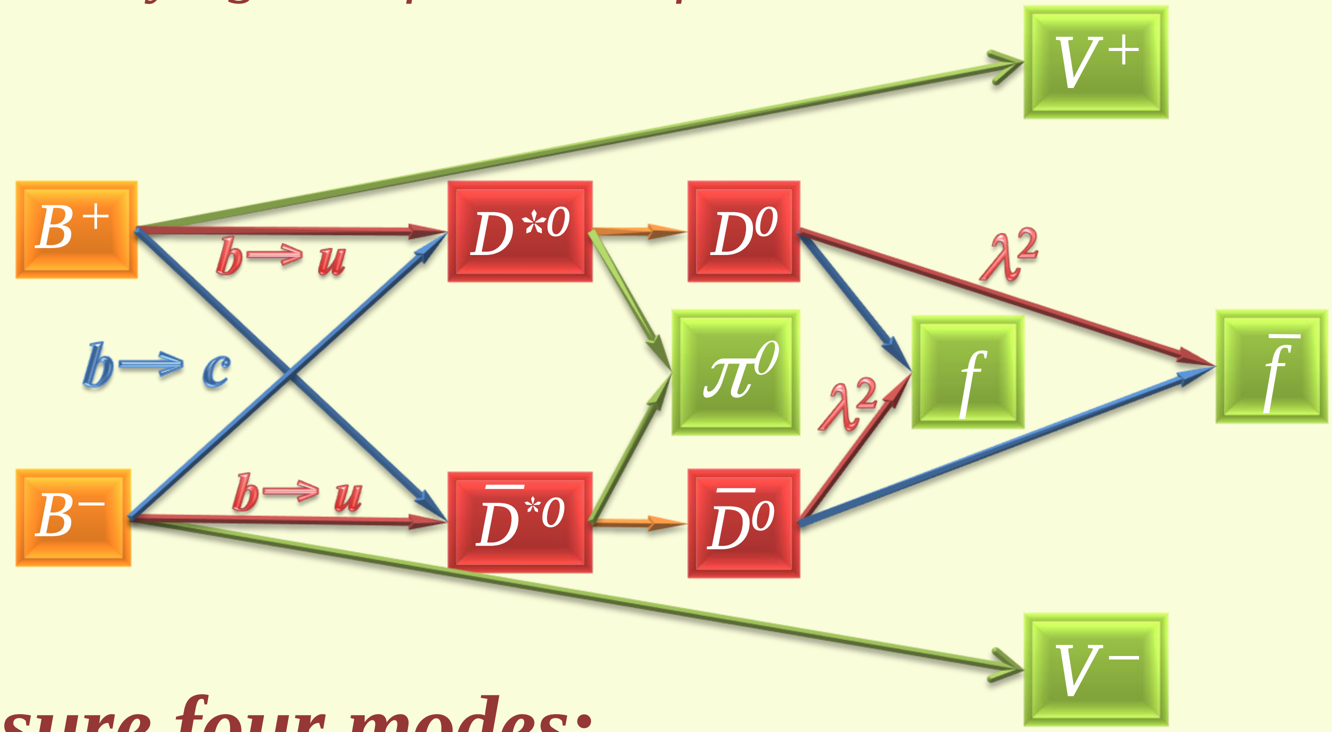
2 equations involve only 2 unknown quantities $\tan\phi$ and $a_i a_\perp \cos\Delta_i$, can easily be solved up to a sign ambiguity in each of these quantities.

$\tan^2\phi$ or, equivalently, $\sin^2\phi$ can be obtained from angular analysis

ϕ_3 *using* B$^\pm$ →D* V$^\pm$ *mode*

Phys.Rev.Lett.80:3706,1998.

Consider $D^{*0}/\bar{D}^{*0}$ *decaying into* $D^0\pi/\bar{D}^0\pi$ *with* $D^0/\bar{D}^0$ *meson further decaying to a final state f that is common both* $D^0/\bar{D}^0$



Measure four modes:

$$B^\pm \rightarrow (f\,\pi^0)\ V^\pm,\ B^\pm \rightarrow (\bar{f}\,\pi^0)\ V^\pm$$

The partial decay rate for $B \to D^* K^* \to (D\pi)(K\pi)$ can be written as

$$\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} \propto \Big(|A_0|^2 \cos^2\theta_1 \cos^2\theta_2 + \frac{|A_\perp|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \sin^2\phi + \frac{|A_{||}|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \cos^2\phi + \frac{\mathrm{Re}(A_0 A_{||}^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos\phi - \frac{\mathrm{Im}(A_\perp A_0^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin\phi - \frac{\mathrm{Im}(A_\perp A_{||}^*)}{2} \sin^2\theta_1 \sin^2\theta_2 \sin 2\phi \Big),$$

$$Amp(B^+ \to D^{*0} K^{*+})_\lambda = a_\lambda e^{i\delta_\lambda^a} e^{i\phi_3}$$

$$Amp(B^+ \to \bar{D}^{*0} K^{*+})_\lambda = b_\lambda e^{i\delta_\lambda^b}$$

$$Amp(B^- \to D^{*0} K^{*-})_\lambda = b_\lambda e^{i\delta_\lambda^b}$$

$$Amp(B^- \to \bar{D}^{*0} K^{*+})_\lambda = a_\lambda e^{i\delta_\lambda^a} e^{-i\phi_3}$$

Strong phase difference

$D^0/\bar{D}^0$ decays further to f, define $\mathcal{R} e^{i\Delta} = \frac{Amp(\bar{D}^0 \to f)}{Amp(D^0 \to f)}$

$Amp(B^+, B^- \to f/\bar{f})$ contributions from sum of D^0 and $\bar{D}^0$

$$\begin{aligned}
A_\lambda^f &= A_\lambda(B^+ \to [\,[f]_D \pi]_{D^*} K^{*+}) \\
&= \sqrt{B}(a_\lambda e^{i\delta_\lambda^a} e^{i\phi_3} + b_\lambda e^{i\delta_\lambda^b} \mathcal{R} e^{i\Delta}) \\
\bar{A}_\lambda^{\bar{f}} &= A_\lambda(B^- \to [\,[\bar{f}]_D \pi]_{D^*} K^{*-}) \\
&= \sqrt{B}(a_\lambda e^{i\delta_\lambda^a} e^{-i\phi_3} + b_\lambda e^{i\delta_\lambda^b} \mathcal{R} e^{i\Delta}) \\
\bar{A}_\lambda^{f} &= A_\lambda(B^- \to [\,[f]_D \pi]_{D^*} K^{*-}) \\
&= \sqrt{B}(a_\lambda e^{i\delta_\lambda^a} e^{-i\phi_3} \mathcal{R} e^{i\Delta}) + b_\lambda e^{i\delta_\lambda^b}) \\
A_\lambda^{\bar{f}} &= A_\lambda(B^+ \to [\,[\bar{f}]_D \pi]_{D^*} K^{*+}) \\
&= \sqrt{B}(a_\lambda e^{i\delta_\lambda^a} e^{i\phi_3} \mathcal{R} e^{i\Delta}) + b_\lambda e^{i\delta_\lambda^b}),
\end{aligned}$$

May not be distinguishable,
Still enough observables,
Can determine ϕ_3.

Observables $\to$ 23

Parameters $\to$ 14

$a_\lambda, b_\lambda, \delta_\lambda^a, \delta_\lambda^b, \phi_3, \Delta, \mathcal{R}$

5

$\mathcal{R} \ll 1$, $a_\lambda \ll b_\lambda$, help reduce ambiguities

Usual Signatures: $|A_\lambda^f|^2 - |\bar{A}_\lambda^{\bar{f}}|^2 = 4\mathcal{R}Ba_\lambda b_\lambda \sin(\delta_b^\lambda - \delta_a^\lambda + \Delta)\sin\phi_3$,

Additional signatures from $B + \bar{B}$:

$$\mathrm{Im}[(A_\perp A_i^*)^f - (\bar{A}_\perp \bar{A}_{i*})^{\bar{f}}] = 2\mathcal{R}B\sin\phi_3\,\big(a_\perp b_i \cos(\delta_\perp^a - \delta_i^b - \Delta) - b_\perp a_i \cos(\delta_\perp^b - \delta_i^a + \Delta)\big)$$

Interference terms-essential: *If only* $|A_\lambda^f|^2, |\bar{A}_\lambda^{\bar{f}}|^2, |\bar{A}_\lambda^f|^2, |A_\lambda^{\bar{f}}|^2$ *measured just like doing additional modes. The azimuthal angle* ϕ, *plays a crucial role.*

CP Violating Asymmetries

It is easy to see that $\Lambda_{\perp 0}$ *can be isolated using the asymmetry*

$$A_1 = \frac{\left(\int_0^{\pi} - \int_{\pi}^{2\pi}\right) d\phi \int_D d\cos\theta_1 \int_D d\cos\theta_2 \frac{d\Gamma_{sum}}{d\cos\theta_1 d\cos\theta_2 d\phi}}{\int_0^{2\pi} d\phi \int_S d\cos\theta_1 \int_S d\cos\theta_2 \frac{d\Gamma_{sum}}{d\cos\theta_1 d\cos\theta_2 d\phi}},$$

where as, $\Lambda_{\perp\parallel}$ can be isolated using the asymmetry

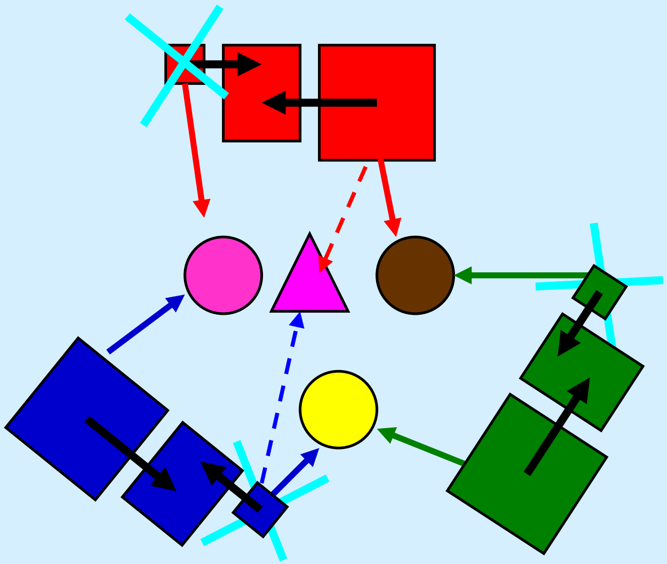
$$A_2 = \frac{\int_Q d\phi \int_S d\cos\theta_1 \int_S d\cos\theta_2 \frac{d\Gamma_{sum}}{d\Omega}}{\int_0^{2\pi} d\phi \int_S d\cos\theta_1 \int_S d\cos\theta_2 \frac{d\Gamma_{sum}}{d\Omega}}$$

$$g_{\perp 0} = \sin 2\theta_1 \sin 2\theta_2 \sin\phi$$

$$g_{\perp\parallel} = \sin^2\theta_1 \sin^2\theta_2 \sin 2\phi,$$

$$\Gamma_{sum} = \Gamma(B \to V_1 V_2) + \Gamma(\bar{B} \to \bar{V}_1 \bar{V}_2)$$

- *Signals of CP Violation can be obtained by adding* B *and* $\bar{B}$
- *Signals of CP Violation not diluted by sine of strong phases.*



Adapted from a figure in talk of Abi Soffer

Multiple helicities provide powerful technique. Due to interference between different helicities, enough observables, without even measuring the tiny b^2 amplitudes.

Examples presented can be extended to many modes:

Michael Gronau, Dan Pirjol, and Daniel Wyler, $B^0 \to D^{*-} a_1^{\,+}$

Phys. Rev. Lett. 90, 051801 (2003)

Look for other modes where helicities can be used to our advantage.

Conclusions

- *Modes with multiple helicities enable clean measurement of weak phases.*
- *Experimentally challenging.*