Measurement(s) of γ/ϕ_3

K.Trabelsi
(karim.trabelsi@kek.jp)
on behalf of the CKMfitter group

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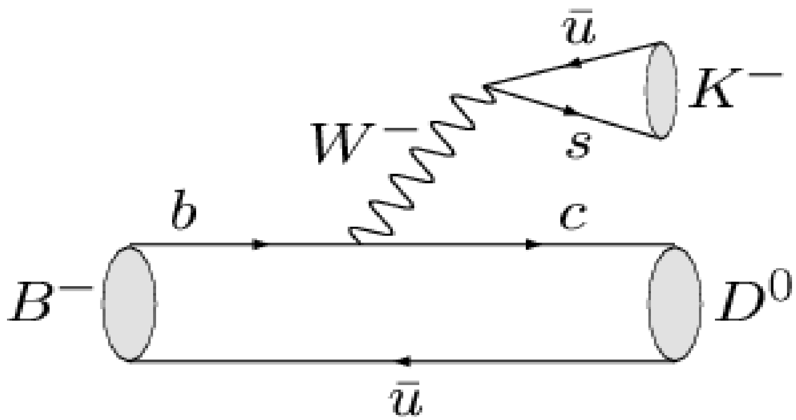
γ measurements from $B^{\pm} \rightarrow DK^{\pm}$

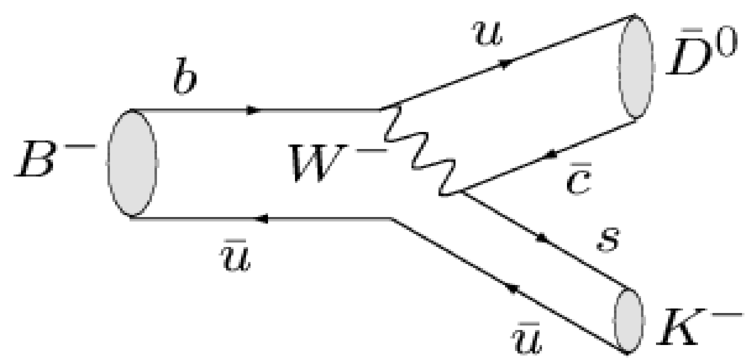
- Access γ via interference between $B^{-} \rightarrow D^{0} K^{-}$ and $B^{-} \rightarrow \overline{D}^{0} K^{-}$



color allowed

$B^{-} \rightarrow D^{0} K^{-} \sim V_{cb} V_{us}^{*}$

$\sim \mathbf{A\lambda^{3}}$



color suppressed

$B^{-} \rightarrow \overline{D}^{0} K^{-} \sim V_{ub} V_{cs}^{*}$

$\sim \mathbf{A\lambda^{3}(\rho + i\eta)}$

$$r_B = \frac{|A_{suppressed}|}{|A_{favoured}|} \sim \frac{|V_{ub} V_{cs}^{*}|}{|V_{cb} V_{us}^{*}|} \times [\text{color supp}] = 0.1\text{-}0.2$$

- Reconstruct D in final states accessible to both D^{0} and $\overline{D}^{0}$

$D = D_{CP}$ **GLW method (Gronau-London-Wyler)**

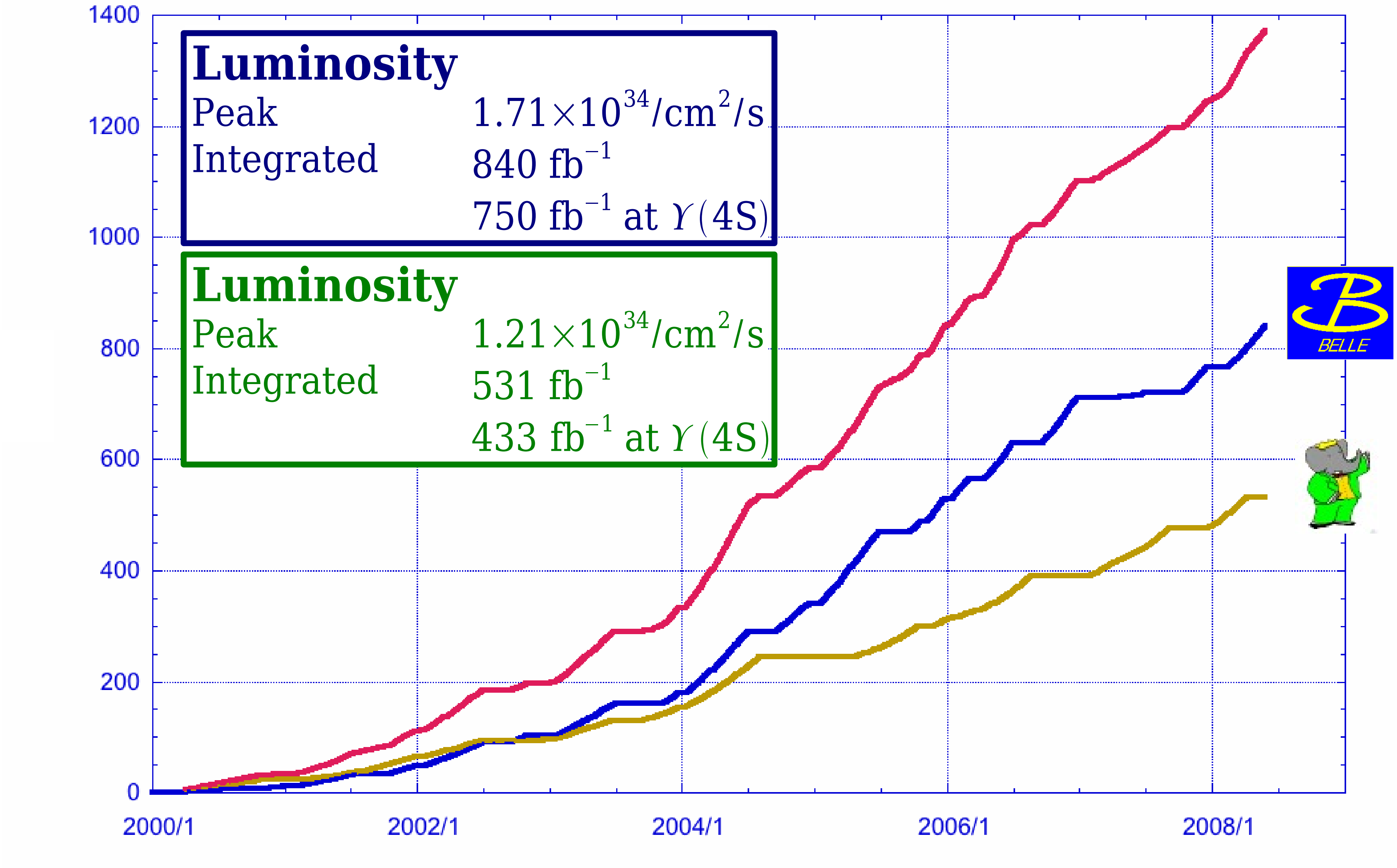
$D = D_{WS}$ (doubly-Cabbibo suppressed) **ADS method (Atwood-Dunietz-Soni)**

$D \rightarrow K_S \pi^{+} \pi^{-}$, $K_S K^{+} K^{-}$ **GGSZ (Dalitz) method (Giri-Grossman-Soffer-Zupan)**

B factories: BaBar and Belle

Luminosity (fb^{-1})

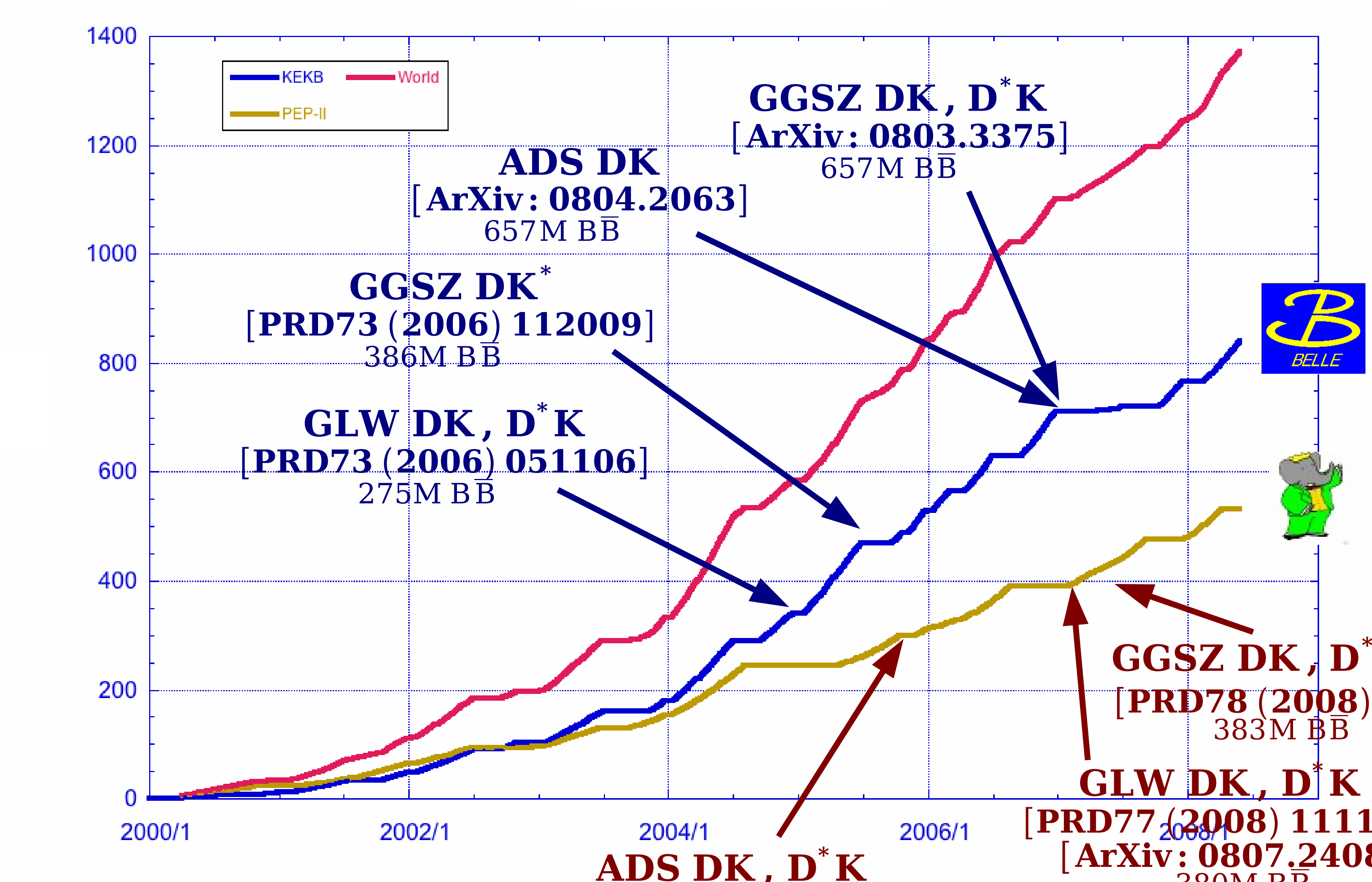
cumulated stat: $\sim 1400\,fb^{-1}$!!



B factories: BaBar and Belle

and CDF (GLW) !

Luminosity (fb^{-1})



γ and r_B... (BaBar example)

analysis with $227\times10^6 B\overline{B}$ **[PRL95, 121802 (2005)]**

$r_B(DK) = 0.12 \pm 0.08 \pm 0.03 \pm 0.04$ $\quad$ $r_B(D^*K) = 0.17 \pm 0.10 \pm 0.03 \pm 0.03$

$$\boldsymbol{\gamma = 70^\circ \pm 31^\circ (stat)^{+12^\circ}_{-10^\circ}(syst)^{+14^\circ}_{-11^\circ}(model)}$$

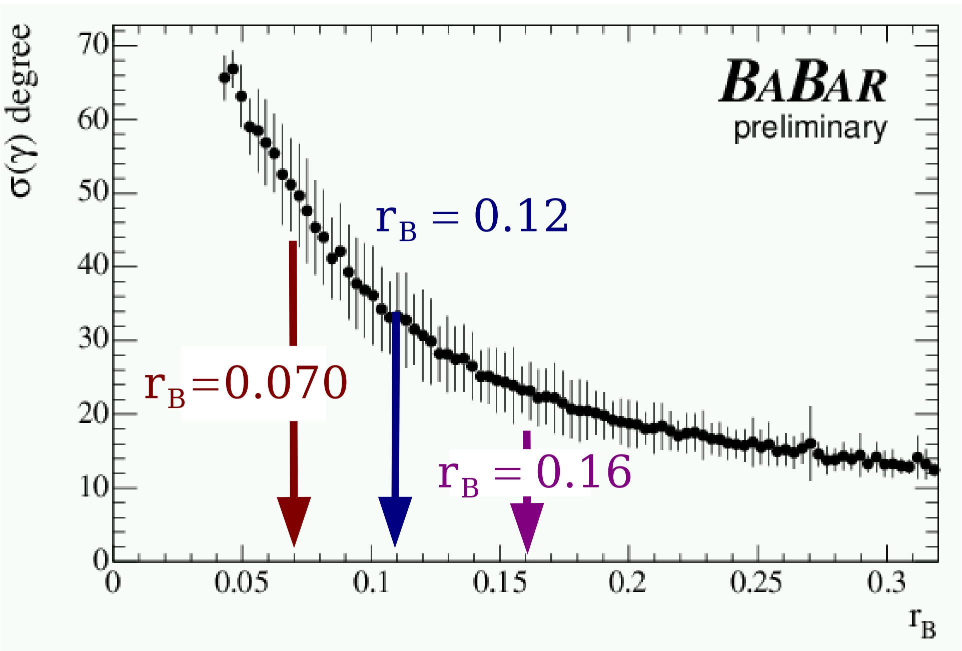
analysis with $347\times10^6 B\overline{B}$ **[hep-ex/0607104]**

$r_B(DK) < 0.140$ $\quad$ $0.017 < r_B(D^*K) < 0.203$

$$\boldsymbol{\gamma = 92^\circ \pm 41^\circ (stat) \pm 11^\circ (syst) \pm 12^\circ (model)}$$

uncertainty on γ scales as $1/r_B$! ($r_B = 0 \Rightarrow$ no constraint on γ)

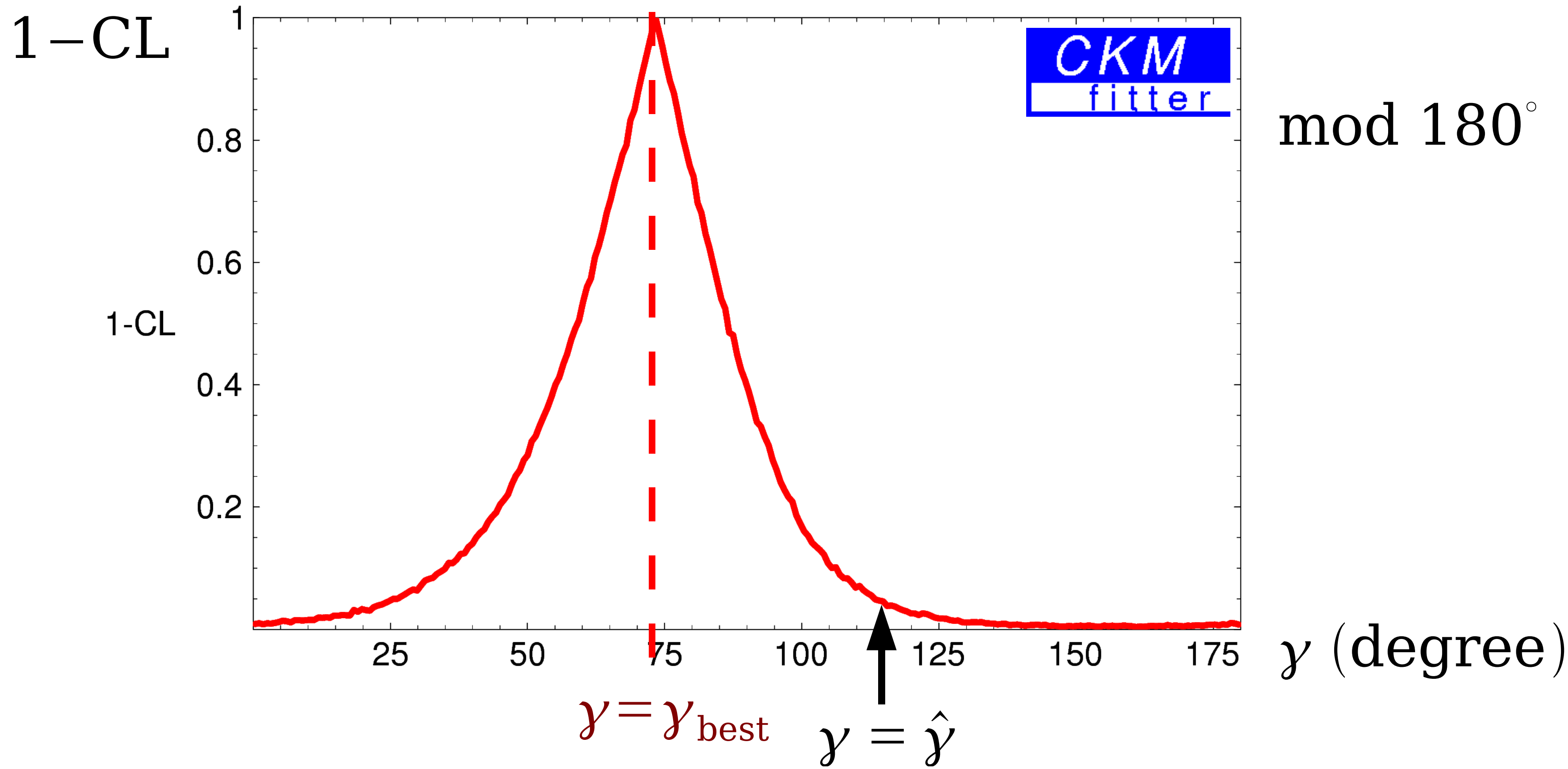
DK case



if r_B is small, the r_B(true) found is biased to higher values and the error on γ is biased toward small values

Determination of γ (central value and intervals)

from different measurements, independently of other parameters



γ parameter of interest, μ ($= r_B, \delta_B$) nuisance parameters

$$\Delta\chi^2(\hat{\gamma}) = \chi^2(\hat{\gamma}, \hat{\mu}) - \chi^2(\gamma_{best}, \mu_{best})$$

best parameters for the given $\hat{\gamma}$ and actual z measurements

N_γ minimisations

get the CL:

- if the sampling PDF of $\Delta\chi^2$ is a χ^2 law

$\Rightarrow$ cumulative distribution function (prob)

(true asymptotically)

if not: the sampling PDF depends, in general, on the nuisance parameters

what to do with the nuisance parameters ?

$\rightarrow$ plug-in principle ($\hat{\mu}$ method): nuisance parameters are fixed to $\hat{\mu}$ values

$\rightarrow$ supremum method: least favored values of the nuisance parameters

(sure to never undercover)

Interval definition and coverage tests:

A parameter p belongs to an interval $[p_1, p_2]$ at α% confidence level, if the probability that the interval $[p_1, p_2]$ contains the true value p_{true}, when we repeat the experiment many times, is α

if this probability is larger than α: **overcoverage**
if it's smaller: **undercoverage**

why it's important to check the coverage ?!
that's the meaning of the intervals provided
(of course, you would like to use a method which has an exact coverage for any true values of (γ, μ) but...)

why it's better to have overcoverage than undercoverage ?
to avoid too quick conclusion ?

→ powerful check that you can perform with MC toys (many experiments !)
→ only way to check if your method ensures coverage

Coverage tests:

Procedure:

- fix the true values of $\gamma_{\rm true}$ and $\mu_{\rm true}$
- generate large number of experiments $(x_\pm, y_\pm)$ around the true values
- for each experiment, we calculate CL at $\gamma = \gamma_{\rm true}$
- the fraction of experiments for which $CL(\gamma_{\rm true})$ is larger than a given value α gives the coverage

N.B: the coverage depends of $\gamma_{\rm true}$ and $\mu_{\rm true}$, also of α

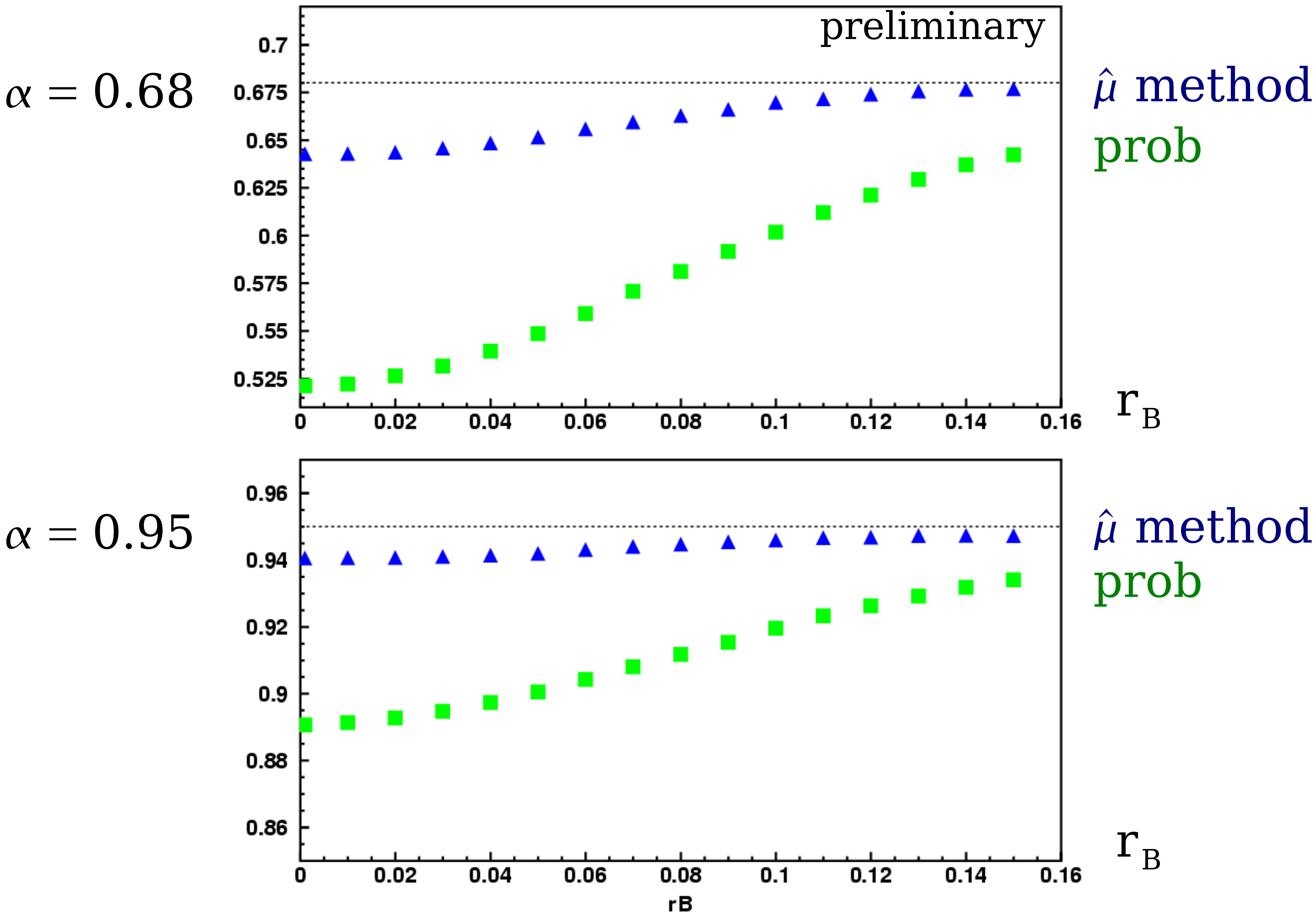
Simple example for GGSZ (one mode DK)

$\gamma_{\rm true}$, $\mu_{\rm true}$: $\gamma=60^\circ$, $\delta_B=140^\circ$, r_B from 0.001 to 0.15

observables $(x_\pm, y_\pm)$ generated with $\sigma_x = \sigma_y = 0.07$

large number of experiments: you estimate $CL(\mu=\mu_{\rm true})$, and estimate the fraction of experiments for which $CL(\mu=\mu_{\rm true}) > \alpha$

Coverage tests: Simple example for GGSZ DK

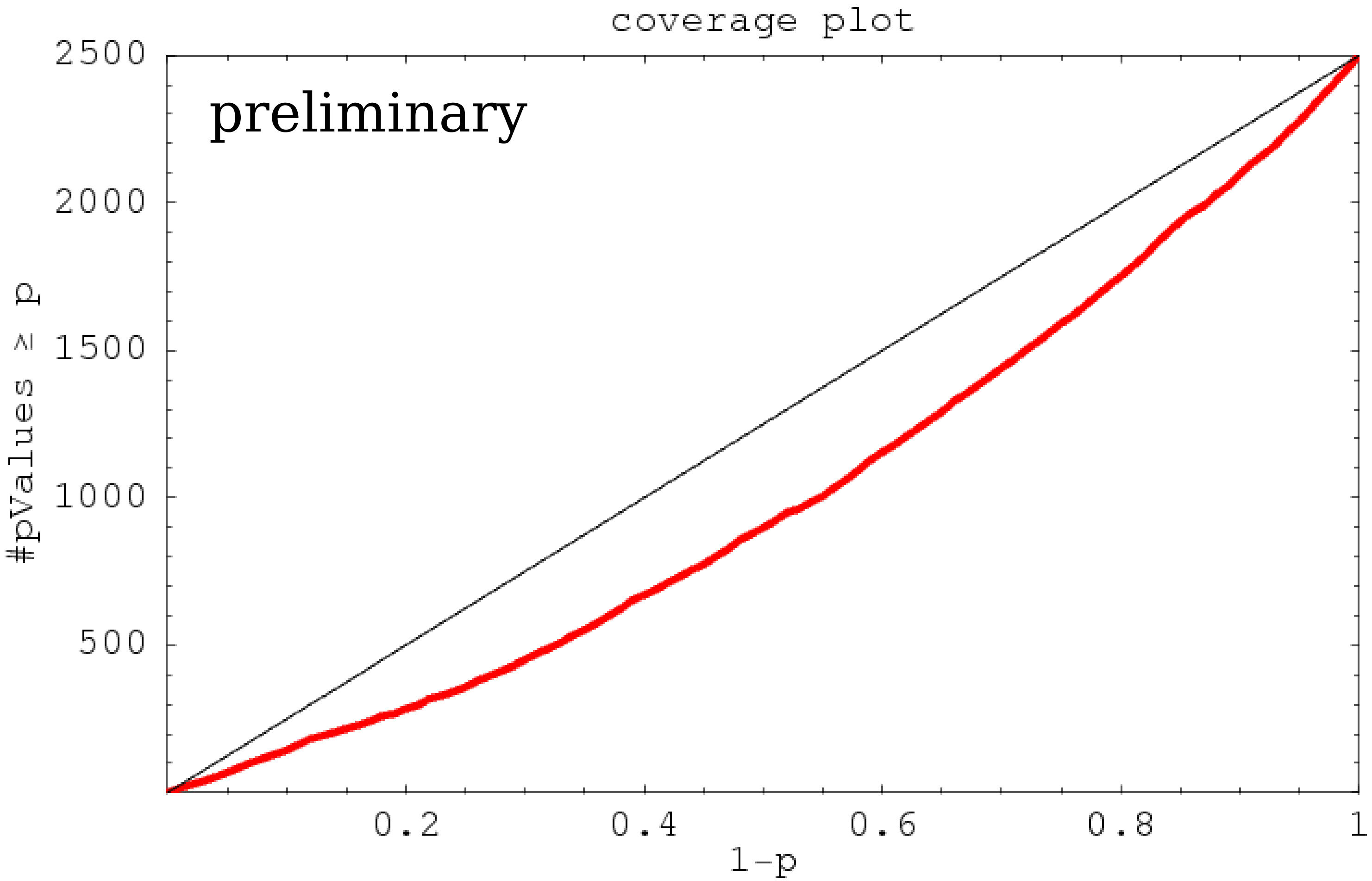


More complete example:

GGSZ/GLW/ADS: $r_B(DK) = r_B(D^* K) = r_B(DK^*) = 0.04$

$\hat{\mu}$ method: coverage is 55% and 92% instead of 68% and 95%

prob method: coverage is 39% and 79%



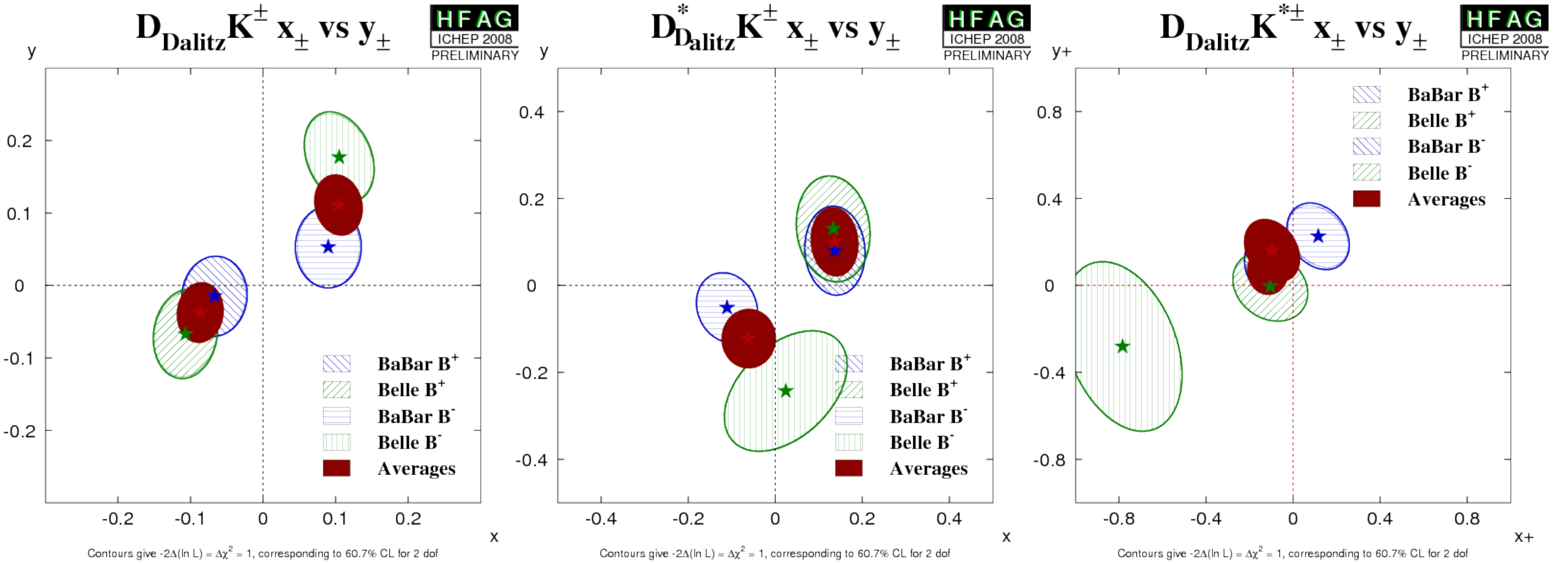
→ we have undercoverage for $\hat{\mu}$ method !

GGSZ with DK, D*K, DK*

D$^{(*)}$ decays to multibody final states

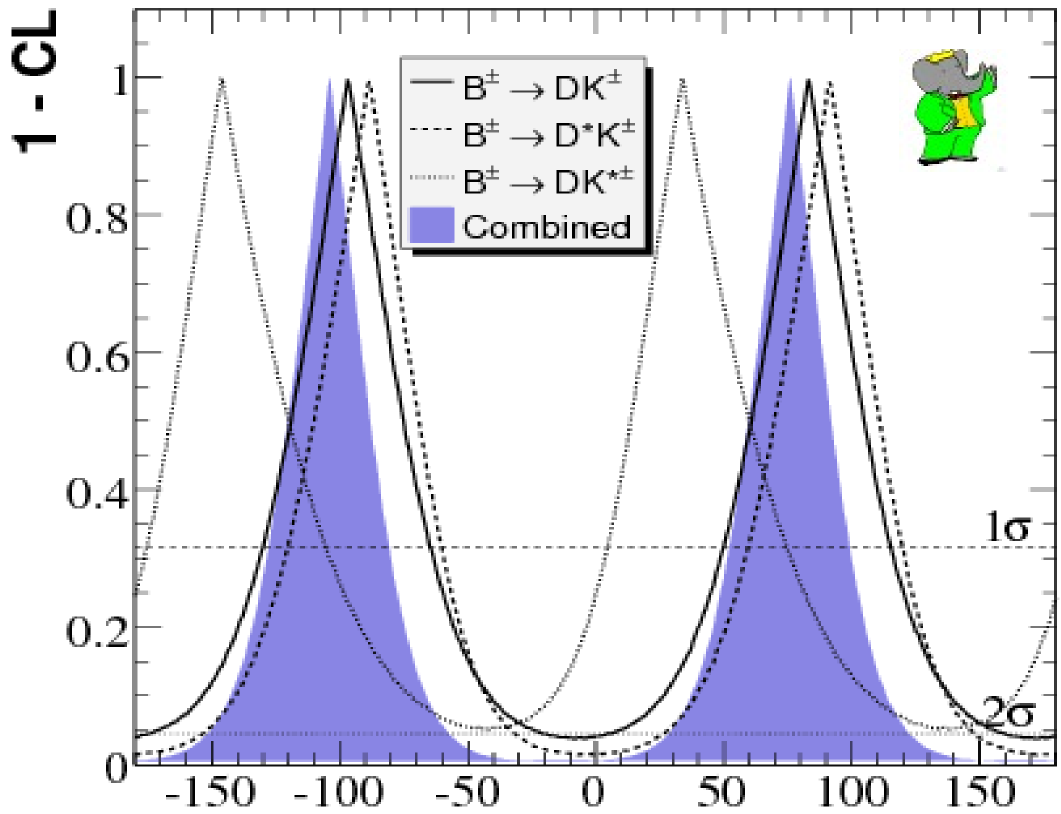
Relation between $(x_\pm, y)$ and (γ, r_B, δ_B)

$$(x_{+-}, y_{+-}) = (r_B \cos(\delta_B \pm \gamma), r_B \sin(\delta_B \pm \gamma))$$



GGSZ results (BaBar)

[PRD78 (2008) 034023]



$$\boldsymbol{\gamma = 76^\circ \pm 22^\circ (\text{stat}) \pm 5^\circ (\text{syst}) \pm 5^\circ (\text{model})}$$

$r_B(DK) = 0.086 \pm 0.035$

$r_B(D^* K) = 0.135 \pm 0.051$

$r_B(DK^*) = 0.163^{+0.088}_{-0.105}$

$\delta_B(DK) = 109^{\circ}{}^{+28^\circ}_{-31^\circ}$

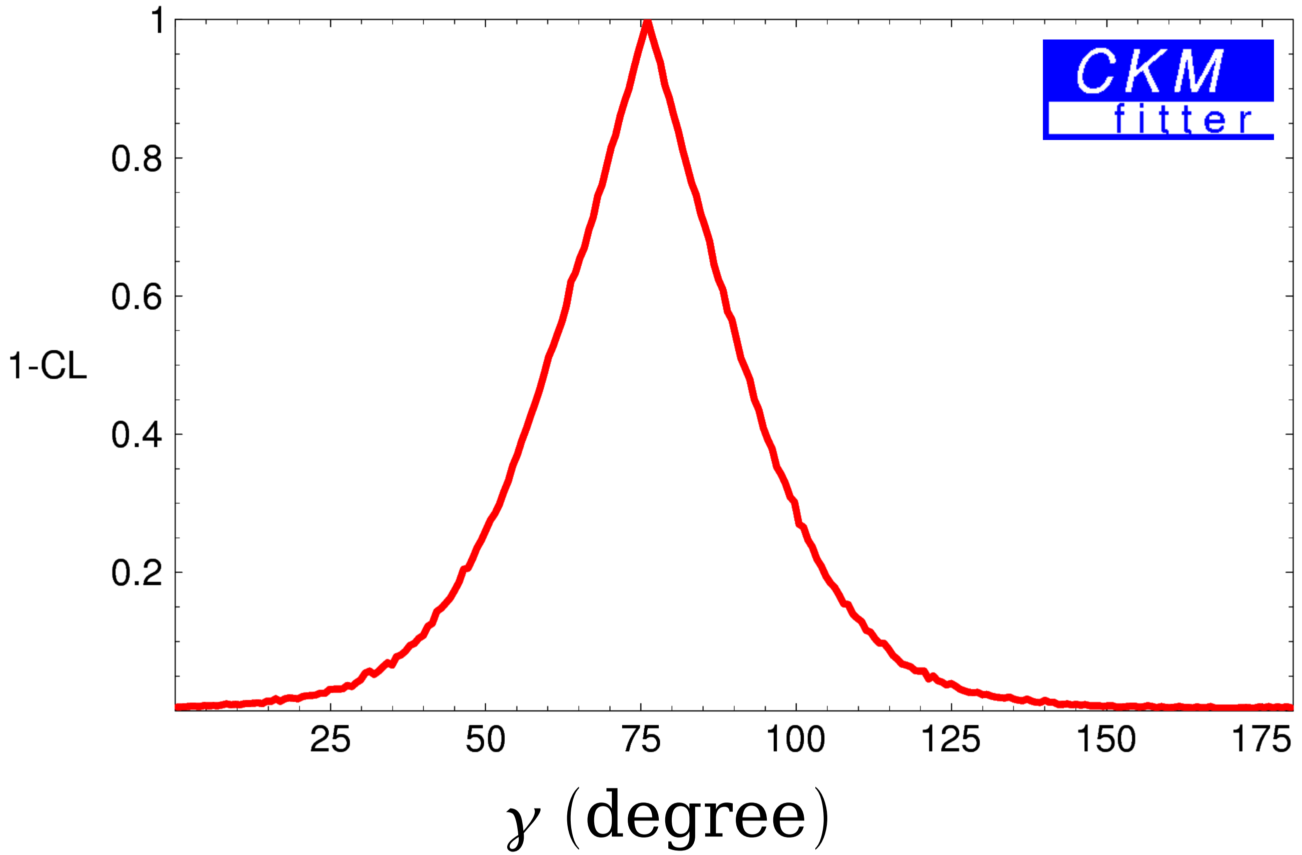
$\delta_B(D^* K) = -63^{\circ}{}^{+28^\circ}_{-30^\circ}$

$\delta_B(DK^*) = 104^{\circ}{}^{+43^\circ}_{-41^\circ}$

- all information (stat, syst, model) provided in terms of x, y (error matrices in the paper)
- but error matrix for model part combined $K_S \pi \pi$ and $K_S KK$

γ result from GGSZ BaBar ($\hat{\mu}$ method)

all inputs available:



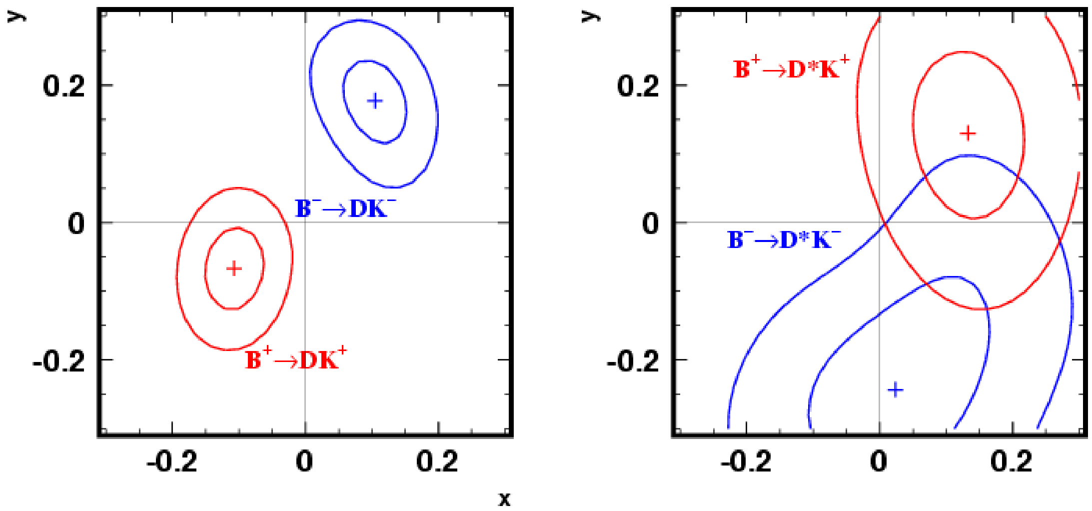
$\hat{\mu}$ method: $(76 \pm 23)^\circ$, $(\pm 2\sigma: {}^{+47}_{-46})^\circ$

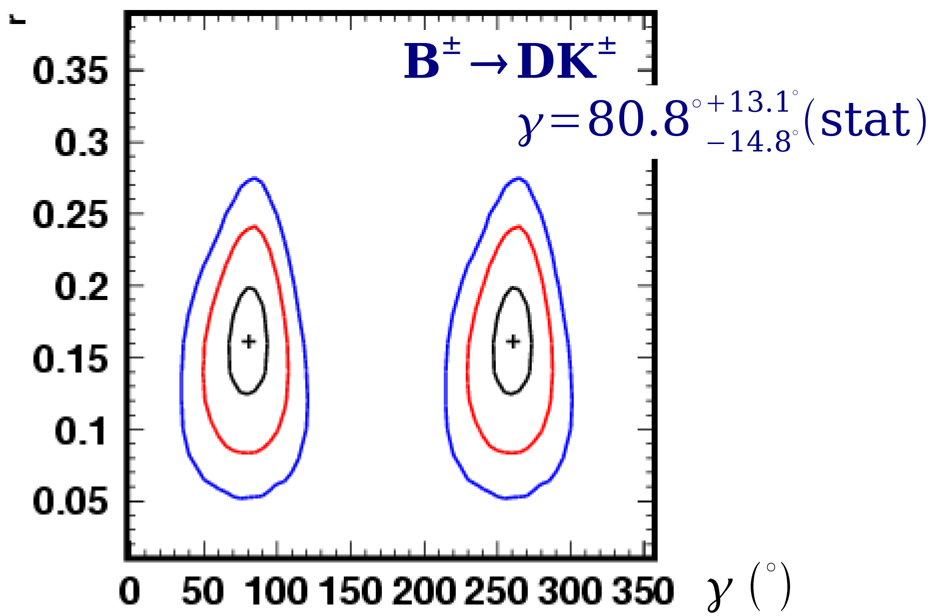
this is exactly what BaBar found... (for all central values, $\pm 1\sigma$ and $\pm 2\sigma$)

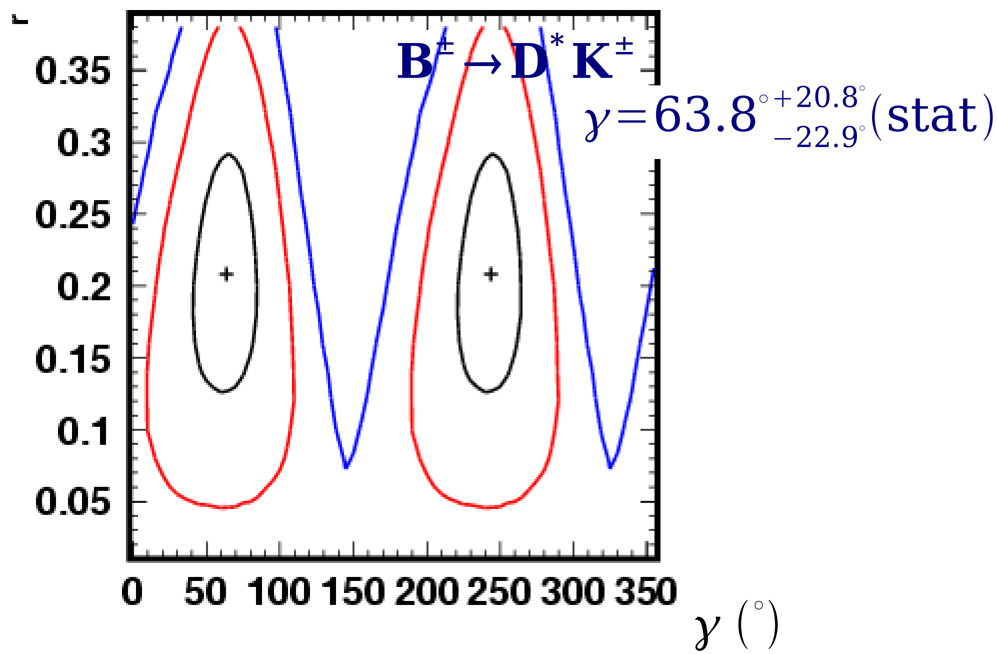
→ in their last paper, that's the method they are using !!

GGSZ results (Belle)

[ArXiv: 0803.3375]







$$\gamma = 76^\circ{}^{+12^\circ}_{-13^\circ}(\text{stat}) \pm 4^\circ(\text{syst}) \pm 9^\circ(\text{model})$$

$r_B(DK) = 0.16 \pm 0.04 \pm 0.01 \pm 0.05$

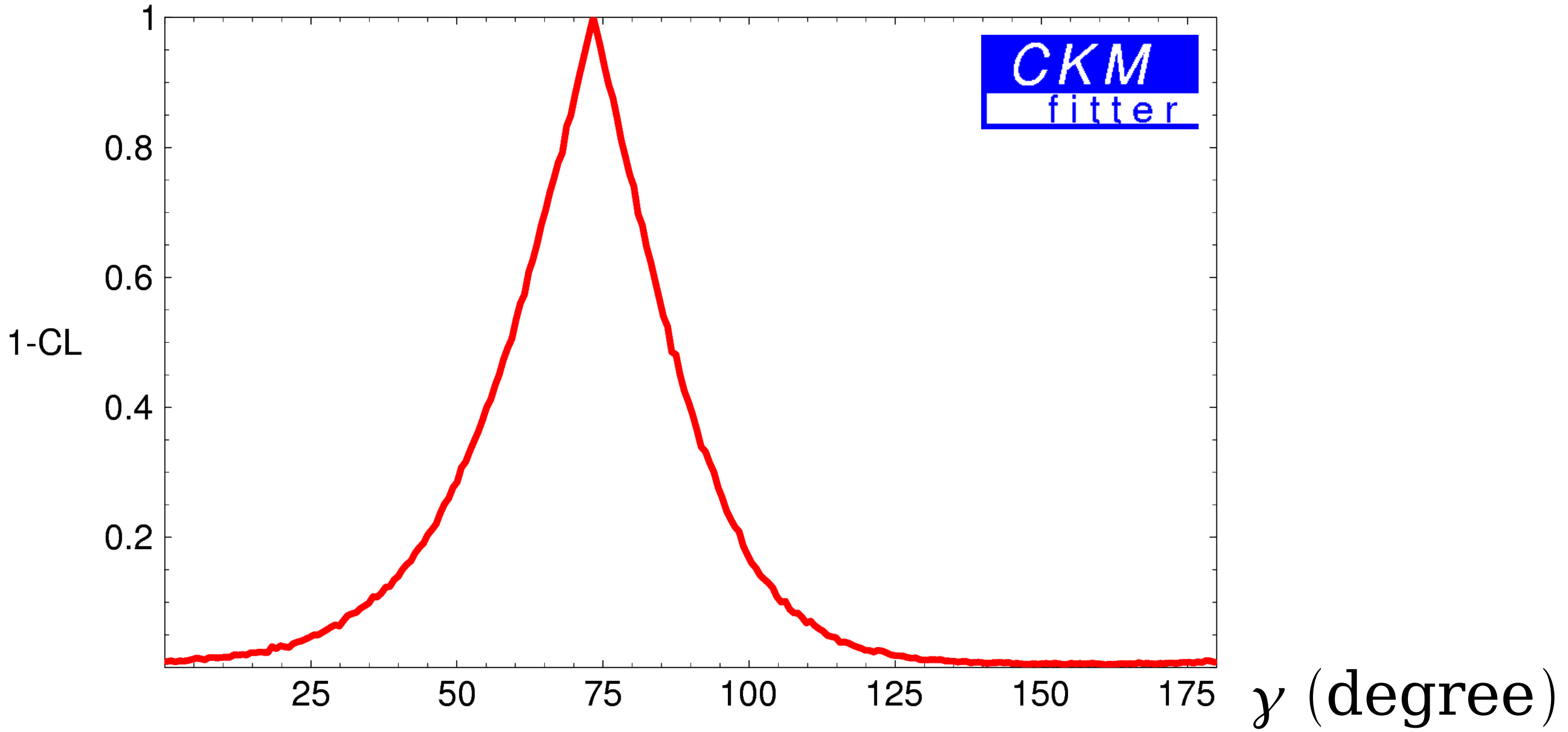
$r_B(D^*K) = 0.21 \pm 0.08 \pm 0.02 \pm 0.05$

$\delta_B(DK) = 136^\circ{}^{+14^\circ}_{-16^\circ} \pm 4^\circ \pm 23^\circ$

$\delta_B(D^*K) = 343^\circ{}^{+20^\circ}_{-22^\circ} \pm 4^\circ \pm 23^\circ$

γ result from GGSZ Belle ($\hat{\mu}$ method)

conversion $\sigma(\gamma)$, $\sigma(r_B)$, $\sigma(\delta_B)$ (for syst and model) in terms of $\sigma(x)$, $\sigma(y)$ from Belle



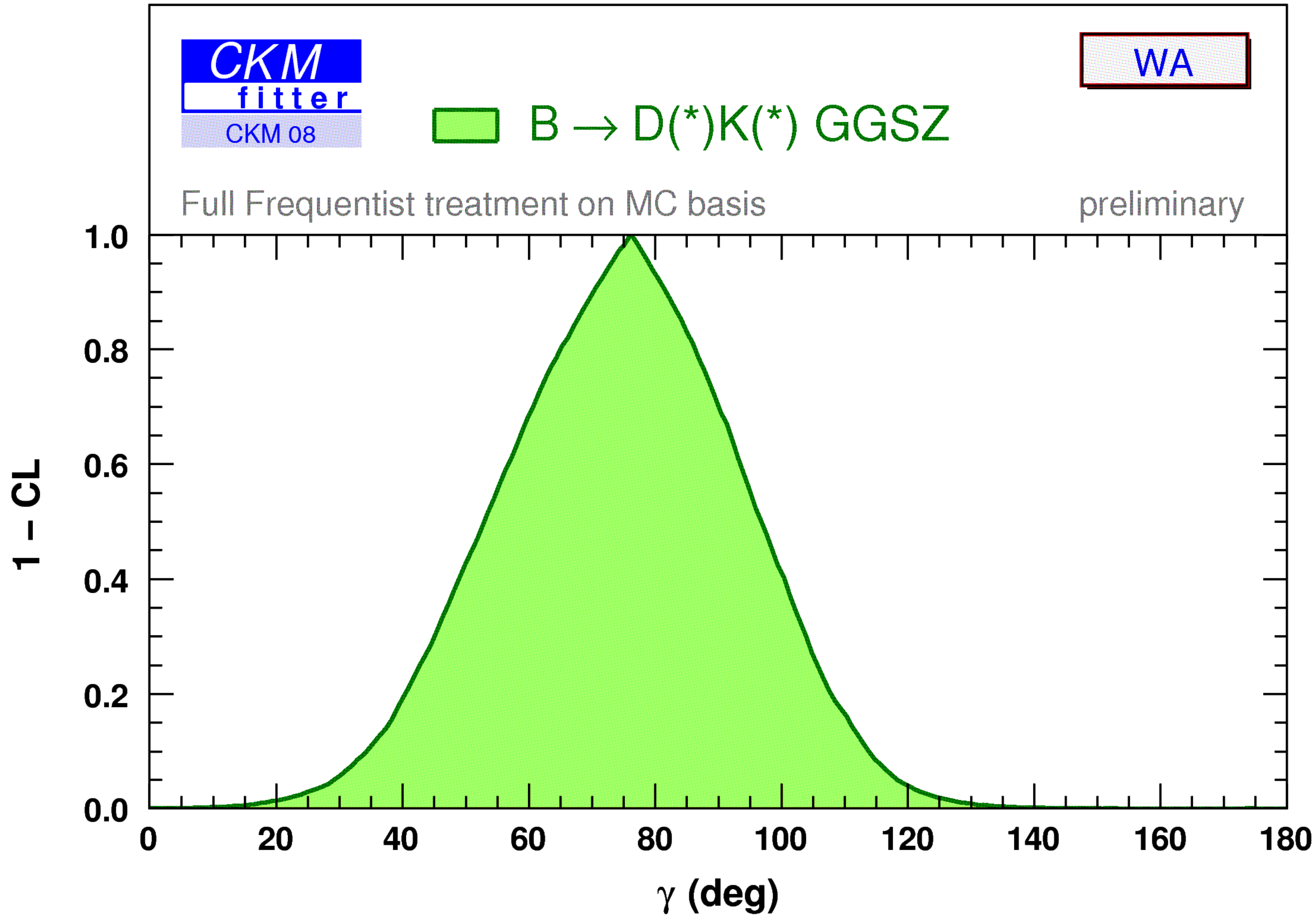
$$\hat{\mu} \text{ method}: \ (73^{+20}_{-22})^\circ, \ (\pm 2\sigma: {}^{+42}_{-49})^\circ$$

- here the statistical method is different
- we combined stat, syst and model errors to get Belle $\sigma(x_\pm)$, $\sigma(y_\pm)$

$\rightarrow$ the goal is to provide an average of the experiments and the modes...
(if we consider only stat errors: the central value is reproduced)

γ result from GGSZ BaBar+Belle

combined the $(x_{\pm}, y_{\pm})$ from Belle and BaBar



$\hat{\mu}$ method: $\gamma = (76^{+16}_{-18})^{\circ}, \ (\pm 2\sigma: {}^{+32}_{-36})^{\circ}$

supremum method: $\gamma = (76^{+27}_{-30})^{\circ}, \ (\pm 2\sigma: {}^{+43}_{-47})^{\circ}$

GLW with $D_{CP}K$, $D^*_{CP}K$, $D_{CP}K^*$

$D^{(*)}$ decays to CP eigenstates

Relation between $(A_{CP+}, A_{CP-}, R_{CP+}, R_{CP-})$ and (γ, r_B, δ_B)

$$A_{CP+} = \frac{2 r_B \sin\delta_B \sin\gamma}{1 + r_B^2 + 2 r_B \cos\delta_B \cos\gamma} \qquad A_{CP-} = \frac{-2 r_B \sin\delta_B \sin\gamma}{1 + r_B^2 - 2 r_B \cos\delta_B \cos\gamma}$$

$$R_{CP+} = 1 + r_B^2 + 2 r_B \cos\delta_B \cos\gamma \qquad R_{CP-} = 1 + r_B^2 - 2 r_B \cos\delta_B \cos\gamma$$

ADS with DK, D^*K, DK^*

$D^{(*)}$ decays to doubly-Cabibbo-suppressed states

Relation between (A_{ADS}, R_{ADS}) and $(\gamma, r_B, \delta_B + \delta_D, r_D)$

$$A_{ADS} = \frac{2 r_B r_D \sin(\delta_B + \delta_D) \sin\gamma}{r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos\gamma}$$

$$R_{ADS} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos\gamma$$

r_D : ratio of the magnitudes of the suppressed and favoured decays

GLW with $D_{CP}K$, $D^*_{CP}K$, $D_{CP}K^*$

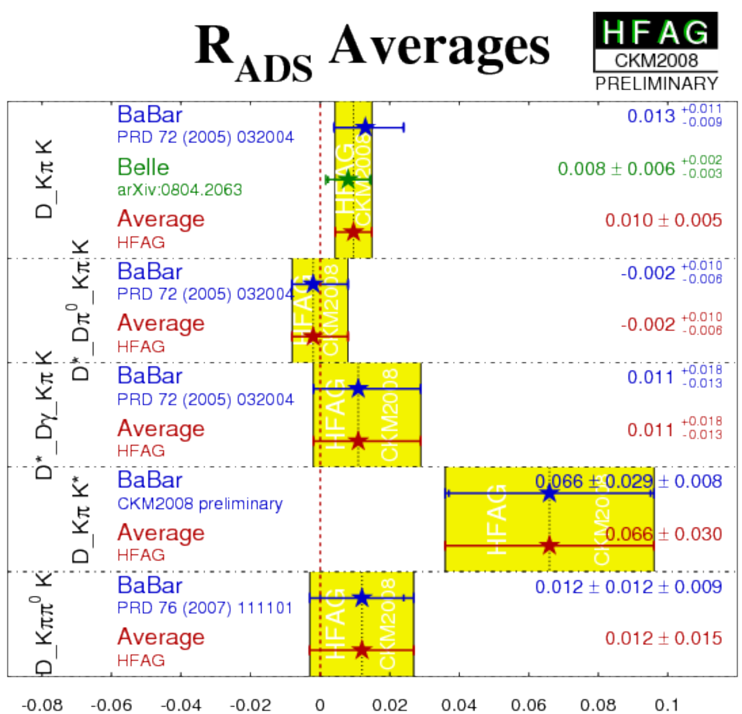
Mode	Experiment	A_{CP+}	A_{CP-}	R_{CP+}	R_{CP-}	Reference
$D_{CP}K^-$	BaBar N(BB)=382M	0.27 ± 0.09 ± 0.04	−0.09 ± 0.09 ± 0.02	1.06 ± 0.10 ± 0.05	1.03 ± 0.10 ± 0.05	PRD 77 (2008) 111102
	Belle N(BB)=275M	0.06 ± 0.14 ± 0.05	−0.12 ± 0.14 ± 0.05	1.13 ± 0.16 ± 0.08	1.17 ± 0.14 ± 0.14	PRD 73, 051106 (2006)
	CDF	0.39 ± 0.17 ± 0.04	-	1.30 ± 0.24 ± 0.12	-	ICHEP 2008 preliminary
	Average	0.24 ± 0.07 χ^2 = 2.3 (CL=0.32 ⇒ 1.0σ)	−0.10 ± 0.08 χ^2 = 0.03 (CL=0.86 ⇒ 0.2σ)	1.10 ± 0.09 χ^2 = 0.7 (CL=0.70 ⇒ 0.4σ)	1.06 ± 0.10 χ^2 = 0.4 (CL=0.54 ⇒ 0.6σ)	HFAG
Figures:		eps.gz png	eps.gz png	eps.gz png	eps.gz png	.
$D^*_{CP}K^-$	BaBar N(BB)=383M	−0.11 ± 0.09 ± 0.01	0.06 ± 0.10 ± 0.02	1.31 ± 0.13 ± 0.03	1.09 ± 0.12 ± 0.04	arXiv:0807.2408
	Belle N(BB)=275M	−0.20 ± 0.22 ± 0.04	0.13 ± 0.30 ± 0.08	1.41 ± 0.25 ± 0.06	1.15 ± 0.31 ± 0.12	PRD 73, 051106 (2006)
	Average	−0.12 ± 0.08 χ^2 = 0.14 (CL=0.71 ⇒ 0.4σ)	0.07 ± 0.10 χ^2 = 0.05 (CL=0.83 ⇒ 0.2σ)	1.33 ± 0.12 χ^2 = 0.12 (CL=0.73 ⇒ 0.4σ)	1.10 ± 0.12 χ^2 = 0.03 (CL=0.87 ⇒ 0.2σ)	HFAG
Figures:		eps.gz png	eps.gz png	eps.gz png	eps.gz png	.
$D_{CP}K^{*-}$	BaBar N(BB)=379M	0.09 ± 0.13 ± 0.05	−0.23 ± 0.21 ± 0.07	2.17 ± 0.35 ± 0.09	1.03 ± 0.27 ± 0.12	CKM2008 preliminary
	Belle	**NO RESULTS AVAILABLE** (*)				-
	Average	0.09 ± 0.14	−0.23 ± 0.22	2.17 ± 0.36	1.03 ± 0.30	HFAG

(*) We do not include a preliminary result from Belle on $D_{CP}K^{*-}$ (BELLE-CONF-0316) which is more than two years old.

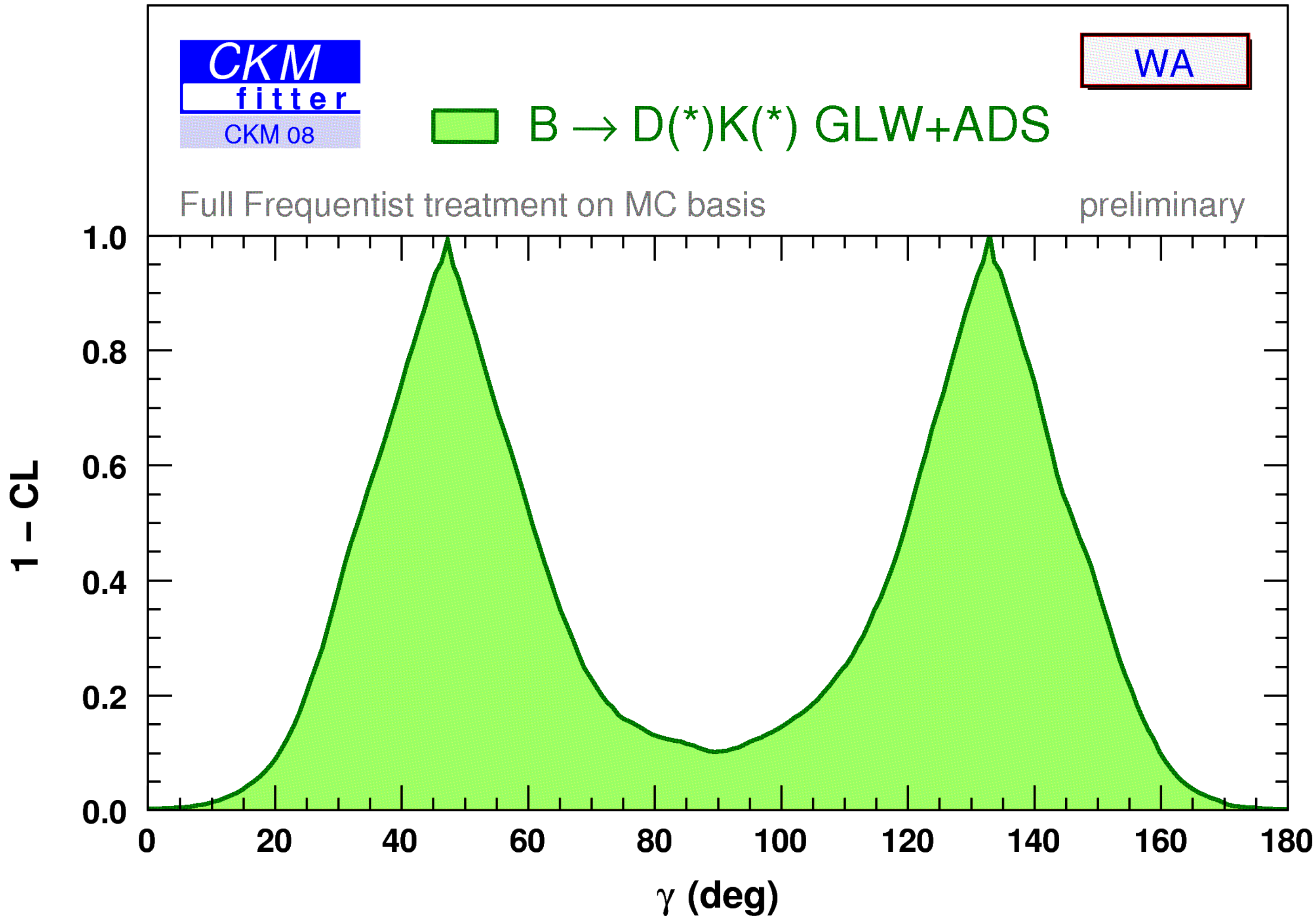
include last BaBar results on DK^*

ADS with DK, D*K, DK*

Mode	Experiment	R_{ADS}	A_{ADS}	Reference
DK^- $D \to K\pi$	BaBar N(BB)=232M	$0.013\ ^{+0.011}_{-0.009}$	-	PRD 72 (2005) 032004
	Belle N(BB)=657M	$0.008 \pm 0.006\ ^{+0.002}_{-0.003}$	$-0.13\ ^{+0.97}_{-0.88} \pm 0.26$	arXiv:0804.2063
	Average	0.010 ± 0.005 $\chi^2 = 0.19$ (CL=0.67 ⇒ 0.4σ)	$-0.13\ ^{+1.00}_{-0.92}$	HFAG
D^*K^- $D^* \to D\pi^0$ $D \to K\pi$	BaBar N(BB)=232M	$-0.002\ ^{+0.010}_{-0.006}$	-	PRD 72 (2005) 032004
	Average	$-0.002\ ^{+0.010}_{-0.006}$	-	HFAG
D^*K^- $D^* \to D\gamma$ $D \to K\pi$	BaBar N(BB)=232M	$0.011\ ^{+0.018}_{-0.013}$	-	PRD 72 (2005) 032004
	Average	$0.011\ ^{+0.018}_{-0.013}$	-	HFAG
DK^{*-} $D \to K\pi$	BaBar N(BB)=379M	$0.066 \pm 0.029 \pm 0.008$	$-0.34 \pm 0.43 \pm 0.16$	CKM2008 preliminary
	Average	0.066 ± 0.030	-0.34 ± 0.46	HFAG

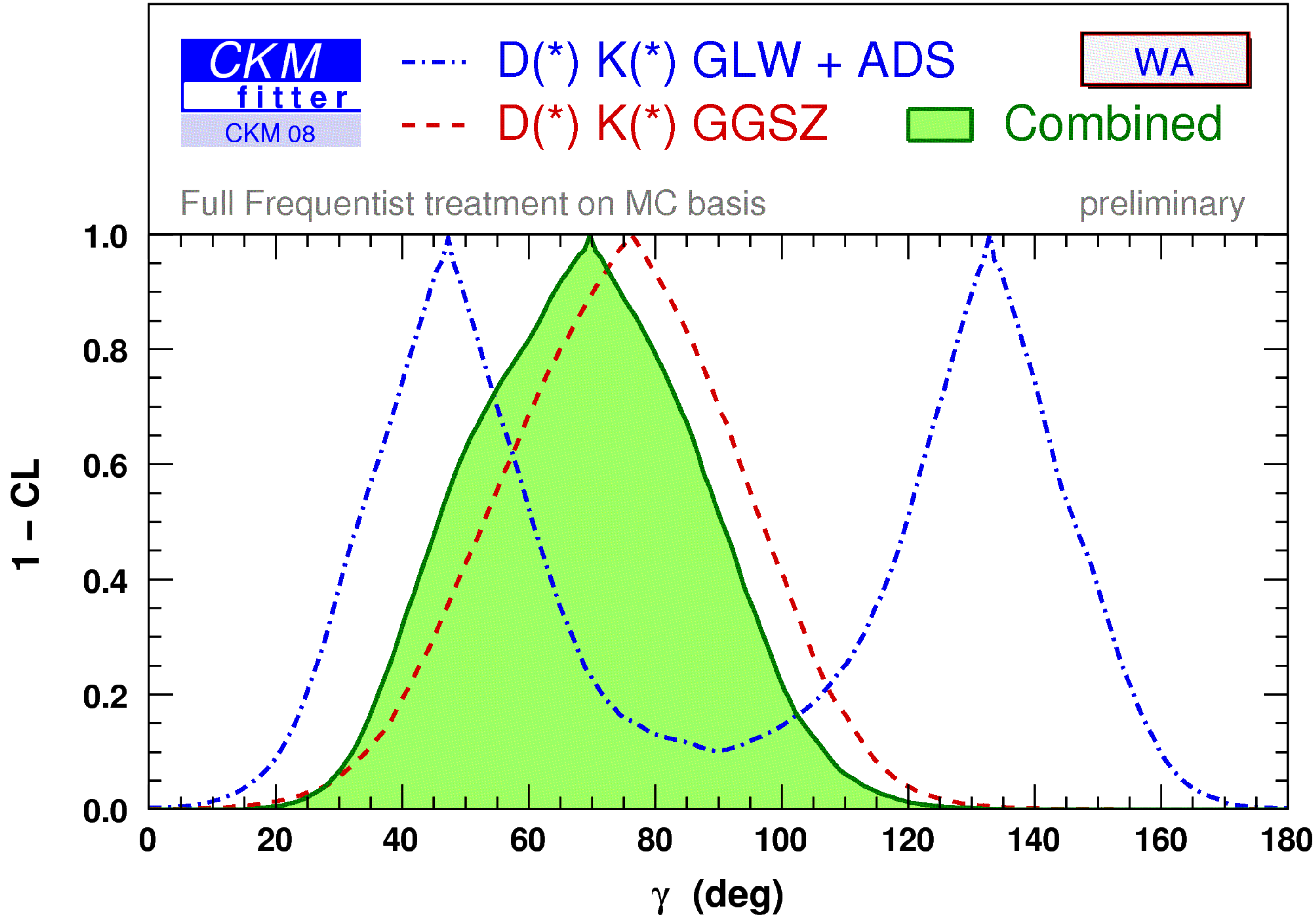


γ result from GLW+ADS BaBar+Belle



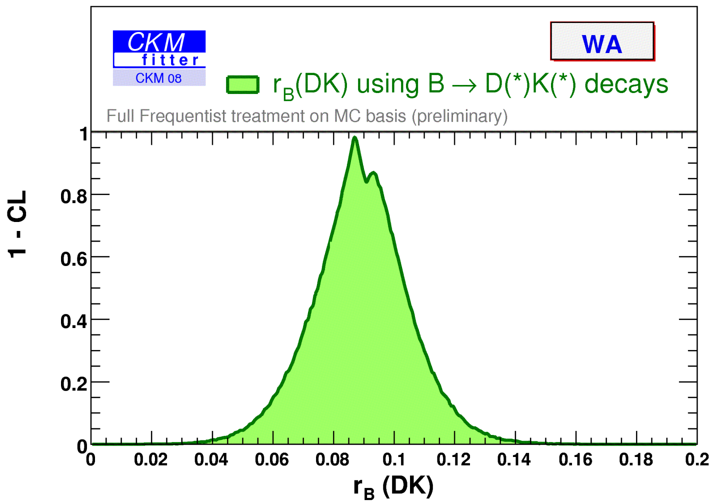
supremum method: $\gamma = (47 \pm 19)^\circ$ and $(133 \pm 19)^\circ$, $(\pm 2\sigma : {}^{+31}_{-116})^\circ$

γ result from GGSZ+GLW+ADS BaBar+Belle

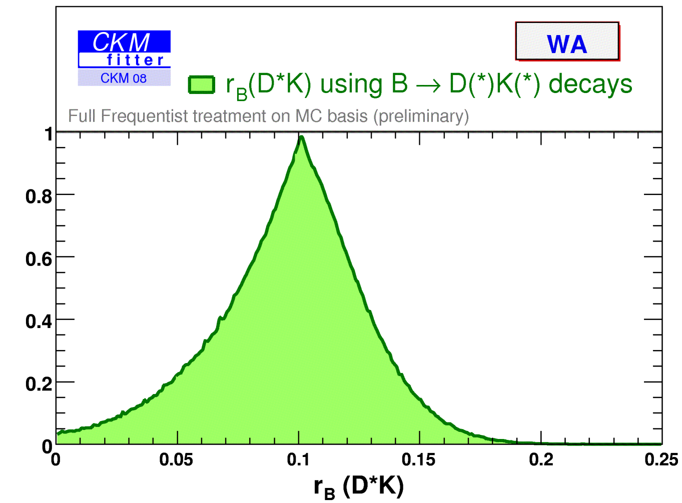


$$\text{supremum method}: \ \gamma = (70^{+27}_{-29})^\circ, \ (\pm 2\sigma: {}^{+43}_{-41})^\circ$$

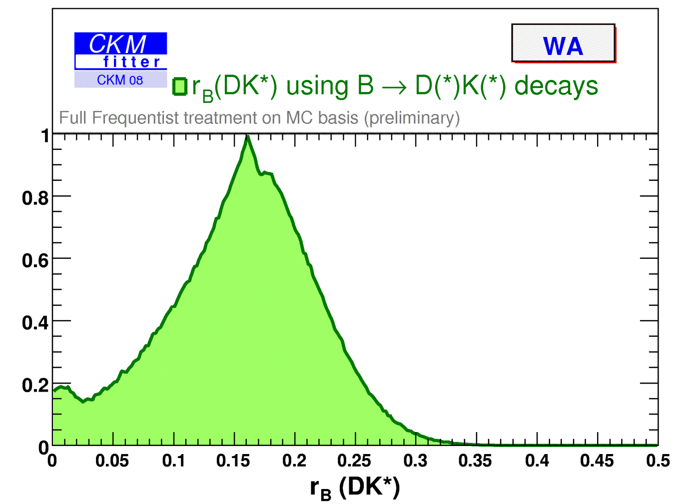
results from GGSZ+GLW+ADS BaBar+Belle



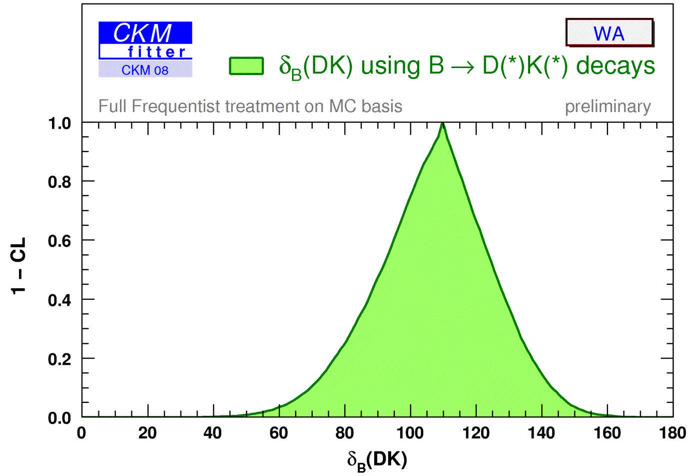
$$r_B(DK)=(0.087^{+0.022}_{-0.028})$$



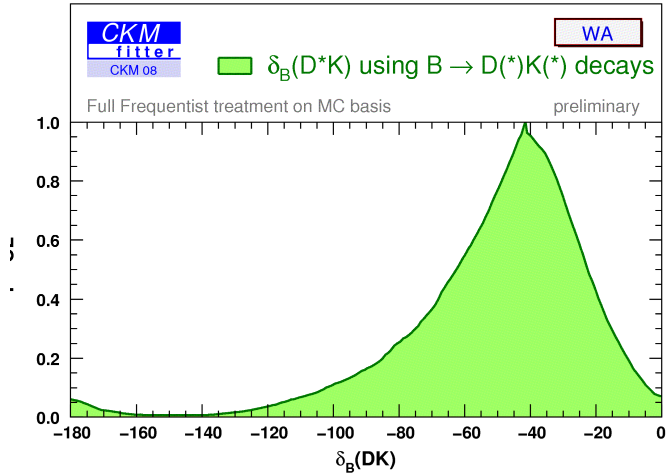
$$r_B(D^*K)=(0.101^{+0.034}_{-0.040})$$



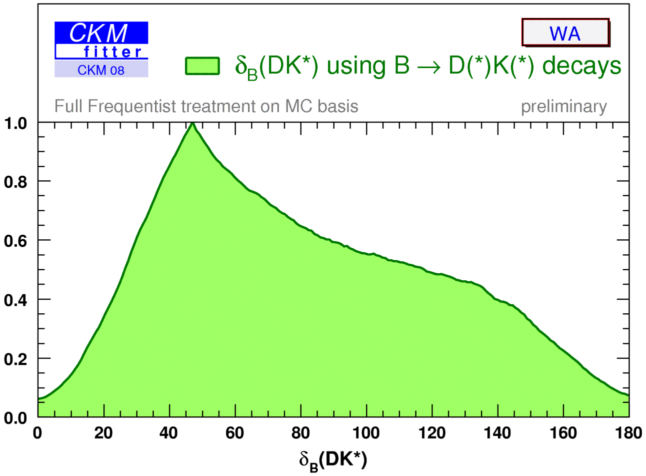
$$r_B(DK^*)=(0.161^{+0.079}_{-0.084})$$



$$\delta_B(DK)=(110^{+22}_{-27})^\circ$$



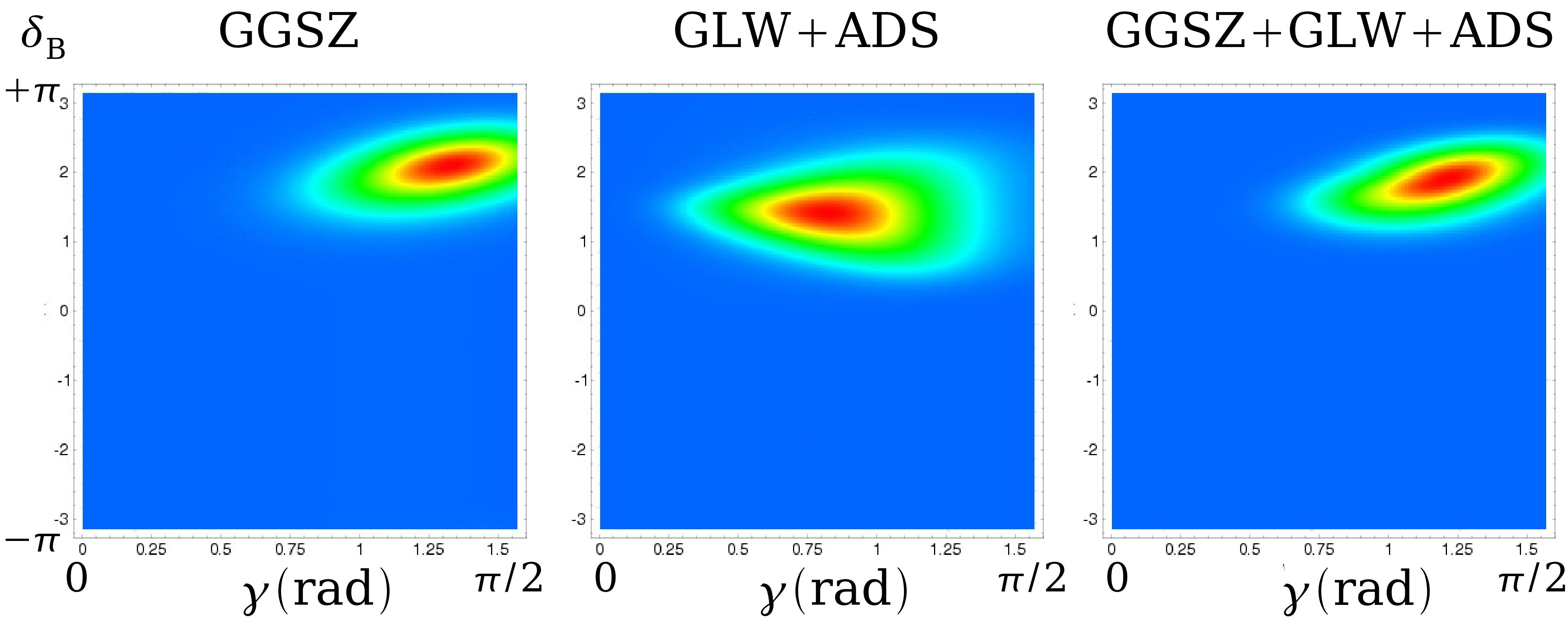
$$\delta_B(D^*K)=(-42^{+26}_{-32})^\circ$$



$$\delta_B(DK^*)=(47^{+103}_{-28})^\circ$$

$\delta_B(DK)$ vs γ

naive statistical treatment
(for illustration purpose only)



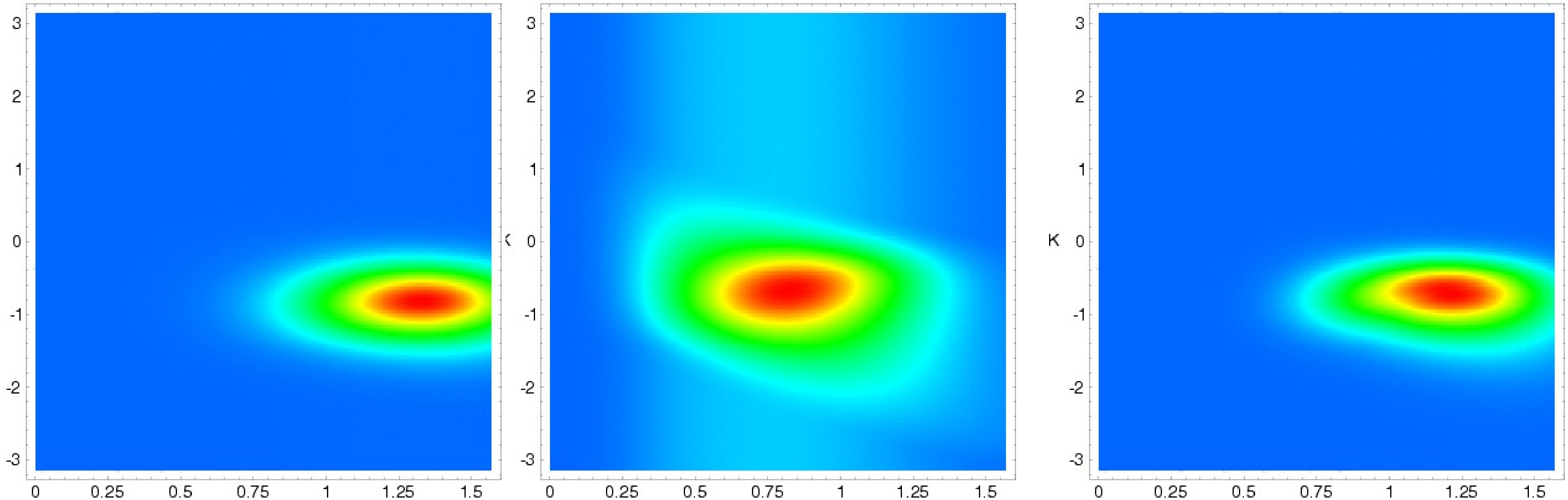
$\delta_B(D^*K)$ vs γ

naive statistical treatment
(for illustration purpose only)

δ_B

GGSZ | GLW+ADS | GGSZ+GLW+ADS

$+\pi$

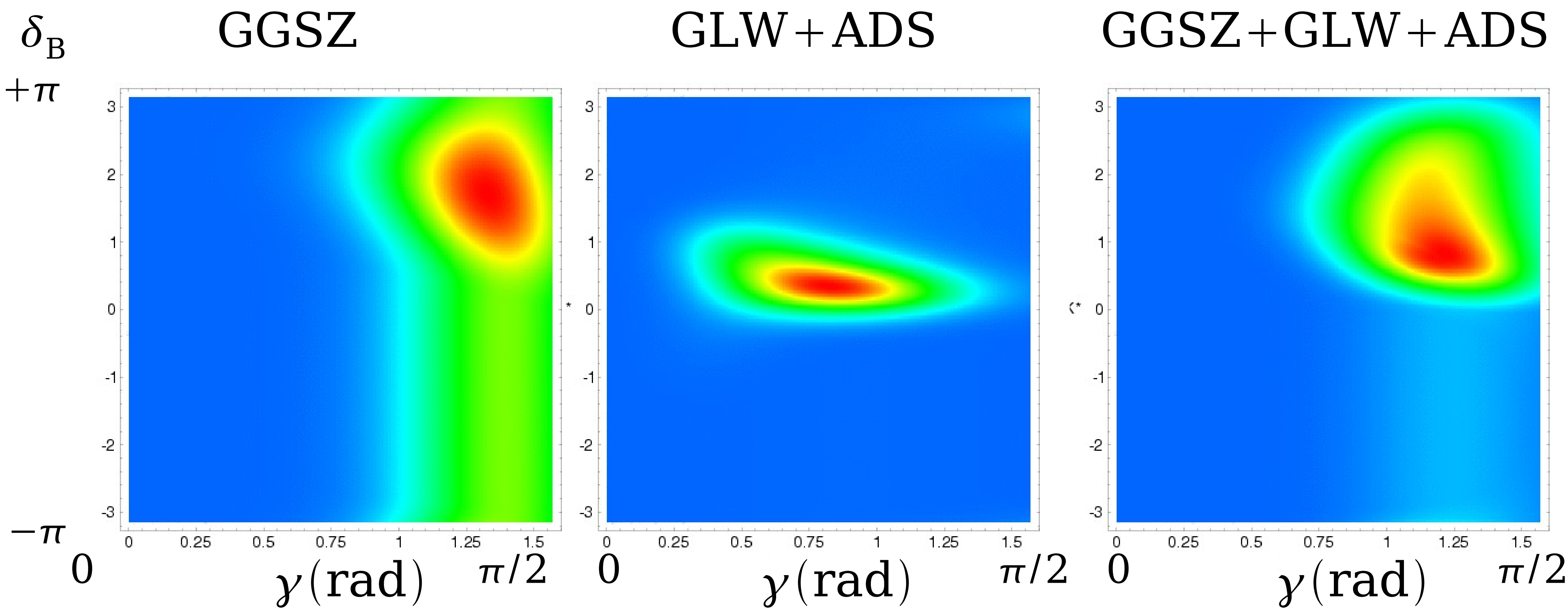


$-\pi$

0 γ(rad) $\pi/2$ 0 γ(rad) $\pi/2$ 0 γ(rad) $\pi/2$

$\delta_B(DK^*)$ vs γ

naive statistical treatment
(for illustration purpose only)



Conclusion

$$\boldsymbol{\gamma = (70^{+27}_{-29})^\circ, \ (\pm 2\sigma : {}^{+43}_{-41})^\circ}$$

- continue the coverage tests
  (also comparison with Belle results)
- new possible methods: adjusted p-value (bootstrap), ...
- add δ_D information !
  (or even better 26 observables, 8 parameters ?)

Backup slides

GLW with $D_{CP}K$, $D^*_{CP}K$, $D_{CP}K^*$