Lattice calculations for exclusive $b \to s$ decays

Matthew Wingate
Dept of Applied Math and Theoretical Physics
University of Cambridge

CKM 2008, Roma

in collaboration with

L Khomskii, R R Horgan, S Meinel (Cambridge)
A Hart, E H Müller (Edinburgh)

(A Subsidiary of the HPQCD Collaboration)

Motivation

- Why calculate exclusive $b \to s$ form factors using LQCD?
- Interest in FCNC obvious here
- Ratios of form factors required for some observables (*e.g.* V_{td} from radiative decays)
- Consistency check of other quantities
- Lesson from saga of V_{ub}: $B \to X_u \ell\nu$ vs. $B \to \pi\ell\nu$
  - Complementary measurements required!
- Difficult, not yet "gold-plated," but worth pursuing

The ultimate goal

Reduce theoretical uncertainties in exclusive $b \to s$ decays

Decays

$$B \to K^* \gamma$$
$$B_s \to \phi \gamma$$
$$B \to (\rho/\omega) \gamma$$

$$B \to K^{(*)} \ell^+ \ell^-$$
$$B_s \to \phi \, \ell^+ \ell^-$$

$$\Lambda_b \to \Lambda \gamma$$
$$\Lambda_b \to \Lambda \, \ell^+ \ell^-$$

SM operators

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \, \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) b_i F_{\mu\nu}$$

$$Q_{9V} = \frac{e}{8\pi^2} (\bar{s} \, b)_{V-A} \, (\bar{\ell} \, \ell)_V$$

$$Q_2 = (\bar{s} \, c)_{V-A} \, (\bar{c} \, b)_{V-A}$$

Some f.f. also have an impact on hadronic decays through QCDF/SCET

Full set of form factors

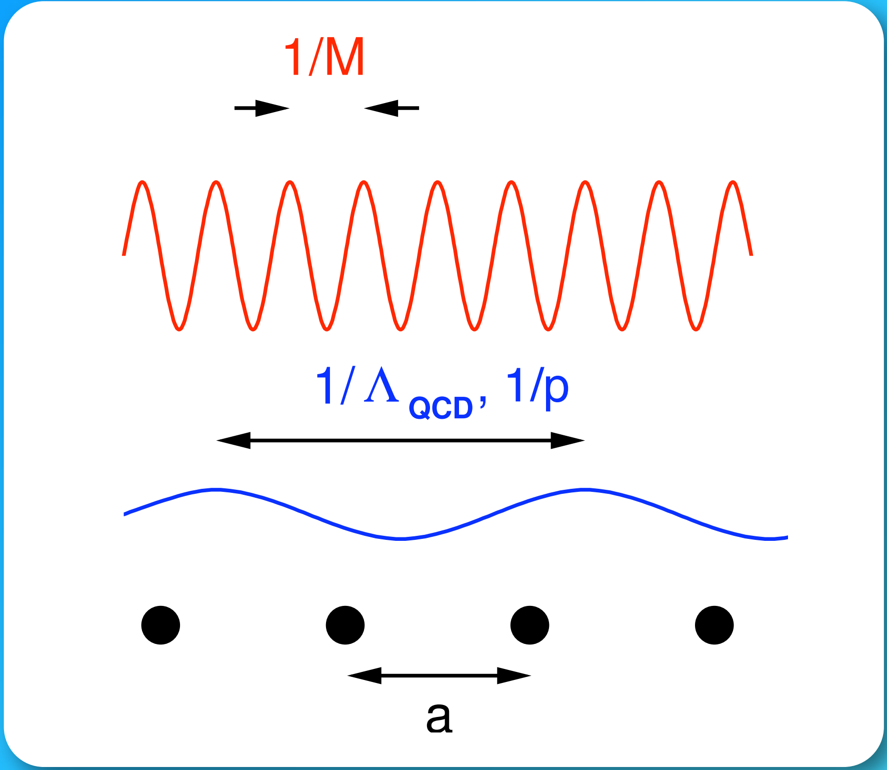
Matrix element	Form factor	Relevant decay(s)
$\langle P\vert\bar{q}\gamma^\mu b\vert B\rangle$	f_+, f_0	$B\to\pi\ell\nu$ $B\to K\ell^+\ell^-$
$\langle P\vert\bar{q}\sigma^{\mu\nu}q_\nu b\vert B\rangle$	f_T	$B\to K\ell^+\ell^-$
$\langle V\vert\bar{q}\gamma^\mu b\vert B\rangle$	V	$\left\{\begin{array}{l} B\to(\rho/\omega)\ell\nu \\ B\to K^*\ell^+\ell^- \end{array}\right.$
$\langle V\vert\bar{q}\gamma^\mu\gamma^5 b\vert B\rangle$	A_0, A_1, A_2	
$\langle V\vert\bar{q}\sigma^{\mu\nu}q_\nu b\vert B\rangle$	T_1	$\left\{\begin{array}{l} B\to K^*\gamma \\ B\to K^*\ell^+\ell^- \end{array}\right.$
$\langle V\vert\bar{q}\sigma^{\mu\nu}\gamma^5 q_\nu b\vert B\rangle$	T_2, T_3	

Full set of form factors

Matrix element	Form factor	Relevant decay(s)
$\langle P \vert \bar{q}\gamma^\mu b \vert B\rangle$	f_+, f_0	$B \to \pi\ell\nu$ $B \to K\ell^+\ell^-$
$\langle P \vert \bar{q}\sigma^{\mu\nu} q_\nu b \vert B\rangle$	f_T	$B \to K\ell^+\ell^-$
$\langle V \vert \bar{q}\gamma^\mu b \vert B\rangle$	V	$\left\{ \begin{array}{l} B \to (\rho/\omega)\ell\nu \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V \vert \bar{q}\gamma^\mu \gamma^5 b \vert B\rangle$	A_0, A_1, A_2	
$\langle V \vert \bar{q}\sigma^{\mu\nu} q_\nu b \vert B\rangle$	T_1	$\left\{ \begin{array}{l} B \to K^*\gamma \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V \vert \bar{q}\sigma^{\mu\nu}\gamma^5 q_\nu b \vert B\rangle$	T_2, T_3	

... also make the spectator an s quark for B_s decays

Difficulties



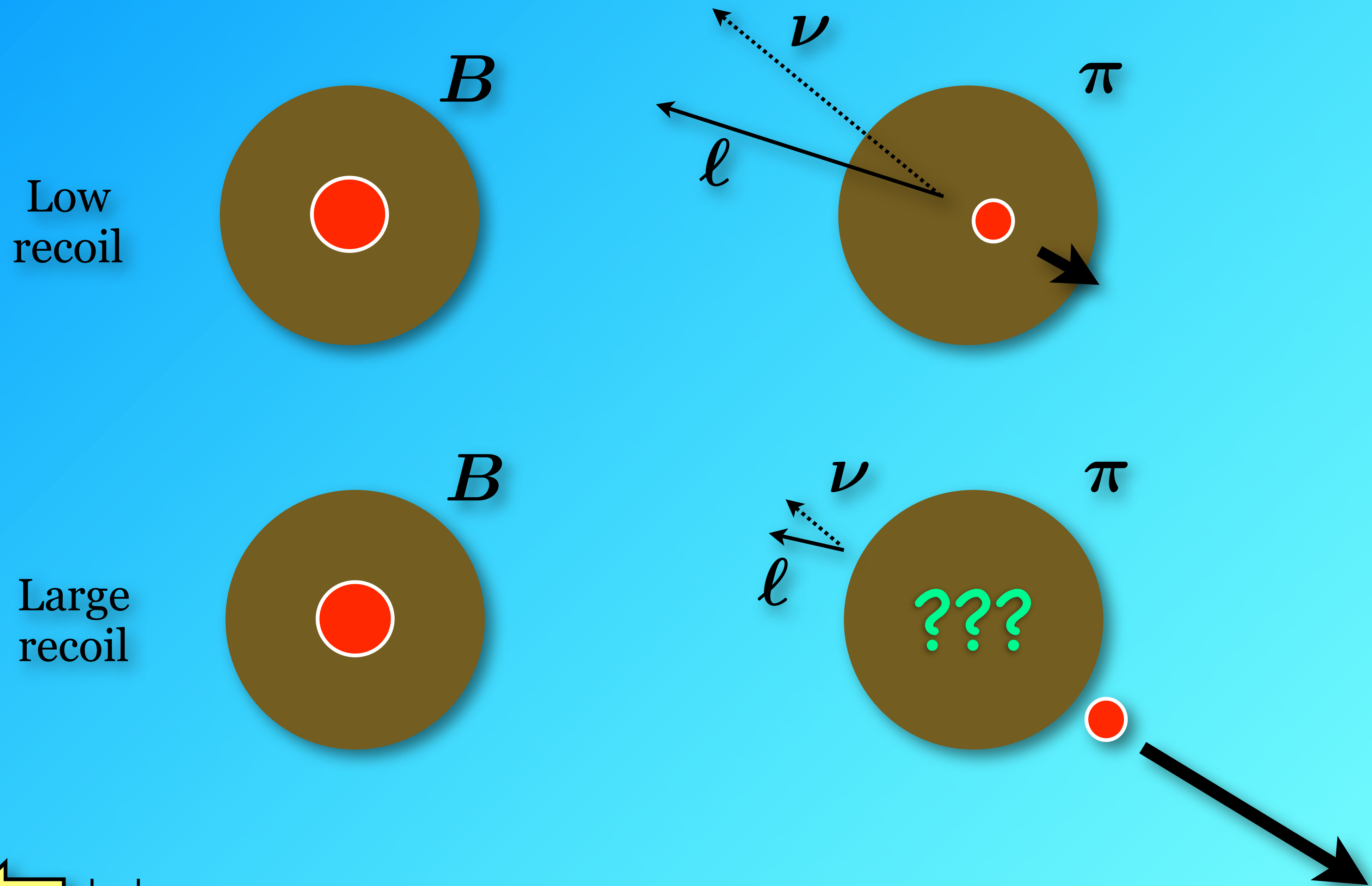
- Long distance effects -- small only away from $\bar{c}c$ resonances
- Breakdown of HQET at large recoil. Need $p_B \cdot p_M \ll m_B^2$

cartoon

- Discretization errors at large recoil. Need $ap_j \ll 1$
- Statistical noise

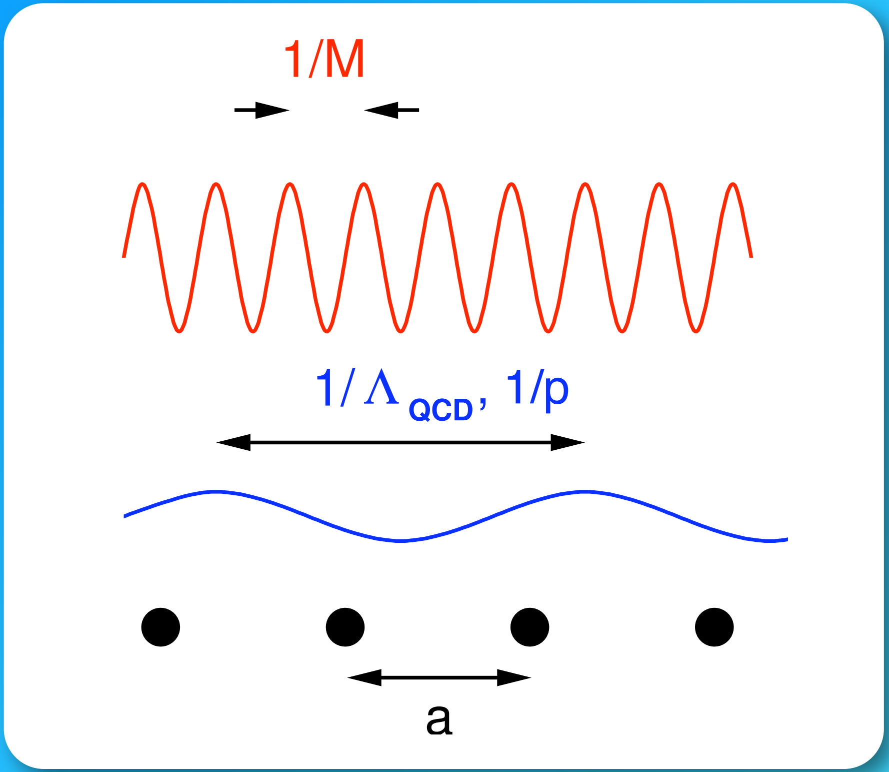
$$\sigma^2(t) = \frac{1}{N}\left(\langle|M\,Q\,B^\dagger|^2\rangle(t) - |\langle M\,Q\,B^\dagger\rangle|^2(t)\right)$$

Breakdown of HQET at large recoil



back

Difficulties



- Long distance effects -- small only away from *cc* resonances

- Breakdown of HQET at large recoil. Need $p_B \cdot p_M \ll m_B^2$

- Discretization errors at large recoil. Need $ap_j \ll 1$

- Statistical noise

$$\sigma^2(t) = \frac{1}{N}\left(\left\langle (M\,Q\,B^\dagger)^2\right\rangle(t) - \left\langle M\,Q\,B^\dagger\right\rangle^2(t)\right)$$

Early LQCD efforts for $B \to K^*\gamma$

- Bowler, *et al.* (UKQCD) (1994)
- Bernard, Hsieh, Soni (1994)
- Abada, *et al.* (APE) (1996)
- Bhattacharya and Gupta (1995)
- Del Debbio, *et al.* (UKQCD) (1998)

Bećirević-Lubicz-Mescia

Nucl. Phys. B769, 31 (2007), hep-ph/0611295

- Most recent study of form factor for $B \to K^* \gamma$
- Calculate with heavy quarks such that $m_H \approx m_D$
  - Allows calculation with $q^2 = 0$
  - Extrapolate using

$$T^{H \to V}(0) \times m_{H_s}^{3/2} = c_0 + c_1 m_{H_s}^{-1} + c_2 m_{H_s}^{-2}$$

- Quenched result:

$$T^{B \to K^*}(q^2 = 0; \mu = m_b) = 0.24 \pm 0.03^{+0.04}_{-0.01}$$

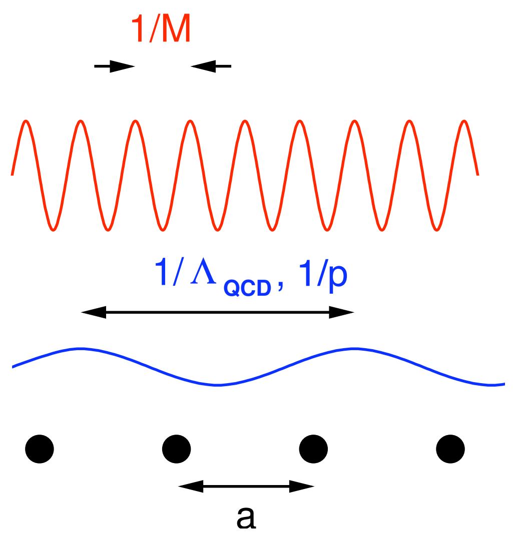
$$T^{B \to K^*}(0)/T^{B \to \rho}(0) = 1.2 \pm 0.1$$

Our plan

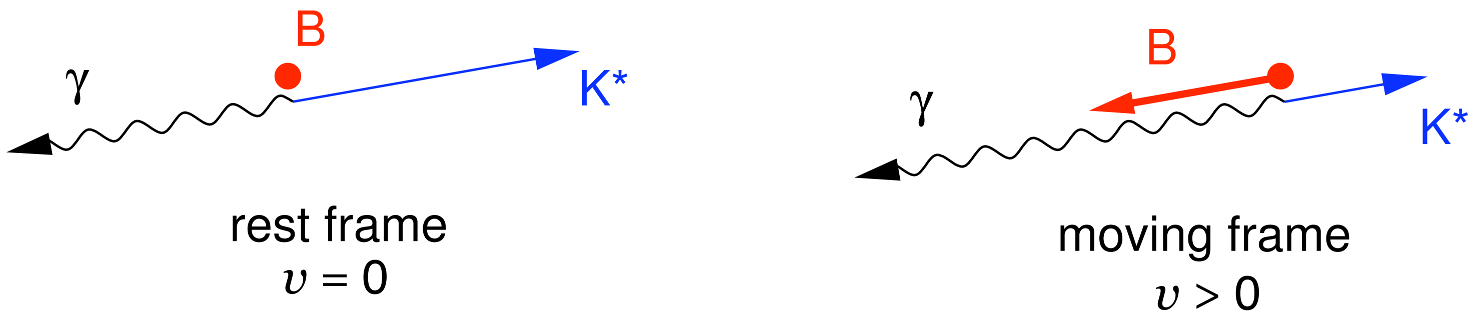
- Use unquenched MILC lattices (impr staggered light quarks)
- Compute directly with b quark using NRQCD
- Large q^2. Extend toward smaller q^2 using
  - Moving-NRQCD
  - Random wall sources (all-to-all propagators)
- All values of q^2 are relevant for semileptonic decays
  - (Must neglect long distance effects)

Lattice dynamics and kinematics

Discretize EFT which treats HQ physics as short distance physics: lattice NRQCD with HQET power counting



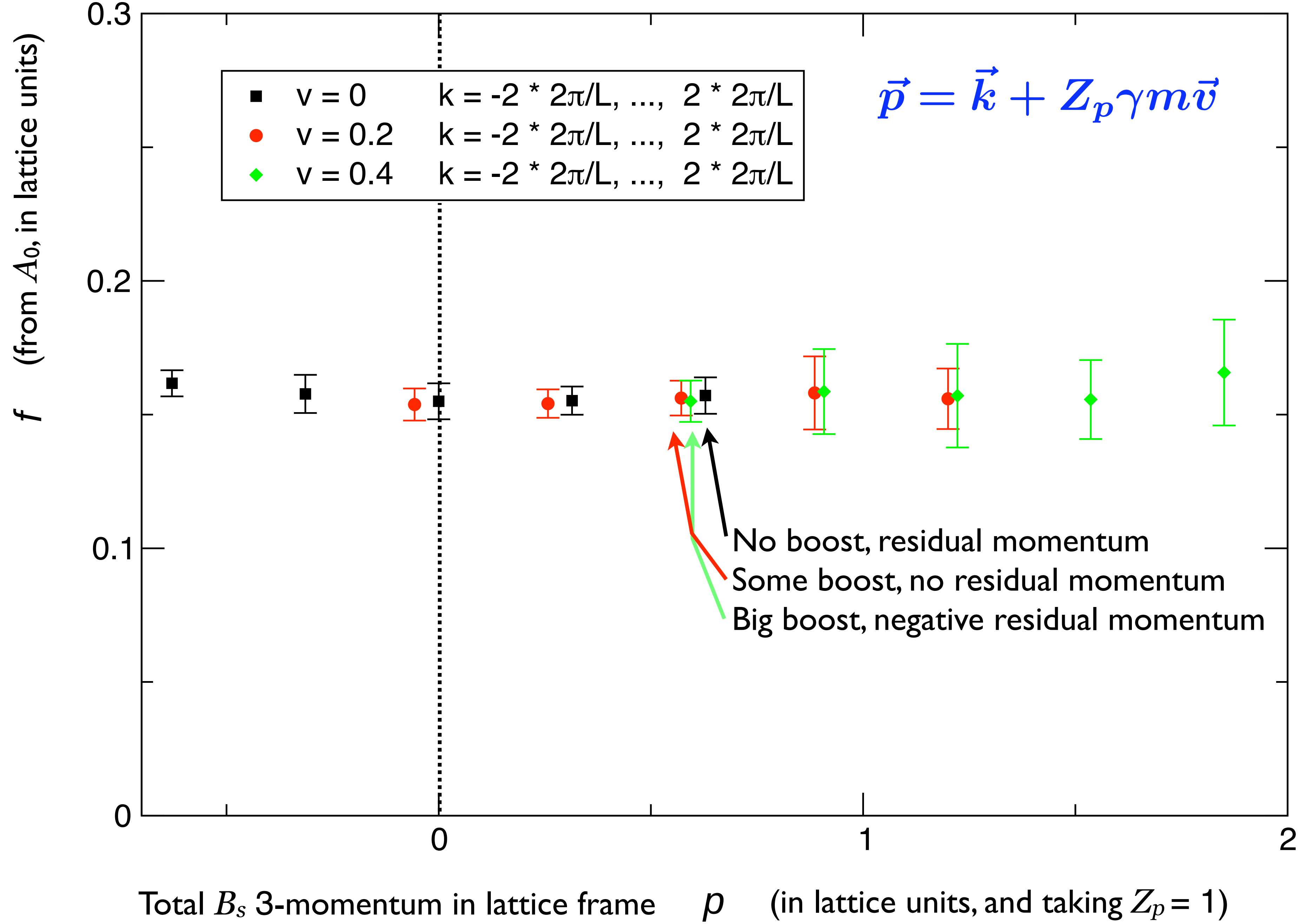
Generalize to discretizing in frame moving relative to *B* (mNRQCD)



We can also give *B* small residual momentum *k* in either frame

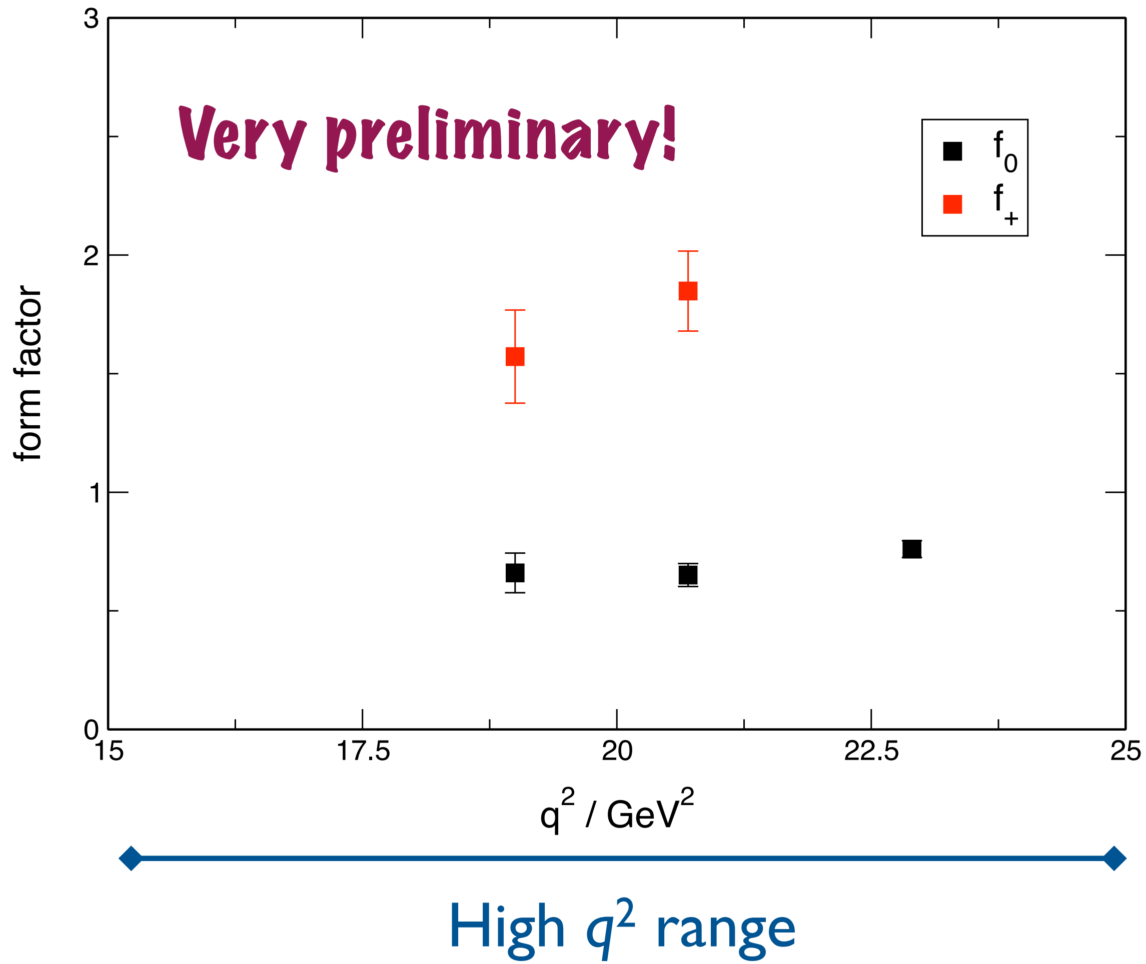
$$\vec{p} = \vec{k} + Z_p \gamma m_B \vec{v}$$

Test of mNRQCD: B_s decay constant



Thanks: S Meinel

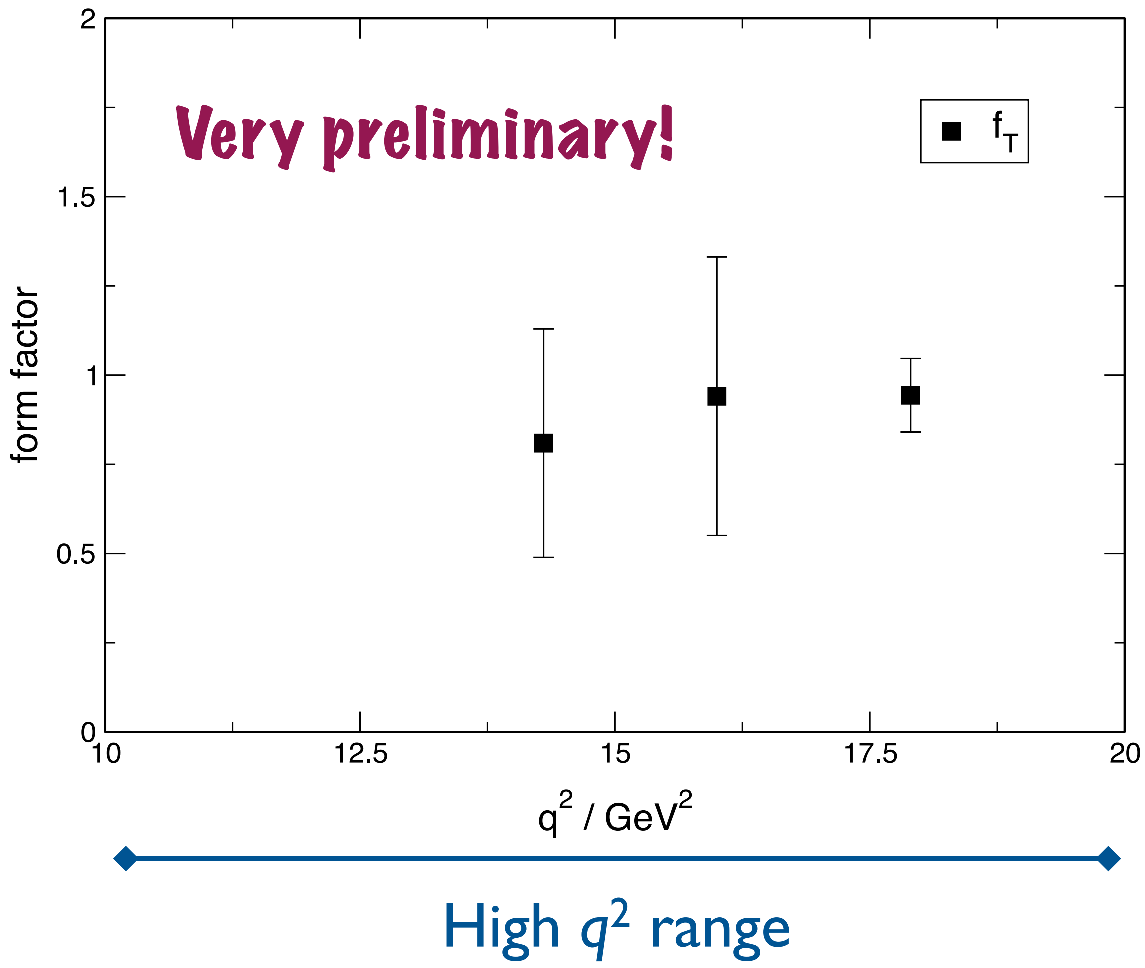
$B \to P$ form factors (V)



- Lowest q^2 points use boost v = 0.2
- Test: comparison to existing $B \to \pi$ form factors
- Try to resolve difference between $B \to \pi$ and $B \to K$

Thanks: S Meinel

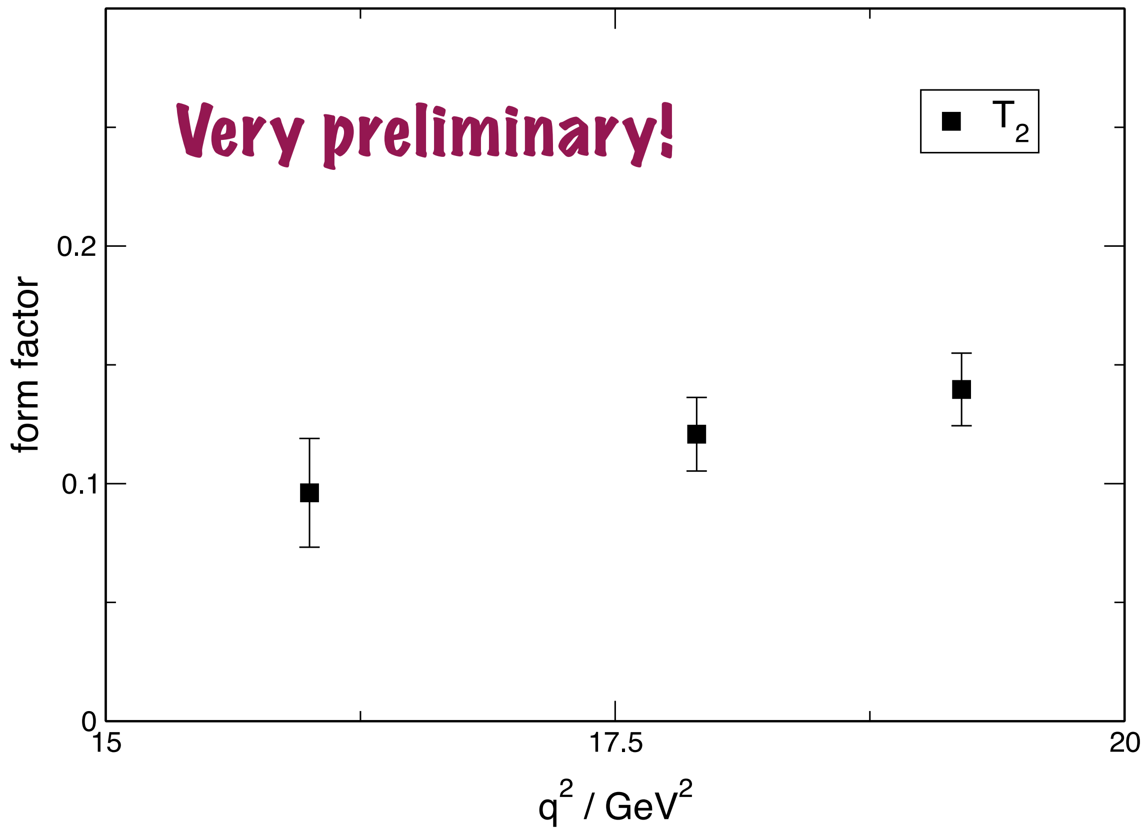
$B \to K$ form factor (T)



- Lowest q^2 points use boost $v = 0.4$
- Note q^2 is lower than in previous and following plot
- Tensor operator intrinsically noisier than vector current

Thanks: S Meinel

$B \to V$ form factor (T)



High q^2 range

- Lowest q^2 points use boost v = 0.2
- Still work to do to improve statistics, lower q^2

Thanks: S Meinel

Perturbative matching

In the continuum, at leading order in $1/m_b$

$$\langle s|Q_7^{\mu\nu}|b\rangle \;=\; (1+\alpha_s\,\delta Z_7)\langle s|Q_7^{\mu\nu}|b\rangle_{\rm tree}$$

with

$$\delta Z_7 \;=\; \frac{1}{3\pi}\left(-\frac{11}{4}-\frac{3}{2}\log\hat\lambda^2\right)$$

On the lattice with boost

$$Q_{7,1}^{\mu\nu} \;=\; \frac{e}{16\pi^2}m_b\sqrt{\frac{1+\gamma}{2\gamma}}\,\bar q\sigma^{\mu\nu}\tilde\Psi_v^{(+)}$$

$$Q_{7,2}^{\mu\nu} \;=\; -\frac{e}{16\pi^2}m_b\,v\sqrt{\frac{\gamma}{2(1+\gamma)}}\bar q\sigma^{\mu\nu}\hat{\vec v}\cdot\vec\gamma\gamma^0\tilde\Psi_v^{(+)}$$

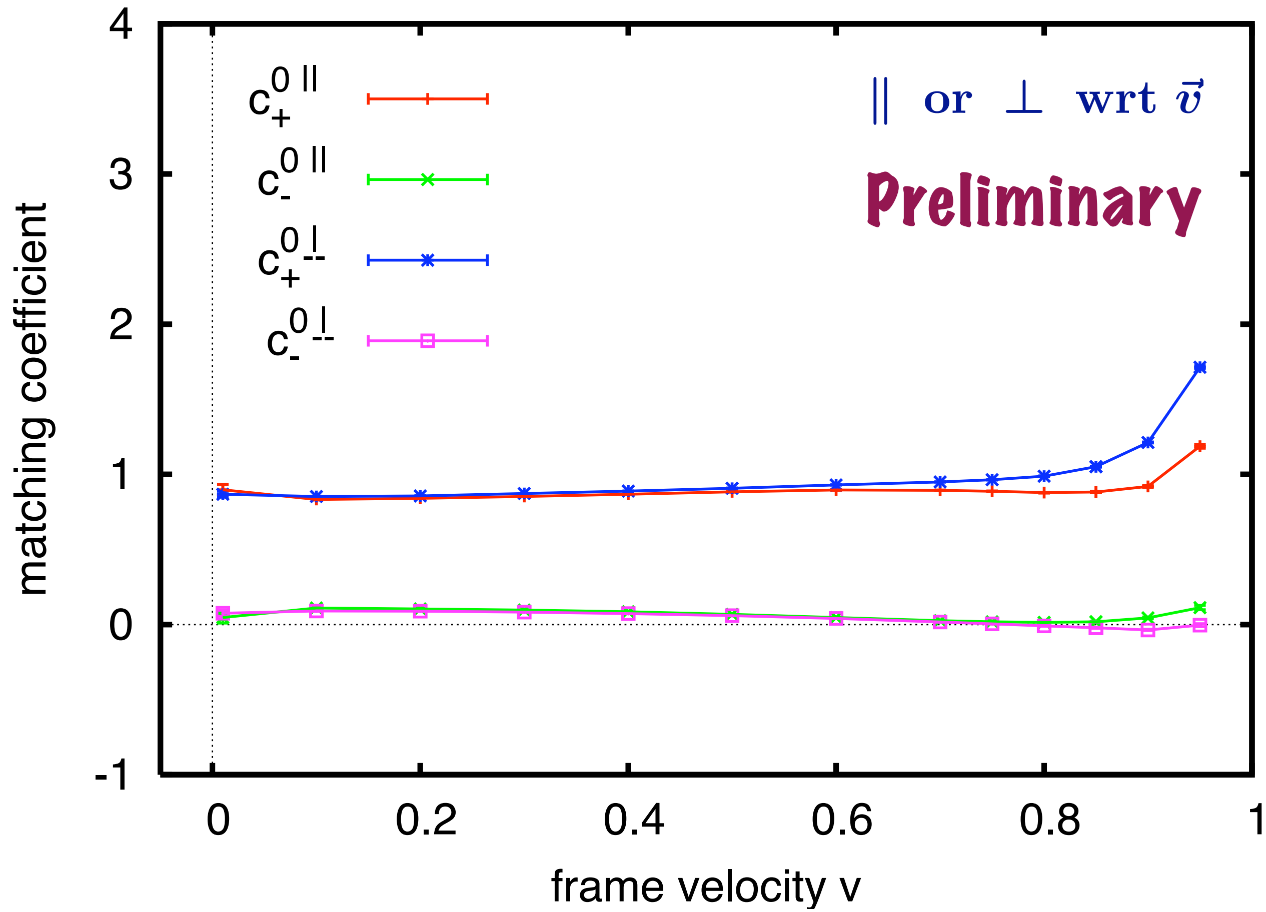
are renormalized separately

$$Q_{7,\pm}^{\mu\nu} \;=\; Q_{7,1}^{\mu\nu}\pm Q_{7,2}^{\mu\nu}$$

$$Q_7^{\mu\nu} \;=\; (1+\alpha_s c_+^{\mu\nu})Q_{7,+}^{\mu\nu}+\alpha_s c_-^{\mu\nu}Q_{7,-}^{\mu\nu}$$

Perturbative matching

$$Q_7^{\mu\nu} \;=\; (1+\alpha_s c_+^{\mu\nu})Q_{7,+}^{\mu\nu} + \alpha_s c_-^{\mu\nu} Q_{7,-}^{\mu\nu}$$



$c_+ \approx 1$

$c_- \approx 0$

Λ_{QCD}/m action, $am_b = 2.8, n = 2$

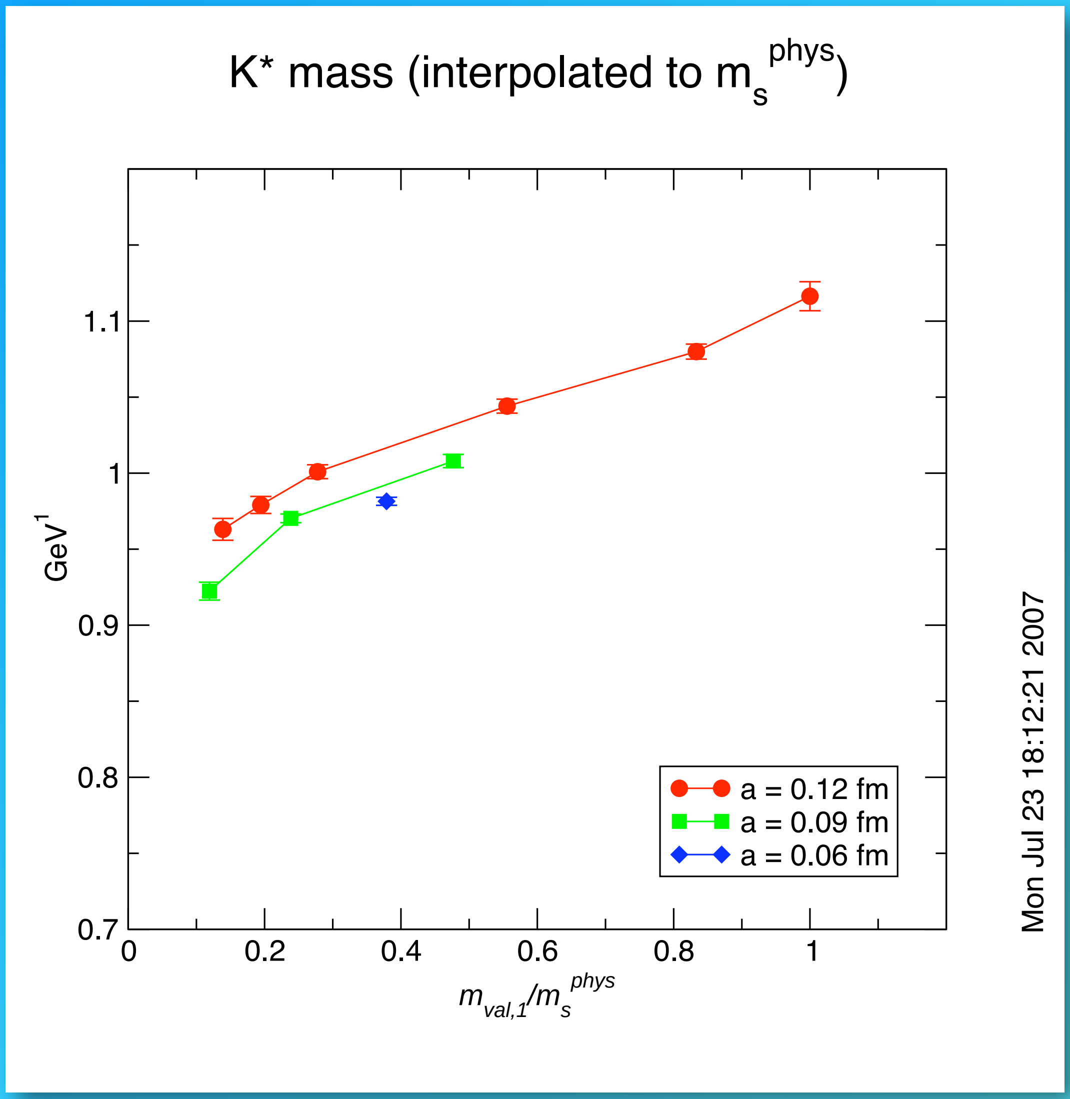
Thanks: E H Müller

Outlook

- mNRQCD implemented and tested (paper in preparation)
- Perturbative matching essentially done (E H Müller, *et al.* (T), L Khomskii (V, A))
- Necessary 3-point correlation functions calculated on single lattice
- Further improve statistics
- Explore systematics
- Too soon for forecasts -- still checks to make, tricks to try
- Forecasts for the book, perhaps

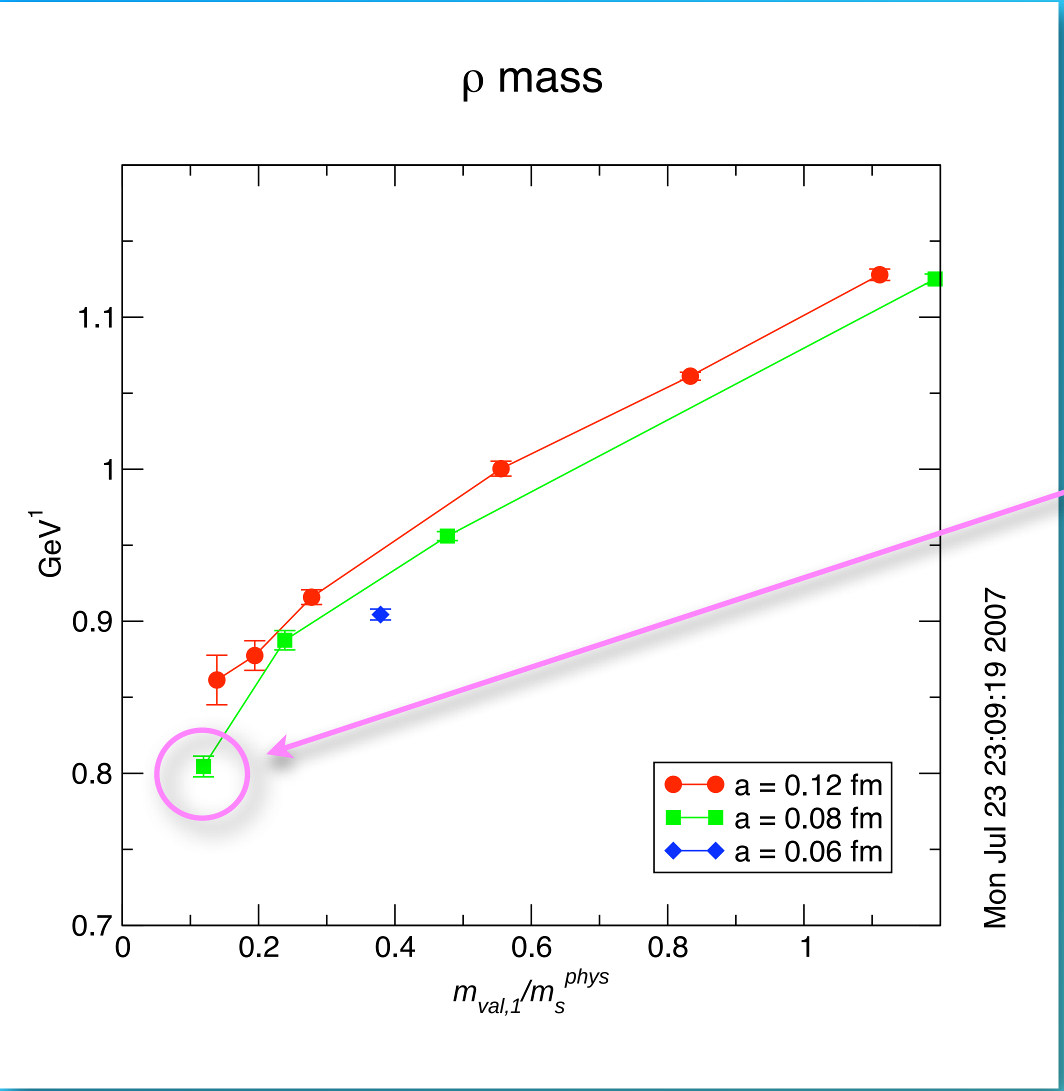
Beyond here,
there be dragons

K^* mass on MILC lattices



- Unquenched data
- Communicated by D. Toussaint, MILC
- Interpolated to (m_1, m_s) using (m_1, m_1) & (m_1, m_2)
- Discretization errors small (for our purposes)
- Negligible taste splitting between local and 1-link tastes (not shown)

ρ mass on MILC lattices



- Unquenched data
- Communicated by D. Toussaint, MILC
- Effected by π-π threshold
- Discretization errors small (for our purposes)
- Negligible taste splitting between local and 1-link tastes (not shown)