$B \to X_c \ell \nu$ Theory: Status and Prospects

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Outline

- Introduction: OPE calculation and moment analysis
- Progress since CKM '06
  - $\mathcal{O}(\alpha_s^2)$ calculation of leading power rate and moments.
  - $\mathcal{O}(\alpha_s)$ calculation of $1/m_b^2$ power corrections.
  - leading-order calculation of $1/m_b^4$.
- Beyond NNLO and $1/m_b{}^4$
  - "Intrinsic charm",
  - quark-hadron duality
  - $B \to X_c \ell \nu$ vs. $B \to X_s \gamma$

Operator Product Expansion

- OPE corresponds to expansion of the rate in $1/m_\mathrm{b}$

$$\Gamma(\bar{B} \to X_c \ell \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left\{ f(\rho) + k(\rho)\frac{\mu_\pi^2}{2m_b^2} + g(\rho)\frac{\mu_G^2}{2m_b^2} \right.$$

$$\left. + d(\rho)\frac{\rho_D^3}{m_b^3} + l(\rho)\frac{\rho_{\mathrm{LS}}^3}{m_b^3} + \mathcal{O}(m_b^{-4}) \right\}$$

with $\rho = \dfrac{m_c^2}{m_b^2}$.

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- Wilson coefficients f, g, k, d, l can be calculated in perturbation theory, e.g.

$$f = f^{(0)}(\rho) + \frac{\alpha_s}{\pi} f^{(1)}(\rho) + \left(\frac{\alpha_s}{\pi}\right)^2 f^{(2)}(\rho) + \mathcal{O}(\alpha_s^3)$$

  - Note: $f = -k$ for total rate.

Operator Product Expansion

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- Non-perturbative parameters $\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ are matrix elements of local operators in HQET, e.g.

$$\frac{1}{2M_B} \langle \bar{B}(p_B) | \, \bar{h}_v (iD)^2 h_v \, | \bar{B}(p_B) \rangle = \lambda_1 = -\mu_\pi^2 \qquad \frac{1}{2M_B} \langle \bar{B}(p_B) | \, \frac{g}{2} \, \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v \, | \bar{B}(p_B) \rangle = 3\lambda_2 = \mu_G^2$$

- scale like $(\Lambda_{\rm QCD})^n$

DETERMINATION OF $|V_{cb}|$

- Need the values of the quark masses m_c and m_b and matrix elements μ_π, μ_G, ρ_D, ρ_{LS} to obtain a precise value of $|V_{cb}|$.

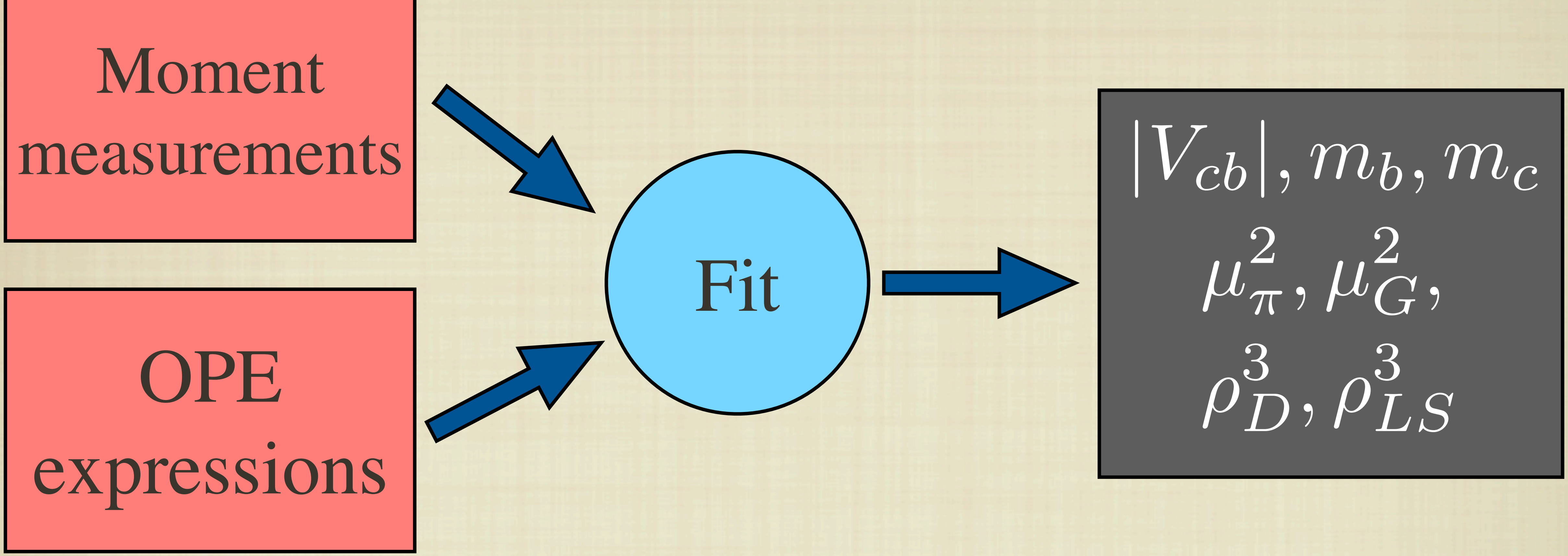
- Can be obtained from spectral moments

$$\langle E_l^n\, E_X^m (M_X^2)^l \rangle = \frac{1}{\Gamma(E_l > E_0)} \int_{E_0}^{E_{\max}} dE_l \int dE_X \int dM_X \, \frac{d\Gamma}{dE_X\, dM_X\, dE_l} \, E_l^n \, E_X^m \, (M_X^2)^l$$

  - OPE for moments involves *same* matrix elements
  - and different, calculable Wilson coefficients.

Moment analysis

see Ch. Schwanda's talk



- Two independent implementations:
  - Bauer, Ligeti, Luke, Manohar and Trott '01 and '04
  - Buchmüller and Flächer '05 (fit) using Benson, Bigi, Gambino, Uraltsev '04 (calculations)
- + improved codes used by Babar, Belle and HFAG.

FIT RESULT

- HFAG, ICHEP '08

Fit Results in the Kinetic Scheme

Input	$\|V_{cb}\|$ (10^{-3})	m_b^{kin} (GeV)	$mu^2{}_{pi}$ (GeV^2)		chi^2/ndf.
all moments ($X_c$lnu and $X_s$gamma)	41.67 +/- 0.43(fit) +/- 0.08(tau_B) +/- 0.58(th)	4.601 +/- 0.034	0.440 +/- 0.040	details	29.7 / (64-7)
$X_c$lnu only	41.48 +/- 0.47(fit) +/- 0.08(tau_B) +/- 0.58(th)	4.659 +/- 0.049	0.428 +/- 0.044	details	24.1 / (53-7)

- Most precise value of $|V_{cb}|$. Exclusive determination gives $|V_{cb}| = (38.7 \pm 0.9_{exp} \pm 1.0_{theo}) \times 10^{-3}$ (J. Laiho's talk),
- and precise value of m_b, crucial input for $|V_{ub}|$.

Scheme Choice

- Pole-scheme for HQET parameters leads to large perturbative corrections, use improved definitions: kinetic-, 1S- , potential-subtracted or shape function scheme, ...
  - common goal of these schemes: reduce IR sensitivity by removing Λ_{QCD} renormalon of pole mass.
- Currently two fits are performed by HFAG, based
  - *1S-scheme* and *kinetic scheme*
  - Seizable two-loop effects in conversion of m_b between schemes! → Two-loop effects in moments will also be important, at least in one of the schemes.
- Note: m_b is an important input for $|V_{ub}|$ determination .
  - Performed in the *shape-function scheme*. Bosch, Neubert, Lange and Paz '05 and in kinetic scheme. *Gambino, Giordano, Ossola and Uraltsev '07*

Recent Progress

NNLO

- NEW: Full two-loop calculation of leading power rate!
  - $\beta_0\,\alpha_s^2$ known before Aquila, Gambino, Ridolfi and Uraltsev '05 + many earlier partial results
- Agreement between two independent calculations
  - Analytical, expansion in m_c/m_b. Czarnecki and Pak '08
    - total rate and lowest two moment in lepton and hadron-energy.
  - Numerical Melnikov '08
    - arbitrary moments with arbitrary cuts
  - Corrected earlier estimate of the correction to the rate.

Czarnecki, Dowling and Pak '08

NNLO RESULTS

- Lepton energy moments, normalized to tree-level rate

$$L_0 = \quad 1 - 1.78\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2\left[-1.92_{BLM}\beta_0 + 3.40\right]$$

$$L_1 = \quad 0.307\left\{1 - 1.79\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2\left[-2.01_{BLM}\beta_0 + 3.61\right]\right\}$$

$$L_2 = \quad 0.102\left\{1 - 1.82\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2\left[-2.11_{BLM}\beta_0 + 3.83\right]\right\}$$

- The BLM-corrections were known before and are included in fit [$\beta_0 = 8.33$]. The new, non-BLM, terms have opposite sign.
- Leads to a ~ 1% reduction in $|V_{cb}|$ (pole scheme).
  - Fit in kinetic scheme includes an estimate of non-BLM terms, further reduction is 0.25 x 10^{-3} ~ 0.6%.
  - Shift in 1S scheme is -0.14 x 10^{-3}.

Lepton Moments

K. Melnikov, arXiv:0803.0951

- Lepton moments, but now normalized to the total rate

$$
\begin{aligned}
L_0 &= 1 \\
L_1 &= 0.307\left\{ 1 - 0.02\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-0.09_{BLM}\beta_0 + 0.18\right]\right\} \\
L_2 &= 0.102\left\{ 1 - 0.04\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-0.19_{BLM}\beta_0 + 0.35\right]\right\}
\end{aligned}
$$

- with a cut E_l>1 GeV

$$
\begin{aligned}
L_0 &= 0.815\left\{1 + 0.01\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-0.05_{BLM}\beta_0 - 0.15\right]\right\} \\
L_1 &= 0.278\left\{1 - 0.01\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-0.12_{BLM}\beta_0 + 0.18\right]\right\} \\
L_2 &= 0.098\left\{1 - 0.04\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-0.21_{BLM}\beta_0 + 0.42\right]\right\}
\end{aligned}
$$

- In either case, the corrections are small! Looks like small correction for m_b.

Power Corrections at NLO

$$\Gamma(\bar{B} \to X_c \ell \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left\{ f(\rho) + k(\rho) \frac{\mu_\pi^2}{2m_b^2} + g(\rho) \frac{\mu_G^2}{2m_b^2} + \mathcal{O}(m_b^{-3}) \right\}$$
$$\text{with } \rho = m_c^2/m_b^2$$

- The one-loop corrections to the Wilson coefficient of the power suppressed kinetic- and chromo-magnetic contribution, $k(\varrho)$ and $g(\varrho)$, give effects of the same order of magnitude as two-loop $f(\varrho)$.
  - Have evaluated one-loop kinetic corrections to moments, TB, Boos and Lunghi, arXiv:0710.0680
  - chromo-magnetic corrections in progress.

TB, Lange, Lunghi; Mannel et al.

One-loop kinetic corrections

- To get kinetic correction, one needs to expand one-loop rate in residual momentum $p_b^\mu = m_b\, v^\mu + r^\mu$.
- Two ways of doing this matching calculation:
  - expand partonic rate in r_μ *before* or *after* loop and phase-space integration. Since on-shell HQET matrix elements are trivial, the expansion commutes with loop integration.
- As a check on our result and its numerical accuracy, we have performed the calculation both ways.
  - Expanding after integration is more elegant and more subtle.
  - Expanding before integration somewhat tedious but a good warm up for calculation of chromo-magnetic corrections.

Results for Partonic Moments

	1	$\frac{\alpha_s}{\pi}$	$\frac{\mu_\pi^2}{2m_b^2}$	$\frac{\alpha_s}{\pi}\,\frac{\mu_\pi^2}{2m_b^2}$	%
1	0.5149(3)	−0.910(3)	−0.5692(6)	0.987(8)	0.1
$\hat{E}_l$	0.1754(1)	−0.314(1)	0.0109(3)	−0.024(3)	0.
$\hat{E}_l^2$	0.06189(5)	−0.1128(5)	0.1105(1)	−0.202(1)	−0.2
$\hat{E}_l^3$	0.02251(2)	−0.0418(2)	0.09269(5)	−0.1722(7)	−0.6
$\hat{E}_x$	0.2111(1)	−0.365(1)	−0.5694(2)	1.010(3)	0.4
$\hat{E}_x^2$	0.08917(7)	−0.1482(7)	−0.3378(1)	0.576(1)	0.5
$\hat{E}_x^3$	0.03867(4)	−0.0606(4)	−0.16898(6)	0.2639(7)	0.5

with cut $\hat{E}_l = E_l/m_b > 1/4.6$ and $\frac{m_c}{m_b} = 1/4$

$$\frac{\alpha_s}{\pi} \approx 0.07$$

$$\frac{\mu_\pi^2}{2m_b^2} \approx 0.01$$

- Small correction for moments which are of “natural size”.
- Expect chromo-magnetic corrections to be more important, since tree-level corrections are few times larger.

Results for Partonic Moments

	1	$\frac{\alpha_s}{\pi}$	$\frac{\mu_\pi^2}{2m_b^2}$	$\frac{\alpha_s}{\pi}\,\frac{\mu_\pi^2}{2m_b^2}$	$\%$
$(\hat{p}_x^2-\rho)$	0	0.03618(2)	−0.6855(2)	1.213(2)	−25.5
$(\hat{p}_x^2-\rho)^2$	0	0.002808(2)	0.15198(4)	−0.4388(5)	−21.6
$(\hat{p}_x^2-\rho)^3$	0	0.0004053(3)	0	0.020998(4)	32.9
$\hat{E}_x(\hat{p}_x^2-\rho)$	0	0.01801(1)	−0.20707(6)	0.2961(8)	−39.2
$\hat{E}_x(\hat{p}_x^2-\rho)^2$	0	0.0015307(10)	0.06794(2)	−0.1897(3)	−20.1
$\hat{E}_x^2(\hat{p}_x^2-\rho)$	0	0.009147(6)	−0.05271(2)	0.0304(3)	12.4

with cut $\hat{E}_l = E_l/m_b > 1/4.6$ and $\frac{m_c}{m_b} = 1/4$

$$\frac{\alpha_s}{\pi} \approx 0.07$$

$$\frac{\mu_\pi^2}{2m_b^2} \approx 0.01$$

- Large corrections for those moments that vanish at leading power.
  - The corrections will change the extracted value of $\mu_\pi{}^2$ by about 20-30% in the pole scheme (maybe less with a better scheme).

$1/m_b^4$ Corrections

Dassinger, Mannel and Turczyk '06

- Heavy quark expansion has been carried out one order further for rate and moments.
- Five additional hadronic parameters $s_1, \ldots, s_5$.
- Using "naive factorization" estimate for these parameters, the effects are found to be small unless high moments are considered.
  - e.g. $\frac{\delta^{(4)}\Gamma}{\Gamma} \approx 0.25\%$
- Can either try to extract $s_{1, \ldots, s_5}$ from moment fit, or scan over some range to estimate theoretical $1/m_b{}^4$ uncertainty.

Error estimates

- How do the newly calculated results compare to earlier error estimates?
  - Moment fit in kinetic scheme estimated

$$A_{\text{pert}} = \frac{\Gamma(B \to X_c \ell \nu)}{\Gamma(B \to X_c \ell \nu)|_{\text{tree}}} = 0.908 \pm 0.009$$

    - Full NNLO value: $A_{\text{pert}} = 0.919$ ✓
  - Fit in 1S scheme estimated uncertainty from non-BLM two-loop piece to be half of of the BLM piece. This amounts to a 1.5% of the tree-level rate, 3x more than the actual correction. ✓

Power corrections

- For α_s-corrections to $1/m_b^2$ terms:
  - Kinetic scheme fit estimates those by varying μ_π and μ_G by 20%.
  - 1S fit estimates them to be

$$\frac{\alpha_s}{4\pi}\left(\frac{\Lambda_{QCD}}{m_b}\right)^2 \sim 0.0002$$

  - Calculation of kin. corr typically gives values

$$8 \times \frac{\alpha_s}{4\pi}\left(\frac{\Lambda_{QCD}}{m_b}\right)^2 = 0.15\left(\frac{\Lambda_{QCD}}{m_b}\right)^2$$

    - Larger than 1S estimate, as expected in kin. fit.

To Do

- Finish $O(\alpha_s)$ calculation of chromomagnetic Wilson coefficient, $g(\rho)$.
  - if possible evaluate also $O(\alpha_s)$ for $d(\rho)$ the coefficient of ρ_D, which is $1/m_b^3$ but numerically large.
- Implement new corrections into moment fit codes
  - Convert from pole to other schemes: kinetic, 1S, shape-function scheme, ...
  - What form is most convenient? Grid of values, expansion around default values?
- This will largely eliminate theoretical uncertainties from moment fit.
  - increased precision and better consistency check

Beyond α_s^2 and $1/m_b^4$.

"Intrinsic Charm"

Bigi, Uraltsev, Zwicky '05; Breidenbach, Feldmann, Mannel, Turczyk'08

- In the standard OPE, $m_b \sim m_c \gg \Lambda_{QCD}$ charm quarks are integrated out in perturbation theory, such that no operators with c-quarks have to be considered.
- Can instead assume $m_b \gg m_c \gg \Lambda_{QCD}$ and perform the OPE in two steps:
  - First integrate out physics associated with $\mu \sim m_b$. OPE includes operators with c-quarks, e.g.

$$(\bar{b}_v \, \gamma_\nu P_L \, c) \, (\bar{c} \, \gamma_\mu P_L \, b_v)$$

  - Then integrate out charm.
    - Second step is an expansion in $1/m_c$.

“Intrinsic Charm”

- Leading terms from the expansion in $1/m_c$ scale as

$$\propto \frac{\Lambda_{\rm QCD}^5}{m_c^2 m_b^3} \qquad \text{or} \qquad \alpha_s(m_c)\frac{\Lambda_{\rm QCD}^4}{m_c m_b^3}$$

  - In the standard counting $1/m_b^5$ and α_s/m_b^4 , but enhanced compared to other contributions.
  - Numerical estimate gives Bigi, Uraltsev, Zwicky ’05

$$\frac{\delta\Gamma_{\rm s.l.}}{\Gamma_{\rm s.l.}} \approx 0.003$$

- N.B. effect has little or nothing to do with “intrinsic charm” in exclusive decays.

"Intrinsic Charm"

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- Ir
  co

muliplied by $d(\rho)$, can be absorbed into redefinition of ρ_D

Breidenbach, Feldmann, Mannel, Turczyk'08

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Quark-Hadron Duality

- Expanding in $1/m_b$ one looses non-analytic terms of the form

$$\frac{1}{m_b^n}\sin(m_b/b)$$

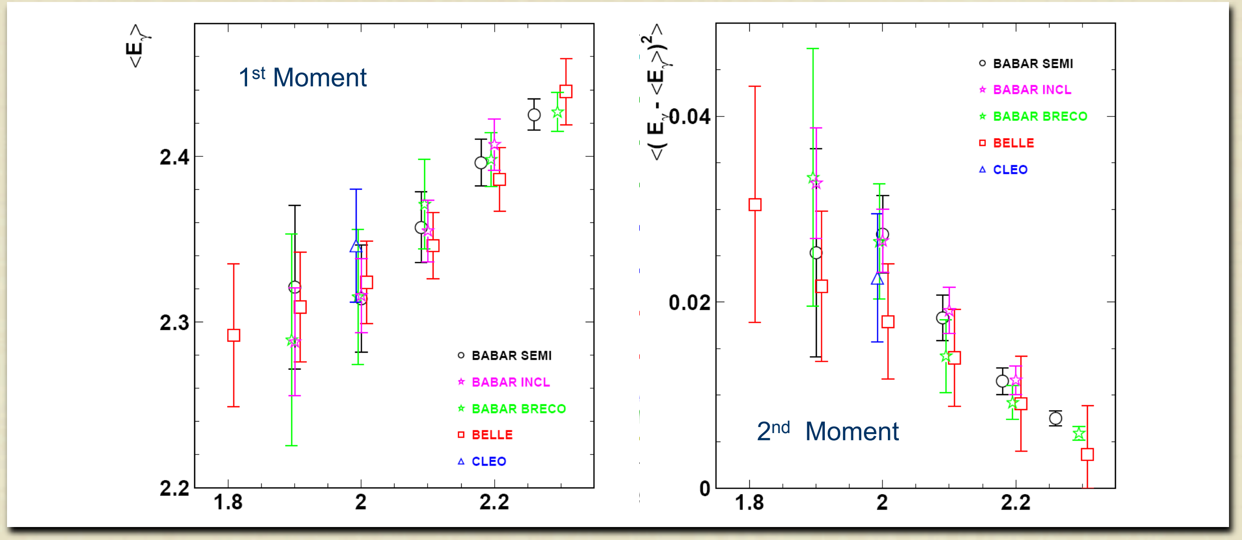
  - Model calculations Shifman '00 give high value n=8 suppression relative to leading term for semileptonic decay rate.
    - if so, these effects are tiny
  - Oscillatory terms due to resonances and get averaged away in sufficiently inclusive quantities.
    - Effects can become bigger once cuts are applied

Duality in $b \to c$

- To my knowledge all model studies were done for *totally inclusive* $b \to u$ rate, but $b \to c$ situation seems quite different.
- Lowest resonances essentially saturate $B \to X_c\, l\, \nu$
  - ~ 80% $X_{\rm c} = D$ or D^*
  - ~ 5-10% $X_{\rm c} = D^{**}$
- Lowest states (π, η, ϱ, ω) amount to ~ 30% of $B \to X_u\, l\, \nu$
- It would be interesting to specifically try to study duality violation in “heavy quark ” → “heavy quark” decays.

$B \to X_s\gamma$ MOMENTS

- Moment fit currently also includes $\bar{B} \to X_s\gamma$ photon energy moments with cut on $E_\gamma > 1.8$ GeV



- Because of the hard cut they cannot be reliably calculated using the standard OPE.
  - Non-perturbative shape function, even at leading power.
  - Current fits either use OPE directly (1S) or use shape function model (kin. scheme) “bias correction”.
  - Until the uncertainty from the shape function is estimated, these moments should not be included in fit.

Summary

- Moment analysis of $B \to X_c\, l\, \nu$ provides most precise value of $|V_{cb}|$ and crucial input (m_b, μ_π^2) for $|V_{ub}|$ determination
- Efforts to push the calculation of inclusive B decays to NNLO are far along.
  - Two-loop calculations of rate and moments complete.
  - One-loop corrections to power suppressed terms:
    - Evaluation of kinetic corrections complete,
    - chromo-magnetic corrections in progress.
  - Corrections need to be implemented into fitting codes.
    - This will eliminate large part of theory uncertainty.
- Theoretical treatment $B \to X_s \gamma$ photon energy moments is model dependent, needs to be improved.

Extra Slides

Theoretical vs. experimental uncertainties

- Theoretical uncertainties are larger than experimental ones.

$B \to X_c \ell \bar{\nu}$			OPE FIT RESULT	
$+ B \to X_s \gamma$	$\|V_{cb}\| \times 10^{-3}$	m_b (GeV)	m_c (GeV)	μ_π^2 (GeV^2)
RESULT	41.91	4.613	1.187	0.408
Δ exp	0.19	0.022	0.033	0.017
Δ HQE	0.28	0.027	0.040	0.031
$\Delta\, \Gamma_{\rm SL}$	0.59			

Buchmüller and Flächer '07 for LP07 (kinetic scheme)

Impact of reduced theory Uncertainties

- Run fit w/o theoretical uncertainties. ($B \rightarrow X_c \nu l$ only)

	with theory uncertainties		w/o theory unc.	
	value	unc.	value	unc.
V_{cb}	41.85	$\pm 0.38 (\pm 0.58)$	41.42	± 0.31
m_b	4.66	± 0.053	4.699	± 0.04
m_c	1.262	± 0.078	1.312	± 0.061
μ_π	0.4169	± 0.0379	0.3736	± 0.0217
ρ_D	0.092	± 0.022	0.066	± 0.012
μ_G	0.237	± 0.046	0.252	± 0.039
ρ_{LS}	-0.139	± 0.089	-0.082	± 0.06

thanks to H. Flächer