

Inclusive moment measurements and determination of $|V_{cb}|$ and m_b

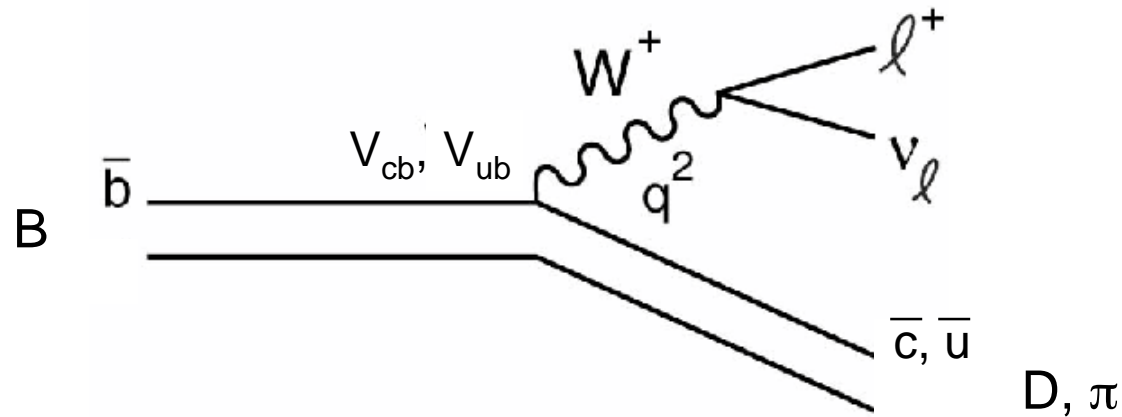
*Christoph Schwanda, HEPHY Vienna
representing the Belle collaboration*



5th Workshop on the Unitarity Triangle
September 9-13, 2008, Roma, Italy

Semileptonic decays

- $|V_{cb}|$ and $|V_{ub}|$ are determined from semileptonic B decays



- “Old physics”: tree diagrams, no ‘pollution’ from New Physics contributions
- A precise determination of $|V_{ub}/V_{cb}|$ is crucial to be able to observe deviations from the CKM mechanism due to New Physics

The semileptonic width

- $\Gamma(B \rightarrow X_c \ell \nu)$ can be systematically calculated with the operator production expansion (OPE)

$$\Gamma_{\text{sl}}(b \rightarrow c) = \frac{G_F^2 m_b^5(\mu)}{192 \pi^3} |V_{cb}|^2 (1 + A_{\text{ew}}) A^{\text{pert}}(r, \mu)$$

$$\left[z_0(r) \left(1 - \frac{\mu_\pi^2(\mu) \mu_G^2(\mu) + \frac{\rho_D^3(\mu) \rho_{LS}^3(\mu)}{m_b(\mu)}}{2m_b^2(\mu)} \right) - 2(1-r)^4 \frac{\mu_G^2(\mu) \frac{\rho_D^3(\mu) \rho_{LS}^3(\mu)}{m_b(\mu)}}{m_b^2(\mu)} + d(r) \frac{\rho_D^3(\mu)}{m_b^3(\mu)} + \dots \right]$$

from [Benson et al.,
Nucl. Phys. B665,
367 (2003)]

$$r = m_c^2(\mu) / m_b^2(\mu)$$

○ ... HQ parameters (non-calculable;
contain soft QCD physics)

- At each order in $1/m_b$, the expectation values of local operator products (heavy quark parameters) are multiplied by perturbatively calculable coefficients

Other observables in B decays

- Moments of the lepton energy spectrum in $B \rightarrow X_c l \nu$

$$R_n(E_{\text{cut}}, \mu) = \int_{E_{\text{cut}}} (E_\ell - \mu)^n \frac{d\Gamma}{dE_\ell} dE_\ell, \quad \langle E_\ell^n \rangle_{E_{\text{cut}}} = \frac{R_n(E_{\text{cut}}, 0)}{R_0(E_{\text{cut}}, 0)}$$

- Moments of the hadronic mass spectrum in $B \rightarrow X_c l \nu$

$$\langle m_X^{2n} \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} (m_X^2)^n \frac{d\Gamma}{dm_X^2} dm_X^2}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dm_X^2} dm_X^2}$$

- Moments of the photon energy spectrum in $B \rightarrow X_s \gamma$

$$\langle E_\gamma^n \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} E_\gamma^n \frac{d\Gamma}{dE_\gamma} dE_\gamma}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\gamma} dE_\gamma}$$

The OPEs of these inclusive observables contain the same HQ parameters

Global analysis of B decays

- Dedicated predictions for each observable
 - $\langle E_l^n \rangle_{E_l > E_{\text{cut}}} = f^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
 - $\langle M_X^{2n} \rangle_{E_l > E_{\text{cut}}} = g^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
 - $\langle E_\gamma^n \rangle_{E_\gamma > E_{\text{cut}}} = h^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
- Determine HQ parameters by performing a **minimum χ^2 fit to all available moment measurements**
- Take into account correlated experimental and theoretical errors
- External input: average B lifetime $\tau_B = (1.585 \pm 0.006)$ ps

Available calculations

- Kinetic running mass
 - [P.Gambino, N.Uraltsev, Eur.Phys.J. C34, 181 (2004)]
 - [D.Beson, I.Bigi, N.Uraltsev, Nucl.Phys. B710, 371 (2005)]
- 1S mass
 - [C.Bauer, Z.Ligeti, M.Luke, A.Manohar, M.Trott, Phys.Rev. D70, 094017 (2004)]
- Non-perturbative parameters in the $1/m_b$ expansion

both calculations up to $O(1/m_b^3)$

	Kinetic scheme	1S scheme
$O(1)$	m_b, m_c	m_b
$O(1/m_b^2)$	μ_π^2, μ_G^2	λ_1, λ_2
$O(1/m_b^3)$	ρ_D, ρ_{LS}	ρ_1, τ_{1-3}

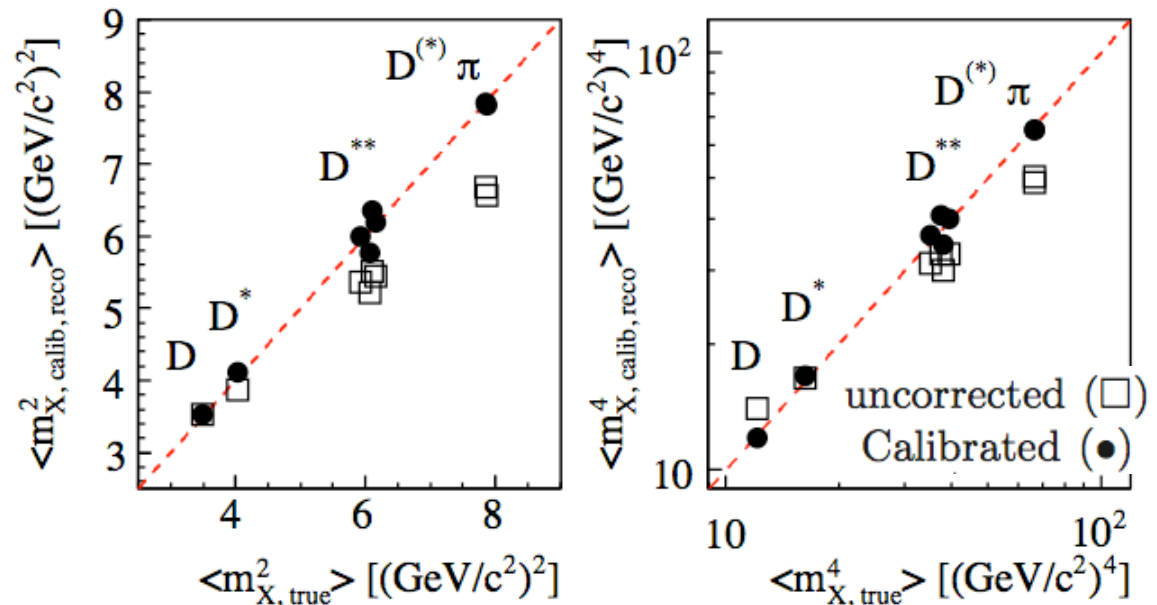
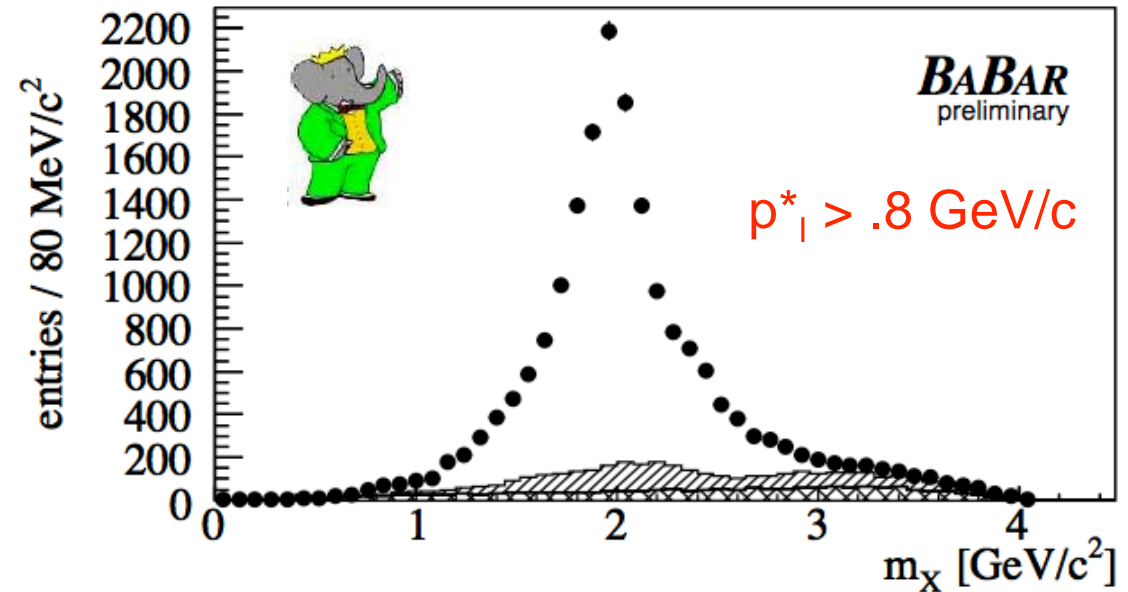
Available measurements

- Belle E_γ , 605/fb [[arXiv:0804.1580](#)] preliminary
- BaBar E_l , M^2_X , 210/fb [[arXiv:0707.2670](#)] preliminary
- Belle E_l , 140/fb [[PRD 75, 032001 \(2007\)](#)]
- Belle M^2_X , 140/fb [[PRD 75, 032005 \(2007\)](#)]
- DELPHI E_l , M^2_X , 3.4M Z [[EPJ C45, 35 \(2006\)](#)]
- BaBar, E_γ , 82/fb [[PRL 97, 171803 \(2006\)](#)]
- BaBar, E_γ , 82/fb [[PRD 72, 052004 \(2005\)](#)]
- CDF, M^2_X , 180/pb [[PRD 71, 051103 \(2005\)](#)]
- Belle, E_γ , 140/fb [[PRL 93, 061803 \(2004\)](#)]
- CLEO, M^2_X , 9/fb [[PRD 70, 032002 \(2004\)](#)]
- BaBar, E_l , 47/fb [[PRD 69, 111104 \(2004\)](#)]
- BaBar, M^2_X , 89M BB [[PRD 69, 111103 \(2004\)](#)]
- CLEO, E_γ , 9/fb [[PRL 87, 251807 \(2001\)](#)]

BaBar M_X^2 moments

[arXiv:0707.2670] preliminary

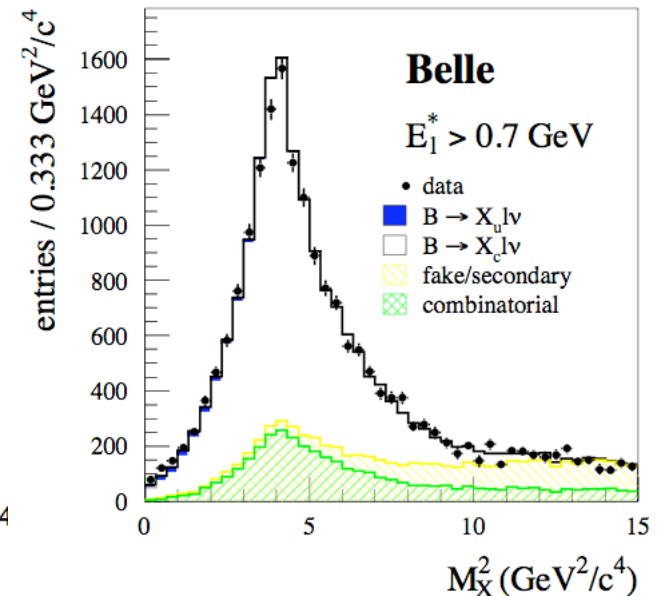
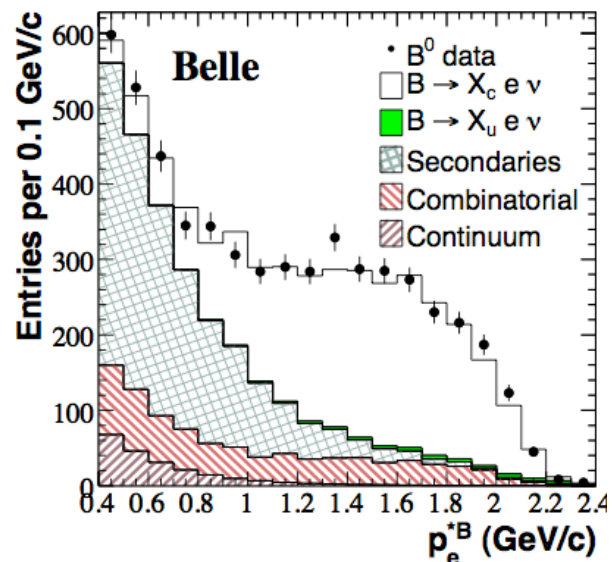
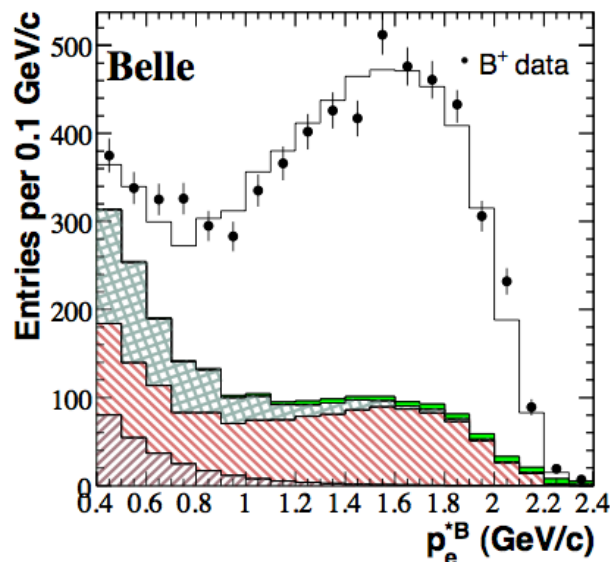
- 210/fb of Y(4S) data
- Hadronic decay of one B meson fully reconstructed
- Semileptonic decay of other B selected by requiring identified lepton (e/ μ)
- Reconstructed moments corrected event-by-event for detector effects
- $\langle M_X^k \rangle$ measured for $k=1, \dots, 6$ and p_{\perp}^* from 0.8 to 1.9 GeV/c



Belle E_1 and M_X^2 moments

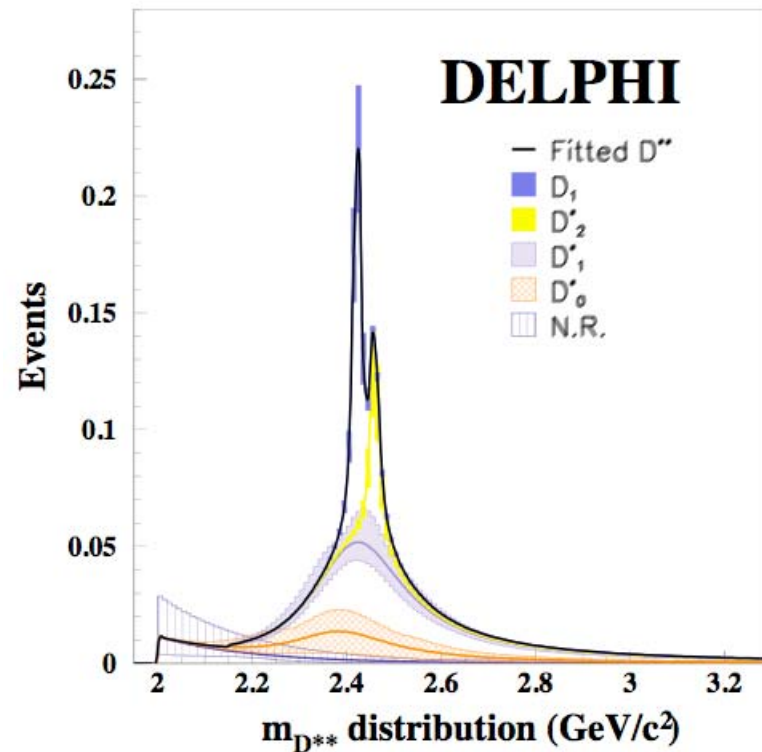
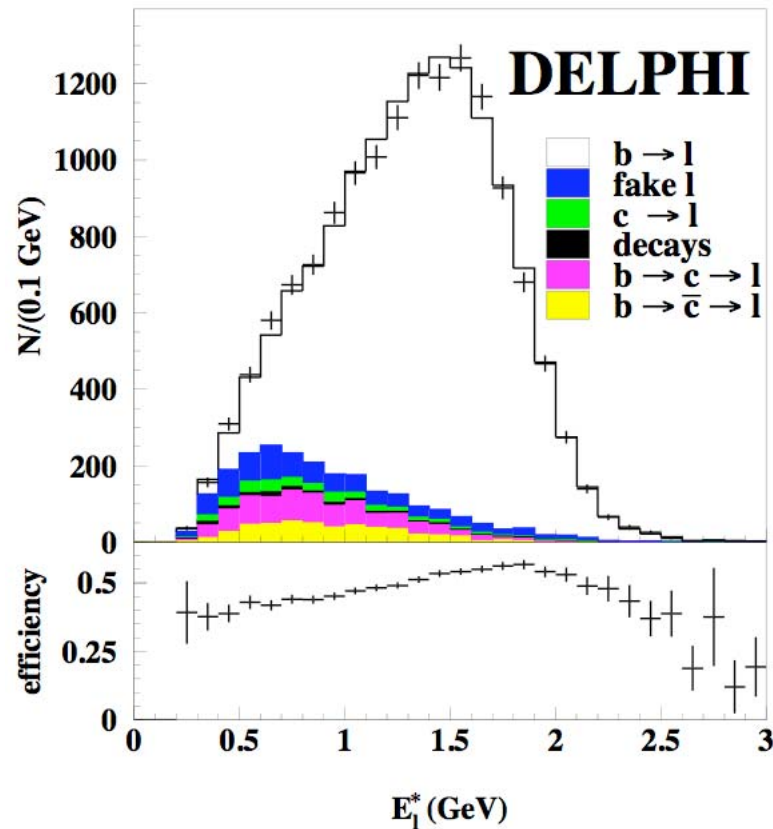
[PRD 75, 032001 (2007)]
 [PRD 75, 032005 (2007)]

- 140/fb of Y(4S) data
- Measurement also done with fully reconstructed events
- The finite detector resolution is unfolded with SVD algorithm [NIM A372, 469 (1996)]
- $\langle E_e^n \rangle$ measured for $n=0, \dots, 4$ and $E_{\text{cut}}=0.4-2.0$ GeV
- $\langle M_X^{2n} \rangle$ measured for $n=1, 2$ and $E_{\text{cut}}=0.7-1.9$ GeV



DELPHI E_1 and M^2_x moments

[EPJ C45, 35 (2006)]



- $\langle E_1^n \rangle$, $n=1, \dots, 3$ and $\langle M_x^{2n} \rangle$, $n=1, \dots, 5$ measured at $E_{\text{cut}} = 0$ as in Z events the b -quark is produced with a boost
- The hadronic moments are derived from the fitted D^{**} mass spectrum; assumptions on the D^{**} decay are made

$|V_{cb}|$ and m_b from the fit to
the Belle moment data



[Phys. Rev. D78, 032016 (2008)]

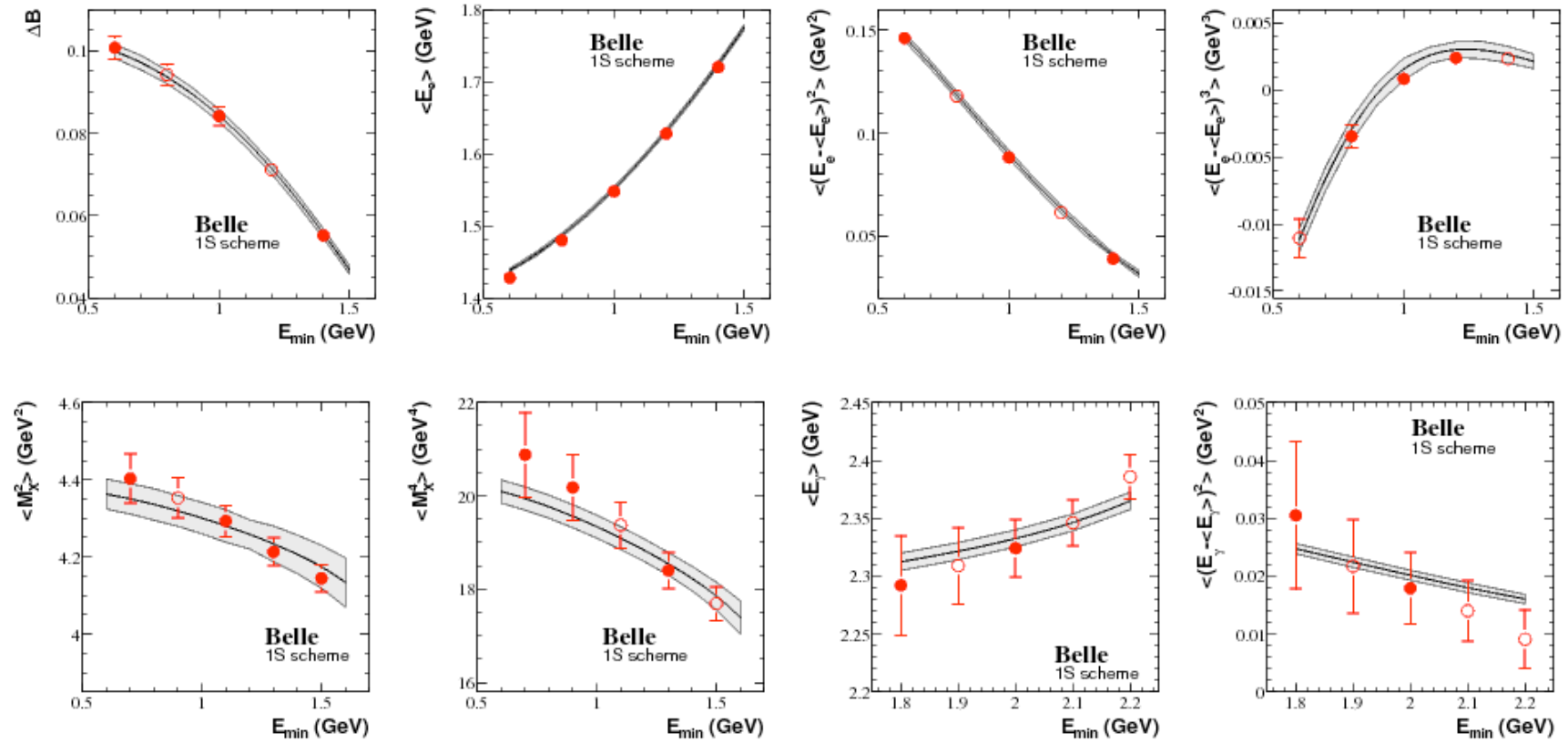
Similar analysis recently done on the BaBar
moment data [[arXiv:0707.2670](https://arxiv.org/abs/0707.2670)] preliminary

Belle measurements used

Electron moments $\langle E^n_l \rangle$	n=0: $E_{\text{cut}}=0.6, 1.0, 1.4$ GeV n=1: $E_{\text{cut}}=0.6, 0.8, 1.0, 1.2, 1.4$ GeV n=2: $E_{\text{cut}}=0.6, 1.0, 1.4$ GeV n=3: $E_{\text{cut}}=0.8, 1.0, 1.2$ GeV
Hadron moments $\langle M^{2n}_X \rangle$	n=1: $E_{\text{cut}}=0.7, 1.1, 1.3, 1.5$ GeV n=2: $E_{\text{cut}}=0.7, 0.9, 1.3$ GeV
Photon moments $\langle E^n_\gamma \rangle$	n=1: $E_{\text{cut}}=1.8, 2.0$ GeV n=2: $E_{\text{cut}}=1.8, 2.0$ GeV

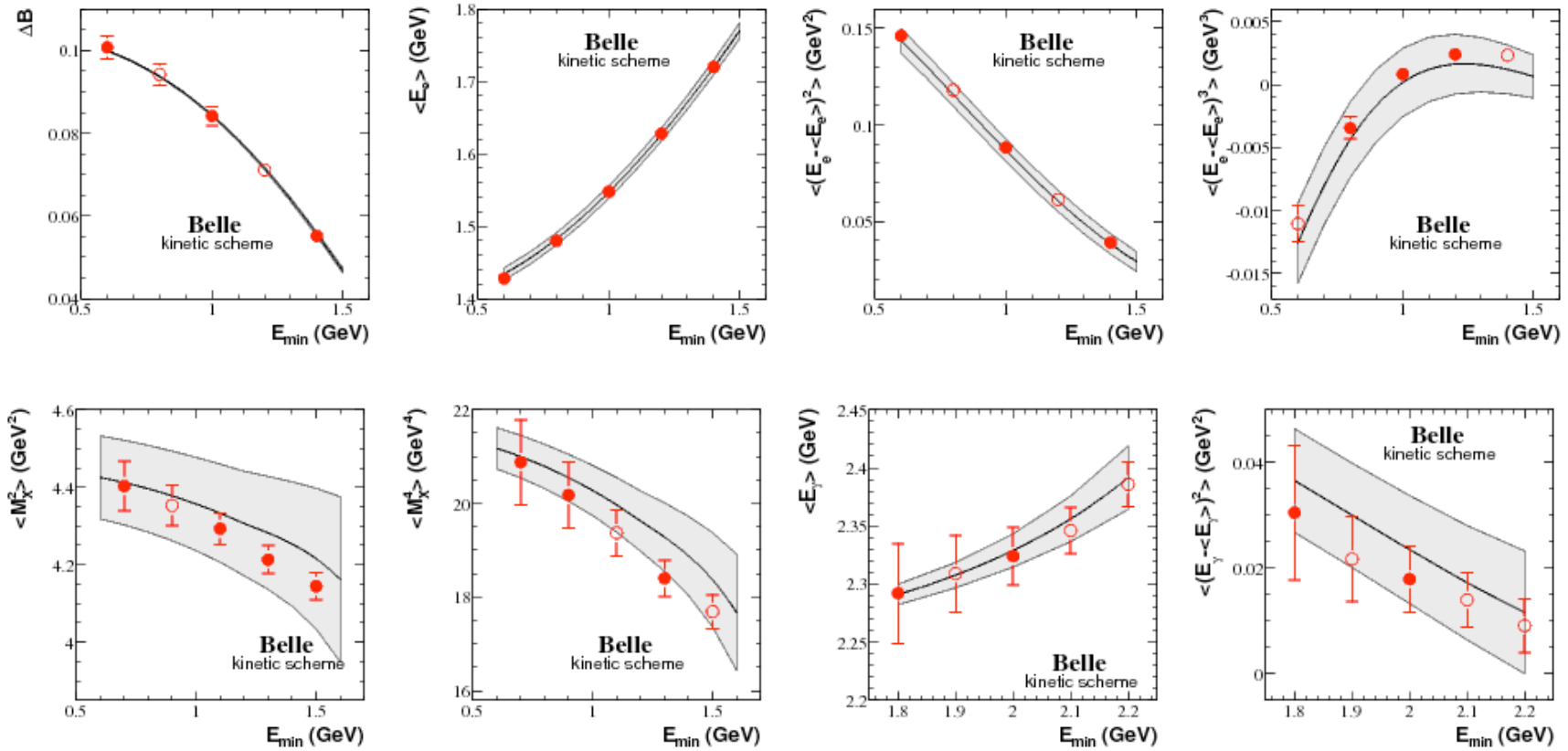
- Exclude measurements
 - with no (reliable) theory prediction
 - with excessive correlations

Fit result in the 1S scheme



$$\chi^2/\text{ndf.} = 7.3 / (25-7)$$

Fit result in the kinetic scheme



$$\chi^2/\text{ndf.} = 4.7 / (25-7)$$

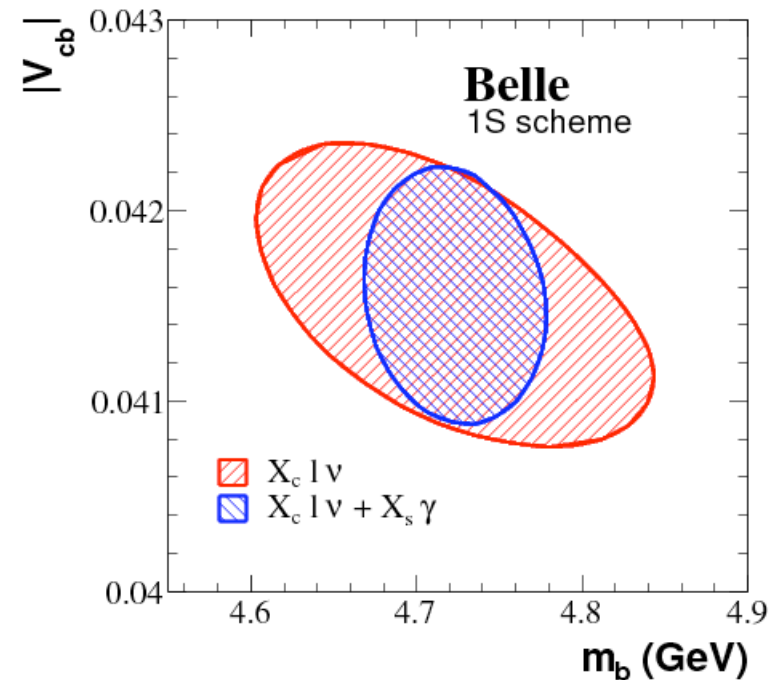
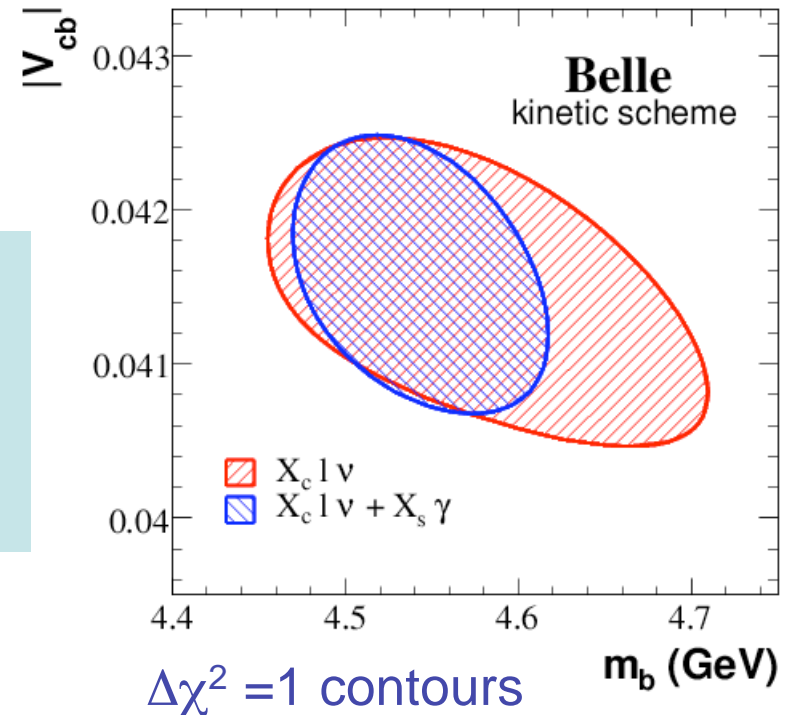
Kinetic scheme ($X_c l \nu + X_s \gamma$ data)

$$|V_{cb}| = (41.58 \pm 0.69_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}) \times 10^{-3}$$
$$m_b^{\text{kin}} = 4.543 \pm 0.075 \text{ GeV}$$
$$m_c^{\text{kin}} = 1.055 \pm 0.118 \text{ GeV}$$

Results for m_b compatible after
scheme translation

1S scheme ($X_c l \nu + X_s \gamma$ data)

$$|V_{cb}| = (41.56 \pm 0.68_{\text{fit}} \pm 0.08_{\tau_B}) \times 10^{-3}$$
$$m_b^{1S} = 4.723 \pm 0.055 \text{ GeV}$$



HFAG ICHEP08 fit in the kinetic scheme

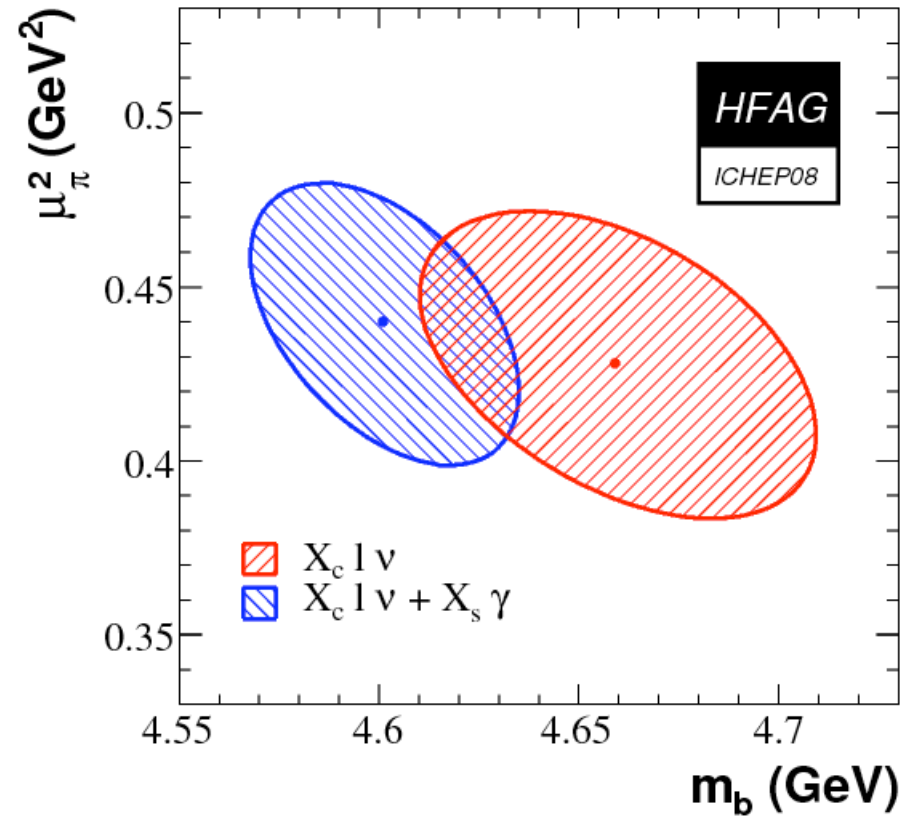
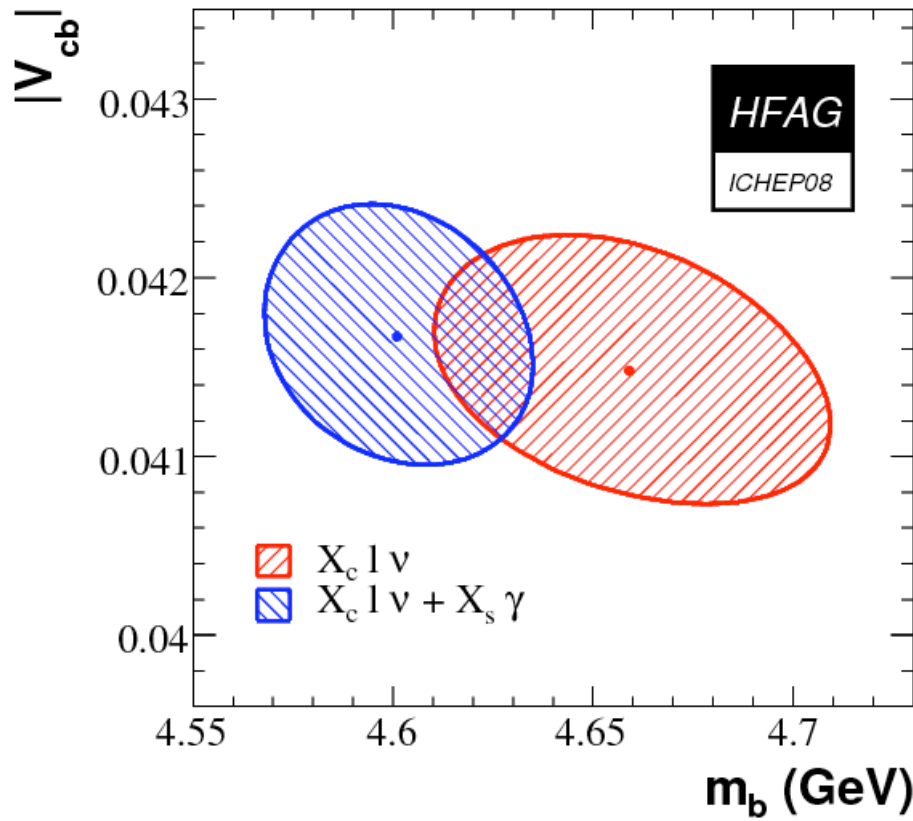


http://www.slac.stanford.edu/xorg/hfag/semi/ichep08/gbl_fits/kinetic/

Measurements used

BaBar	$\langle E_1^n \rangle$: $n=0,1,2,3$ [PRD 69, 111104 (2004)] $\langle M^{2n}_X \rangle$: $n=1,2$ [arXiv:0707.2670] preliminary $\langle E_\gamma^n \rangle$: $n=1,2$ [PRL 97, 171803 (2006)] and [PRD 72, 052004 (2005)]
Belle	$\langle E_1^n \rangle$: $n=0,1,2,3$ [PRD 75, 032001 (2007)] $\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 75, 032005 (2007)] $\langle E_\gamma^n \rangle$: $n=1,2$ [arXiv:0804.1580] preliminary
CDF	$\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 71, 051103 (2005)]
CLEO	$\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 70, 032002 (2004)] $\langle E_\gamma^n \rangle$: $n=1$ [PRL 87, 251807 (2001)]
DELPHI	$\langle E_1^n \rangle$: $n=1,2,3$ $\langle M^{2n}_X \rangle$: $n=1,2$ [EPJ C45, 35 (2006)]

- 27 moments from BaBar, 25 moments from Belle and 12 moments from other experiments



Input	$ V_{cb} (10^{-3})$	m_b (GeV)	μ_π^2 (GeV ²)	χ^2/ndf
All moments	41.67 \pm 0.43(fit) \pm 0.08(τ_B) \pm 0.58(th)	4.601 \pm 0.034	0.440 \pm 0.040	29.7/57
$X_c lv$ only	41.48 \pm 0.47(fit) \pm 0.08(τ_B) \pm 0.58(th)	4.659 \pm 0.049	0.428 \pm 0.044	24.1/46 18

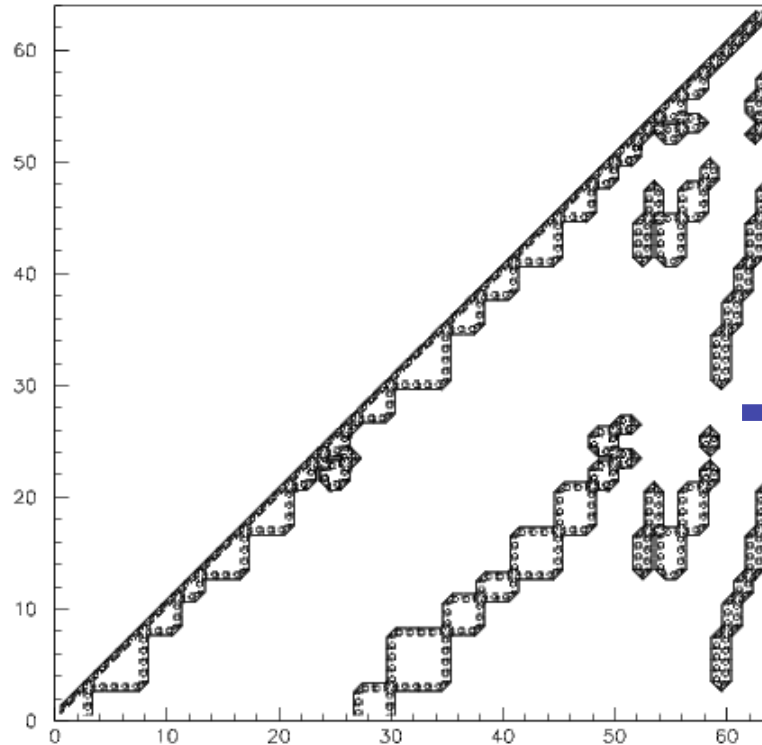
Outlook

Open issues in global fits

- χ^2 puzzle
 - $\chi^2/\text{ndf.}$ of global fits comes out much too low
 - Are (theory) errors overestimated?
 - Are correlations underestimated?
- $B \rightarrow X_s \gamma$ bias
 - Inclusion of the $B \rightarrow X_s \gamma$ data lowers the b-quark mass m_b

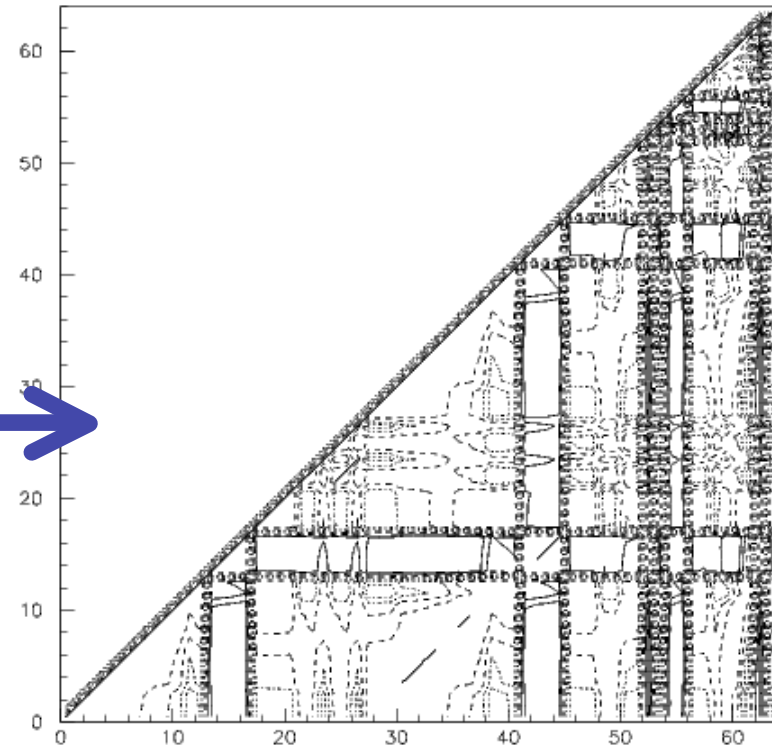
Theory correlation coefficients (HFAG data, kinetic scheme)

default

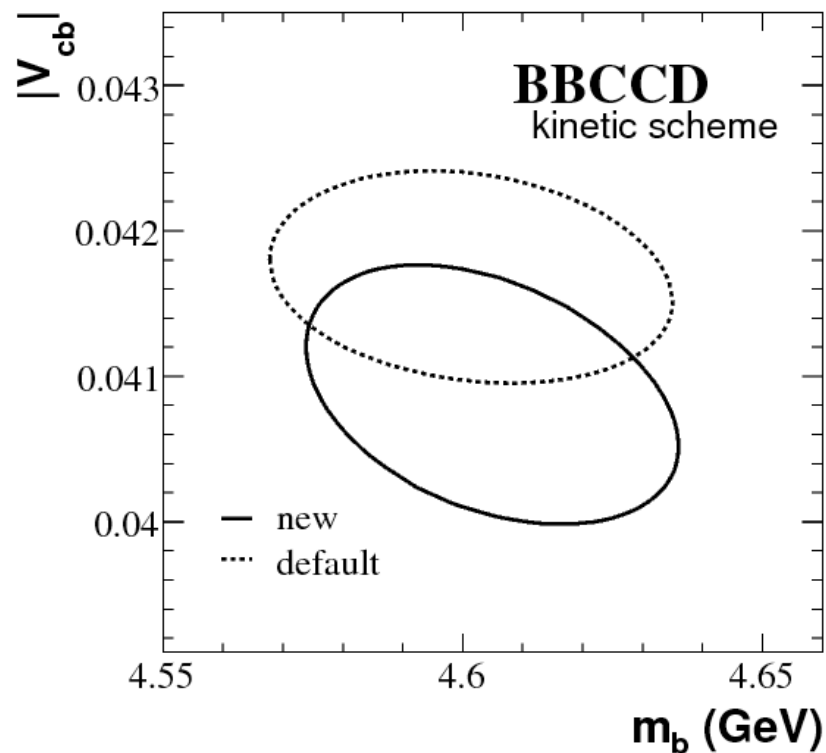
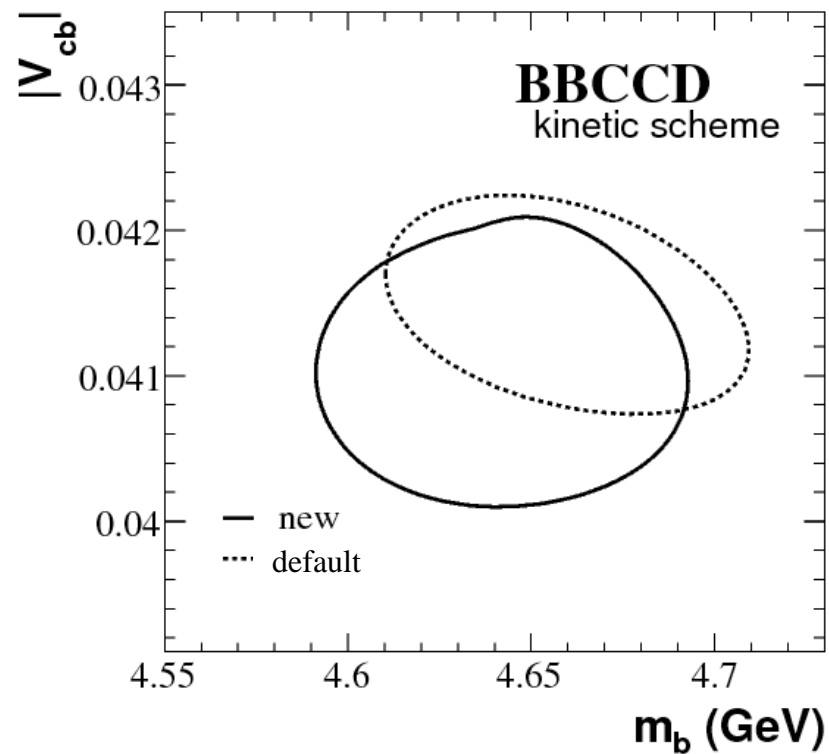


theory correlation coefficients
in the present HFAG fit

new



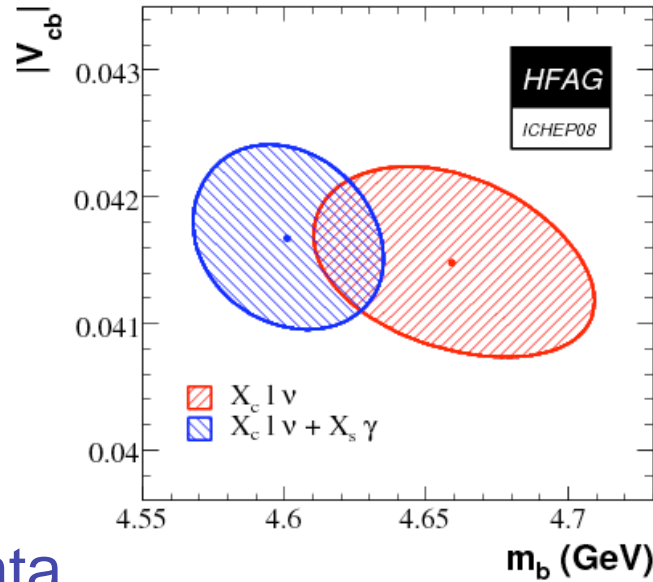
“actual correlations”, i.e.,
correlation coefficients derived
from theory expressions using a
toy MC approach

$X_c l\nu + X_s \gamma$  $X_c l\nu$ only

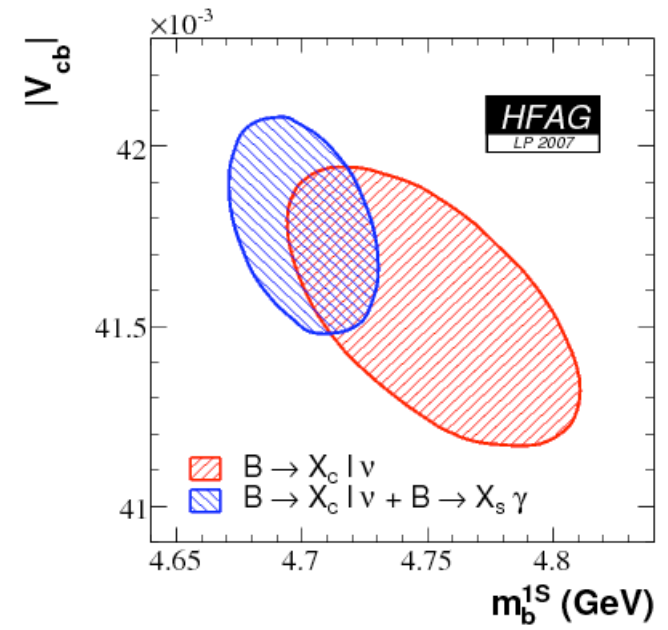
	$ V_{cb} (10^{-3})$	m_b (GeV)	μ_{π}^2 (GeV ²)	χ^2/ndf
default	41.67 \pm 0.43(fit) \pm 0.08(τ_B) \pm 0.58(th)	4.601 \pm 0.034	0.440 \pm 0.040	29.7/57
new	40.85 \pm 0.68(fit) \pm 0.08(τ_B) \pm 0.57(th)	4.605 \pm 0.031	0.312 \pm 0.060	54.2/57

$B \rightarrow X_s \gamma$ bias

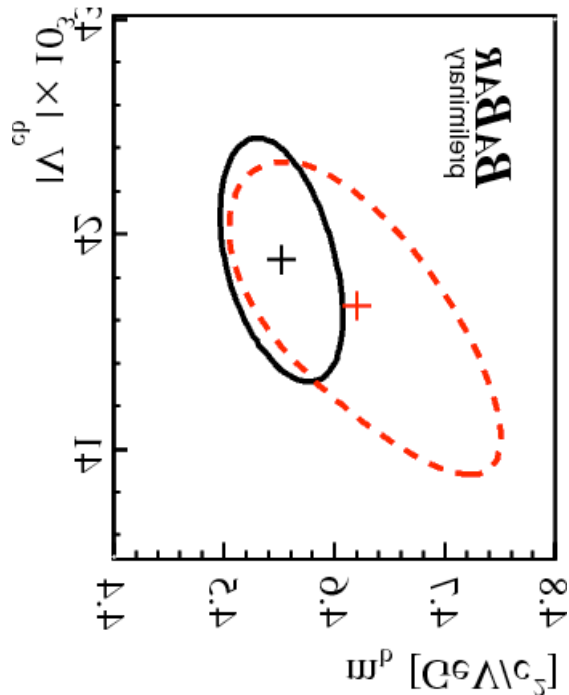
kinetic scheme



1S scheme

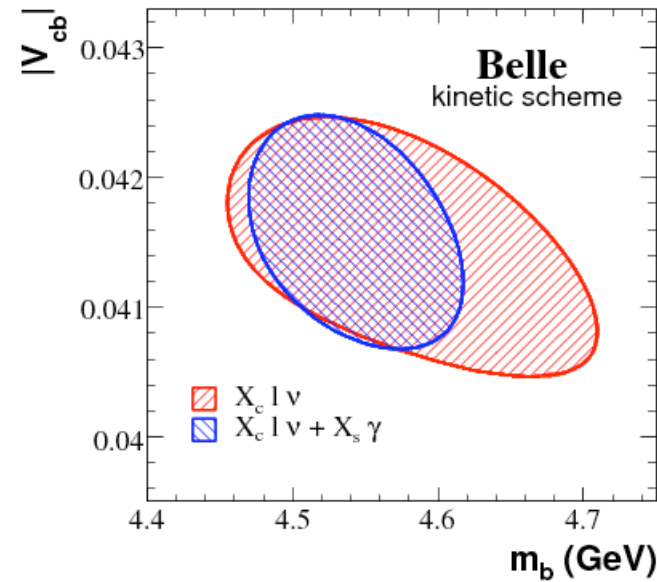


BaBar data



[arXiv:
0707.2670]

Belle data



[PRD78,
032016]

Summary and conclusions

- Calculations based on heavy quark effective theory and operator product expansion can reproduce inclusive observables in B decays to a high degree of precision
- Fits to the Belle/BaBar data in the kinetic scheme

	$ V_{cb} (10^{-3})$	m_b (GeV)
BaBar [arXiv:0707.2670]	$41.88 \pm 0.56_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.59_{\text{th}}$	4.552 ± 0.055
Belle [PRD78, 032016]	$41.58 \pm 0.69_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}$	4.543 ± 0.075

- Fits to Belle data in the kinetic/1S schemes [PRD78, 032016]

	$ V_{cb} (10^{-3})$
kinetic scheme	$41.58 \pm 0.69_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}$
1S scheme	$41.56 \pm 0.68_{\text{fit}} \pm 0.08_{\tau_B}$

- ICHEP08 HFAG result in the kinetic scheme

	$ V_{cb} (10^{-3})$	m_b (GeV)
HFAG ICHEP08	$41.67 \pm 0.43_{\text{fit}} \pm 0.08_{\tau_B} \pm 0.58_{\text{th}}$	4.601 ± 0.034

- Open issues

- Theory error correlations

- $B \rightarrow X_s \gamma$:

Can we safely use this data in the fit?

Backup

Theory error in the kinetic scheme

- **Non-perturbative corrections:** consider the following variations in the HQ parameters
 - m_b/m_c : +/- 20 MeV
 - μ_π^2/μ_G^2 : +/- 20%
 - ρ_D^3/ρ_{LS}^3 : +/- 30%
- **Perturbative corrections**
 - $\alpha_S = 0.22$ +/- 0.04
- **Bias correction uncertainty ($B \rightarrow X_s \gamma$)**
 - 30% of the absolute value of the bias correction

Toy MC for theory correlations

1. For α_S , m_b , m_c and each HQ parameter, draw a Gaussian random number within the allowed range (previous slide)
2. Calculate the moment predictions x_i , $i=1, \dots, 64$ (HFAG data) for this set of parameters
3. Repeat step 1) and 2) 100,000 times
4. Calculate the theory correlations as:

$$\rho(x_i, x_j) = \langle (x_i - \langle x_i \rangle)(x_j - \langle x_j \rangle) \rangle / \sigma_{x_i} \sigma_{x_j}$$