Vud from Nuclear Decays

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Advantages of Nuclear Decays

- select pure vector transitions: $0^+ \rightarrow 0^+$ decays
  - 10 cases, experimental precision $\leq 0.1\%$
  - 3 cases, experimental precision $\leq 0.4\%$
- conserved vector current (CVC) hypothesis
  - $G_V = G_F V_{ud}$ is a 'true' constant, nucleus-independent
  - $\mathcal{F}t$ values are constant, nucleus-independent
  - provides consistency checks

Disadvantages of Nuclear Decays

- $SU(2)$ -symmetry breaking correction needed
  - requires nuclear-structure calculation
  - typically $\sim 0.5\%$ – <u>small</u> and <u>testable</u>

Radiative correction in nuclear decays

In total: $RC \sim 4\%$

- Nucleus-independent component

$$\Delta_R = 2.361 \pm 0.038\%$$

error reduction, Marciano-Sirlin PRL 96, 032002 (2006)

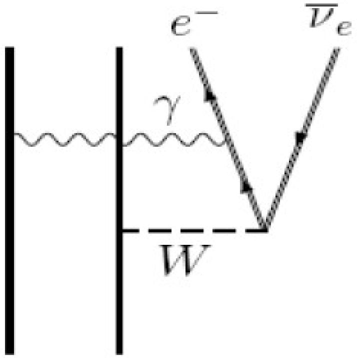
- Trivially nucleus-dependent component

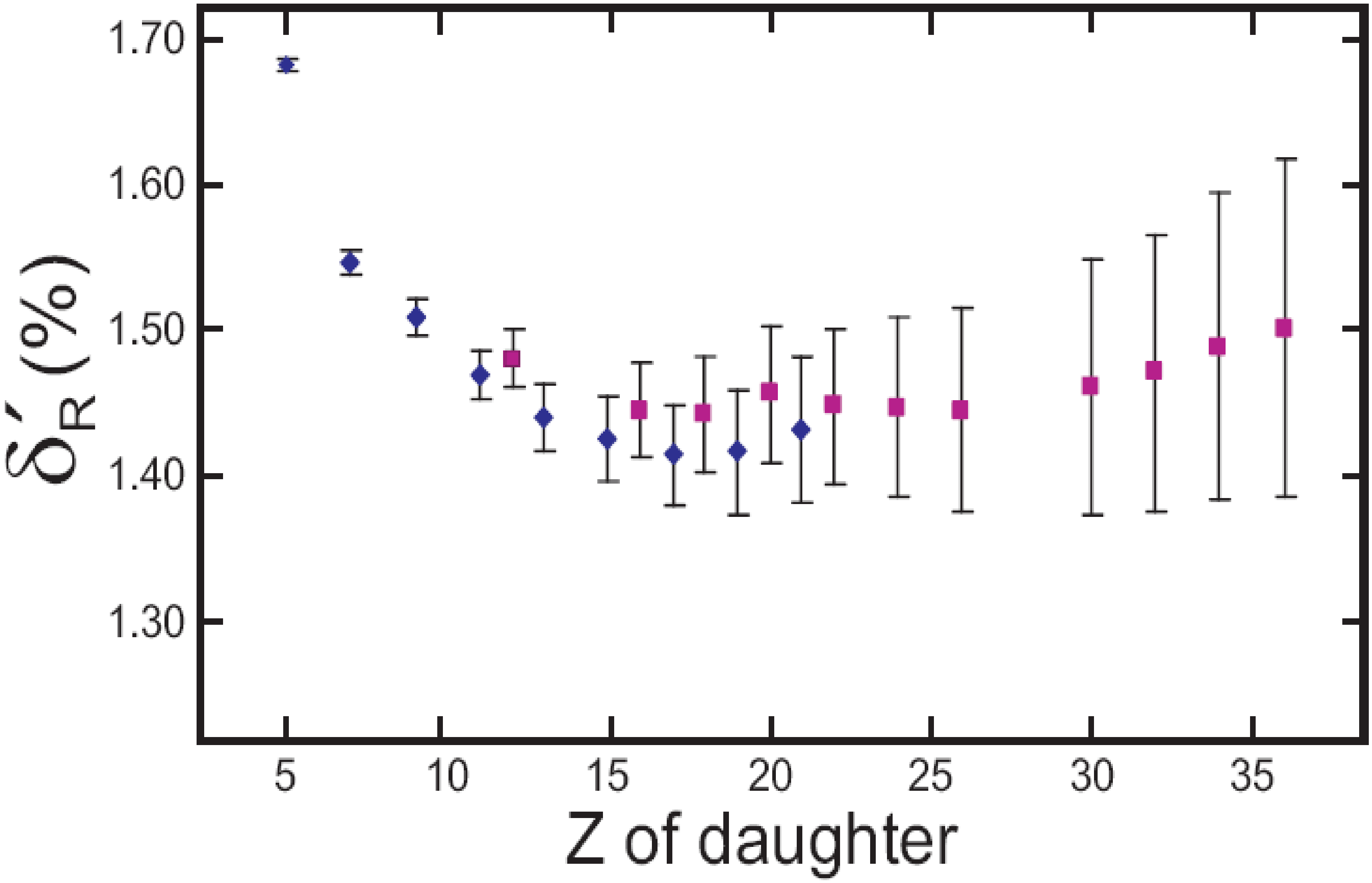
$$\delta'_R \simeq 1.44 \pm 0.05\% \qquad \text{(typically)}$$

principally a QED calculation, depending on nuclear charge, Z, and electron energy, E_e.

- Small nuclear-structure dependent component

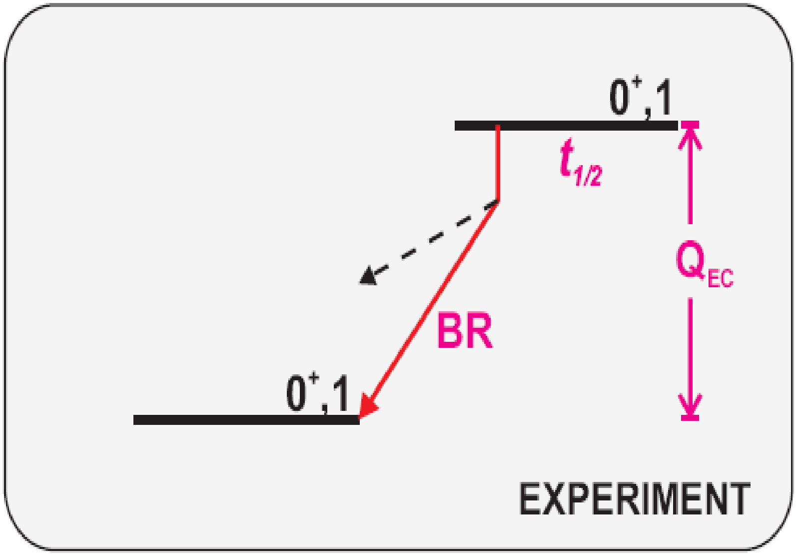
$$\begin{aligned} \delta_{NS} &= -0.20 \pm 0.02\% \quad T_z = -1 \text{ nuclei} \\ &= -0.05 \pm 0.02\% \quad T_z = 0 \text{ nuclei} \end{aligned}$$




δ'_R (%)
1.70
1.60
1.50
1.40
1.30
5
10
15
20
25
30
35
Z of daughter

V_{ud} from nuclear beta decay

$$ft = \frac{K}{G_V^2 \langle \tau_+ \rangle^2}$$



f = statistical rate function $f(Z, Q_{ec})$

$t = t_{1/2}/BR =$ partial half life

$$\begin{aligned}\langle \tau_+ \rangle &= \text{isospin ladder operator matrix element} \\ &= \sqrt{2} \text{ for isospin } T = 1 \text{ states}\end{aligned}$$

Including radiative and isospin-symmetry breaking corrections

$$\begin{aligned}\mathcal{F}t &= ft(1+\delta'_R)(1-(\delta_C-\delta_{NS})) = \frac{K}{2G_V^2(1+\Delta_R)} \\ G_V^2 &= G_F^2 V_{ud}^2\end{aligned}$$

MASTER EQUATIONS

$$\mathrm{CVC:}\ \ \mathcal{F}t = ft(1+\delta_R')(1-(\delta_C-\delta_{NS})) = \mathrm{constant}$$

$$V_{ud}^2 = \frac{K}{2G_F^2\overline{\mathcal{F}t}(1+\Delta_R)} \qquad \frac{K}{(\hbar c)^6} = \frac{2\pi^3\hbar\ln 2}{(m_ec^2)^5}$$

where

ft = experimental nuclear ft values.

$\overline{\mathcal{F}t}$ = average corrected ft values (13 cases).

G_F = weak interaction coupling constant (from muon lifetime).

$\left.\begin{array}{r}\Delta_R\\ \delta_R'\\ \delta_{NS}\end{array}\right\}$ = calculated radiative correction.

δ_C = calculated isospin symmetry breaking correction.

EXPERIMENTS PUBLISHED SINCE 2005

Q_{EC} values:

Argonne (Canadian Penning trap)

^{46}V Savard *et al.*, PRL 95, 102501 (2005)

Jyvaskyla (JYFLTRAP)

^{62}Ga Eronen *et al.* PLB 636, 191 (2006)

^{26}Al^{m}, ^{42}Sc, ^{46}V Eronen *et al.*, PRL 97, 232501 (2006)

^{50}Mn, ^{54}Co Eronen *et al.*, PRL 100, 132502 (2008)

NSCL (LEBIT)

^{38}Ca Bollen *et al.*, PRL 96, 152501 (2006)

^{66}As Schury *et al.*, PRC 75, 055801 (2007)

Munich Tandem

^{46}V Faestermann *et al.*, Progress Report

ISOLTRAP

^{38}Ca George *et al.*, PRL 98, 162501 (2007)

^{26}Al^{m} George *et al.*, EPL 82, 50005 (2008)

Half-lives:

Auckland/Canberra

^{50}Mn Barker & Byrne, PRC 73, 064306 (2006)

LBNL

^{14}O Burke *et al.*, PRC 74, 025501 (2006)

Texas A&M

^{34}Cl, ^{34}Ar Iacob *et al.*, PRC 74, 055502 (2006)

^{10}C Iacob *et al.*, PRC 77, 045501 (2008)

TRIUMF

^{18}Ne Grinyer *et al.*, PRC 76, 025503 (2007)

^{62}Ga Grinyer *et al.*, PRC 77, 015501 (2008)

Jyvaskyla

^{26}Si Matea *et al.*, EPJA to be pub. (2008)

Melbourne

^{26}Al^{m} Scott *et al.*, NIMPRA 539, 191 (2005)

Branching ratios:

TRIUMF

^{62}Ga Finlay *et al.*, PRC 78, 025502 (2008)

^{38}K^{m} Leach *et al.*, PRL 100, 192504 (2008)

Jyvaskyla

^{26}Si Matea *et al.*, EPJA to be pub. (2008)

^{62}Ga Bey *et al.*, EPJA 36, 121 (2008)

Texas A&M

^{14}O Towner & Hardy, PRC 72, 055501 (2005)

WHAT CAN WE LEARN?

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

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Test Conservation of the Vector current (CVC)

Test for presence of a Scalar current

$\mathcal{F}t$ values constant

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Test for presence of a Scalar current

$$\boxed{\mathcal{F}t \text{ values constant}}$$

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

Obtain precise value of $G_V^2(1+\Delta_R)$

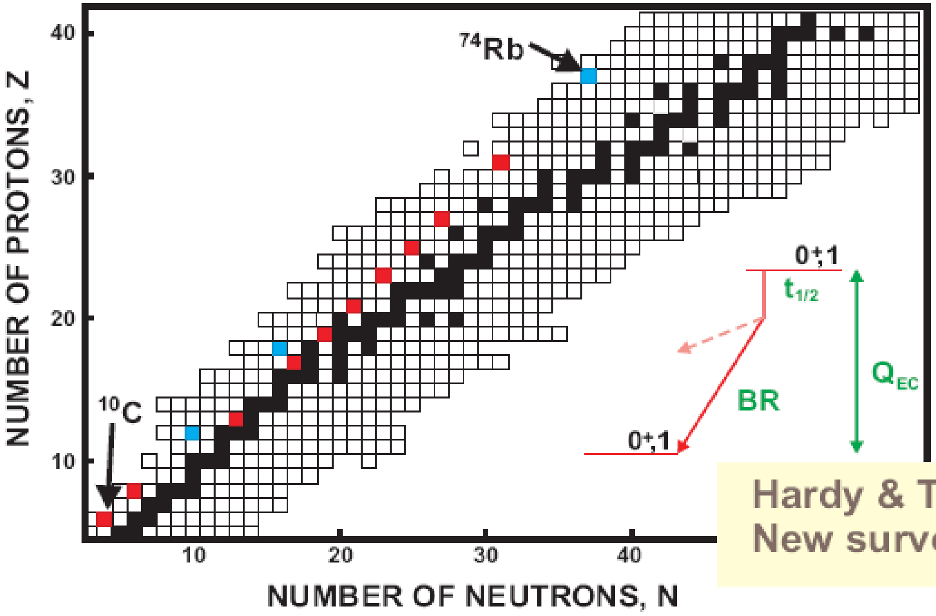
Determine V_{ud}^2

$$\boxed{V_{ud}^2 = G_V^2/G_\mu^2}$$

Test CKM unitarity

$$\boxed{V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1}$$

WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2008



Hardy & Towner
New survey (2008)

- 10 cases with *ft*-values measured to ~0.1% precision; 3 more cases with <0.3% precision.
- ~150 individual measurements with compatible precision

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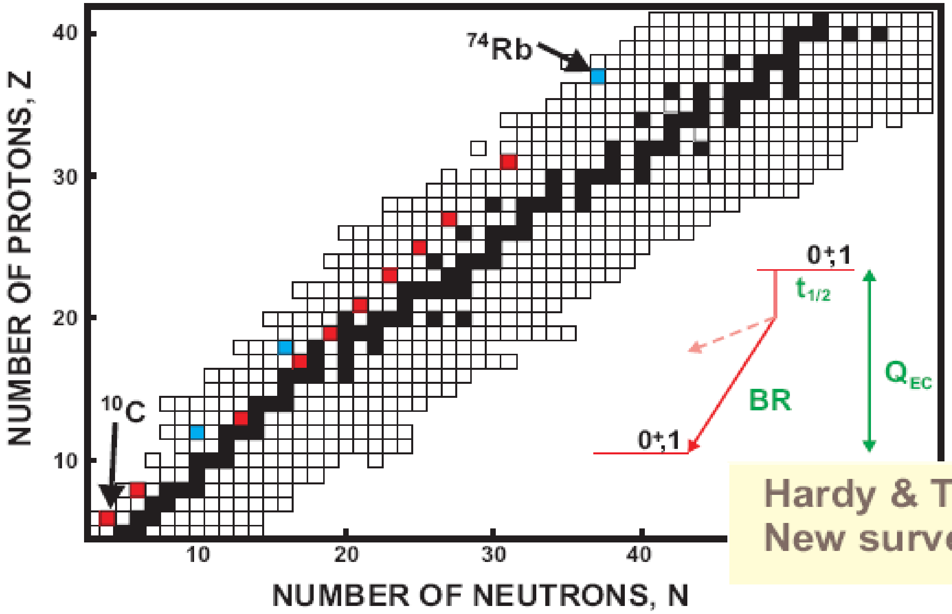

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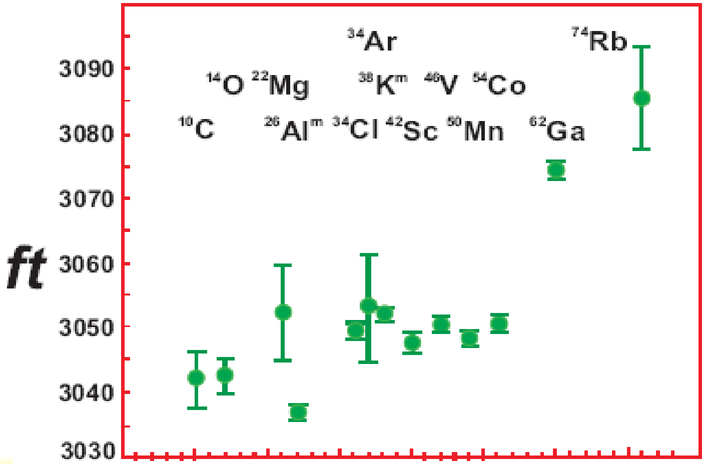

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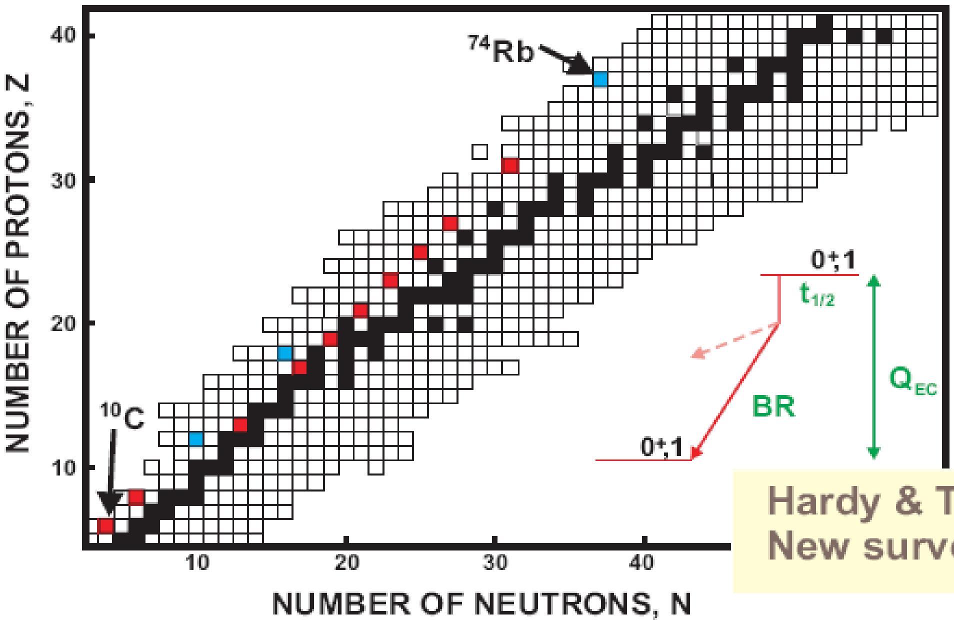

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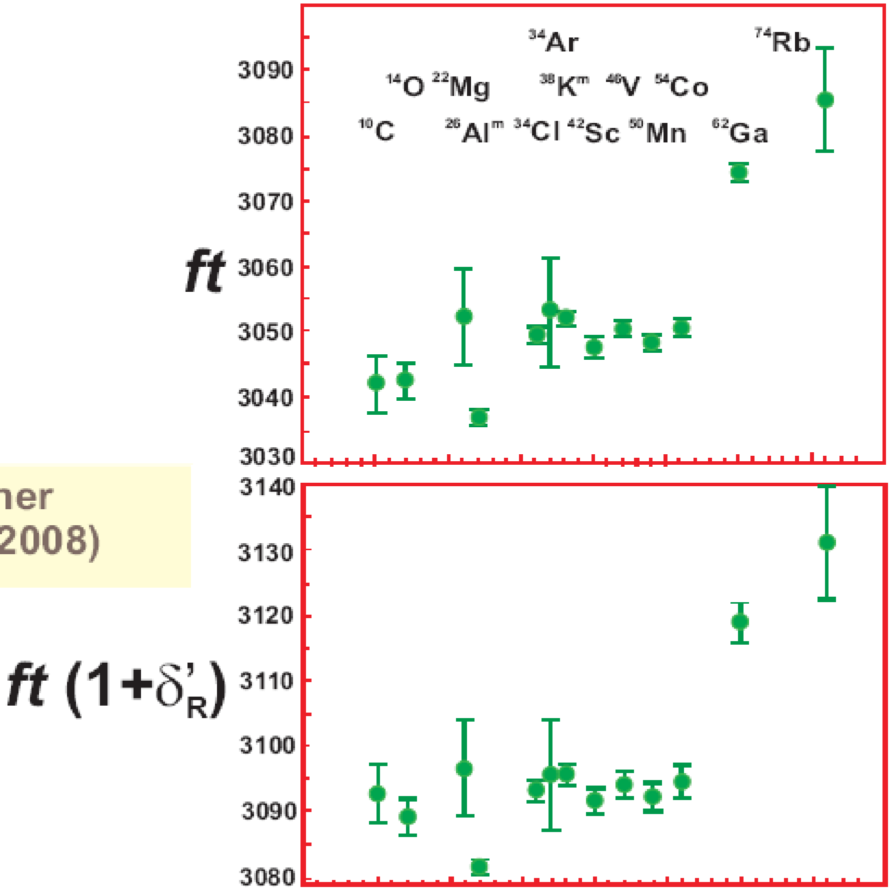

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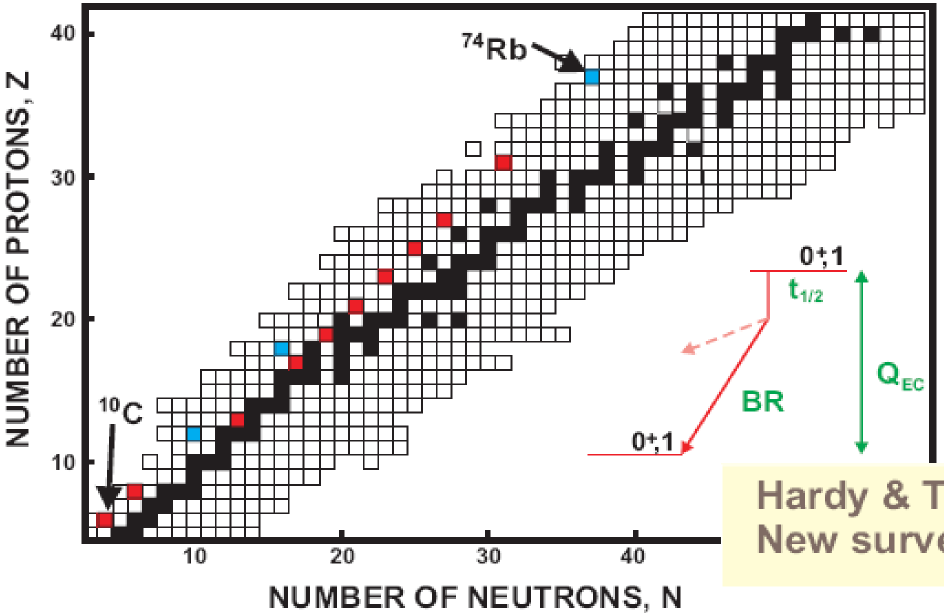

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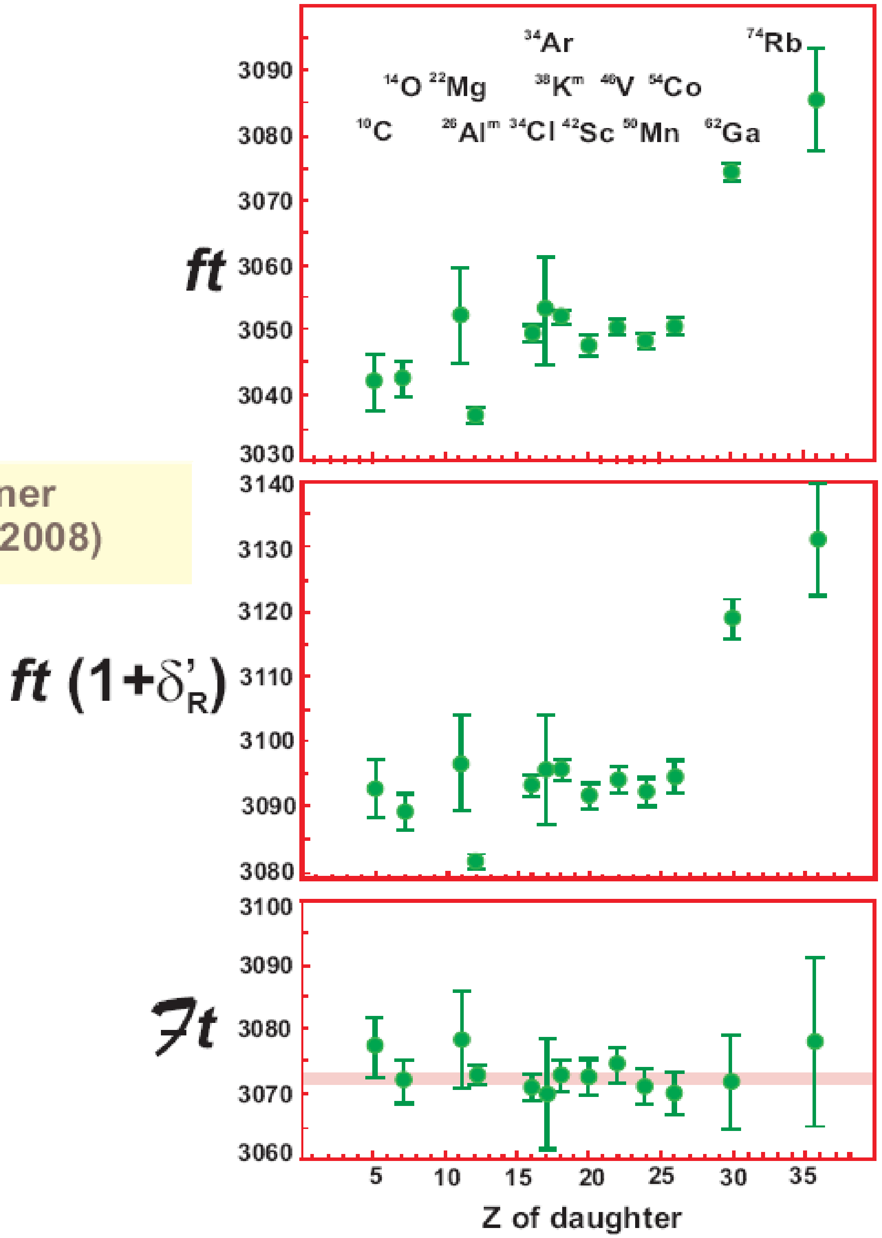

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY IN 2008

1) G_V constant

$$\boxed{\mathcal{F}t = \frac{K}{2G_V^2\,(1 + \Delta_R)}}$$

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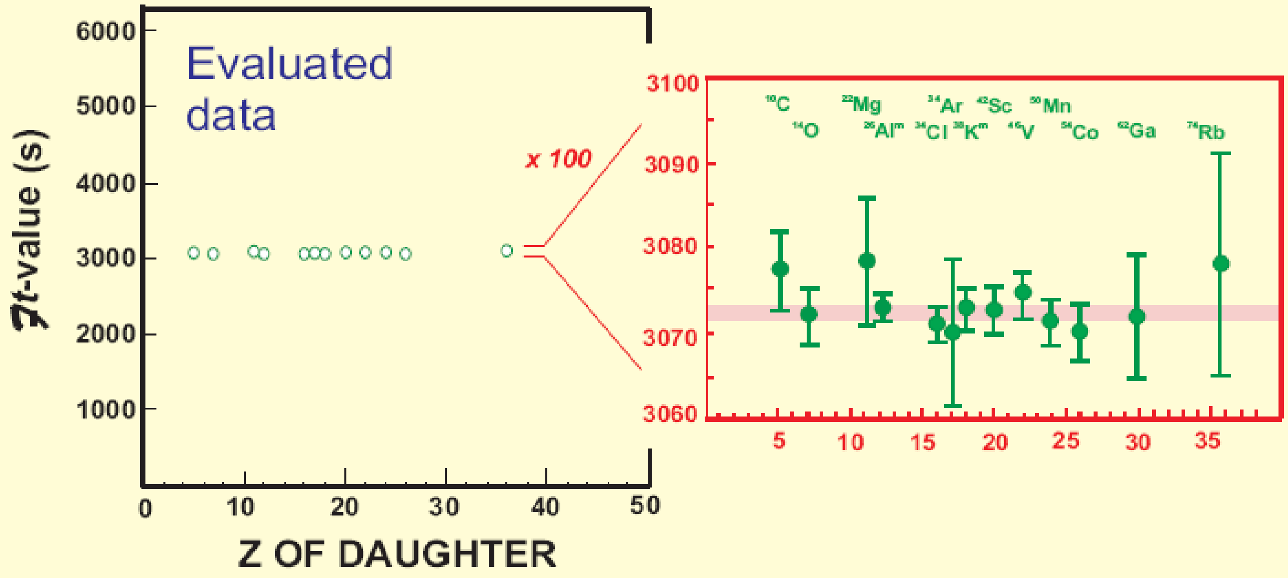
✔ verified to ± 0.013%



$\overline{\mathcal{F}t}$ = 3072.2(8)

$G_V(1+\Delta_R)^{1/2}/(\hbar c)^3$
= 1.14961(15)
x10^{-5} GeV^{-2}

$\chi^2/\nu = 0.3$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY IN 2008

1) G_V constant

$$\boxed{\mathcal{F}t = \frac{K}{2G_V^2\,(1 + \Delta_R)}}$$

✔ verified to ± 0.013%

2) Scalar current zero

Jackson, Treiman, Wyld: weak-interaction Hamiltonian:

$$\begin{aligned} H &= (\overline{\psi}_p \psi_n)(C_S \overline{\phi}_e \phi_\nu + C'_S \overline{\phi}_e \gamma_5 \phi_\nu) \\ &\quad + (\overline{\psi}_p \gamma_\mu \psi_n)(C_V \overline{\phi}_e \gamma_\mu \phi_\nu + C'_V \overline{\phi}_e \gamma_\mu \gamma_5 \phi_\nu) \\ &\quad + \; A \; + \; T \; + \; P \end{aligned}$$

Time-reversal invariance: C_S, C'_S, C_V, C'_V real

Standard Model: $C'_S = C_S = 0 \quad ; \quad C'_V = C_V$

We assume: $C_S \neq C'_S \neq 0$ and $C'_V = C_V$

Beta decay rate

$$\Gamma = \Gamma_0 \left(1 + \frac{b}{E_e}\right) \qquad b \simeq \frac{1}{C_V}\left(C_S + C'_S\right)$$

Note: electron energy dependence spoils $\mathcal{F}t =$ constant

Beta-neutrino correlation coefficient

$$\Gamma = \Gamma_0 \left(1 + a \cos \theta_{e\nu}\right) \qquad a \simeq 1 - \frac{1}{2}\left(C_S^2 + C_S'^2\right)$$

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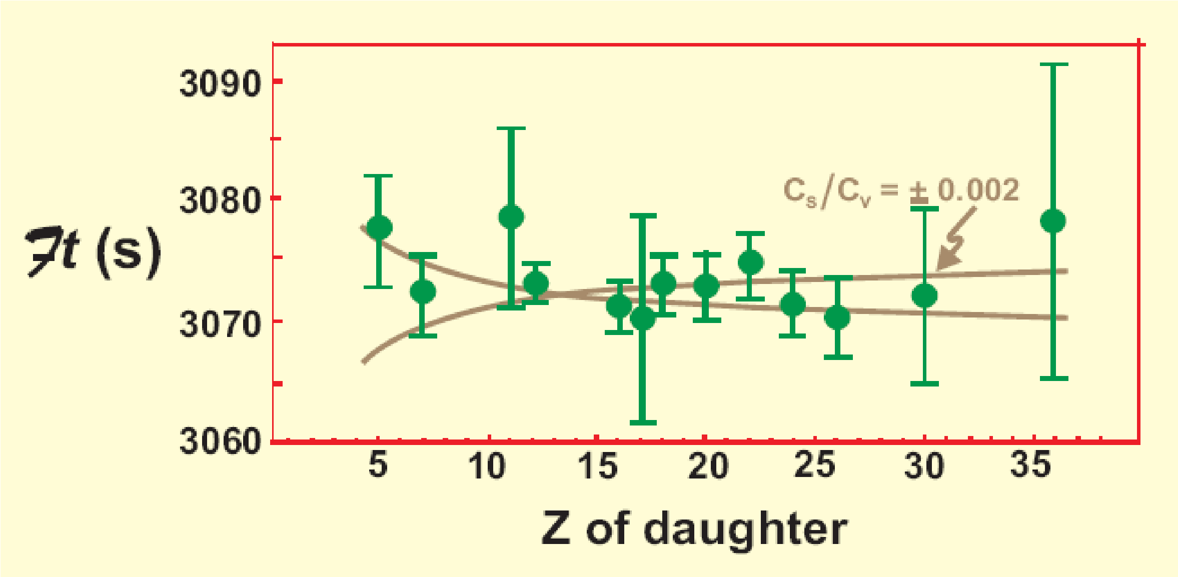
1) G_V constant $\boxed{\mathcal{F}t = \frac{K}{2G_V^{\,2}\,(1 + \Delta_R)}}$ ✓ verified to ± 0.013%

2) Scalar current zero ✓ limit, $C_S/C_V = 0.0011\ (14)$



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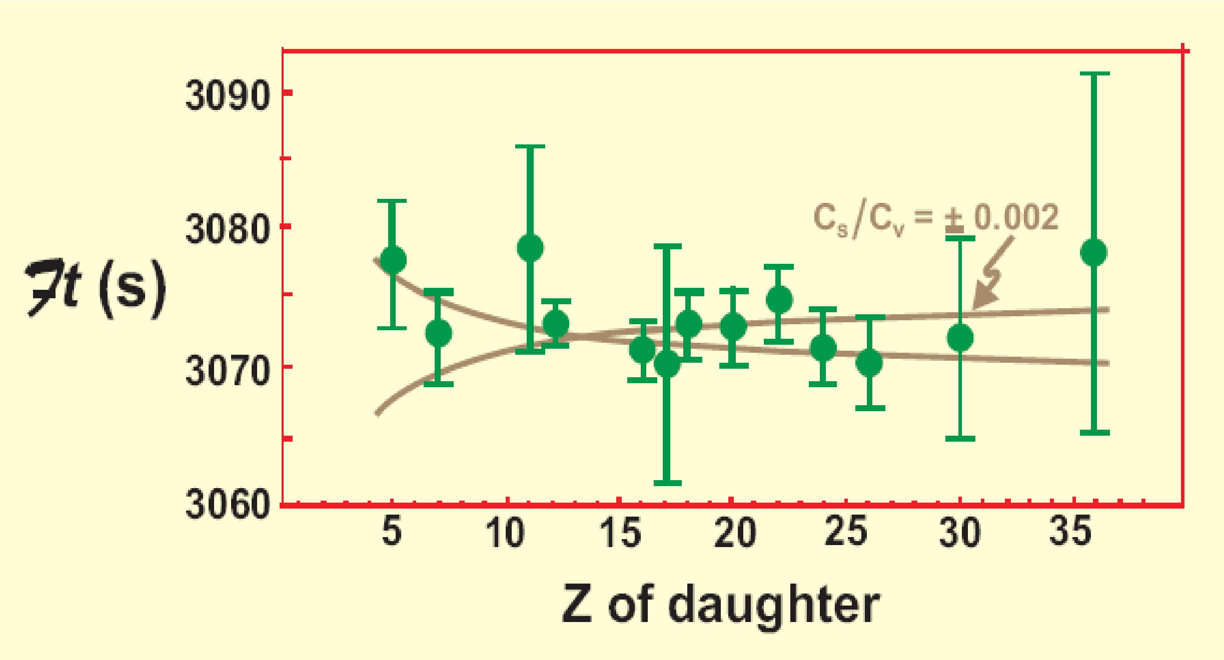

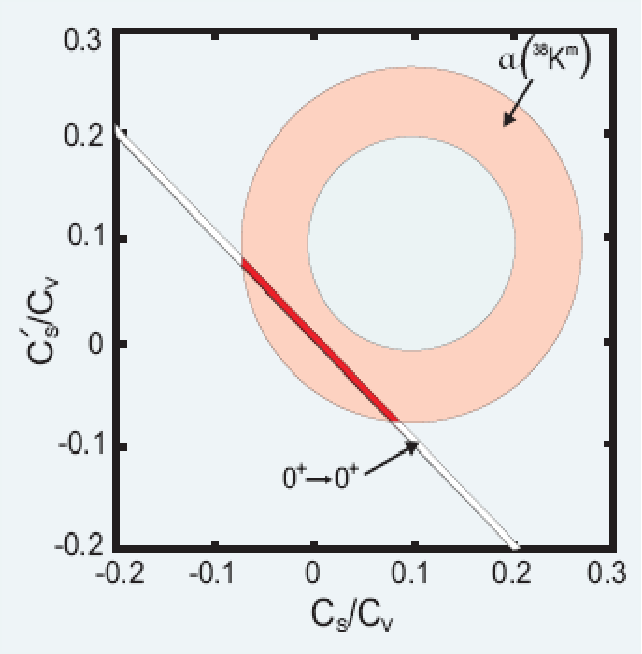

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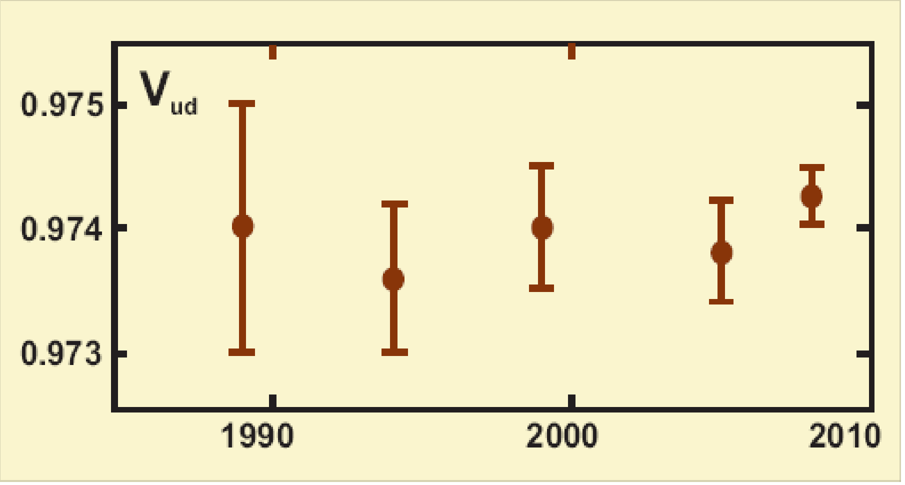
$$\boxed{V_{ud} = 0.97425 \pm 0.00023}$$



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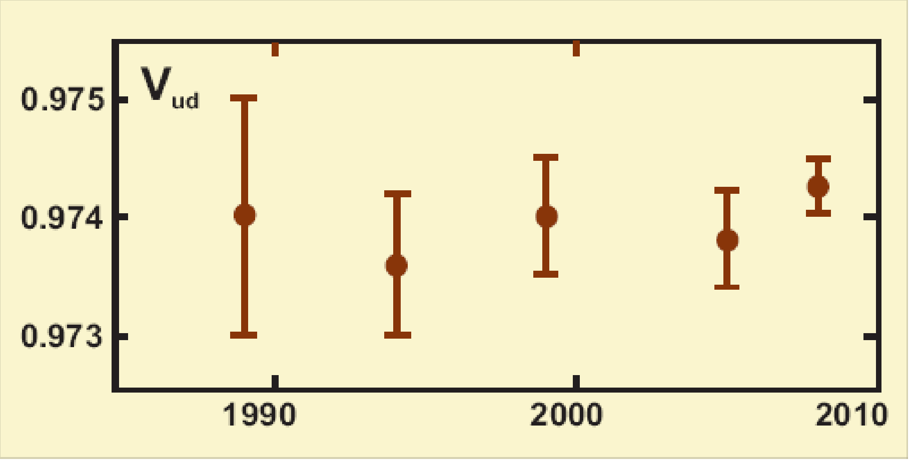
Compare:

neutron $V_{ud} = 0.9746 \pm 0.0019$

pion $V_{ud} = 0.9749 \pm 0.0026$



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$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9996(7)$$

0.9491(4)

0.0504(6)

<0.0001

Value of V_{us}

Reference: arXiv:0801.1817v1 (2008)

$$f_+(0)|V_{us}| = 0.21664(48) \quad \text{FlaviaNet '08}$$

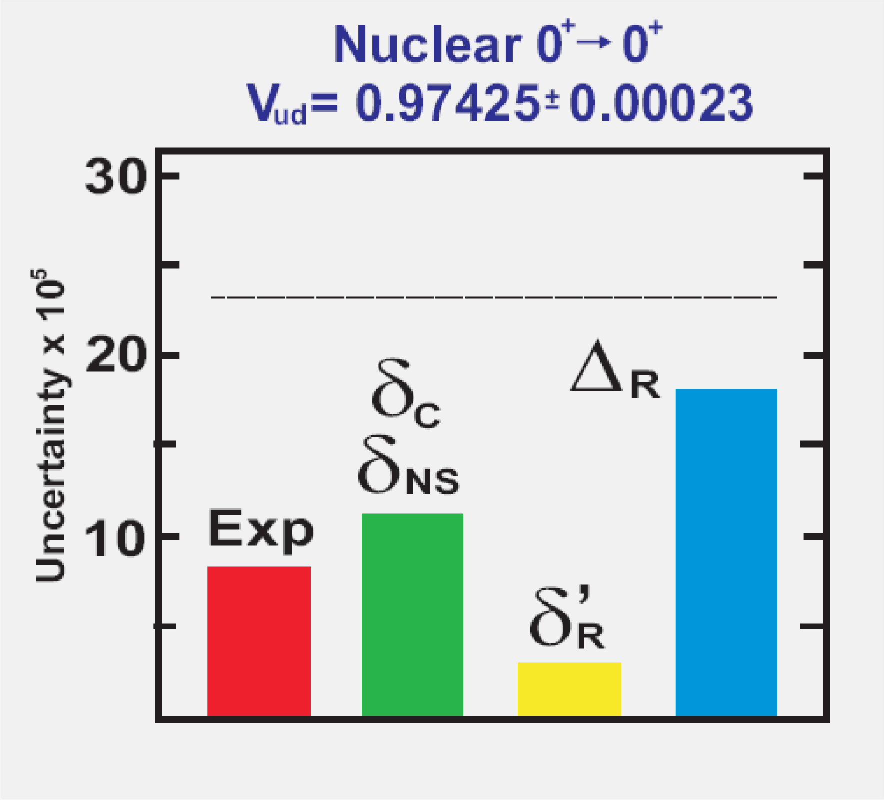
$$f_+(0) = 0.9644(49) \quad \text{Lattice: RBC-UKQCD '07}$$

$$|V_{us}| = 0.2246(12) \quad K_{\ell 3} \text{ only}$$

$$V_{us}^2 = 0.05046(56)$$

OPPORTUNITIES FOR IMPROVEMENT

- Goal remains to tighten the window for new physics by reducing the uncertainty on V_{ud}.



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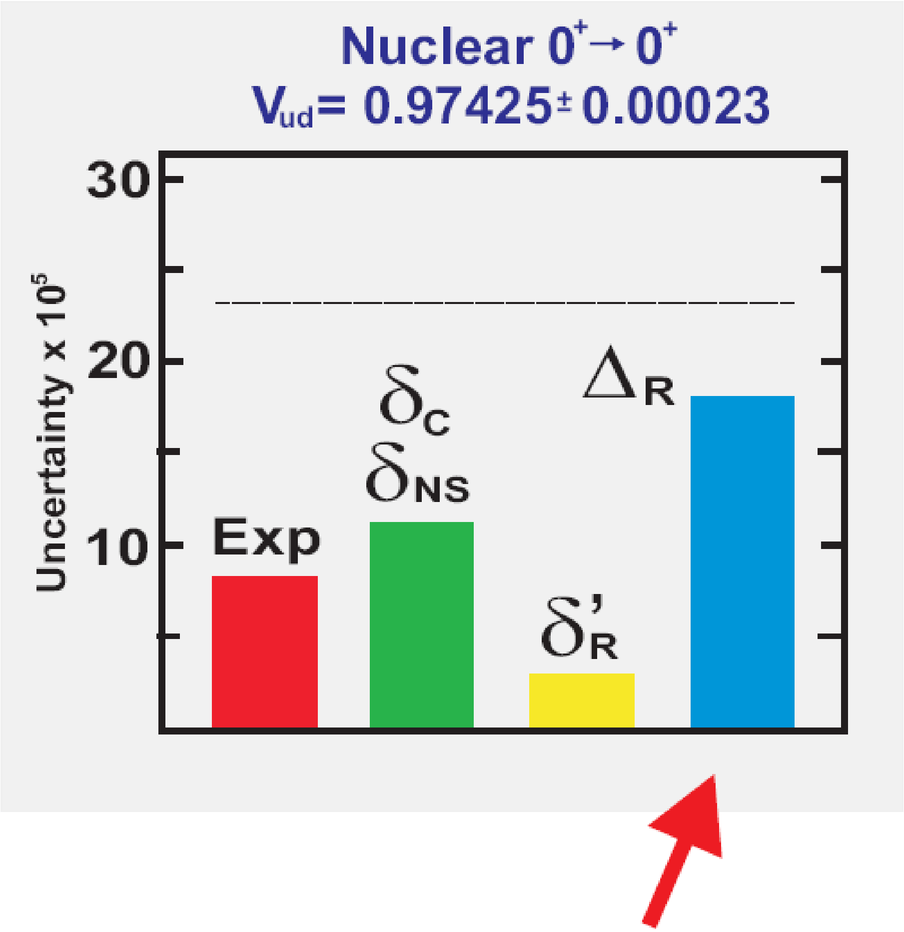
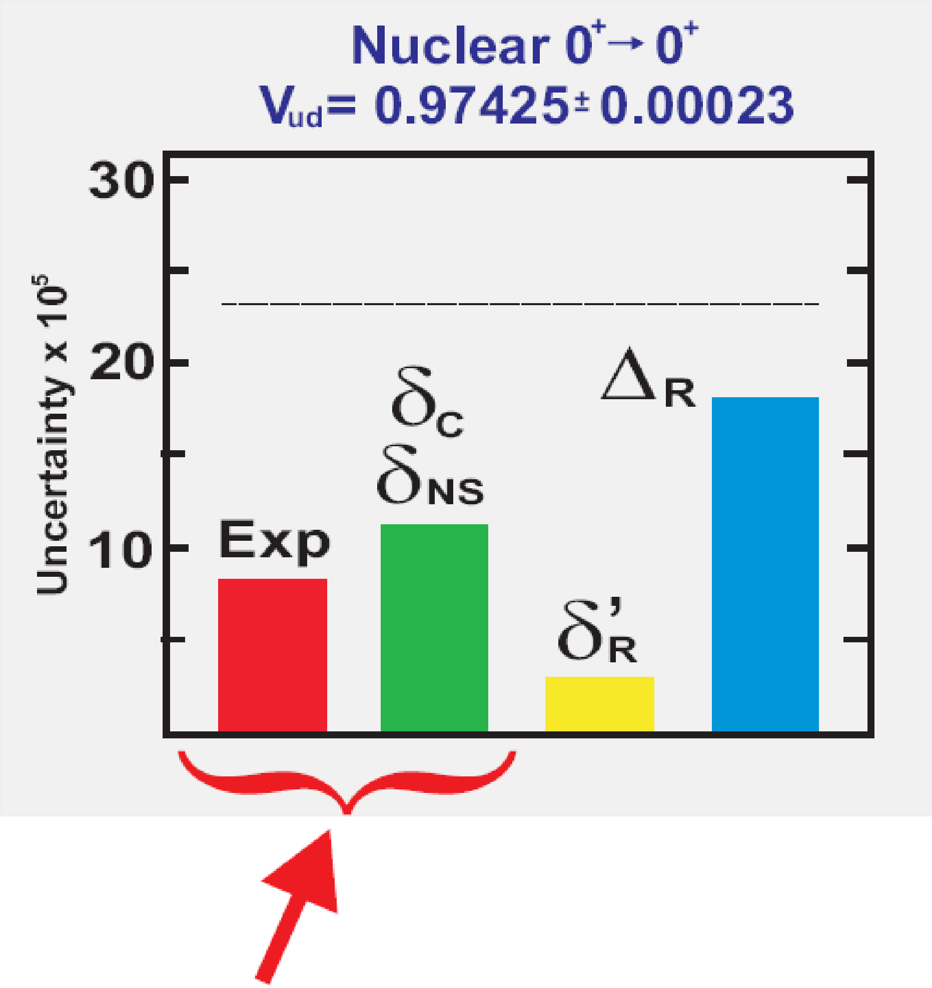


Nuclear $0^+ \rightarrow 0^+$
$V_{ud} = 0.97425 \pm 0.00023$

OPPORTUNITIES FOR IMPROVEMENT

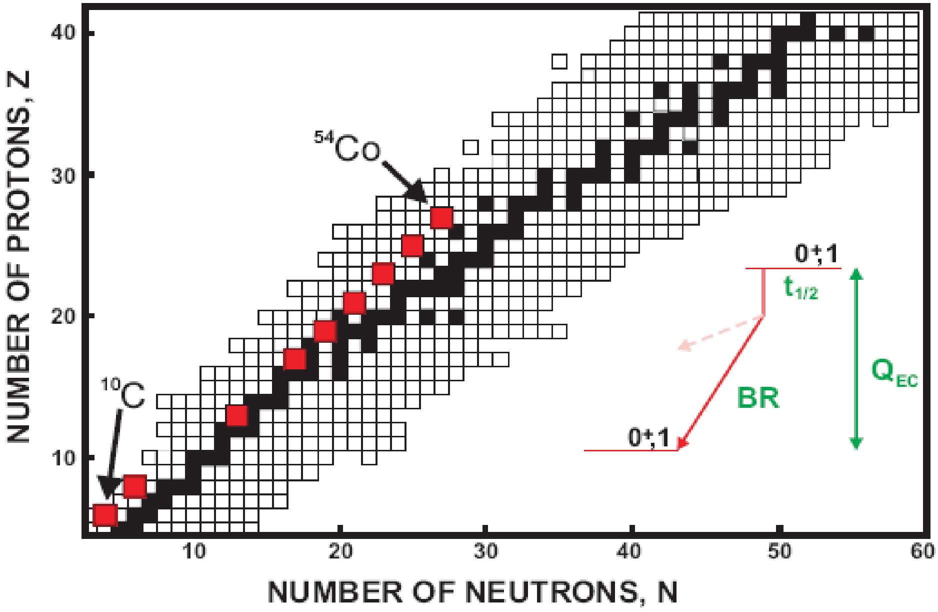
- Goal remains to tighten the window for new physics by reducing the uncertainty on V_{ud}.
- Uncertainty on calculated radiative correction Δ_R is the dominant contribution to the error budget.
- Nuclear-structure-dependent corrections, δ_C and δ_{NS}, can be tested by experiment; this has already led to improvements, but more are still possible.

Data on "well known" transitions can be made more precise, and new cases can be measured.



TESTING NUCLEAR-STRUCTURE-DEPENDENT CORRECTIONS

Strategy is to probe the nucleus-to-nucleus variation in δ_C - δ_{NS}.



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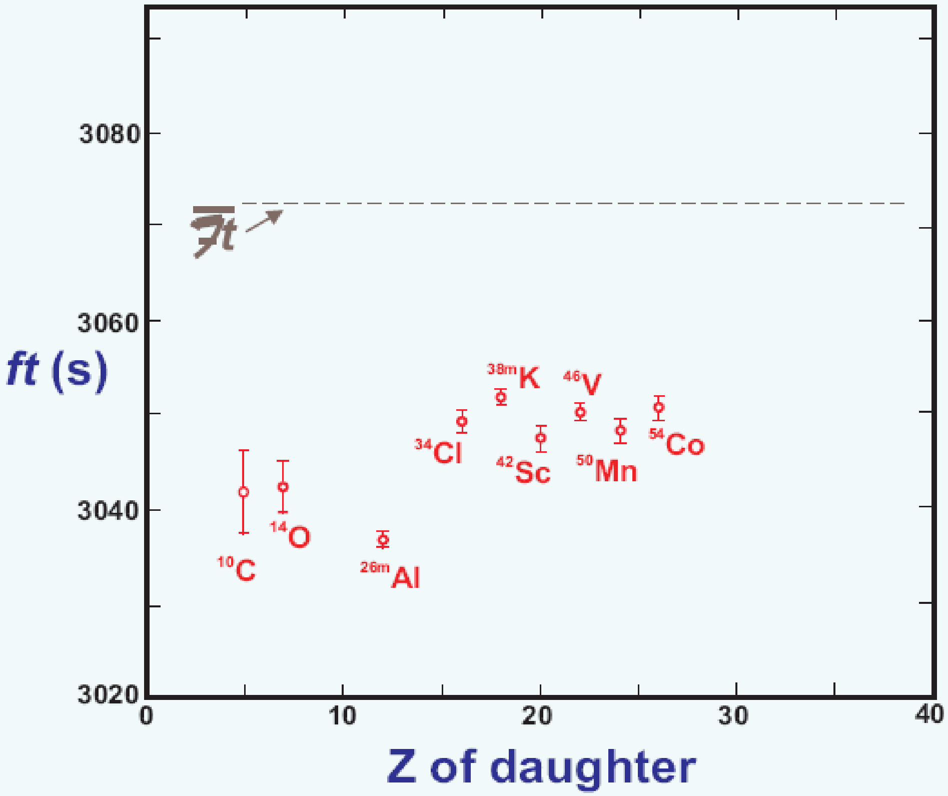

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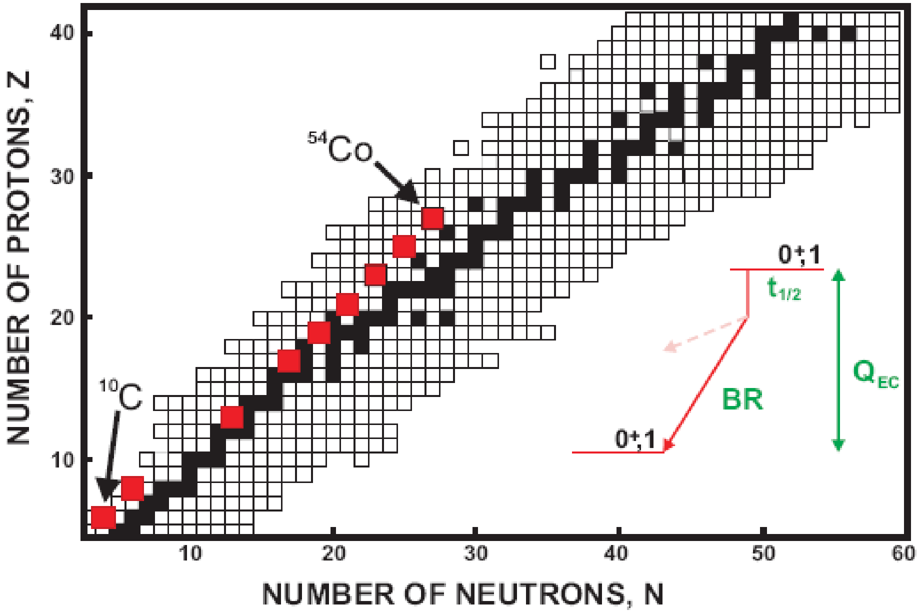
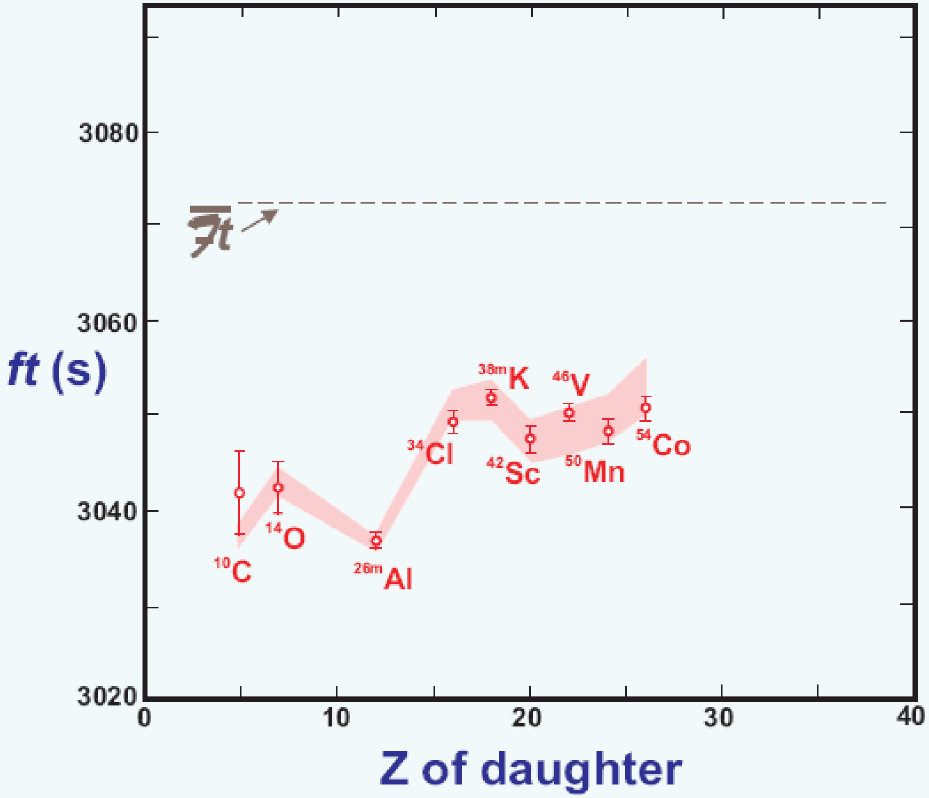


- Increase measured precision on “traditional” nine *ft*-values

Strategy is to probe the nucleus-to-nucleus variation in δ_C - δ_{NS}.

Calculated *ft*-value = $\dfrac{\overline{\mathcal{F}t}}{(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})]}$



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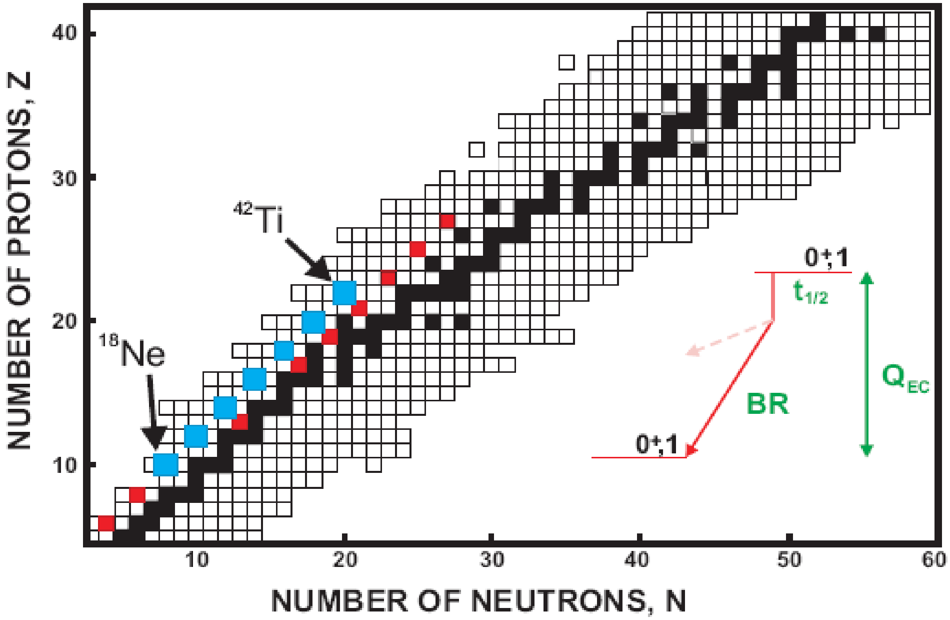
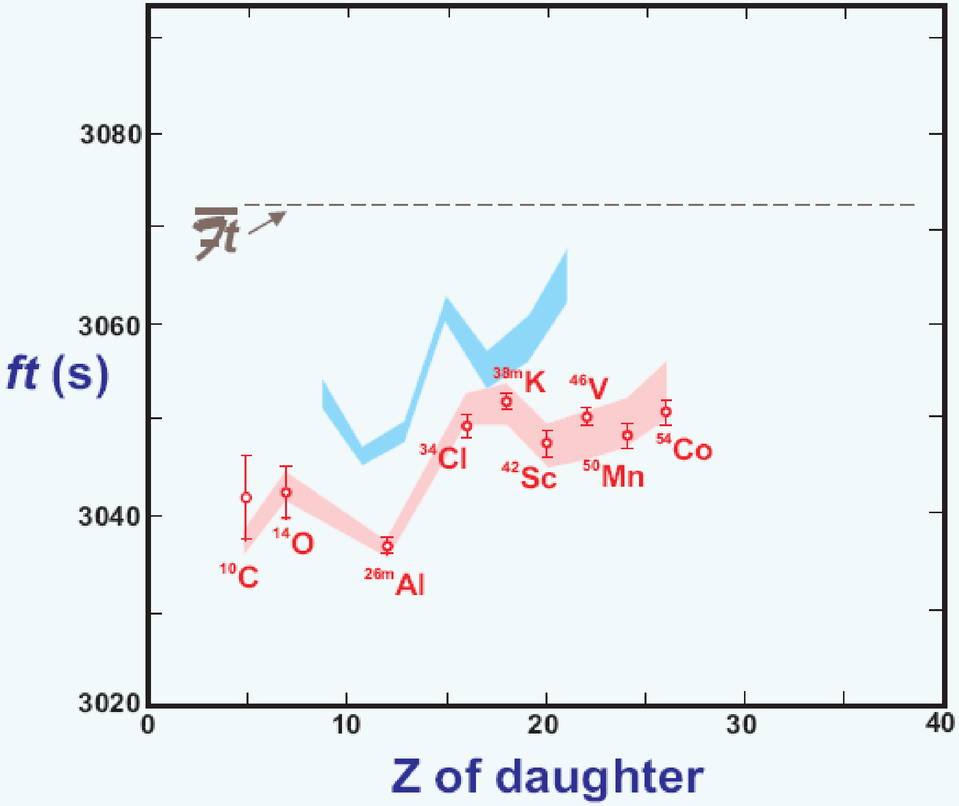


Strategy is to probe the nucleus-to-nucleus variation in δ_C - δ_{NS}.

- **Increase measured precision on "traditional" nine *ft*-values**
- **measure new $0^+ \rightarrow 0^+$ decays with $18 \leq A \leq 42$ ($T_Z = -1$)**

Calculated *ft*-value = $\dfrac{\mathcal{F}t}{(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})]}$



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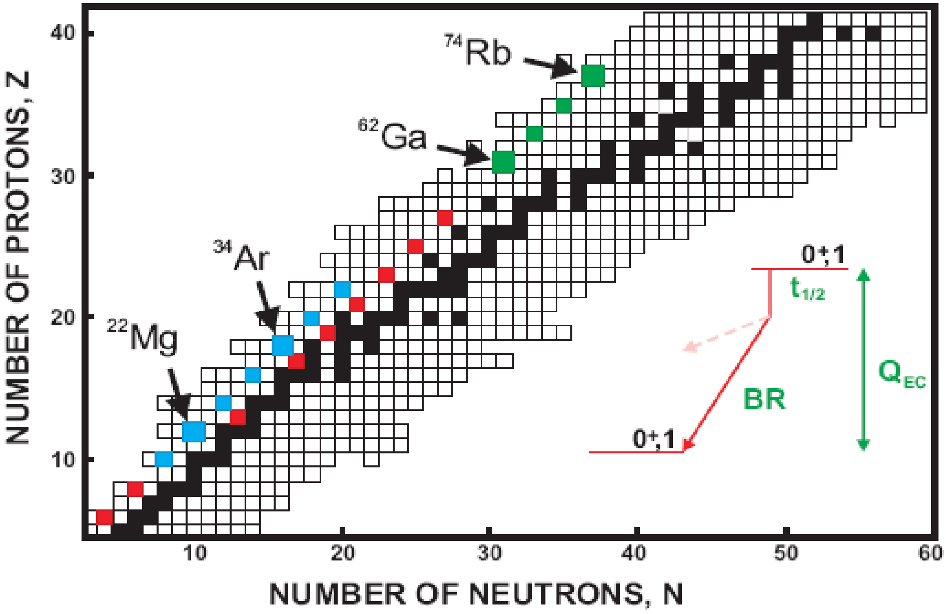
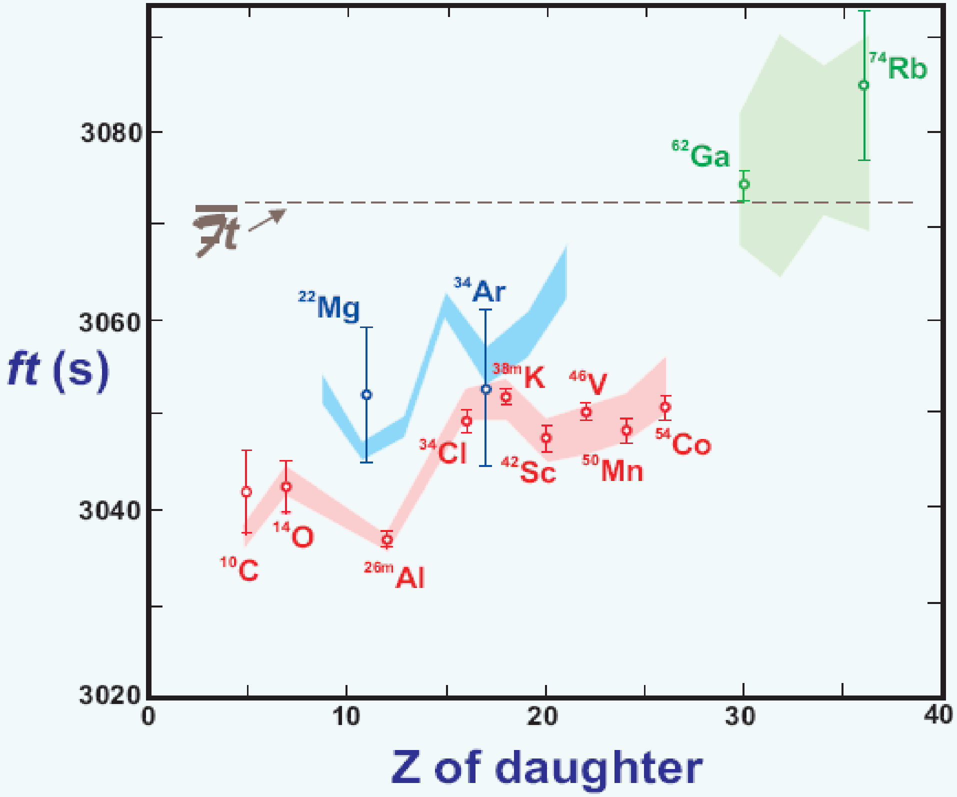


Strategy is to probe the nucleus-to-nucleus variation in $\delta_C - \delta_{NS}$.

$$\text{Calculated } ft\text{-value} = \frac{\overline{\mathcal{F}t}}{(1 + \delta_R')[1 - (\delta_C - \delta_{NS})]}$$



- Increase measured precision on “traditional” nine *ft*-values

- measure new $0^+ \rightarrow 0^+$ decays with $18 \leq A \leq 42$ ($T_Z = -1$)

- measure new $0^+ \rightarrow 0^+$ decays with $A \geq 62$ ($T_Z = 0$)

Summary

- The average corrected $\overline{\mathcal{F}t}$ value is reduced

$$\text{Review '05}: \quad 3072.7 \pm 0.8 \quad \chi^2/\nu = 0.42$$

$$\text{Sept '08}: \quad 3072.2 \pm 0.8 \quad \chi^2/\nu = 0.31$$

principally due to Penning-trap Q-value measurements and a recalculation of the isospin-symmetry breaking correction (PR C77, 025501).

- This increases the value of V_{ud}

$$\text{Review '05}: \quad V_{ud} = 0.97380(40)$$

$$\text{Sept '08}: \quad V_{ud} = 0.97425(23)$$

error reduction mainly due to Marciano-Sirlin reevaluation of radiative correction.

- CKM unitarity sum

$$\text{Review '05}: \quad V_{ud}^2 + V_{us}^2 + V_{ud}^2 = 0.9987(10)$$

$$\text{Sept '08}: \quad V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9996(7)$$

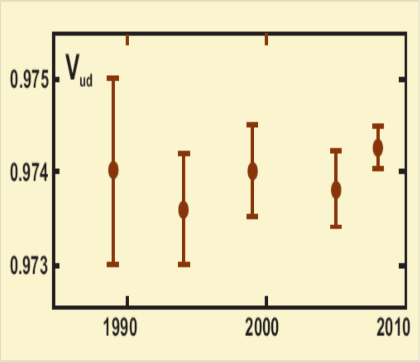
in <u>both</u> cases with FlaviaNet '08 value of V_{us}

Conclusions

- Superallowed β decay currently yields most precise value of V_{ud}, limited by theory uncertainties.

$$\boxed{V_{ud} = 0.97425 \pm 0.00023}$$

- Value of V_{ud} proving to be very robust.



- Neutron and pion decays yield V_{ud} consistent with nuclear result, but with larger experimental errors. This will change in $3 - 5$ years.

- Much activity in nuclear physics focussed on reducing errors still further via tests of structure-dependent corrections.

Conclusions (continued)

- CKM Unitarity now verified to 0.07% – dominant errors are from theory.

$$\underset{0.9491(4)}{V_{ud}^2} + \underset{0.0504(6)}{V_{us}^2} + \underset{<0.0001}{V_{ub}^2} = 0.9996(7)$$